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# ADVANCED MODELLING OF VIBRATORY MACHINES

Mariusz Giergiel

*AGH University of Science and Technology. Cracow*
*e-mail: giergiel@agh.edu.pl*

Józef Giergiel

*Rzeszow University of Technology*

Along with increasing dimensions, speed, and output of modern machines, many newly appearing problems are being encountered in design and manufacturing of such machines. Vibrations occuring in certain technological processes create undesirable phenomena which limit the durability, cause excessive dynamic reactions with surroundings or increase level of emitted sound. For a certain class of machines, however, vibration constitutes the primary factor ensuring that a desired technological process is performed correctly. This class is hereafter referred to as vibratory machines. Based on the considerations made in this paper, a conclusion can be drawn that from the point of view of mechatronical design of vibratory machines it is necessary to determine precise models of machines including the driving system, especially the induction motor.

*Key words:* vibratory machines, machine dynamics, modelling

## 1. Introduction

Machines which fulfil a manufacturing or a transport process based on transmission of vibrations of the body of a machine to the machined medium, i.e. so-called vibrating machines, are the only, and perhaps the most advantageous form of the realization of a manufacturing process in many branches of industry. The mode of working of such a machine, which is based on causing intensive polytonal vibrations, often results in transmission of vibration to the ground, which, in turn, has a disturbing effect to the surroundings and is detrimental to health of people and buildings they live in. Amplitudes of dynamic forces transmitted to the ground achieving up to 20k kN, or even more, together with frequencies up to 100 Hz are the major source of vibration in industrial plants.

CAE systems became the most necessary and essential instrument of today designer's work. Computer and applicable software offers a lot of helpful functions, especially in analysis, modeling and simulation. Today design engineer with CAE system can create a model of a machine and observe it by performing necessary analysis and simulations as well. What is more, it is possible to make durability calculations, analysis and even dynamical simulations of the working machine.

To realize all these purposes, in the design of vibratory machines it is necessary to use precise models of dynamic events in the electromechanical system: vibratory machine - inductive drive.

## 2. The mathematical model of a vibratory machine

### 2.1. Model of the electrical part

The propulsion of vibratory machines is usually built with single run three-phased asynchronous engines. It is indispensably to use suitable model of propulsion, when performing simulation. The asynchronous machine is the most popular electromechanical converter used in systems with various dynamical structures. Dynamical advances that exist during the starting phase, braking, voltage-hesitation are the basic gauge during mechanical, electrical and thermal selection of engines for designed propulsion.

Nonlinear differential systems determine the mathematical model of an engine. The nonlinearity results from relationships between generalized coordinates describing mechanical and electric quantities. The structure of engine electromagnetic configuration classifies the type of a machine to holonomical systems, where equations of constraints are integrable. The dependence between electromagnetic induction in a slit and voltage is linear.

It is possible to use double or single-frame models of an asynchronous machine. What is more, we are able to explore the Kloss model or a static characteristic as well. However, from exchanged models, only single and double-frame models suit dynamics of the asynchronous machine.

The use of a double-frame model permits one to describe a wide group of asynchronous machines. This model gives extraordinary precision of calculations including static characteristics. The single-framed model has quite big precision for annular and single-framed machines with cages ring-shaped bars about round rods (Puchała, 1977).

Approaching the problem of mathematical modelling of an asynchronous machine, it is necessary to take care of basic purposes, which should be executed. The first thing is that we expect mathematical models in form of relations between voltages, currents, torques and rotation speed. We are especially interested in dynamical conditions like the starting phase, braking, charge and discharge phases, changes during the switching of voltages and resistance or finally short-circuit. The most important conditions required to obtain a useful mathematical model are the verifiability of its adequacy and the possibility to fix parameters through measurements.

The double-frame model of an asynchronous machine described in terms of stator streams and two frames divided by the carriage voltage, denoted by $\varPhi_s$, $\varPhi_1$, $\varPhi_2$, can be expressed by following equations (Rams, 1977)

$$\begin{aligned}
\dot{\varPhi}_s &= -a_s\Big(\varPhi_s - \frac{\sigma_2}{\sigma}\varPhi_1 - \frac{\sigma_1}{\sigma}\varPhi_2\Big) + \mathrm{j}\omega_x\varPhi_s + U \\
\dot{\varPhi}_1 &= -a_1\Big(-\frac{\sigma_2}{\sigma}\varPhi_s + \frac{\sigma_{\varDelta s} + \sigma_{s2}}{\sigma}\varPhi_1 - \frac{\sigma_{\varDelta s}}{\sigma}\varPhi_2\Big) - \mathrm{j}(\omega_x - \omega_e)\varPhi_1 \\
\dot{\varPhi}_2 &= -a_2\Big(-\frac{\sigma_1}{\sigma}\varPhi_s + \frac{\sigma_{\varDelta s} + \sigma_{s1}}{\sigma}\varPhi_2 - \frac{\sigma_{\varDelta s}}{\sigma}\varPhi_1\Big) - \mathrm{j}(\omega_x - \omega_e)\varPhi_2 \\
M_{el} &= -C \cdot \mathrm{Im}\Big[\varPhi_s\Big(\frac{\sigma_2}{\sigma}\varPhi_1 + \frac{\sigma_1}{\sigma}\varPhi_2\Big)\Big] \\
C &= \frac{2M_u\omega_o^2 p}{I_w}
\end{aligned} \tag{2.1}$$

where

$M_{el}$ – electromagnetic torque
$M_u$ – breakdown torque
$\sigma_w, \sigma_1, \sigma_2, \sigma_{s1}, \sigma_{s2}, \sigma_{\varDelta s}$ – parameters of the motor model
$C$ – electromechanical constant of the motor
$p$ – number of pairs of poles of the asynchronous engine
$U$ – normalized excitation voltage
$\omega_o$ – synchronous speed
$\omega_e$ – electrical angular velocity
$\omega_x$ – angular velocity of the stator coordinate system
$I_w$ – mass moment of inertia of the rotor

Because of difficulty in the identification of parameters of the system and restricted precision, especially when interested in electromechanical worth, we should use the simplest model as it is possible. The parameters describing the double-frame model must be consistent with engine characteristics and its structural parameters. Usually, even if all data of the engine are known, it is necessary to use optimization methods to obtain right parameters. Moreover, a double-frame machine has parameters which are not defined in literature, as it goes with single-frame models. Summing up, there is no possibility to get accurate parameters even if catalogue data of the engine are well known. The simplicity and possibility of using catalogue data of engines are the main reasons why the single-frame model has become the most popular. The stator of a machine has $p$ pole pairs of clutches and the rotor is single-frame. Such a model is described by equations (Puchała, 1977)

$$\begin{aligned} \dot{\varPhi}_s &= -a_s\Big(\varPhi_s - \frac{1}{k_w}\mp_w\Big) + U \\ \dot{\varPhi}_w &= -a_w\Big(\varPhi_w\frac{k_s}{k_w} - \frac{1}{k_w}\varPhi_s\Big) - \mathrm{j}\omega_e\varPhi_w \\ M_{el} &= -C\cdot\mathrm{Im}(\varPhi_s\varPhi_w) \qquad\qquad C = \frac{2M_u\omega_o^2p}{I_w} \end{aligned} \tag{2.2}$$

The model of an asynchronous machine is expressed by equations, where the linked streams of the rotor and stator divded by the worth of carriage voltage, marked as $\varPhi_w$ and $\varPhi_s$, are condition co-ordinates.

The following denote

$k_w, k_s$ – dissipation of the rotor and the stator, respectively
$a_w, a_s$ – proportional parameters of the engine model

### 2.2. The model of the mechanical part

For investigation purposes, the mathematical models describing relations between the machine body and its propulsion were used for digital simulation of dynamical effects (ultra-resonance machine). These models make it possible to examine a major part of difficult dynamical problems unable to explore analytically. These dynamical effects have considerable influence on the rotation speed of the system.

Some popular vibratory machines are equipped with the pendular vibrator, which makes them incline by more degrees of latitude than it is observed in machines with strong axis vibrator. If we consider a machine with the pendular vibrator, we should remember that the model of the mechanical part include flat motion of the body, pendulum and pulp of the vibrator. Moreover, we should take care of relations between the string suspension and the body of the machine. Our calculation takes also into account the susceptibility of the

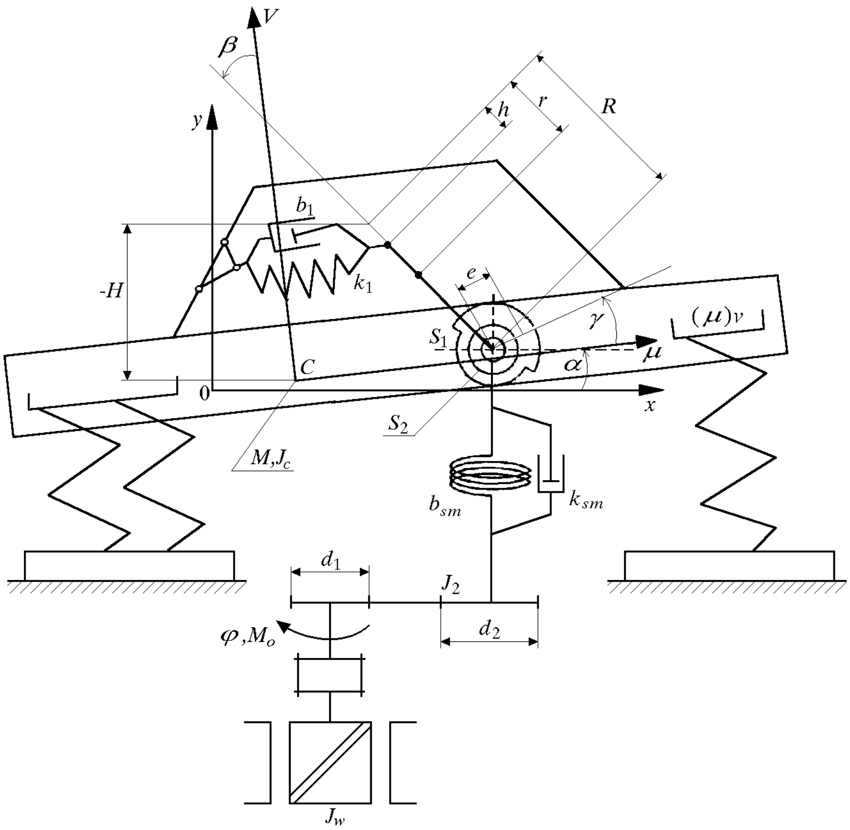


Fig. 1. A physical model of a vibratory machine with the pendulum inertial vibrator

clutch that keeps the propulsion-vibrator system tighter

$$
\begin{aligned}
&(M+m_h+m_n)x+(m_h+m_n)H\alpha+(m_hr+m_nR)\beta-m_ne\sin\gamma=\\
&=-n\sum[k_{xx}(x-\gamma_i\alpha)+k_{xy}(y+\mu_i\alpha)+k_{x\alpha}\alpha]-b_xx+m_ne\gamma^2\cos\gamma\\
&(M+m_h+m_n)y+m_ne\gamma\cos\gamma=\\
&=-n\sum[k_{yx}(x-\gamma_i\alpha)+k_{yy}(y-\mu_i\alpha)+k_{y\alpha}\alpha]-b_yy+m_ne\gamma^2\sin\gamma\\
&(m_h+m_n)Hx+[I_c+(m_h+m_n)H^2]\alpha+(m_hr+m_nR)H\beta-m_neH\gamma\sin\gamma=\\
&=-n\sum\{k_{\alpha x}(x-\gamma_i\alpha)+k_{\alpha y}(y-\mu_i\alpha)+k_{\alpha\alpha}\alpha+\\
&+\mu_i[k_{yx}(y-\gamma_i\alpha)+k_{yy}(y-\mu_i\alpha)+k_{y\alpha}\alpha]+\\
&-\gamma_i[k_{xx}(x-\gamma_i\alpha)+k_{xy}(y-\mu_i\alpha)+k_{x\alpha}\alpha]\}+\\
&-b_\alpha+m_neH\gamma^2\cos\gamma+b_1h^2(\beta-\alpha)+k_1h^2(\beta-\alpha)
\end{aligned}
\tag{2.3}
$$

$$
\begin{aligned}
&(m_hr + m_nR)x + (m_hr + m_nR)H\alpha + (m_hr^2 + m_nR^2 + I_h)\beta - m_neR\gamma\sin\gamma = \\
&= m_neR\gamma^2\cos\gamma - b_1h^2(\beta-\alpha) - k_1h^2(\beta-\alpha) +
\end{aligned}
$$

$$
\begin{aligned}
&-m_ne\sin\gamma + m_ne\cos\gamma - m_neH\alpha\sin\gamma - m_neR\beta\sin\gamma + (I_n + m_ne^2)\gamma = \\
&= -k_{sm}\Big(\gamma - \phi\frac{d_1}{d_2}\Big) - b_{sm}\Big(\gamma - \phi\frac{d_1}{d_2}\Big) - b_o\gamma^2\,\mathrm{sgn}\,(\gamma-\beta)
\end{aligned}
$$

$$
\begin{aligned}
&\Big[I_w + I_1 + I_2\frac{d_1^2}{d_2^2}\Big]\varphi = -k_{sm}\Big(\phi\frac{d_1}{d_2} - \gamma\Big)\frac{d_1}{d_2} - b_{sm}\Big(\phi\frac{d_1}{d_2} - \gamma\Big)\frac{d_1}{d_2} + \\
&+M_{el} - M_o\,\mathrm{sgn}\,\varphi
\end{aligned}
$$

$$
\begin{aligned}
\mathrm{Re}\dot{\varPhi}_s &= -a_s\Big(\mathrm{Re}\varPhi_s - \frac{1}{k_w}\mathrm{Re}\varPhi_w\Big) + \mathrm{Re}U \\
\mathrm{Im}\dot{\varPhi}_s &= -a_s\Big(\mathrm{Im}\varPhi_s - \frac{1}{k_w}\mathrm{Im}\varPhi_w\Big) + \mathrm{Im}U \\
\mathrm{Re}\dot{\varPhi}_w &= -a_w\Big(\mathrm{Re}\varPhi_w\frac{k_s}{k_w} - \frac{1}{k_w}\mathrm{Re}\varPhi_s\Big) - \dot{\varphi}ip\mathrm{Im}\varPhi_w \\
\mathrm{Im}\dot{\varPhi}_w &= -a_w\Big(\mathrm{Im}\varPhi_w\frac{k_s}{k_w} - \frac{1}{k_w}\mathrm{Im}\varPhi_s\Big) - \dot{\varphi}ip\mathrm{Re}\varPhi_w \\
M_{el} &= -C(\mathrm{Re}\varPhi_s\mathrm{Im}\varPhi_w - \mathrm{Im}\varPhi_s\mathrm{Re}\varPhi_w)I_w
\end{aligned}
\tag{2.4}
$$

Equations $(2.3)_{1,2,3}$ describe flat motion of the machine body. Equation $(2.3)_4$ describes motion of the pendulum, while equation $(2.3)_5$ expresses motion of the vibrator. Equation $(2.3)_6$ describes rotary motion of the engine and equations (2.4) define dynamical effects in the propulsion system.

The following denote:

| | | |
|---|---|---|
| $x, y, \alpha$ | – | horizontal coordinates of the mass center of mass and angle of deviation from the machine body level |
| $\beta$ | – | angle of inclination of the pendulum measured with respect to machine level |
| $\varphi$ | – | angle of rotation of the engine |
| $\gamma$ | – | angle of rotation of the vibrator |
| $\varPhi_s, \varPhi_w$ | – | fluxes of the stator and rotor divided by reference voltage |
| $M$ | – | mass of the machine body |
| $m_h$ | – | mass of the pendulum |
| $m_n$ | – | unbalanced mass of the vibrator |
| $I$ | – | mass moment of inertia of the driving system with respect to the rotor axis |

| | | |
|---|---|---|
| $I_c, I_h, I_n$ | – | central moment of inertia of the machine body, the pendulum and the unbalanced mass, respectively |
| $I_w$ | – | moment of inertia of the rotor |
| $I_1, I_2$ | – | moments of belt pulleys |
| $H$ | – | distance between the axle of the pendulum and the center of mass of the machine body |
| $h$ | – | distance between the point of fastening of the pendulum stabilization system and its rotation axis |
| $e$ | – | eccentricity of the vibrator |
| $R$ | – | distance between the pendulum axis and vibrator axis |
| $r$ | – | distance between the center of mass of the pendulum and its rotation axis |
| $d_1, d_2$ | – | diameters of belt pulleys |
| $\mu_i, \nu_i$ | – | coordinates of the fixing points of elastic elements in the central coordinate system $C_{\mu\nu}$ |
| $k_1, b_1$ | – | coefficients of elasticity and viscous damping of the pendulum stabilization system |
| $k_{sm}, b_{sm}$ | – | coefficients of elasticity and damping of the coupling |
| $\alpha_s, \alpha_w$ | – | proportional parameters of the engine model |
| $n$ | – | number of elastic elements in the set |
| $M_o$ | – | anti-torque of the driving system |
| $i$ | – | transmission ratio of the belt drive between the motor and vibrator |
| $b_o$ | – | resistance coefficient of the bearings. |

During simulation carried out on the presented model of the ultra-resonance vibration machine, we take care of precision in the determination of parameters of the string suspension system. Temporary resonance simulations demand high standards of the string suspension system and are dependent on flexibility and damping features of the system as well as relations between thr co-ordinates.

The model of the ultra-resonance vibration machine, which is still in the building phase, enables one to describe the string-damping reaction of the suspension. If we are able to qualify the elasticity matrix $\mathbf{K}$, we can also generally describe the string element in a matrix form as follows

$$\begin{bmatrix} k_{xx} & k_{xy} & k_{x\alpha} \\ k_{yx} & k_{yy} & k_{y\alpha} \\ k_{\alpha x} & k_{\alpha y} & k_{\alpha\alpha} \end{bmatrix} \begin{bmatrix} x_i \\ y_i \\ \alpha_i \end{bmatrix} = \begin{bmatrix} p_{xi} \\ p_{yi} \\ p_{\alpha i} \end{bmatrix} \tag{2.5}$$

where

$x_i, y_i, \alpha_i$ – displacements of the upper end of the elastic element
$p_{xi}, p_{yi}, p_{\alpha i}$ – components of forces and moments applied to it.

Because of constant character of relations between the co-ordinates, we may use an equivalent viscosity damping for motion along co-ordinates marked as: $b_x$, $b_y$, $b_\alpha$. In most of popular types of the string suspension (steel strings with mounted endings or a multi-layer metal –rubber block, the matrix of damping and elasticity coefficients should be automatically generated in a special model, which concerns Clayperon's (Michalczyk, 1995) influence of string forces on the flexibility in all angular and linear directions of the deformation.

## 3. Conclusions

To realize all the above mentioned purposes in the design of vibratory machines, it is necessary to use a precise model of dynamic effects in the electromechanical system: vibratory machine – inductive drive. The model was verified by M. Giergiel (2002) and to the best author's knowledge it can be used for analysis and simulation of dynamics of vibratory machines in the design process, virtual prototyping and mechatronical design.

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**Zaawansowane modelowanie maszyn wibracyjnych**

Streszczenie

Wraz ze zwiększeniem wymiarów, prędkości oraz wydajności współczesnych maszyn i urządzeń występują coraz większe problemy w projektowaniu i konstrukcji, wynikające z występującymi w nich drganiami. W procesie projektowania maszyn wibracyjnych niezwykle interesujący jest aspekt możliwości wydatnego skrócenia fazy projektowej maszyn poprzez szczegółowe określenie w fazie koncepcji projektowej własności ruchowych maszyny. W pracy przedstawiono zagadnienia związane z budową modelu fizycznego maszyny wibracyjnej zgodnego z obiektem rzeczywistym i użytecznego w procesie komputerowego wspomagania prac inżynierskich. Szczególną uwagę zwrócono na modelowanie silnika napędzającego wibrator, co z jednej strony zapewnia prawidłowe odzwierciedlenie istotnych zjawisk fizycznych, z drugiej możliwość identyfikacji parametrów modelu.

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