

# Some aspects of vibration control

## Part I: Active and passive correction

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### ABSTRACT



The paper presents a general approach to mechanical system modification aimed at controlling the steady harmonic vibrations by means of passive and active methods. The relative decrease of harmonic vibration amplitudes of selected elements of the mechanical system has been chosen as a measure of the quality of the introduced modification. The proposed theoretical method enables to determine the parameters of the system's dynamic flexibility matrix, which show the most remarkable effect on the dynamic behaviour of the whole system. When active control is considered the method is useful in designing the structure and choosing the parameters of the control system. In certain cases of self-excited vibration the approach helps examining the elements of the system, most responsible for this kind of excitation.

**Key words :** harmonic vibrations, passive control, active control

### PROBLEM DESCRIPTION

Theoretical investigations into the problem of active control of mechanical vibrations have been carried out for many years, but real-life mechanical systems making use of this idea are still rare. However recently the progress in control methods and technology has had an impact on the development of novel practical solutions, used to reduce vibration of turbomachinery rotor systems. Application of magnetic and pressurized bearings reveals new possibilities for the control of rotor behaviour. With the advent of nanotechnology the practical application of active control methods may increase in the future. The application of distributed sensor and actuator systems is tightly linked with the theory of multidimensional system control.

Let us consider a mechanical system with  $n$  degrees of freedom, Fig.1a. The inertial elements of the system can perform translational, bending and torsional movements. The forced harmonic vibration of the system is given by the matrix equation :

$$\mathbf{J} \cdot \ddot{\mathbf{Q}} + \mathbf{B} \cdot \dot{\mathbf{Q}} + \mathbf{K} \cdot \mathbf{Q} = \mathbf{F} \quad (1)$$

where :

- J** - the matrix of the moments of inertia of the system
- B** - the matrix of the damping coefficients of the system
- K** - the matrix of the stiffness coefficients of the system
- F** - the vector of the harmonic forces or moments acting upon the inertial elements of the system
- Q** - the vector of displacement of the inertial elements of the system.

The matrices **J**, **B**, **K** are of  $n \times n$  dimension, and the vectors **F**, **Q** - of  $n \times 1$  dimension.

The forces **F** acting on the system are assumed to have an identical frequency  $\omega$ , and amplitudes varying within the system to be described by the vector **f**. They may therefore be expressed as a function of time  $t$  :

$$\mathbf{F} = \mathbf{f} \cdot e^{j\omega t} \quad (2)$$

where :  
 $j = \sqrt{-1}$  - the imaginary unit.

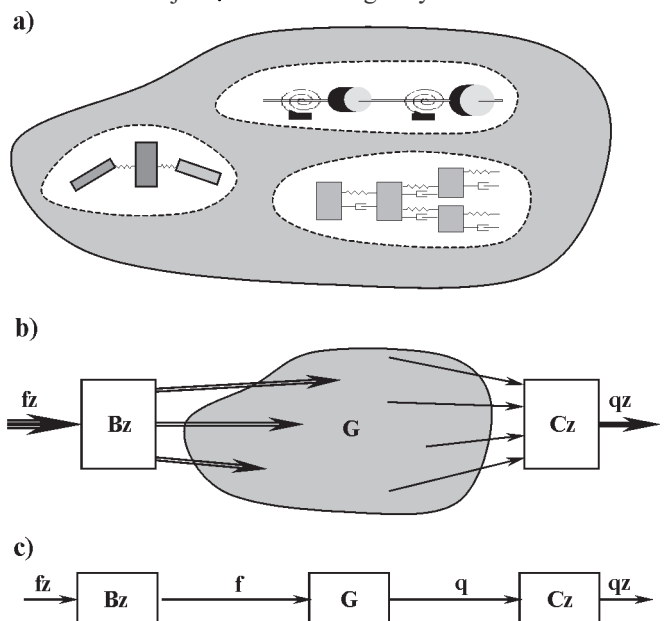


Fig 1. Schematic diagram of a mechanical system and its block diagram describing mechanical vibrations

The solution  $\mathbf{Q}$  of equation (1) describing the displacement of the elements of the system may be expected to have the form :

$$\mathbf{Q} = \mathbf{q} \cdot e^{j\omega t} \quad (3)$$

where :

$\mathbf{q}$  - the vector of displacement amplitudes of the inertial elements of the system.

After accounting for relations (2) and (3), equation (1) takes the form :

$$(-\omega^2 \mathbf{J} + j\omega \mathbf{B} + \mathbf{K})\mathbf{q} = \mathbf{f} \quad (4)$$

All matrices and vectors bearing the subscript  $i$  are assumed to refer to the vibrations of the mechanical system for a given frequency  $\omega_i$ . For the sake of clarity it is helpful to introduce the following notation:  $\mathbf{D}_i \equiv (\mathbf{K} - \omega_i^2 \mathbf{J} + j\omega_i \mathbf{B})$ . The matrix  $\mathbf{D}_i$  is of  $n \times n$  dimension. Equation (4) may be rewritten to include  $\mathbf{D}_i$  :

$$\mathbf{D}_i \cdot \mathbf{q}_i = \mathbf{f}_i \quad (5)$$

or equivalently (assuming the equation has a solution) :

$$\mathbf{q}_i = \mathbf{G}_i \cdot \mathbf{f}_i \quad (6)$$

where :

the matrix  $\mathbf{G}_i \equiv \mathbf{D}_i^{-1}$  stands for the dynamic flexibility matrix of the system for the frequency  $\omega_i$ .

The external forces  $\mathbf{f}_i$  are assumed to act on selected inertial elements in a way which can be described by the binary matrix  $\mathbf{Bz}_i$ . The matrix  $\mathbf{qz}_i$  in turn represents the vibration amplitudes of the elements whose behaviour has to be controlled. The selection of these amplitudes is performed by means of the binary matrix  $\mathbf{Cz}_i$ . This is shown (for any vibration frequency) in Fig.1b and Fig.1c.

### Active control

The behaviour of the system shown in Fig.1c is controlled by adding a feedback loop. The schematic block diagram of the active control of mechanical vibrations is presented in Fig.2.

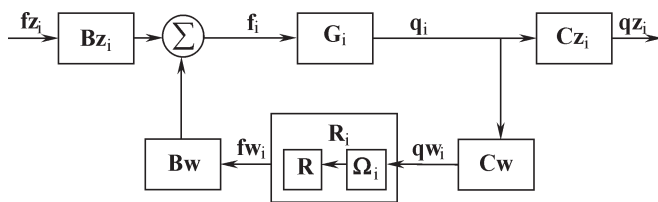


Fig 2. Block diagram of active control of mechanical vibrations

It is here assumed that only the amplitudes  $\mathbf{qw}_i$  of a certain number  $s$  of inertial elements (selected by the  $s \times n$  binary matrix  $\mathbf{Cw}$ ) can be measured. They are treated as an input value for the controller. The output vector  $\mathbf{fw}_i$  of the controller consists of  $r$  elements and is a function of the vector  $\mathbf{qw}_i$  of measured displacement, as well as of the derivative and calculus (of any order) of the displacement. The transfer matrix  $\mathbf{R}_i$  of the controller action is a  $r \times s$  matrix with complex elements, which fulfils the relation:

$$\mathbf{fw}_i = \mathbf{R}_i \cdot \mathbf{qw}_i \quad (7)$$

In the case when a PID controller is used in the system, the matrix  $\mathbf{R}_i$  takes the following form :

$$\mathbf{R}_i = \mathbf{K}_p + j \cdot (\mathbf{K}_D \omega_i - \mathbf{K}_I \omega_i^{-1}) \quad (8)$$

where :

$\mathbf{K}_p$ ,  $\mathbf{K}_I$  and  $\mathbf{K}_D$  -  $r \times s$  matrices of the proportional (P), integrating (I) and differentiating (D) controller, respectively.

Let us define the  $r \times 3s$  matrix  $\mathbf{R}$  of the controller parameters as :

$$\mathbf{R} = [\mathbf{K}_p \mid \mathbf{K}_I \mid \mathbf{K}_D]$$

The relation between matrices  $\mathbf{R}_i$  and  $\mathbf{R}$  may be written in the form :

$$\mathbf{R}_i = \mathbf{R} \cdot \mathbf{U}_i$$

where :

$$\mathbf{U}_i = \begin{bmatrix} \text{---} & \text{---} & \text{---} & \mathbf{I}_s \\ \text{---} & \text{---} & \text{---} & \mathbf{I}_s \\ -j\omega_i^{-1} & \text{---} & \text{---} & \mathbf{I}_s \\ \text{---} & \text{---} & \text{---} & \mathbf{I}_s \\ j\omega_i & \text{---} & \text{---} & \mathbf{I}_s \end{bmatrix}$$

When taking into consideration controller models with differentiation or integration of a higher order it is sufficient to extend the parameter matrix  $\mathbf{R}$  horizontally, and appropriately extend the frequency multiplier matrix  $\mathbf{U}$  vertically.

The steering force signal  $\mathbf{fw}_i$  is passed onto the active elements of the control feedback loop. The locations of the elements upon which they act are given by the  $n \times r$  matrix  $\mathbf{Bw}$ .

The aim of the controller  $\mathbf{R}$  is to minimize the vibration amplitudes of selected elements described by the matrix  $\mathbf{qz}_i$ .

### Passive control

Passive control is here understood as the modification of parameters of the mechanical system. This may be achieved by introducing the changes  $\mathbf{P} = [\Delta \mathbf{J} \mid \Delta \mathbf{B} \mid \Delta \mathbf{K}]$  to certain system parameters (i.e. inertia, damping or stiffness coefficients), selected by the binary matrices  $\mathbf{Bw}$  and  $\mathbf{Cw}$ . For a given frequency  $\omega_i$  the changes introduced to the matrix  $\mathbf{D}_i$  lead to the following changes in equation (5) :

$$(\mathbf{D}_i + \mathbf{Bw} \cdot \mathbf{P}_i \cdot \mathbf{Cw})\mathbf{q}_i = \mathbf{f}_i \quad (9)$$

where :

$$\mathbf{P}_i = \mathbf{P} \Omega_i$$

From equation (9) one obtains :

$$\mathbf{q}_i = \mathbf{D}_i^{-1} \mathbf{f}_i - \mathbf{D}_i^{-1} \mathbf{Bw} \cdot \mathbf{P}_i \cdot \mathbf{Cw} \cdot \mathbf{q}_i \quad (10)$$

By taking into account the notation:  $\mathbf{G}_i \equiv \mathbf{D}_i^{-1}$ , equation (10) may be written in the form :

$$\mathbf{q}_i = \mathbf{G} \cdot \mathbf{f}_i - \mathbf{G} \cdot \mathbf{Bw} \cdot \mathbf{P}_i \cdot \mathbf{Cw} \cdot \mathbf{q}_i \quad (11)$$

Relation (11) is presented in the form of the block diagram shown in Fig 3.

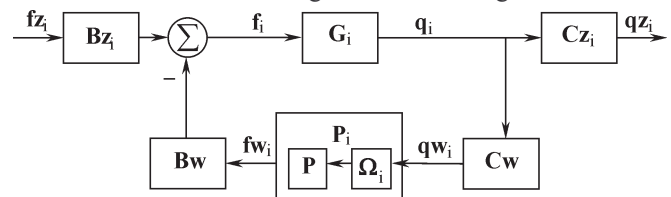


Fig 3. Block diagram of harmonic vibrations with correction of system parameters

The aim of selecting the matrix  $\mathbf{P}_i$  (which describes changes of system parameters) is the minimization of the vibration amplitudes of chosen elements described by the matrix  $\mathbf{qz}_i$ . It should be emphasized that harmonic vibrations with the correction of system parameters and the process of active control of mechanical vibrations may be represented by the same general form of block diagram (compare Fig.2 and 3).

### Some cases of self-excited vibrations

In many cases the so-called self-excited vibrations depend on the behaviour of the mechanical system itself. Let us consider, as an example, self excited vibrations of a turbomachinery

rotor system due to aerodynamic forces. Rotor-stator eccentricity or rotor-stator misalignment changes the clearance distribution above the blade shroud (and in glands) which results in aerodynamic forces and moments acting on the turbine rotor. As a result, self-excited vibrations of the whole rotor system may be observed. Due to inaccuracy of manufacture and assembly, the value and the distribution of the clearance in particular seals may differ significantly, and seals of different types can be used in the same machine, thus the aerodynamic forces acting on a rotor in each stage can vary remarkably. Various theoretical models have been elaborated to describe the fluid motion in the seals and to determine the aerodynamic forces generated in a shroud clearance. Usually the aerodynamic forces and moments are expressed in the form of rotodynamic coefficients [1÷ 6, 8, 9, 12, 13] which can be written in the form of the following vector equation:

$$\mathbf{F} = \mathbf{J}_u \ddot{\mathbf{Q}} + \mathbf{B}_u \dot{\mathbf{Q}} + \mathbf{K}_u \mathbf{Q} \quad (12)$$

where :

- $\mathbf{F}$  - vector of the components of the aerodynamic forces and moments
- $\mathbf{Q}$  - rotor displacement and rotation in horizontal and vertical directions
- $\mathbf{J}_u, \mathbf{B}_u$  and  $\mathbf{K}_u$  - matrices with the so called “inertia”, “damping” and “stiffness” coefficients of the shroud (or gland), respectively.

The symbol  $\mathbf{S}$  is used to denote the matrix of rotodynamic coefficients :  $\mathbf{S} = [\mathbf{J}_u | \mathbf{B}_u | \mathbf{K}_u]$ .

Because the self-excited vibrations occur at a certain frequency  $\omega_i$ , equation (12) takes a form similar to that of equation (4) :

$$\mathbf{Bw} (-\omega_i^2 \mathbf{J}_u + j\omega_i \mathbf{B}_u + \mathbf{K}_u) \mathbf{Cw} \cdot \mathbf{q}_i = \mathbf{f}_i \quad (13)$$

The matrix  $\mathbf{Cw}$  selects the displacements responsible for aerodynamic excitations, while the matrix  $\mathbf{Bw}$  determines the places where these forces are applied.

The equation (13) can also be written in the form :

$$\mathbf{Bw} \cdot \mathbf{S}_i \cdot \mathbf{Cw} \cdot \mathbf{q}_i = \mathbf{f}_i$$

where :

$$\mathbf{S}_i \equiv -\omega_i^2 \mathbf{J}_u + j\omega_i \mathbf{B}_u + \mathbf{K}_u = \mathbf{S} \cdot \mathbf{\Omega}_i$$

When taking into consideration only the aerodynamic excitation forces, the behaviour of the rotor system can be represented by means of the block diagram shown in Fig.4.

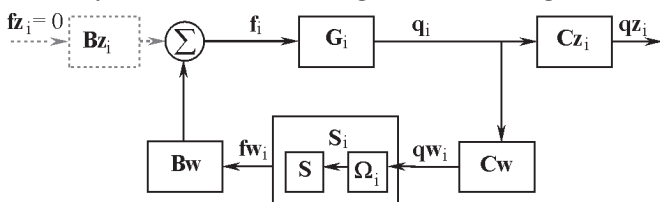


Fig 4. Block diagram of harmonic self-excited vibrations

The comparison of Fig.2 , 3 and 4 leads to the conclusion that harmonic vibrations with the correction of system parameters, the process of active control of mechanical vibrations as well as some cases of self-excited vibrations can be represented by one and the same general form of block diagram. Thus all the three cases may be generalized to the form presented in Fig.5. The matrix  $\mathbf{U}_i$  can represent any of the matrices  $\mathbf{R}_i, \mathbf{P}_i, \mathbf{S}_i$ , depending on the context. All further considerations are conducted with the use of this general form, and the attention is focused on determining the value of the coefficients of the matrix  $\mathbf{U}$  for which the amplitudes  $\mathbf{qz}$  of chosen elements of the system (described by the dynamic flexibility matrix  $\mathbf{G}$ ) will be minimum.

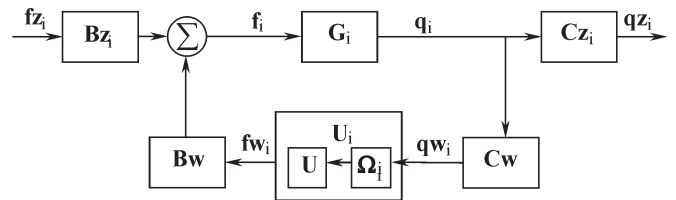


Fig 5. Block diagram of the system performing mechanical vibrations

When taking into consideration vibrations of a mechanical system with  $k$  different values of frequency  $\omega$  it is possible to generalize the model given in Fig.5 to the form shown in Fig.6. This is performed by introducing generalized matrices  $\mathbf{G}, \mathbf{\Omega}, \tilde{\mathbf{U}}, \bar{\mathbf{U}}, \tilde{\mathbf{Bw}}, \tilde{\mathbf{Cw}}, \mathbf{Bz}, \mathbf{Cz}$ , and vectors  $\mathbf{f}, \mathbf{fz}, \mathbf{fw}, \mathbf{q}, \mathbf{qz}, \mathbf{qw}$  (Fig.6), which correspond, respectively, to the matrices  $\mathbf{G}_i, \mathbf{\Omega}_i, \mathbf{U}, \mathbf{U}_i, \mathbf{Bw}, \mathbf{Cw}, \mathbf{Bz}_i, \mathbf{Cz}_i$ , and vectors  $\mathbf{f}_i, \mathbf{fz}_i, \mathbf{fw}_i, \mathbf{q}_i, \mathbf{qz}_i, \mathbf{qw}_i$  in the case of vibrations with a single frequency  $\omega_i$  (Fig. 5).

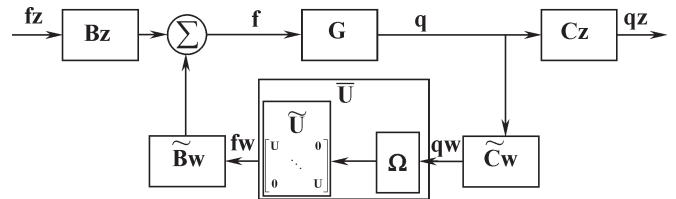


Fig 6. Block diagram of active control of mechanical vibrations

All of the vectors  $\mathbf{f}, \mathbf{fz}, \mathbf{fw}, \mathbf{q}, \mathbf{qz}, \mathbf{qw}$  in the generalized scheme are column vectors composed of the corresponding vectors for a single frequency. In other words, these vectors may be written in the form :

$$\mathbf{f} = \begin{bmatrix} \mathbf{f}_1 \\ \mathbf{f}_2 \\ \vdots \\ \mathbf{f}_k \end{bmatrix} \quad \mathbf{fz} = \begin{bmatrix} \mathbf{fz}_1 \\ \mathbf{fz}_2 \\ \vdots \\ \mathbf{fz}_k \end{bmatrix} \quad \mathbf{fw} = \begin{bmatrix} \mathbf{fw}_1 \\ \mathbf{fw}_2 \\ \vdots \\ \mathbf{fw}_k \end{bmatrix}$$

$$\mathbf{q} = \begin{bmatrix} \mathbf{q}_1 \\ \mathbf{q}_2 \\ \vdots \\ \mathbf{q}_k \end{bmatrix} \quad \mathbf{qz} = \begin{bmatrix} \mathbf{qz}_1 \\ \mathbf{qz}_2 \\ \vdots \\ \mathbf{qz}_k \end{bmatrix} \quad \mathbf{qw} = \begin{bmatrix} \mathbf{qw}_1 \\ \mathbf{qw}_2 \\ \vdots \\ \mathbf{qw}_k \end{bmatrix}$$

The matrices  $\mathbf{G}, \mathbf{\Omega}, \tilde{\mathbf{U}}, \bar{\mathbf{U}}, \tilde{\mathbf{Bw}}, \tilde{\mathbf{Cw}}, \mathbf{Bz}, \mathbf{Cz}$  are described by the following formulas :

$$\mathbf{G} = \begin{bmatrix} \mathbf{G}_1 & \mathbf{0} & \dots & \mathbf{0} \\ \mathbf{0} & \mathbf{G}_2 & \dots & \mathbf{0} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{0} & \mathbf{0} & \dots & \mathbf{G}_k \end{bmatrix} \quad \mathbf{\Omega} = \begin{bmatrix} \mathbf{\Omega}_1 & \mathbf{0} & \dots & \mathbf{0} \\ \mathbf{0} & \mathbf{\Omega}_2 & \dots & \mathbf{0} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{0} & \mathbf{0} & \dots & \mathbf{\Omega}_k \end{bmatrix}$$

$$\tilde{\mathbf{U}} = \begin{bmatrix} \mathbf{U} & \mathbf{0} & \dots & \mathbf{0} \\ \mathbf{0} & \mathbf{U} & \dots & \mathbf{0} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{0} & \mathbf{0} & \dots & \mathbf{U} \end{bmatrix} \quad \bar{\mathbf{U}} = \begin{bmatrix} \mathbf{U}_1 & \mathbf{0} & \dots & \mathbf{0} \\ \mathbf{0} & \mathbf{U}_2 & \dots & \mathbf{0} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{0} & \mathbf{0} & \dots & \mathbf{U}_k \end{bmatrix}$$

$$\tilde{\mathbf{Bw}} = \begin{bmatrix} \mathbf{Bw} & \mathbf{0} & \dots & \mathbf{0} \\ \mathbf{0} & \mathbf{Bw} & \dots & \mathbf{0} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{0} & \mathbf{0} & \dots & \mathbf{Bw} \end{bmatrix} \quad \tilde{\mathbf{Cw}} = \begin{bmatrix} \mathbf{Cw} & \mathbf{0} & \dots & \mathbf{0} \\ \mathbf{0} & \mathbf{Cw} & \dots & \mathbf{0} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{0} & \mathbf{0} & \dots & \mathbf{Cw} \end{bmatrix}$$

$$\mathbf{Bz} = \begin{bmatrix} \mathbf{Bz}_1 & \mathbf{0} & \dots & \mathbf{0} \\ \mathbf{0} & \mathbf{Bz}_2 & \dots & \mathbf{0} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{0} & \mathbf{0} & \dots & \mathbf{Bz}_k \end{bmatrix} \quad \mathbf{Cz} = \begin{bmatrix} \mathbf{Cz}_1 & \mathbf{0} & \dots & \mathbf{0} \\ \mathbf{0} & \mathbf{Cz}_2 & \dots & \mathbf{0} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{0} & \mathbf{0} & \dots & \mathbf{Cz}_k \end{bmatrix}$$

It is useful to note that all the generalized matrices may be described as a linear combination of the corresponding matrices for particular frequencies and certain other fixed matrices. In particular, for the matrix  $\tilde{U}$  it can be written :

$$\tilde{U} = \sum_{i=1}^k [(\mathbf{e}_i \times \mathbf{I}_s) \cdot \mathbf{U} \cdot (\mathbf{e}_i^T \times \mathbf{I}_{3r})] \quad (14)$$

where :

$\mathbf{e}_i$  - the  $i$ -th versor of dimension  $k$ , the symbol  $\cdot$  stands for the cartesian product of matrices.

### METHOD OF RESPONSE CIRCLES

The following set of equations can be written for the system presented in Fig.6 :

$$\mathbf{q} = \mathbf{G} \cdot \mathbf{f} \quad (15)$$

$$\mathbf{f} = \tilde{\mathbf{B}}\mathbf{z} \cdot \mathbf{f}\mathbf{z} + \mathbf{B}\mathbf{w} \cdot \mathbf{f}\mathbf{w} \quad (16)$$

$$\mathbf{q}\mathbf{z} = \tilde{\mathbf{C}}\mathbf{z} \cdot \mathbf{q} \quad (17)$$

$$\mathbf{q}\mathbf{w} = \mathbf{C}\mathbf{w} \cdot \mathbf{q} \quad (18)$$

$$\mathbf{f}\mathbf{w} = \bar{\mathbf{U}} \cdot \mathbf{q}\mathbf{w} \quad (19)$$

$$\bar{\mathbf{U}} = \tilde{\mathbf{U}} \cdot \boldsymbol{\Omega} \quad (20)$$

By combining equations (14) ÷ (22) the following relation can be derived :

$$\mathbf{q}\mathbf{z} = \mathbf{C}\mathbf{z}(\mathbf{I}_n - \mathbf{G} \cdot \tilde{\mathbf{B}}\mathbf{w} \sum_{i=1}^k [(\mathbf{e}_i \times \mathbf{I}_s) \cdot \mathbf{U} \cdot (\mathbf{e}_i^T \times \mathbf{I}_{3r})] \boldsymbol{\Omega} \cdot \tilde{\mathbf{C}}\mathbf{w})^{-1} \cdot \mathbf{G} \cdot \mathbf{f}\mathbf{z} \quad (21)$$

In the case without any feedback ( $\mathbf{U}=\mathbf{0}$ ) the vector  $\mathbf{q}\mathbf{z}_0$  of amplitudes of selected elements may be written in the form :

$$\mathbf{q}\mathbf{z}_0 = \mathbf{C}\mathbf{z} \cdot \mathbf{G} \cdot \mathbf{f}\mathbf{z} \quad (22)$$

By using equations (15) ÷ (21) it is possible to investigate how the real or imaginary part  $u$  of a particular coefficient of the matrix  $\mathbf{U}$  influences the amplitudes in vector  $\mathbf{q}\mathbf{z}$ . The vibrations of particular elements represented by the matrix  $\mathbf{q}\mathbf{z}$  can play a different role in the dynamic behaviour of the mechanical system. Therefore in some cases it is useful to use a weighted sum of amplitudes as a measure of the vibration level :

$$\sigma_z = \sum_i \mathbf{a}[i] \cdot \mathbf{q}\mathbf{z}[i] \quad (23)$$

where :

$\mathbf{a}[i]$  - the weight coefficient corresponding to amplitude  $\mathbf{q}\mathbf{z}[i]$ .

For the case of no feedback it can be similarly written :

$$\sigma_{z0} = \sum_i \mathbf{a}[i] \cdot \mathbf{q}\mathbf{z}_0[i] \quad (24)$$

After some transformation it is possible to show that the ratio  $\zeta$  of the indexes  $\sigma_z$  and  $\sigma_{z0}$  may be written in the general form :

$$\zeta(u) = \frac{\sigma_z(u)}{\sigma_{z0}(u)} = 1 + \sum_{i=1}^k \frac{u(\mathbf{a}_i + j\mathbf{b}_i)}{1 - u(\mathbf{c}_i + j\mathbf{d}_i)} \quad (25)$$

where :

$\mathbf{a}_i, \mathbf{b}_i, \mathbf{c}_i, \mathbf{d}_i, (1 \leq i \leq k)$  are real numbers.

The module  $|\zeta|$  shows the effect of the feedback loop on the weighted vibration amplitude of the elements of  $\mathbf{q}\mathbf{z}$ . In general, when  $\forall_{1 \leq i \leq k} [\mathbf{d}_i \neq 0 \wedge (\mathbf{a}_i \neq 0 \vee \mathbf{b}_i \neq 0)]$  equation (25) represents a closed smooth curve which is described by the end of the vector  $\zeta$  in the complex coordinate system ( $r, i$ ) when  $u$

varies from  $-\infty$  to  $+\infty$ . An example of such curve is presented for  $k = 4$  in Fig.7.

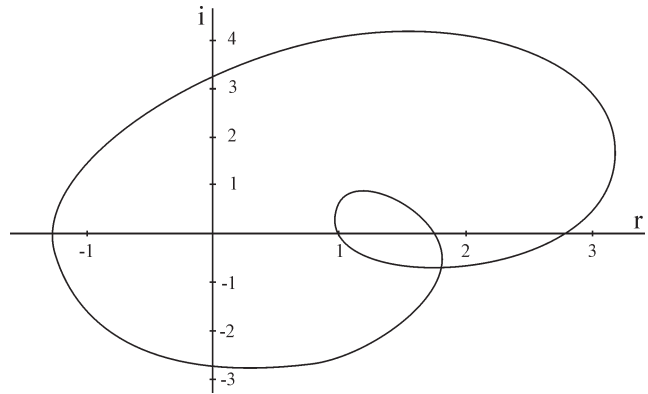


Fig 7. Graphical interpretation of equation (25) with exemplary coefficients for  $k$  different frequencies ( $k = 4$ )

When taking into consideration only a particular frequency  $\omega$ , equation (25) may be written in the form :

$$\zeta(u) = 1 + \frac{u(\mathbf{a} + j\mathbf{b})}{1 - u(\mathbf{c} + j\mathbf{d})} \quad (26)$$

where :

$\mathbf{a}, \mathbf{b}, \mathbf{c}, \mathbf{d}$ , are real numbers.

In the usual case when  $\mathbf{d} \neq 0 \wedge (\mathbf{a} \neq 0 \vee \mathbf{b} \neq 0)$ , equation (26) represents a circle which is described by the end of the vector  $\zeta$  in the complex coordinate system ( $r, i$ ) when  $u$  varies from  $-\infty$  to  $+\infty$  [7] :

$$\left[ r - \left( 1 - \frac{\mathbf{b}}{2\mathbf{d}} \right) \right]^2 + \left[ i - \frac{\mathbf{a}}{2\mathbf{d}} \right]^2 = \frac{\mathbf{a}^2 + \mathbf{b}^2}{4\mathbf{d}^2} \quad (27)$$

A graphical interpretation of equations (26, 27) is shown in Fig.8.

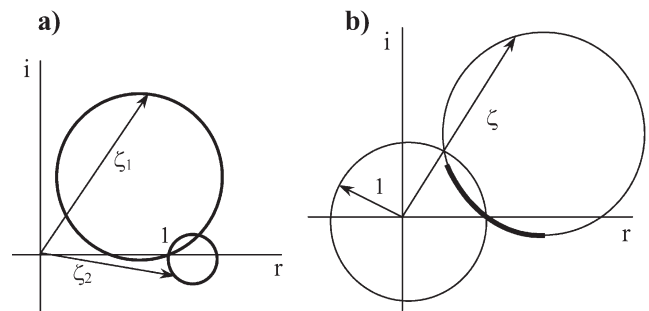


Fig 8. Graphical interpretation of equation (27)

The diameter of the closed curve (in particular – a circle) described by the vector  $\zeta$  enables to assess the influence of the parameter  $u$  on the weighted amplitude of vibrations. The large diameter of the curve (the circle described by the vector  $\zeta_1$  in Fig.8a) may be interpreted as a significant influence of the parameter  $u$ , while the small diameter (the circle determined by  $\zeta_2$  in Fig.8a) – as an insignificant influence of the parameter  $u$ . However in practice the parameter  $u$  has a reasonably limited range of values. Thus only a part of the curve drawn by the vector  $\zeta$  can be applied in practice (for example only the marked part of the circle shown in Fig. 8b). Moreover, only the values of parameter  $u$ , for which the module  $|\zeta|$  is lesser than 1, result in the decrease of the weighted amplitude level. In this way it is possible to estimate the effect of all parameters  $u$  on the vibrations and thus to choose :

- ★ the controller structure and its parameters for the best active control of mechanical vibrations

- ★ the changes of system parameters leading to the most effective reduction of vibrations
- ★ the shrouds and the glands which play the most important part in generating forces and moments in the case when rotor self-excited vibrations of aerodynamic type are considered.

## EXAMPLES

### A ship propulsion system

Ship propulsion systems equipped with flexible couplings are very sensitive to disturbances caused by unsteady engine operation. The disturbances have the form of shaft torque periodical changes which lead to torsional vibrations of the whole propulsion system. In some cases resonance vibrations resulting in damages to flexible couplings, were observed. This situation very often occurs when the engine works with one misfiring cylinder. In Fig.9a an example ship propulsion system is presented.

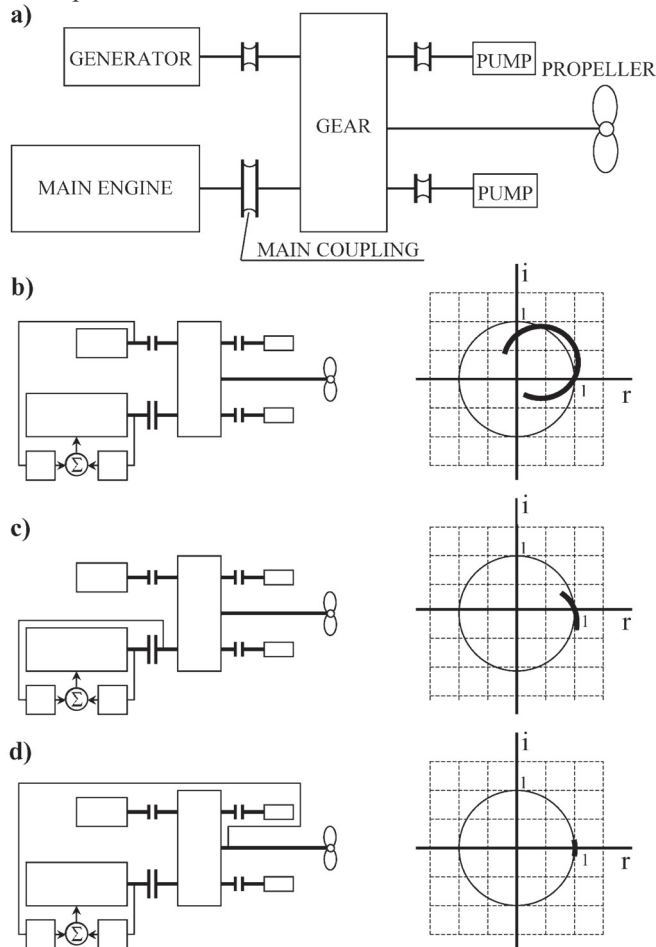


Fig 9. Variants of additional control systems and their response circles

It consists of a main medium-speed diesel engine which – through a main coupling and a mechanical gear – drives a ship propeller, an electric generator and two hydraulic pumps. All the couplings are flexible. The linear model of this system and the analysis of its behaviour was elaborated [11] by using the engine producer’s data and results of some additional investigations. The reduction of the main coupling torsional vibration was performed by modifying the main engine governor. Three system variants were considered for the following different correction input signals :

- angular velocity of the generator (Fig.9b)
- angular velocity of the main coupling (before the gear) (Fig.9c)

- angular velocity of the propeller shaft (measured directly after the gear) (Fig.9d).

In all three cases the presented method of response circles was applied to the analysis of the structure and parameters of the additional correction controller. The reduction ratio  $\zeta$  of the main coupling vibration amplitude for the case of a proportional controller, 270 rpm shaft speed and 14 Hz fundamental harmonic frequency of forced vibrations, is shown in Fig.9b ÷ 9d for the above mentioned system variants, respectively.

From Fig.9 it is evident that the controller using the generator’s angular velocity as its correction signal offers the largest possibilities of reducing torsional vibration amplitude in the main coupling.

### Turbine rotor self-excited vibrations

The method of response circles was used to select the seals which play the most important part in generating aerodynamic forces leading to self-excited vibrations of the rotor system. The forces were described by means of the rotordynamic coefficient matrix  $\mathbf{S} = [\mathbf{J}_u | \mathbf{B}_u | \mathbf{K}_u]$ , and calculated from relation (13). The relevant schematic diagram is shown in Fig.10.

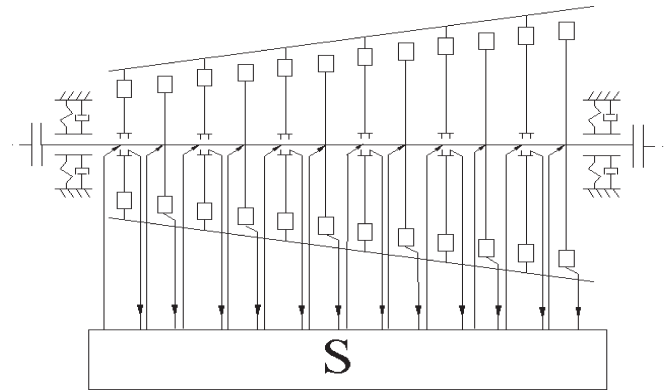


Fig 10. Schematic diagram for rotordynamic coefficient analysis

In the case of a double-cylinder medium-power steam turbine the performed analysis proved that the seals of the first stages of the HP cylinder had the greatest influence on self-excited vibrations of the aerodynamic type. It was enough to change the seals of the shrouds and shaft in first four turbine stages to achieve the desired effect of vibration reduction.

A currently conducted work is concentrated on active control of rotor vibrations of a steam turbine by means of pressurized bearings. The method of response circles is used to detect the bearings which have the greatest influence on active control. Results of the work in question will be presented in a separate paper in due course.

## CONCLUSIONS

- An analytical method for the investigation of linear mechanical systems performing harmonic motion was presented. This approach was successfully applied for the analysis and improvement of the dynamic behaviour of a ship propulsion system.
- The proposed theoretical method makes it possible to determine the parameters of the system’s dynamic flexibility matrix which show the most remarkable effect on the dynamic behaviour of the whole system.
- When active control is considered the method is useful in the designing of the structure and choice of parameters of the control system.
- In certain cases of self-excited vibrations the approach helps examining the elements of the system which are most responsible for this kind of excitation.

In Part II of the paper (to be published) a theoretical method of optimum vibration control by active means is described and illustrated by some examples, including an approach to pressurized bearing application.

### Acknowledgements

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### NOMENCLATURE

<b>B</b>	- matrix of damping coefficients
<b>C</b>	- selection matrix
<b>Bw, Bz, Cw, Cz</b>	- binary matrices
<b>D</b>	- transfer matrix
<b>f</b>	- vector of force amplitudes
<b>fw</b>	- controller output vector
<b>fz</b>	- vector of external forces
<b>F</b>	- vector of harmonic forces (or moments)
<b>G</b>	- dynamic flexibility matrix
<b>I</b>	- unitary matrix
<b>j</b>	- imaginary unit
<b>J</b>	- matrix of inertia moments
<b>k, n, r, s</b>	- dimensions of matrices and vectors
<b>K</b>	- matrix of stiffness coefficients
<b>P</b>	- matrix of changes of system parameters
<b>q</b>	- vector of displacement amplitudes
<b>qw</b>	- vector of measured amplitudes
<b>qz</b>	- vector of amplitudes of controlled elements
<b>Q</b>	- vector of displacements
<b>R</b>	- controller matrix
<b>S</b>	- matrix of rotordynamic coefficients $J_u, B_u, K_u$
<b>t</b>	- time
<b>U</b>	- general symbol for the matrices <b>P, R, S</b>
$\sigma$	- weighted sum of amplitudes
$\omega$	- frequency
$\Omega$	- frequency multiplier matrix

**Indices :** P - proportional controller  
I - integrating controller  
D - differentiating controller  
u - of rotordynamic coefficients of turbine seals

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## Miscellanea

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