

Location of ship rolling axis

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ABSTRACT



In the paper is presented a definition of ship rolling axis and a method for determination of location of the axis. Location of rolling axis depends not only on ship mass distribution but also on the so-called „added masses” which are equivalents of the hydrodynamic forces acting on ship hull during its motion. It has been assumed that the rolling axis is fixed with respect to ship's hull, and its location is determined with accounting for the added masses accompanying ship rolling, whereas the influence of damping forces is neglected. For a representative group of ships appropriate coefficients of hydrodynamic forces were calculated on the basis of the experimental test results available in the subject-matter literature. It has been assumed that the rolling axis passes through a common centre of ship's mass and relevant added masses. As a result proposed are simple approximate formulas which make it possible to calculate location of ship rolling axis on the basis of the typical ship data available at the preliminary design stage.

Key words : ship seaworthiness, ship safety, ship hydromechanics, stability of floating units

INTRODUCTION

The ship rolling axis is a conventional notion which serves mainly to simplified representation of ship motions in the case when ship rolling prevails. Under the assumption that the location of ship rolling axis is known it is easy to determine translations and accelerations in any ship's point, as then they are functions of distance of a given point from the axis in question. In some of the points the placing of some cargoes, measuring instruments, passenger cabins should be avoided or – in reverse – they may appear most favourable for installing given devices. In order to estimate rolling axis location a ship physical model of two degrees of freedom (dof) was assumed. However if the location is assumed permanent the model can be taken as that of one dof only. In this model it is assumed that the considered ship rolls in calm water due to an external excitation free moment of appropriately selected parameters; it can also roll freely. In this paper the method of determining location of the rolling axis is described and – on this basis – submitted are simplified formulas for calculation of its location, applicable to typical merchant ships.

DEFINITION OF SHIP ROLLING AXIS

It is usually assumed that the ship rolling axis coincides with X-axis of the coordinate system whose origin is fixed with respect to the ship in its mass centre [3,5]. It is also assumed that the axis which lies on the ship plane of symmetry and initially is parallel to ship waterline, can be considered as the ship's main axis of inertia. With respect to it, calculated is the ship mass moment of inertia I_x , an element appearing in

equations of motion. The axis is assumed motionless with respect to both the ship and space. The approximate character of the calculations makes it possible to use values of the inertia moments of the assumed form, i.e. $(I_x + I_{xx})$, in spite of the fact, that the inertia moments are not corrected with taking into account a real location of rolling axis. It was this way assumed because the magnitudes of the added mass inertia moments of water, I_{xx} , are often determined on the basis of the measured free-rolling period of ship, hence the correct location of rolling axis has been already accounted for. This also concerns the approximate calculation formulas for both I_x and I_{xx} values.

LOCATION OF SHIP ROLLING AXIS

In order to determine location of ship rolling axis in accordance with the above specified assumptions it is postulated that the ship oscillates in calm water, having two degrees of freedom : roll and sway. In this case the line on the ship's plane of symmetry, where lateral translations in the motionless, earth-fixed coordinate system are equal to zero, can be considered as the rolling axis. The similar approach is presented in [2, 7].

The following equation of ship's rolling is taken for considerations :

$$m \ddot{y}_G + m_{22} \ddot{y} + 2\mu_{22} \dot{y} + m_{24} \ddot{\Phi} + 2\mu_{24} \dot{\Phi} = 0$$

and after changing notation :

$$m \ddot{y}_G + m_{yy} \ddot{y} + 2\mu_{yy} \dot{y} + m_{y\Phi} \ddot{\Phi} + 2\mu_{y\Phi} \dot{\Phi} = 0 \quad (1)$$

where :

- y_G - lateral translation of ship mass centre
- y - lateral translation of the point O (Fig.1)
- Φ - ship rolling angle
- m - ship mass
- m_{yy} - added mass of water for ship sway
- $m_{y\Phi}$ - added mass of water for ship sway in result of rolling with respect to the point O_0
- μ_{yy} - damping coefficient for ship sway
- $\mu_{y\Phi}$ - damping coefficient for ship sway due to rolling with respect to the point O_0

This is the equation which represents the sum of projections – on Y-axis – of the forces acting upon the ship, due to sway. In the equation (1) there are no expressions for restoring forces which really do not appear during sway.

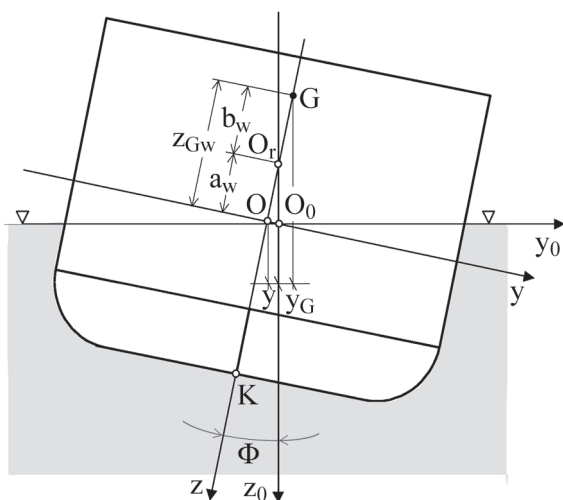


Fig. 1. The coordinate systems and location of rolling axis

In Fig.1 are presented : the ship in a heeling position, the motionless, earth-fixed coordinate system O_0, Y_0, Z_0 , and the movable, ship-fixed coordinate system O, X, Y . In the initial position of the ship both the coordinate systems coincide.

In Fig.1 the point O_r represents the trace of the rolling axis and in accordance with the assumed definition it always remains on Z_0 - axis of the motionless coordinate system. If in the ship-fixed reference system the quantity z_{Gw} stands for the distance of the mass centre G from the point O , i.e. from the initial ship waterline, then the trace of the rolling axis, O_r , splits the segment OG into two parts :

$$z_{Gw} = a_w + b_w$$

For small heeling angles it can be written as follows :

$$y_G = -b_w \Phi \quad \text{and} \quad y = a_w \Phi$$

The quantities y determine shifts of the origin O of the movable coordinate system. The added mass $m_{y\Phi}$ and the damping coefficients μ_{yy} , $\mu_{y\Phi}$ are determined with respect to the origin O . The here considered damping coefficients μ_{yy} and $\mu_{y\Phi}$ take into account only dissipation of the energy generated by the radiation forces with neglecting the viscosity forces.

As the segment $a_w = OO_r$, and $b_w = GO_r$, hence $a_w < 0$ when the trace of the rolling axis, O_r , lies over the initial waterline, i.e over the point O , and $b_w < 0$ when O_r lies below the ship mass centre G (Fig.1). Now the equation (1) can be written as follows :

$$-m b_w \ddot{\Phi} + m_{yy} a_w \ddot{\Phi} + 2\mu_{yy} a_w \dot{\Phi} + m_{y\Phi} \ddot{\Phi} + 2\mu_{y\Phi} \dot{\Phi} = 0 \quad (2)$$

Knowing that $a_w = z_{Gw} - b_w$, and introducing appropriate transformations one obtains :

$$-b_w [(m + m_{yy}) \ddot{\Phi} + 2\mu_{yy} \dot{\Phi}] = - (m_{yy} z_{Gw} + m_{y\Phi}) \ddot{\Phi} - 2(\mu_{yy} z_{Gw} + \mu_{y\Phi}) \dot{\Phi} \quad (3)$$

If the influence of damping is now neglected, i.e. $\mu_{yy} = 0$ and $\mu_{y\Phi} = 0$, then the following expressions for the distance b_w of the rolling axis point O_r from the ship mass centre G , and for the distance a_w of the point O_r from the initial waterline (i.e. from the point O , Fig.1) are obtained:

$$b_w = \frac{m_{yy} z_{Gw} + m_{y\Phi}}{m + m_{yy}} \quad (4)$$

$$a_w = \frac{z_{Gw} m - m_{y\Phi}}{m + m_{yy}}$$

Therefore, with the lack of damping the rolling axis location is fixed for a given rolling period, and it accounts for the influence of the inertia forces resulting from the water surrounding the ship, here represented by the added masses m_{yy} and $m_{y\Phi}$. If the ship having the same rigidity, rolled in the „free space”, i.e. not being immersed in the water, then m_{yy} and $m_{y\Phi}$ would equal zero. In such case the rolling axis would go through the mass centre G , and then it would be :

$$b_w = 0 \quad \text{and} \quad a_w = z_{Gw}, \quad \text{respectively.}$$

In order to determine rolling axis location for a considered ship it is necessary to calculate the added masses m_{yy} and $m_{y\Phi}$. For the quantities it is hard to find approximate value, which is not the case for e.g. rolling or heaving. The quantities can be calculated analytically by using the „strip theory” and applying the Lewis frame-like forms [3, 5, 7]. However this requires using a special software together with many initial data of a considered ship.

For these reasons a simplified method is proposed below. It can be useful for the approximate determination of rolling axis location on the basis of the ship's data usually available for a designed ship.

A METHOD OF DETERMINING LOCATION OF ROLLING AXIS

In order to determine the location of rolling axis in which an influence of added water mass is accounted for, a simplified ship representing a given one has been assumed. The simplified ship is a cylinder of rectangular frame-section and its parameters are so selected as to obtain its roll and sway motions as close as possible to those of the basic ship. Hence the ship is of the same mass m , B/d ratio and free-rolling period τ . The centre of gravity G of the simplified ship is located, proportionally to its main dimensions, in the same point as that of the real ship. It has also the same transverse moment of inertia of waterplane area, which indirectly gives the same small meta-centric radiuses. The last condition makes it possible to determine the main dimensions, L_s , B_s , d_s , of the simplified ship on the basis of the real ship's parameters. It was further assumed that a change of location of rolling axis of the simplified ship, resulting from added mass influence, correctly describes the corresponding change in the basic ship.

Values of the added masses and damping coefficients for the ship of rectangular frame sections (as well as for some other simple forms) are published in [8] and [3] in the form diagrams of the dimensionless values m'_{yy} , $m'_{y\Phi}$, of the added masses m_{yy} and $m_{y\Phi}$, in function of the dimensionless ship rolling frequencies ω' .

In order to determine m'_{yy} it is also necessary to know B/d ratio.

Now it is possible to describe successive steps for determination of ship's rolling axis location in compliance with the above made assumption.

Step 1.

Selection of ship's data

The demanded ship's data are as follows :

- L_{bp} - length between perpendiculars [m]
- B - breadth [m]
- d - draught [m]
- H - depth to upper deck [m]
- m - ship mass [t]
- d - block coefficient of ship's hull [-]
- a - waterplane coefficient [-]
- z_G - height of the ship mass centre over the ship base plane [m]
- GM - initial transverse metacentric height of the ship, not corrected for free- surface influence [m]

In order to obtain calculation results of a sufficiently general character the data of different ships in various loading conditions were used (Tab.1).

Step 2.

Determination (calculation) of ship's rolling period τ

The best solution is to have a value of the rolling period measured on the real ship. However such data are hard available. An approximate value of ship's free-rolling period can be calculated by using the following relationship :

$$\tau = 2\pi \sqrt{\frac{I_x + I_{xx}}{mgGM}} \quad (5)$$

To determine values of I_x and I_{xx} the expressions given in [6] were used :

$$I_x = \frac{m}{12} (B^2 + 4z_G^2) \quad I_{xx} = 0.3 I_x \quad (6)$$

In further considerations the quantity $(I_x + I_{xx})$ is deemed a sufficient approximation of the total moment of inertia of ship mass and added mass of water, respective to ship rolling axis, as it was postulated in its definition.

Step 3.

Calculation of the main dimensions L_s , B_s , d_s of the simplified ship

The ship breadth B_s is determined by assuming that values of the moment of inertia, I_{wx} , of waterplane areas of the real ship and the simplified one are equal. The I_{wx} of the real ship waterplane is determined by approximating ordinates of its contour, $y_w(x)$, by means of the parabola whose order is expressed by the waterplane coefficient α , [4]:

$$y_w(x) = \frac{B}{2} \left[1 - \left(\frac{x}{L/2} \right)^{1-\alpha} \right] \quad (7)$$

$$I_{wx} = \frac{LB^3}{2} \cdot \frac{\alpha^3}{(1+\alpha)(1+2\alpha)}$$

Basing on I_{wx} given by (7) and in accordance with the following relationships :

$$\frac{L_s B_s^3}{12} = I_{wx} \quad L_s B_s d_s = L B d \delta \quad \frac{B_s}{d_s} = \frac{B}{d}$$

one obtains the expression for the breadth B_s , draught d_s and length L_s of the simplified ship :

$$B_s = \frac{6B}{\delta} \cdot \frac{\alpha^3}{(1+\alpha)(1+2\alpha)} \quad (8)$$

$$d_s = \frac{B_s d}{B} \quad L_s = \left(\frac{B}{B_s} \right)^2 L \delta$$

Step 4 and 5.

Calculation of ω' , read-out of m'_{yy} and $m'_{y\Phi}$, and calculation of m_{yy} and $m_{y\Phi}$

The form of the quantities to be calculated, in compliance with the previously applied symbols, is as follows :

$$\omega' = \omega \sqrt{\frac{B_s}{2g}} \quad m'_{yy} = \frac{m_{yyj}}{\rho A_s}$$

$$m'_{y\Phi} = \frac{m_{y\Phi j}}{\rho A_s B_s} \quad (9)$$

$$m_{yy} = L_s \cdot m_{yyj} \quad m_{y\Phi} = L_s \cdot m_{y\Phi j}$$

The values m_{yyj} [t/m] and $m_{y\Phi j}$ [tm/m] concern the ship of the one-meter length and of the assumed, simplified frame section.

The remaining symbols stand for :

- ρ - water mass density [t/m³]
- g - acceleration of gravity [m/s²]
- B_s - breadth of the simplified ship [m]
- A_s - ship's frame area [m²] (here : $A_s = B_s \cdot d_s$)

Having read values m'_{yy} and $m'_{y\Phi}$ according to w' , one calculates, values m_{yyj} and $m_{y\Phi j}$, and next m_{yy} and $m_{y\Phi}$, by using (9).

Step 6.

Calculation of rolling axis location

The calculations were carried out for the simplified ships. First, by using (4) the segments a_{ws} and b_{ws} were calculated and then applied in their relative form by making use of the following relations:

$$a_{ws}/B_s = a_w/B \quad \text{and} \quad b_{ws}/B_s = b_w/B$$

$$\text{to obtain : } a_w = (a_{ws}/B_s) \cdot B \quad \text{and} \quad b_w = (b_{ws}/B_s) \cdot B$$

Next, the values of the added masses, m_{yy} and $m_{y\Phi}$, segments a_{ws} and b_{ws} , as well as ratios a_{ws}/B_s and b_{ws}/B_s were calculated on the basis of the above presented assumptions and formulas.

The calculations were performed for 9 ships in different loading conditions. Tab.1 contains the input data of the ships, and the calculation results of the relative values of rolling axis location are shown in Tab.2 (according to the increasing sequence of z_{Gw}/B values).

The segment $a_w = OO_r$ was selected as the quantity determining location of rolling axis. The segment a_w being the distance

Tab. 1. Input data of 9 selected ships in different loading conditions

No.	Ship	Loading cond.	L_{bp} [m]	B [m]	d [m]	m [t]	α [-]	δ [-]	z_{GW} [m]	τ [s]
1	ferryship	full	154.20	28.50	6.65	20660	0.915	0.672	-6.92	15.6
2	STENA	empty	154.20	28.50	5.87	16160	0.815	0.672	-8.56	18.2
3	ferryship	full	115.00	19.50	5.15	6680	0.755	0.546	-3.27	14.4
4	ROGALIN	empty	115.00	19.50	4.36	5532	0.725	0.529	-4.82	16.1
5	ferryship		120.46	21.70	5.64	8786	0.809	0.596	-3.10	11.5
6	POMERNIA	empty	120.46	21.70	4.68	6786	0.718	0.555	-4.75	13.6
7	semi-container	full	140.00	22.00	9.14	20767	0.855	0.718	0.73	19.0
8	ship	ballast I	140.00	22.00	3.60	7235	0.712	0.635	-6.58	14.8
9	WARSZAWA	ballast II	140.00	22.00	5.75	12231	0.745	0.669	-1.85	12.8
10	dry cargo ship	full	134.00	21.40	8.96	19971	0.800	0.754	0.17	16.7
11	tanker	full	272.00	43.40	15.20	152599	0.884	0.826	4.55	12.0
13	GIEWONT II	heavy ballast	272.00	43.40	10.45	102540	0.851	0.807	-0.28	10.9
14		full	205.00	30.50	12.09	62450	0.871	0.826	2.64	13.7
15	bulk carrier	ore in 5 th hold	205.00	30.50	7.12	35620	0.818	0.801	-1.60	10.4
16	UNIWERSYTET	heavy ballast	205.00	30.50	8.64	43616	0.834	0.807	-0.13	11.5
17		grain	205.00	30.50	11.33	58226	0.861	0.822	1.17	16.1
18	Ro-Ro ship	ballast	182.00	31.00	9.30	34442	0.774	0.637	-3.30	28.8
19	POZNAN	ballast; departure	182.00	31.00	6.80	24012	0.696	0.607	-3.16	12.0
20	fishing trawler	loaded	28.47	6.70	2.76	281	0.77	0.533	0.29	5.8

Tab. 2. Calculation results of rolling axis location for the selected ships

Ship no.	z_{GW}/B [-]	a_w/B [-]	b_w/B [-]
2	-0.300	-0.042	-0.241
8	-0.299	-0.042	-0.258
4	-0.247	-0.010	-0.208
1	-0.243	-0.002	-0.237
6	-0.219	0.012	-0.191
3	-0.168	0.040	-0.231
5	-0.143	0.049	-0.072
18	-0.106	0.054	-0.257
19	-0.102	0.067	-0.171
9	-0.084	0.087	-0.087
15	-0.052	0.092	-0.036
13	-0.007	0.091	-0.097
16	-0.004	0.118	-0.042
10	0.008	0.095	-0.145
7	0.033	0.105	-0.122
17	0.038	0.121	-0.083
20	0.043	0.105	-0.161
14	0.087	0.128	-0.169
11	0.105	0.140	-0.062

of rolling axis from waterline, correctly describes the influence of the added masses determined relative to the point O, (Fig.1), on location of rolling axis.

It was also assumed that the relative quantity a_w/B was most tightly connected with the relative distance of the ship mass centre from the same waterline, z_{GW}/B .

The relationship $a_w/B = f(z_{GW}/B)$ is shown in Fig.2. This diagram reveals good correlation between the selected quantities. A significant part of the irregularities observed in the diagram can be deemed as a result of the accepted simplifying assumptions, as well as inaccuracy in reading-out intermedia-

te values. Though the diagram has been obtained for the simplified ships, it can be considered as that correctly describing the selected relationships also for the real ships. The straight line described as :

$$a_w/B = 0.432 (z_{GW}/B) + 0.102 \quad (10)$$

fits into the relationship $a_w/B = f(z_{GW}/B)$ the best.

After a simple analysis of (10) it can be demonstrated that the rolling axis is located approximately 0.1B under the water when the ship mass centre lies on its surface, and that the rolling axis is located on the water surface when the ship mass centre is abt. 0.24B over the water, and finally, that the rolling axis goes through the ship mass centre when it is located abt. 0.18B under the water surface. To make the diagram more clear, also the straight line $z_{GW}/B = f(z_{GW}/B)$ was drawn in it.

By taking into account that $z_{GW} = d - z_G$ the following approximate expressions for the distance of ship rolling axis from ship's waterline and ship mass centre, respectively, can be obtained :

$$a_w \cong 0.43 (d - z_G) + 0.1B \quad (11)$$

or

$$b_w \cong 0.57 (d - z_G) - 0.1B$$

This is a simplified form of the expressions (4) in which the hydrodynamic coefficients m_{yy} and $m_{y\phi}$ functionally depend also on the rolling frequency ω . The, observed in Fig.2, small scatter of the points, i.e. values determined by (4), indicates that rolling axis location weakly depends on other para-

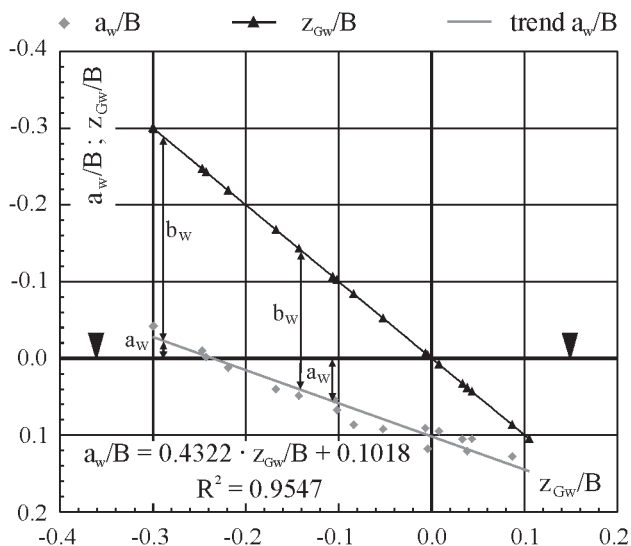


Fig. 2. Location of ship rolling axis respective to ship waterline

meters than those represented by the coordinates shown in the diagram in question. In the obtained formulas the dependence on the rolling frequency has place indirectly only because the calculations for all the selected ships have been carried out with taking into account their free-rolling frequencies, hence the diagram of Fig.2 and the formulas (10) and (11) obtained on its basis, concerns just such frequencies.

FINAL REMARKS AND CONCLUSIONS

- Ship rolling motions are of a great importance in considering the problems of ship stability, strength, cargo lashing, comfort and safety of passengers and crew members; they are also important for correct operation of ship equipment.
- For considering and solving many of the above mentioned problems it is usually sufficient to assume the ship to be an object of one degree of freedom. When such approach is to be used correct determination of location of ship rolling axis is always crucial.
- Due to the water supporting the ship during its motions, which forms constraints of a special type, it is possible to assume the free ship rotation axis to be a straight line passing through – or close to – the ship mass centre. However the total ship mass consists of the mass of the ship itself and the supplementary, „added masses” of water, associated with relevant motions. These conventional masses affect location of ship rolling axis in accordance with places of their occurrence.
- If the ship rolling axis is so defined as it has been proposed in this paper the usually assumed location of rolling axis can be improved; the corrections might reach even $0.1 \div 0.2B$ as it was shown in this paper.
- It is also important that the proposed formulas, (10) and (11), concern resonant rolling, as in real conditions the ship responds to instantaneous excitations with motions of natural frequency, and in irregular waves it „selects” – from an encountered wave spectrum – mainly the frequencies close to natural frequencies of its own motions.
- The proposed simple approximate formulas make it possible to calculate location of ship's rolling axis only on the basis of the typical ship data available at the preliminary design stage.

NOMENCLATURE

a_w	- distance from rolling axis to waterline of a real ship [m]
a_{ws}	- distance from rolling axis to waterline of a simplified ship [m]
b_w	- distance from rolling axis to mass centre of a real ship [m]
b_{ws}	- distance from rolling axis to mass centre of a simplified ship [m]
dof	- degrees of freedom
D	- ship displacement [T or kN]
g	- acceleration of gravity [m/s^2]
GM	- initial metacentric height of a ship, not corrected for free-surface influence [m]
I_x	- ship mass moment of inertia relative to X- axis [tm^2]
I_{xx}	- added mass moment of inertia relative to X- axis [tm^2]
L, B, d, H	- main dimensions of a ship [m]
m	- ship mass [t]
$m'_{yy}, m'_{y\Phi}$	- dimensionless coefficients of added mass of water [-]
$m_{yyj}, m_{y\Phi j}$	- unitary coefficients of added mass of water [t/m, tm/m]
$m_{yy}, m_{y\Phi}$	- coefficients of added mass of water [t, tm]
s	- index of a simplified ship
Y_G, Y	- swaying translations of ship mass centre G and point O, respectively [m]
z_G	- height of ship mass centre over base plane [m]
z_{GW}	- height of ship mass centre over waterplane [m]

α	- waterplane coefficient [-]
δ	- hull block coefficient [-]
τ	- period of ship's free rolling [s]
Φ	- heeling angle [rad or deg]
ω	- angular frequency of ship's free rolling [1/s]
ω'	- dimensionless ship rolling frequencies [-]

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Conference

E Regional Group E

On 11 December 2003 at Faculty of Ocean Engineering and Ship Technology, Gdańsk University of Technology, held was the last-in-that-year seminar of the Regional Group of the Section on Exploitation Foundations, Machine Building Committee, Polish Academy of Sciences.

During the seminar three papers were presented :

- *A global model of environmental safety of ship power plant* – by R. Liberacki (Gdańsk University of Technology)
- *Research on water-lubricated bearings of ship main shafting* – by W. Litwin (Gdańsk University of Technology)
- *An attempt to diagnosing the main propulsion systems of the ferry ships "Polonia" and "Pomerania" by means of the FAM-C method* – by A. Gębura (Air Force Institute of Technology, Warsaw)

The seminar was a good occasion to offer wishes to Prof. J. Lewitowicz in his 70th birthday and to pass congratulations to him due to his outstanding achievements in the area of exploitation of technical objects, especially in aeronautics.