

**ZESZYTY NAUKOWE NR 2 (74)  
AKADEMII MORSKIEJ  
W SZCZECINIE**

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**EXPLO-SHIP 2004**

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**Steady State Ship Attitude during Manoeuvres  
in a Bank Vicinity**

Key words: manoeuvring, mathematical model, restricted water, bank effect, canal

*The existing regression models on the bank (wall) effect are compared to one another both dynamically (force level) and kinematically (counteracting helm and hull drift attitude during a steady-state passage along a bank). A small chemical tanker of known hull and rudder hydrodynamics is used as an example of computations. A lot of essential qualitative and quantitative discrepancies have been found, which might claim the regression models of little use in an adequate bank effect simulation. Further research is required in this field.*

**Równowaga statku podczas oddziaływania efektu brzegowego**

Słowa kluczowe: manewrowanie, model matematyczny, efekt brzegowy, kanał

*Przeprowadzono symulację efektu brzegowego w kanale dla małego chemikaliowca, wykorzystując dostępne modele regresyjne tego zjawiska. Główny nacisk położono na wielkość dryfu i wychylenie steru potrzebne do zrównoważenia efektu brzegowego podczas ustalonego ruchu jednostki poza osią kanału. Stwierdzono zasadnicze różnice między poszczególnymi modelami, co uniemożliwia uzyskanie prawdziwego ilościowego obrazu zjawiska. Konieczne są dalsze prace nad modelowaniem efektu brzegowego.*

## Introduction

The bank (wall) effect is a phenomenon of the arising sway force and yaw moment while a ship sails under conditions of laterally asymmetric flow around her hull. The latter is caused by the close presence of horizontal water boundaries in form of surface piercing or flooded, vertical or sloping banks. These may be single, on one ship's side (an example of a ship moving along a quay), or double (like in a canal). In the case of double banks, a ship must proceed off-centre of the fairway for the bank effect to occur. The bank effect is mostly often understood as ship stern suction, though another behaviour has been lately stressed as experienced in the towing tanks – the bow repulsion. In either of these situations, the bow-out yaw moment is the matter of fact i.e. turning a ship towards open water.

A research on the bank effect has been seriously done over the last 30 years with different intensity. One of the first significant contribution was that made by [Fujino, 1968], who published data on asymmetry related hull force and moment derivatives for a ship engaged in a canal passage of different width but with regard to a small deviation from the canal centre-line. Some model test studies were performed by e.g. [Norrbin, 1974], [Dand, 1982], and [Li, 2000b] aimed at the quantification of the bank effect excitations and the provision of some structural relationship for them. In [Norrbin, 1974], one can find results on bank transient forces for a ship navigating in the vicinity of a rapidly changing bank horizontal layout.

Vital efforts of systematic model tests were made by e.g. [Norrbin, 1985], [Ch'ng et al., 1993], [Li, 2000ab], [Vantorre et al., 2003] in order to further improve the structural models and build the bank effect regression models, very useful in preliminary simulation analyses of different ship types and dimensions. Unfortunately, [Li, 2000a] does not reveal his regression coefficients and is not taken into consideration hereafter. As it will be seen later in the present study, the other three models differ much from one another – there is even too big a contrast between them. Thus thoroughgoing additional (analytical or experimental) validation projects are necessary here and, perhaps, a redesign of the underlying formulas is required as well. Most of the regression models are based on measurements carried out for not more than two ships, of the fine and full form, which do not seem to be representative ones.

Very interesting results of numerical computations (CFD) of the bank effect transient sway force and yaw moment for a broad range of waterway horizontal elements are shown in e.g. [Hsiung/Gui, 1988], [Gui et al., 1990].

Some aspects of the general force balance during a ship steady-state parallel movement along a bank have been investigated by e.g. [Hess, 1978] (pure theoretical treatment, very rough qualitative trends), [Fuehrer, 1981], [Romisch,

1997], [Norrbin, 1985], and [Li et al., 2003]. The latter reference, though pretending to be comprehensive one, does not give however information about the ship hull drift angle and more details on the hull and rudder models used.

Finally, some bold extrapolation of the bank effect excitations from one vertical section shape to another was brought e.g. by [Kabacinski/Czyszczoń, 1986].

Reverting to the topic of force and moment equilibrium in a steady-state run as mentioned before, it shall be stated that it has many practical implications. Among others, the only way to validate and investigate the bank effect magnitude in full scale is to have a look both at the applied helm angle (also directly perceived by a navigator) and the experienced hull drift angle. Drift angles in the order of one degree are usually reported, but the resultant hull sway force and yaw moment are major factors in the overall force balance.

The present study objectives are:

- to compare the known regression models of the bank effect for a small chemical tanker (the ship investigated in the author's previous studies) and specify their basic properties – some preliminary real-time simulation runs show a few adequacy problems of the formulas from the ship-handling point of view;
- to obtain the steady-state helm and drift attitudes for the hull and rudder hydrodynamics as identified through analysis of full scale deep water manoeuvring trials with shallow water corrections imposed upon the hull excitations – some singularities are observed if the bank effect is dominated by the bow repulsion.

Various combinations of the canal depth, width, ship's offset and speed are also accounted for.

## 1. Manoeuvring equations

The steady-state passage of a ship, including the case of straight-linear ship motion along a bank, enables a crucial simplification of the general ship manoeuvring mathematical model. All yaw (damping) related terms in the hull excitations may be disregarded, as well as the rudder local drift angle essential in the formation of rudder forces. This remark is important in view of the manoeuvring model identification or validation.

In this context, only two rather plain equations are normally examined:

$$\begin{cases} 0 = F_{yH} + F_{yR} + F_{yBE} \\ 0 = \underbrace{(m_{22} - m_{11})v_x v_y + M_{zH}^*}_{M_{zH}} + M_{zR} + M_{zBE} \end{cases} \quad (1)$$

where:

- $F_y, M_z$  – sway force and yaw moment (both positive to starboard),
- $H, R, BE$  – subscripts indicating hull, rudder, and bank effect origins,
- $m_{11}, m_{22}$  – surge and sway added masses,
- $v_x, v_y$  – surge and sway (positive to starboard) velocities,
- $M_{zH}$  – hull hydrodynamic yaw moment as directly measured in towing tanks, being the sum of the Munk moment and the pure hull yaw moment  $M_{zH}^*$ .

The hull hydrodynamic influences are frequently written as:

$$\begin{bmatrix} F_{yH} \\ M_{zH} \end{bmatrix} = 0.5 \rho L T v_{xy}^2 \begin{bmatrix} c_{fyh} \left( \beta, \frac{h}{T}, \frac{b}{B} \right) \\ L c_{mzh} \left( \beta, \frac{h}{T}, \frac{b}{B} \right) \end{bmatrix},$$

$$v_{xy} = \sqrt{v_x^2 + v_y^2} \quad \beta = -\arctg\left(\frac{-v_y}{v_x}\right) \quad (2)$$

where:

- $\rho$  – water density,
- $v_{xy}$  – total linear velocity,
- $\beta$  – drift angle (positive if a ship tends to port side),
- $c_{fyh}, c_{mzh}$  – sway force and yaw moment coefficients,
- $h/T$  – water depth-to-draft ratio,
- $b/B$  – canal width-to-beam ratio.

The rudder excitations may be described in a general form according to:

$$F_{yR} = 0.5 \rho A_R v_R^2 c_{fyR}(\delta, \alpha, c_{Th}) \cdot (1 + a_H) \quad M_{zR} = F_{yR} x_R \quad (3)$$

where:

- $v_R$  – rudder flow reference velocity (the sum of jet and local lateral velocity),
- $c_{fyR}$  – rudder sway force coefficient,
- $\delta, \alpha$  – rudder angle and rudder flow effective incidence angle (both positive to port)
- $c_{Th}$  – propeller loading ratio,
- $a_H$  – hull-rudder interaction coefficient,
- $x_R$  – rudder abscissa (negative to stern).

Adopting a straight-linear motion under reasonably low drift angles (high drift angles are obviously undesirable in a canal), the so-called rudder local drift angle, changing the effective rudder incidence angle, diminishes so that the rudder lateral force coefficient  $c_{fR}$  is equal to the rudder lift coefficient  $c_L$ :

$$F_{yR} = 0.5 \rho A_R v_{PS}^2 c_L(\delta, c_{Th}) \cdot (1 + a_H) \quad \delta = \alpha \quad (4)$$

where  $v_{PS}$  is the propeller slipstream velocity.

Fig. 1 displays the hull sway force and yaw moment coefficients,  $c_{f_{yh}}$  and  $c_{m_{zh}}$ , and the rudder lift chart for the chemical tanker of data as per Tab. 1. The coefficient values are based on the full-scale deep-water identification [Artyszuk, 2003]. Anyhow, the hull coefficients are adjusted for low water depths using the [Kijima et al., 1990] shallow water corrections for hull linear derivatives. According to Fig. 1, the virtual centre of pressure is always positioned near the bow between  $+0.54L$  (deep water) and  $+0.62L$  (shallow water) from the midship section. An additional increase of hull forces due to a finite canal width is not accounted for due to the lack of data and much poorer accuracy of the bank effect estimation.

Table 1

Ship characteristics  
Parametry statku

Dimension	Value
length $L$ [m]	97.4
beam $B$ [m]	16.6
draft $T$ [m]	7.1
block coefficient $c_B$ [-]	0.76
sea speed $v$ [kt]	14.1
rudder area $A_R$ [m <sup>2</sup> ]	12.3
rudder aspect $\lambda$ [-]	1.5
rudder type	Schilling
engine power $P_n$ [kW]	3600
engine revs $n_n$ [rpm]	146
propeller diameter $D$ [m]	4.1
propeller pitch ratio $P/D$ [-]	0.8719

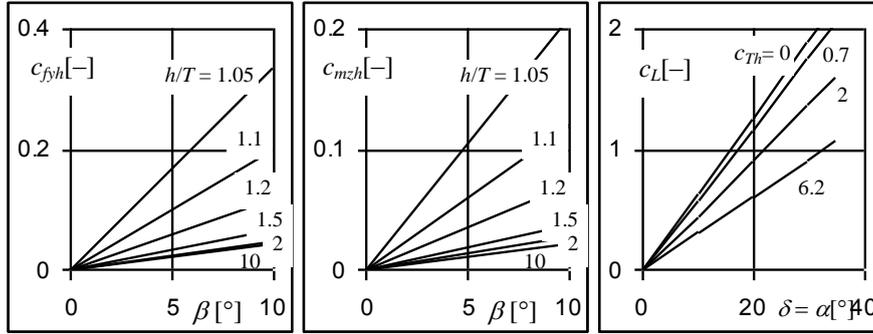


Fig. 1. Hull sway force (left), yaw moment (middle), and rudder lift (right) coefficients  
 Rys. 1. Współczynniki siły poprzecznej i momentu obrotowego kadłuba oraz siły nośnej steru

To keep computations in a non-dimensional way, all excitations, including the bank effect, will be referenced to the above style of the hull hydrodynamics. Therefore, the rudder forces are rearranged to:

$$F_{yR} = 0.5 \rho L T v_{xy}^2 c_{fyr}^*(\delta, c_{Th}) \quad M_{zR} = 0.5 \rho L^2 T v_{xy}^2 c_{mzr}^*(\delta, c_{Th}) \quad (5)$$

where:

$$c_{fyr}^* = \frac{A_R}{LT} (1+w)^2 (1+c_{Th}) c_L(\delta, c_{Th}) \cdot (1+a_H) \quad c_{mzr}^* = -0.5 c_{fyr}^* \quad (6)$$

The bank effect sway force and yaw moment are non-dimensionalised differently by various sources. For mutual comparison purposes, they are converted to the following style:

$$c_{fyBE} = \frac{F_{yBE}(y_{B3}, h/T, F_{nL})}{0.5 \rho L T v_{xy}^2} \quad c_{mzBE} = \frac{M_{zBE}(y_{B3}, h/T, F_{nL})}{0.5 \rho L^2 T v_{xy}^2} \quad (7)$$

where:

- $y_{B3}$  – bank-distance parameter,
- $F_{nL}$  – Froude number (ship's length related).

In all the subsequent bank effect related charts, comprising either the bank effect force and moment or the equilibrium helm/drift attitudes, the  $y_{B3}$  parameter, often used in the bank effect modelling, for this presentation is replaced by the relative distance  $\Delta x'$  from a canal centre-line as much easier to interpret:

$$\Delta x'[-] = \frac{\Delta x[\text{m}]}{0.5(b-B)} \quad \Delta x' \in (0,1) \quad (8)$$

where  $\Delta x' = 0$  indicates a ship in the centre-line, while  $\Delta x' = 1$  stands for a ship touching the port bank. A ship is offset towards the port bank. The double bank case of a canal is solely considered in the present work.

Fig. 2 displays the sway force  $c_{fyBE}$  and yaw moment  $c_{mzBE}$  coefficients calculated for the chemical tanker (Tab. 1) by means of existing three regression models – [Ch'ng et al., 1993], [Norrbin, 1985], and [Vantorre et al., 2003] correspondingly. The sailing conditions are  $b/B = 6$  and  $F_{nL} = 0.075$  (ca. 4.5 [kt]). The influence of propeller loading upon the bank effect itself is omitted here for simplicity. Additionally, a virtual point of the sway force application is included in Fig. 2 according to the following expression:

$$x'_{BE}[-] = \frac{M_{zBE}}{F_{yBE} \cdot L} = \frac{c_{mzBE}}{c_{fyBE}} \quad (9)$$

It is widely known that Norrbin's model (the middle part of Fig. 2) does not encompass a bow repulsion situation, neither at low  $h/T$  nor high  $F_{nL}$ . The Ch'ng's model gives absolutely very low suction sway force, unlike to Norrbin. Its bow repulsion phase starts at  $h/T = 1.2$ , where Norrbin predicts nearly maximum suction force. Except for low water depth-to-draft ratios (1.05, 1.1, 1.2), the magnitude of yaw moment seems to be of the same order in Ch'ng's and Norrbin's models.

The Vantorre's model, at first glance, could be placed incautiously just between the Ch'ng's and Norrbin's models. It seems to inherit all the advantages and hide the deficiencies of the other two models. However, the Vantorre's bank effect yaw moment may be undoubtedly questioned – a large part of  $c_{mzBE}$  lies in a negative range denoting a bow-in moment, which is hard to explain physically. Other investigations of the present author prove that the yaw moment abnormality in the Vantorre's model (probably some inaccuracies may also exist in a sway force) is related with a very high sensitivity of the Vantorre's regression model upon input parameters i.e. ship dimensions. Vantorre's model is rather badly conditioned and requires much more refinement and robustness.

Actually, [Vantorre et al., 2003] briefly mention allowable limits of their formula, but without indicating such a strange divergence of results if the input conditions are slightly violated. It is normal to use regression models sometimes beyond their limits, especially if the latter are quite narrow, otherwise such models are not practical or universal.

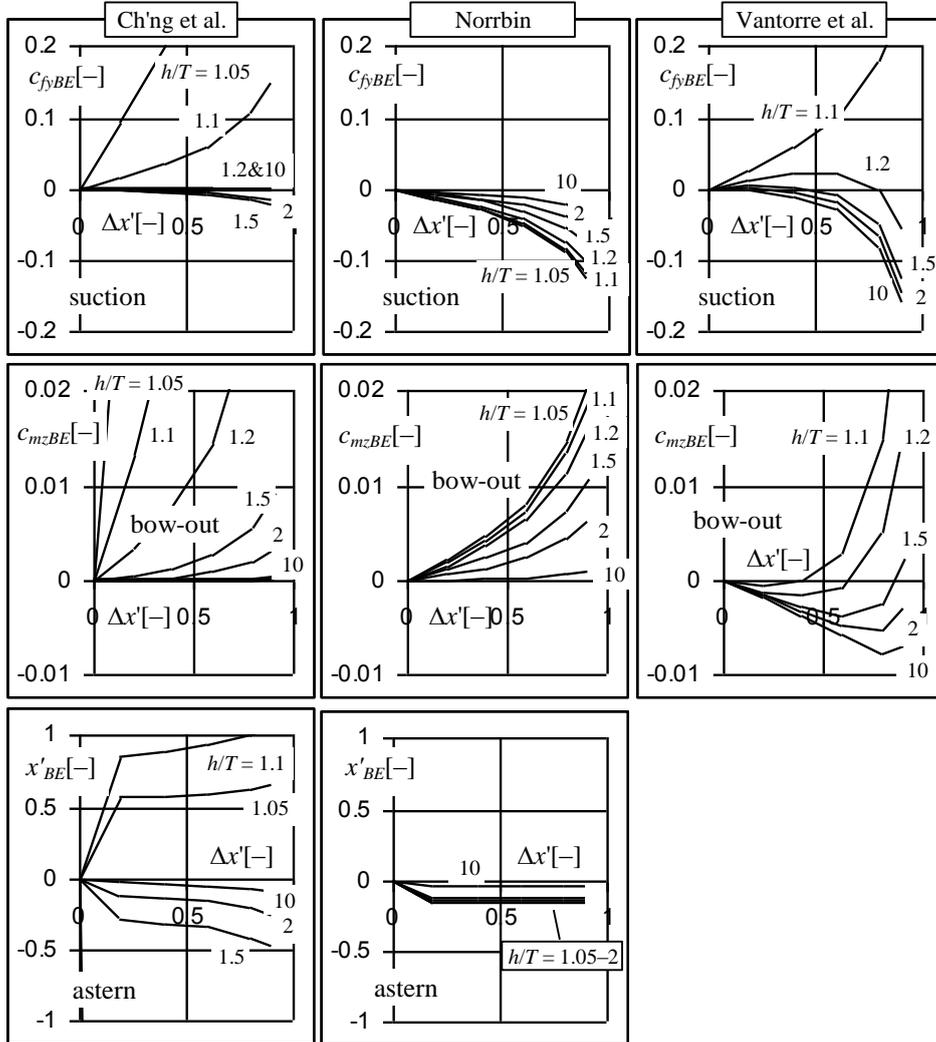


Fig. 2. Bank sway force, yaw moment, pressure centre by various sources  
 – chemical tanker  $L/B = 5.87$ ,  $L/T = 13.7$

Rys. 2. Siła poprzeczna, moment oraz środek parcia efektu brzegowego wg różnych źródeł

For these reasons, the Vantorre's estimation for his original tanker, Fig. 3, will be applied in drift and helm calculations.

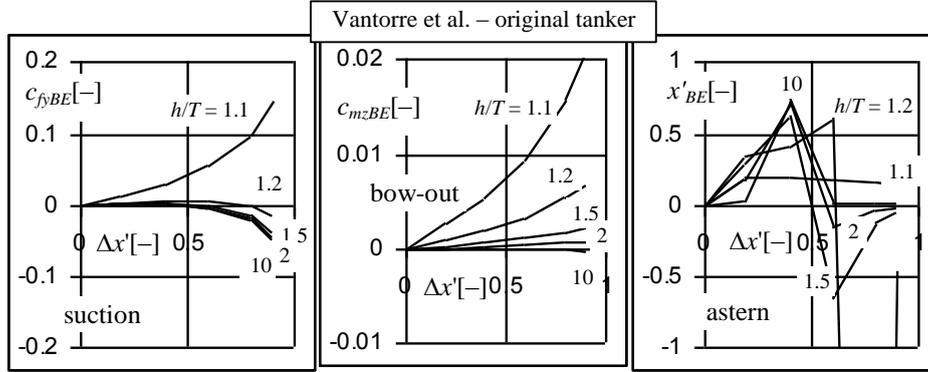


Fig. 3. Vantorre's regression output for his originally measured tanker model  $L/B = 6.03$ ,  $L/T = 17.7$   
 Rys. 3. Wyniki regresji Vantorre'a dla bazowego modelu tankowca

The case  $h/T = 1.05$  in the Vantorre's model is not included as at  $F_{nL} = 0.075$  the under-keel-clearance for the chemical tanker in question is nearly zero – the Vantorre's model 'simulates' a forward speed effect by an effective depth-to-draft ratio i.e. after subtracting a squat magnitude.

## 2. Analytical solutions

The equations (1 – 8) for the force/moment balance are equivalent to the following relationships between the corresponding non-dimensional coefficients:

$$\begin{cases} c_{fyh}(\beta, h/T) + c_{fyr}^*(\delta, c_{Th}) = -c_{fyBE}(\Delta x', h/T, F_{nL}) \\ c_{mzh}(\beta, h/T) + c_{mzr}^*(\delta, c_{Th}) = -c_{mzBE}(\Delta x', h/T, F_{nL}) \end{cases} \quad (10)$$

The hull coefficients, due to their storage in a lookup table being spaced every 10 degrees of drift angle, may be treated as linear functions of this parameter in the assumed range of the allowable drift change (up to 10 degrees each side). This linearity is also physically justified and widely accepted. A similar linear approach is possible for the rudder lift coefficient vs. angle of attack, see Fig. 1, in the whole range of rudder deflection.

Because a speed loss in shallow water, and thus a corresponding propeller load increase, is dependent on  $h/T$ , the eqs. (10) for a given fixed depth-to-draft ratio read:

$$\begin{cases} a_1 \cdot \beta [^\circ] + b_1 \cdot \delta [^\circ] = -c_1(\Delta x', F_{nL}) \\ a_2 \cdot \beta [^\circ] + b_2 \cdot \delta [^\circ] = -c_2(\Delta x', F_{nL}) \end{cases} \quad \square \square \square \square \quad h/T = \text{const} \quad (11)$$

The unknown are drift angle  $\beta$  and rudder angle  $\delta$ . The left side parameters of this system of linear equations (11) are computed in Tab. 2 for the chemical tanker, together with the main determinant  $W$ . In most cases a unique solution will be obtained. The problem is whether the solved  $\beta$  and  $\delta$  would lie within the reasonable range and sign. Figs. 4 and 5 show the results.

Table 2

Parameters of force/moment equilibrium linear equations  
*Parametry równań liniowych równowagi sił i momentów*

$h/T$	$a_1$	$b_1$	$a_2$	$b_2$	$W$
1.05	0.0336	0.0023	0.0210	-0.0011	-8.5E-5
1.1	0.0199	0.0021	0.0122	-0.0011	-4.7E-5
1.2	0.0119	0.0021	0.0071	-0.0010	-2.7E-5
1.5	0.0066	0.0020	0.0038	-0.0010	-1.4E-5
2	0.0049	0.0019	0.0028	-0.0010	-1.0E-5
10	0.0043	0.0018	0.0023	-0.0009	-8.1E-6

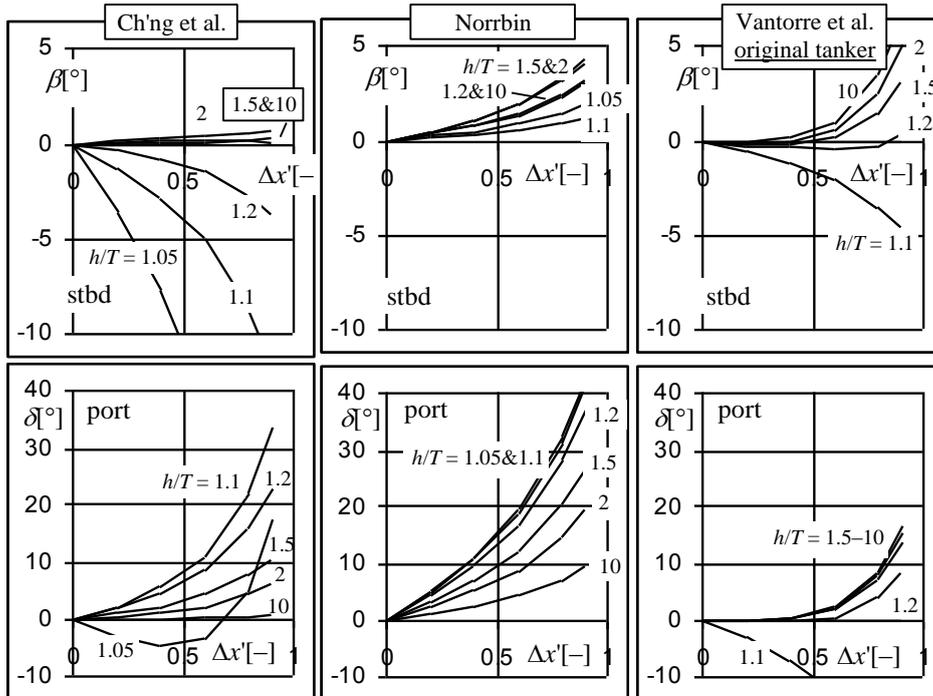


Fig. 4. Drift/helm equilibrium attitudes – chemical tanker  $L/B = 5.87$ ,  $L/T = 13.7$ ,  $b/B = 4$ ,  $F_{nL} = 0.075$   
*Rys. 4. Kąt dryfu i wychylenie steru dla zrównoważenia efektu brzegowego –  $b/B = 4$*

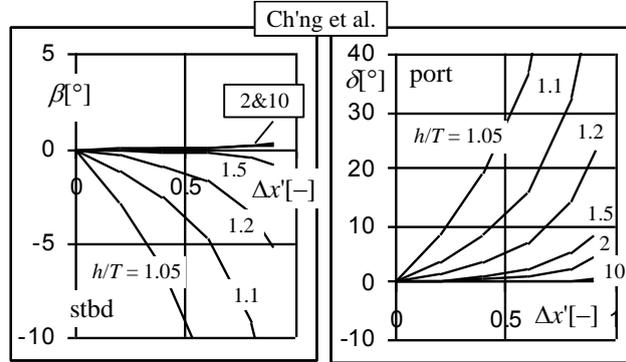


Fig. 5. Drift/helm equilibrium attitudes – chemical tanker  $L/B = 5.87$ ,  $L/T = 13.7$ ,  $b/B = 8$ ,  $F_{nL} = 0.015$   
 Rys. 5. Kąt dryfju i wychylenie steru dla zrównoważenia efektu brzegowego –  $b/B = 8$ ,  $F_{nL} = 0.015$

All regression models generally follow the rule of deflecting the rudder to port i.e. towards the nearest bank – positive  $\delta$ . At very extremely low water depths, both Ch'ng and Vantorre's models lead to the starboard helm accompanying by large negative drift angles  $\beta$ , which means a ship's bow is closer to the bank than her stern.

For medium depth-to-draft ratios ( $h/T \geq 1.5$ ), the Ch'ng model implies the lowest positive drift angles of max. 1 [°], in contrast to the other two models. Both Norrbin and Vantorre reveal here similar drift angles up to 4 – 5 [°]. Moreover, the Norrbin's model causes the strongest helm angles for the chemical tanker.

If  $b/B$  is increased from 4 to 8 (a canal width enlarged twice), the curves of equilibrium drift and helm of Fig. 4 ( $b/B = 4$ ) will remain almost identical, which suggests that the relative offset distance ratio  $\Delta x'$  by eq. (8) is a good predictor of drift-helm patterns. These new charts are deliberately disregarded as redundant.

For higher Froude numbers around 0.15 (~9 [kt]), Fig. 6 (Ch'ng's model only), the drift is almost entirely negative, though the rudder angle seems to receive rather tolerable values. In the light of the regression inaccuracy stated before, Fig. 6 as well as other previous figures shall be interpreted very carefully.

At this stage of research, it is very hard to specify the most adequate regression model for any sailing conditions, if it is possible at all. Further model tests and some full-scale observations (kinematic level) on the ship's equilibrium drift and helm (during accidental sheers towards a bank as encountered in a ship's life) would be helpful to refine and validate them. Planned trials should be considered with caution in order not to unnecessarily endanger a ship.

## Final remarks

On the whole, the existing bank effect regression models may be found unreliable to some extent. Care shall be exercised while applying them in a ship manoeuvring simulation. To achieve typical or characteristic charts of drift and helm, serving as a reference, it is recommended to build general ship manoeuvring models for ships, which are the background of the existing bank effect regression models.

Though the conditions on both the force and moment balance are known, the question is how to reach this equilibrium in ship handling – initially only the yaw moment balance is of major interest, the sway force balance is considered as a side effect.

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*Wpłynęło do redakcji w lutym 2004 r.*

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