ISSN 1733-8670

SCIENTIFIC BULLETIN No 3(75)
MARITIME UNIVERSITY OF SZCZECIN
2004

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Mathematical Model of the Mooring Rope Assisted Ship Manoeuvring

Keywords: *ship manoeuvring, mathematical model, mooring rope*

Benefits and limitations of onboard mooring ropes application to improve a ship manoeuvrability during berthing are discussed. A mathematical model of mooring rope forces is formulated. This has been incorporated in the general ship manoeuvring model for obtaining the real ship motions while a mooring rope is in operation. The simulation output is presented. It appears that the rotational energy absorption performance is largely governed by the location of ship's instant pivot point in relation to the mooring fix point.

Model matematyczny wykorzystania lin cumowniczych w manewrowaniu statkiem

Słowa kluczowe: manewrowanie statku, model matematyczny, liny cumownicze

Przeanalizowano możliwość zastosowania statkowych lin cumowniczych do poprawy sterowności jednostki podczas manewrów cumowania. Sformułowano praktyczny algorytm pozyskiwania oddziaływań lin cumowniczych dla potrzeb symulacji manewrowania. Przedstawiono wyniki symulacji dotyczące redukcji prędkości kątowej wskutek naprężenia liny zamocowanej na dziobie – efektywność takiej operacji zależy w dużym stopniu od wzajemnego położenia chwilowego środka obrotu względem miejsca zamocowania cumy.

Introduction

Due to industry demands, the mooring rope application has undergone so far the greatest progress with reference to station keeping systems of e.g. platforms or to a ship restraint along a berth in rough weather. Various design and analysis methods have been developed, taking account of both static and dynamic loads in the mooring ropes – e.g. [4, 5, 8, 9, 10, 11, 12]. It is not purposeful to cite here all relevant fundamental publications from a really great number. The point is that under static or quasi-static assumptions, as commonly used, the model of mooring rope forces is relatively simple and generally agreed upon. The main attention is usually paid to the other excitations due to e.g. wind, wave, current, and/or upon the resulting ship oscillatory motions. Another area of scientific interest is the mooring rope strength, particularly from the reliability (probabilistic) point of view.

The present paper aims at the feasibility study of mooring rope application (normally the fibre rope) to enhance the ship manoeuvring while approaching a berth. This is a widely recognised practice in ship handling under some circumstances, though it poses some limitations, which are not always properly realised. A real-time and interactive ship manoeuvring software is required to examine and verify the existing recommendations on the mooring rope supported early stages of berthing or unberthing manoeuvres. For this reason, a practical algorithm of mooring rope forces evaluation has been hereafter derived and used to tackle the problem of ship yaw checking by the mooring rope during ship berthing.

1. Mooring rope selection on merchant ships

The main goal of providing a ship with mooring ropes is to secure her stay along a berth, particularly in severe (extreme) weather conditions. The weather conditions comprise wind, wave, and current data. The impact of sea current rapidly rises with the water depth decrease. The most disadvantageous directions of weather factors are assumed in view of exerted excitations. Both static (force balance related) and dynamic (due to ship oscillatory motions) loads in mooring ropes are generally considered. However, there is still no standard concerning the design weather conditions, though some steps are undertaken to clarify this situation on some ship types – e.g. [5, 11]. Adopting arbitrary weather criteria, and even knowing the ship's response to the weather (which is not always possible), some freedom in choosing the rope strength, elasticity, length, or number is a matter of fact. Especially that only general ship's party related items have been discussed above. With regard to shore (berth or terminal) requirements, a minimum safe mooring pattern is often demanded. In some cases, some shore lines are passed to a ship.

Anyhow, the last word belongs to the ship's master, who may tighten up any mooring rules at his discretion.

The responsibility of equipping a ship with necessary mooring ropes is partially taken by the ship classification societies e.g. [6, 7], which set a minimum standard. Nevertheless, the latter is always augmented by industry practice, particularly with reference to a number of ropes, as based on historical experience and the latest scientific research. The input parameter in the classification society recommendations is the widely known so-called equipment number *EN*. Table 1 presents some estimated data by means of [7] formulas for a few real-world tankers.

Table 1

Example of mooring rope equipment
Przykładowe wyposażenie cumownicze

Deadweight [t]	EN [m^2]	Mooring rope data		
		strength (MBL) [kN]	length [m]	number [–]
6 kDWT	1100	166	180	4
87 kDWT	3700	356	220	6
135 kDWT	5200	429	250	8

For the 6 kDWT tanker, referring to marine rope data sheets of leading manufacturers or distributors (freely available in the internet resources), the minimum breaking load (*MBL*) criterion may be accomplished by different types of ropes shown in Table 2.

Table 2

Mooring rope types of 166 [kN] strength
Parametry techniczne lin cumowniczych o wytrzymałości 166 [kN]

Parameter	Fibre ropes			Wire rope
	propylene	nylon	polyester	
diameter[mm]	36	28	28	~15.9 (5/8")
stretch at MBL – $\Delta l_{\%max}$ [%]	18	25	12-20	2
unit weight [kg/m]	0.585	0.485	0.594	1.051

Except for nylon, all other rope types reveal more or less linear load-elongation characteristics over the entire range of stretch. Due to relatively low unit weights and typical short distances between ship's chocks and berth bollards (a deviation is here the occasionally very long head and stern lines at certain terminals), the so-called catenary effects may be disregarded. The lack of catenary shapes is also applied in the hereafter investigations.

The above stated limitations of onboard ropes shall be kept in mind while employing them to control a ship during the berthing or unberthing manoeuvres – with regard to the translational and/or rotational kinematic energy absorption, or initiating a change of the ship's heading, for instance. These additional auxiliary mooring rope applications are of major interest in the present work. Fibre ropes are best suited for this purpose due to their high elasticity and thus energy absorption properties – the wires could be easily broken.

2. Basic ideas

One of good examples how a mooring rope works is the forward (surge) motion checking by the spring line. Because of the negligible ship water resistance at very low speed, the only force is that excited by the mooring line during its stretching. Assuming the linear relationship between the line tension and elongation, the reaction force reads as follows:

$$F_{moor} = \left(\frac{l}{l_0} - 1 \right) \frac{MBL}{\Delta l_{\% \max}} \cdot 100 \tag{1}$$

where:

l – instant rope length (under given tension, being essentially a distance between the shore bollard and ship's fairlead),

l_0 – initial rope length (zero tension),

MBL – minimum breaking load,

$\Delta l_{\%\max}$ – maximum stretch at MBL.

The maximum allowable forward speed is obtained according to the energy equality:

$$\underbrace{\frac{(m + m_{11}) v_{x\max}^2}{2}}_{\text{kinematic energy}} = \underbrace{\int_0^{\Delta l_{\max}} F_{moor}(x) dx \approx \frac{MBL \cdot \Delta l_{\max}}{2}}_{\text{potential energy}} \tag{2}$$

where:

m, m_{11} – ship's mass and surge added mass,

v_x – surge velocity,

x – rope elongation ($= l - l_0$),

Δl_{max} – maximum rope elongation (in [m]).

For the 6 kDWT tanker (Tab. 1) and the spring line of 50 [m] in length and 20% of maximum elongation, the maximum forward speed, which can be hampered without any damage risk to the rope, is approx. 0.43 [m/s]. On the other hand, 10% elasticity at the same rope length produces 0.30 [m/s], while 2% elongation (the wire rope) is associated only with 0.14 [m/s].

Another problem is the rotational energy absorption as for example in a classical portside berthing manoeuvre. This will be deeply considered in the subsequent chapters.

The use of mooring lines for initiating a linear or angular movement is not subject to much breaking risk as comparatively lower forces are developed according to the winch power. Those case studies have been omitted in the present paper, though the mechanisms of motion checking and initiating are similar.

The mooring rope may be operated either manually (fixed rope length) or by means of the winch (fixed rope pretension). In the former case, the instant reaction force, see e.q. (1), is a function of the rope elongation due to the ship motions. In the latter situation, the calculation of current initial rope length l_0 against the maintained tension shall be performed according to:

$$l_0 = \frac{l}{1 + \frac{F_{moor} \cdot \Delta l_{\%\max}}{100 \cdot MBL}} \tag{3}$$

The tension F_{moor} in e.q. (3) may be in turn governed by the manual or automatic control of the mooring winch. The approximate situations mentioned below could be distinguished:

$$\begin{cases} d_{B-F} < l_0 & \text{– no tension, the rope is slack} \\ d_{B-F} \geq l_0 & \text{– the rope is tight} \end{cases}$$

where $d_{B\text{-}F}$ is the distance between the shore bollard (B) and the ship's fairlead (F).

Proper simulation shall also include the usual changeover from the powered (geared) winch to the off gear mode i.e. either to the pure winch brake or directly to the deck bitts.

3. Approximate analytical solutions

In [1], a set of linear ordinary differential equations of the known solution was introduced in relation to the fender effect. This is also partially valid in the mooring line analysis from the ship manoeuvring impact point of view. Those equations read (see Fig. 1):

$$\begin{cases} \dfrac{dv_y}{dt} & = a_{13}y_F \\ \dfrac{d\omega_z}{dt} & = a_{23}y_F \\ \dfrac{dy_F}{dt} & = v_y + \omega_z a_{32} \\ \dfrac{d\psi}{dt} & = \omega_z \end{cases} \qquad \begin{cases} v_y & = B_1 + C_1 \cos t\sqrt{\Delta} \\ \omega_z & = B_2 + C_2 \cos t\sqrt{\Delta} \\ y_F & = D_3 \sin t\sqrt{\Delta} \\ \psi & = A_4 t + D_4 \sin t\sqrt{\Delta} \\ \sqrt{\Delta} & = \sqrt{|a_{13} + a_{23}a_{32}|} \end{cases} \tag{4}$$

where:

y_F – absolute excursion of the ship's fairlead,

v_y, ω_z – ship sway and yaw velocities,

a_{13}, a_{23}, a_{32} – constants dependent among others upon masses and rope elasticity,

B_1, C_1, ... – solution constants dependent upon the above ODE parameters and initial motion state (relating mostly to $v_{y0} \neq 0$ and $\omega_{z0} \neq 0$).

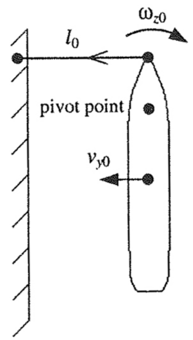

Fig. 1. Yaw checking by a rope – initial conditions

Rys. 1. Zatrzymywanie obrotu statku – warunki początkowe

Beside a simple look into the nature of mooring rope action, the analytical solution provides a few practical data for the general numerical manoeuvring simulation – the necessary integration steps and benchmark data for verifying a computer code. In case of the 6kDWT chemical tanker in deep water, see [2], the analytical results per (4) are given in Tab. 3. This example relates to the 50[m] rope of *MBL* equal to 16[t], and the initial velocities $v_{y0} = -0.29$ [m/s], $\omega_{z0} = +0.01$ [rad/s], as corresponding to the pivot point located $0.3L$, in ship's length units, from the midship. Moreover, the computations are exactly made for the moment of maximum rope elongation (the instant pivot point moves then to the bow).

Table 3

Analytical output for the rope assisted yaw checking manoeuvre
Rozwiązanie analityczne zatrzymywania obrotu statku przy użyciu cumy

rope elasticity (stretch at *MBL*)	20%	10%	2%
time t [s]	27.7	16.0	7.2
sway velocity v_y [m/s]	-0.328	-0.328	-0.328
yaw velocity ω_z [rad/s]	0.0067	0.0067	0.0067
absolute stretch y_F [m]	2.87	1.99	0.89
heading change ψ [°]	11.5	8.1	3.6
pivot point x_{PP} [*L*]	0.5	0.5	0.5
rope tension F_{moor} [t]	4.5	6.4	14.2

The point is the final motion parameters are not dependent upon rope properties (elasticity and length). The stiffer a rope is, the lower the reaction time, rope elongation, and heading deviation are experienced. But the developed tension rapidly increases – the 2% elasticity rope (e.g. wire) is almost to be broken off. Generally, the usual 1 [s] integration step size as used in the numerical manoeuvring simulation is still valid. However, the more critical limitations are imposed by the fender effect.

A rough estimation of the hull hydrodynamic sway force and yaw moment, as normally accompanying the mooring forces but omitted in (4), yields for the above given initial conditions the magnitudes of 2.5 [t] and 330 [tm] correspondingly. These are of the same order as the mooring forces, which evidences why eqs. (4) are rather approximate, though being still very helpful in some aspects.

4. Numerical ship manoeuvring simulation

Let's assume two tables storing coordinates of all ship's fairleads (in body axes) and shore bollards (in earth axes), see Fig. 2:

fairleads: $$\mathbf{r}_F^{(i)} = \left[x_F^{(i)}, y_F^{(i)}, z_F^{(i)},\right]^T, \ i = 1, \ldots, m$$

bollards: $$\mathbf{r}_{BO}^{(j)} = \left[x_{BO}^{(j)}, y_{BO}^{(j)}, z_{BO}^{(j)},\right]^T, \ j = 1, \ldots, n$$

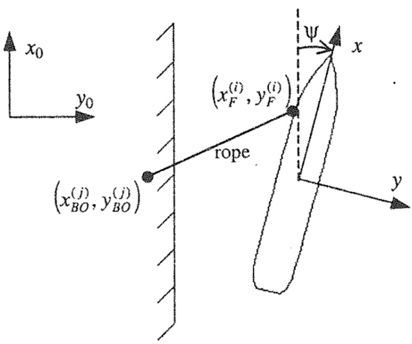

Fig. 2. Systems of reference in mooring rope manoeuvring simulation
Rys. 2. Układy odniesienia w modelu wykorzystania lin cumowniczych

The instant length of the rope, equal to the bollard-fairlead distance (in case of its stretching), reads:

$$l^{(ij)} = \left|\mathbf{r}_{FB}^{(ij)}\right| = \left|\mathbf{r}_B^{(j)} - \mathbf{r}_F^{(i)}\right| = \sqrt{\left(x_B^{(j)} - x_F^{(i)}\right)^2 + \left(y_B^{(j)} - y_F^{(i)}\right)^2 + \left(z_B^{(j)} - z_F^{(i)}\right)^2} \tag{5}$$

where the bollard position in the body axes looks like:

$$\mathbf{r}_B^{(j)} = \begin{bmatrix} x_B^{(j)} \\ y_B^{(j)} \\ z_B^{(j)} \end{bmatrix} = \begin{bmatrix} \cos\psi & \sin\psi & 0 \\ -\sin\psi & \cos\psi & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} x_{BO}^{(j)} - x_{SO} \\ y_{BO}^{(j)} - y_{SO} \\ z_{BO}^{(j)} - z_{SO} \end{bmatrix} \tag{6}$$

in which the vector $[x_{SO}, y_{SO}, z_{SO}]^T$ means the ship's position.

The unit vector of $\mathbf{r}_{FB}^{(ij)}$, very helpful in computing the mooring rope force direction or components, is defined by:

$$\mathbf{r}_{FB-U}^{(ij)} = \frac{\mathbf{r}_{FB}^{(ij)}}{\left|\mathbf{r}_{FB}^{(ij)}\right|} \tag{7}$$

The force magnitude is expressed by the following:

$$F_{moor}^{(ij)} = \left(\frac{l^{(ij)}}{l_0^{(ij)}} - 1\right) \frac{MBL^{(ij)}}{\Delta l_{\%\max}^{(ij)}} \cdot 100 \tag{8}$$

Finally, the particular components of the mooring force and moment in the ship body axes yield:

$$\mathbf{F}_{moor}^{(ij)} = \begin{bmatrix} F_{moor-x}^{(ij)} \\ F_{moor-y}^{(ij)} \\ F_{moor-z}^{(ij)} \end{bmatrix} = F_{moor}^{(ij)} \cdot \mathbf{r}_{FB-U}^{(ij)} \tag{9}$$

$$\mathbf{M}_{moor}^{(ij)} = \begin{bmatrix} M_{moor-x}^{(ij)} \\ M_{moor-y}^{(ij)} \\ M_{moor-z}^{(ij)} \end{bmatrix} = \mathbf{r}_{FB}^{(ij)} \times \mathbf{F}_{moor}^{(ij)} \tag{10}$$

In the simulation of ship manoeuvring planar motions, only the terms $F_{moor-x}^{(ij)}$, $F_{moor-y}^{(ij)}$ and $M_{moor-z}^{(ij)}$ are used. If more than one mooring rope is passed to the shore, appropriate sums should be established:

$$\mathbf{F}_{moor} = \sum_{i,j} \mathbf{F}_{moor}^{(ij)}, \quad \mathbf{M}_{moor} = \sum_{i,j} \mathbf{M}_{moor}^{(ij)} \tag{11}$$

For the aforementioned 6kDWT tanker, under the initial motion conditions as in the previous chapter (Fig. 1), the general manoeuvring simulation has been performed using the newly derived hull hydrodynamic data of [1, 3]. The mooring rope is identically perpendicular to the ship's side and made fast at the bow. The numerical results are presented in Fig. 3 in relation to the yaw motion checking – the 50 [m] rope of various elasticity (2%, 10%, 20%) is operated. Anyhow, due to the fact that e.g. a 10[m] rope of 10% elasticity would be alike in behaviour as the 50 [m] rope of 2%, and so on, the obtained chart is also a good representation of other mooring rope cases.

The left part of Fig. 3 comprises time histories of the yaw velocity in three circumstances (just for comparison): under pure mooring force (equivalent to the former analytical approach), under pure hull forces (as naturally damping the ship sway and yaw motions), and under both excitations (the real case). The aim is to show here a relative influence of mooring force and hull forces upon the change of turning rate.

The right part of Fig. 4 presents the full simulation of other rope elasticities, together with the action of fixed rope tension (e.g. through automatic-tension mooring winch) equal to 30 [kN] and 80 [kN]. The latter figures also correspond to the average tension in mooring ropes of 10% and 2% elasticity as subject to passive stretching. The final yaw velocity, at the moment when the rope is losing tension, is independent from the rope elasticity – this peculiarity is strongly connected with instant pivot point transfer up to the fairlead location. The rope elasticity affects the speed of yaw checking. The constant tension (through the continuous heaving up of the rope) allows diminishing the turn rate down to zero.

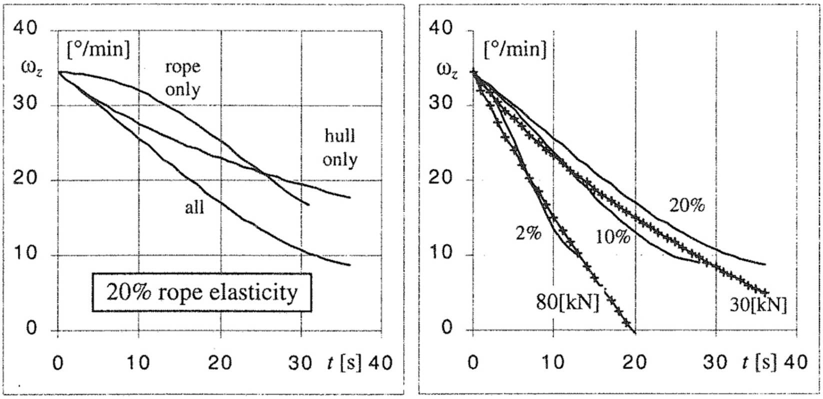

Fig. 3. Ship manoeuvring numerical simulation – yaw checking
Rys. 3. Symulacja numeryczna redukcji prędkości kątowej

Final remarks

The presented mathematical model of mooring rope excitation in view of the manoeuvring motion simulation is rather simple but quite adequate and practically sufficient. As opposed to the fender effect modelling, the reaction forces and the material stiffness of the mooring rope are much lower, and thus the reaction time under reasonable operational conditions is at least a few times higher. So, the usual integration step of 1 [s] in the ship manoeuvring simulation does not require to be shortened in this context.

Some limitations have been discovered with regard to the mooring rope employment in the rotational energy absorption. The efficiency of rope action largely depends upon its relative location vs. the initial pivot point. The residual yaw velocity is not coming down to zero, unless the rope is heaved up onboard.

When using a mooring rope for energy absorption purposes, it shall be borne in mind that any rope is rather ideally elastic i.e. it does not have 'a hysteresis loop' as of great advantage in the fender operation principle. However, this hysteresis may be achieved by the rapid slacking of the rope at the moment of ship's fairlead maximum deviation. Otherwise, the rope would pull a ship in the opposite direction towards a berth.

Further research on the mooring rope effect modelling in the manoeuvring simulation is recommended to cover among others the aspects of the mooring winch and/or ship's personnel performance, and the rope breaking using the probabilistic concepts.

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