ISSN 0209-2069

ZESZYTY NAUKOWE NR 1(73)
AKADEMII MORSKIEJ
W SZCZECINIE

EXPLO-SHIP 2004

Jerzy Girtler

A Model of Ship Propulsion Systems Operating Process

Key words: operation process, semi-Markov process, ship propulsion system

The paper contains a formal description of ship propulsion systems operation process. The model of the process is presented in the form of two-dimensional stochastic process of which coordinates are semi-Markov processes of the final states set. The former process describes the process of changes in technical states of ship propulsion systems, the latter – the process of changes in their operating states. A monovariate model of ship propulsion systems operation process has also been proposed.

Model procesu eksploatacji układu napędowego statku

Słowa kluczowe: proces eksploatacji, proces semi-Markowa, układ napędowy statku

Podano opis formalny procesu eksploatacji układów napędowych statków. Model tego procesu przedstawiono w formie dwuwymiarowego procesu, którego współrzędnymi są procesy semi-Markowa o skończonych zbiorach stanów. Jeden z tych procesów jest procesem zmian stanów technicznych układów napędowych a drugi – procesem zmian ich stanów eksploatacyjnych. Zaproponowano także jednowymiarowy model procesu eksploatacji układów napędowych statków.

Introduction

The most significant problem in operating a propulsion system (e.g. diesel engines, propeller, mechanical transmission, propeller shaft, intermediate shaft) is the problem of rational (especially optimum) control of the machines operating process. Such control may facilitate an employment of the iterative algorithm for determining an optimum strategy, elaborated by R. A. Howard [7]. However, the employment of the algorithm in the control of a machines operating process requires, among other things, a model of a machines operating process [2, 4] as well as a concept of control of a machines operating process on the basis of diagnostic tests [3, 5].

In the paper an attempt has been made to characterize both of these problems.

2. Description of ship propulsion systems operating process

Any ship propulsion systems' operating process can be considered as a combined process of simultaneous changes in technical states $s_i^* \in S^*(i = 1, 2, ...)$ and in operating states $e_j^* \in E^*(j = 1, 2, ...)$ of the machine.

Assuming the performance ability of ship propulsion systems as the division criterion for the set S^*, three classes of technical states can be distinguished [2, 3, 4]. They are referred to:

- state of full ability s_1,
- state of partial ability s_2,
- state of inability s_3.

The process of changes in technical states of the ship propulsion systems which has been distinguished is firmly connected with the process of changes in operating states of the machines. In the period between overhauls, for many ship propulsion systems, the following operating states can be distinguished:

- state of active operating e_1,
- state of passive operating e_2,
- state of planned maintenance e_3,
- state of forced maintenance (non-planned) e_4.

The set of technical states $S = \{s_1, s_2, s_3\}$ can be considered as the set of stochastic process $\{W(t): t \geq 0\}$ of constant intervals and continuous right-hand realizations. This process is the process of changes in ship propulsion systems technical states. The stochastic process $\{W(t): t \geq 0\}$ is a semi-Markov process [1, 2, 3].

The set of operating states $E = \{e_1, e_2, e_3, e_4\}$ can be considered as the set of values of a stochastic process $\{X(t)\colon t \geq 0\}$ of constant intervals and continuous right-hand realizations. This process is the process of changes in ship propulsion systems operating states. The stochastic process $\{X(t)\colon t \geq 0\}$ is a semi-Markov process [1, 2, 3].

Since the processes: $\{W(t)\colon t \geq 0\}$ and $\{X(t)\colon t \geq 0\}$ are interdependent processes, the bivariate process $\{Y(t)\colon t \geq 0\}$ of which components are $W(t)$ and $X(t)$, the processes should be considered as the following process

$$Y(t) = [W(t), X(t)], \quad t \geq 0 \tag{1}$$

This process is the ship propulsion systems operating process. An example of the realization of the two-dimensional (bivariate) operating process $\{Y(t)\colon t \geq 0\}$ of a machine of a propulsion system has been presented in Fig. 1 and Fig 2 [4].

In order to describe the stochastic process $\{Y(t)\colon t \geq 0\}$ it is necessary to find its combined distribution. The combined probability distribution of the bivariate process $Y(t) = [W(t), X(t)]$, $t \geq 0$, can be presented as follows:

$$p(s_i, e_j, t) = P\{W(t) = s_i, X(t) = e_j\} \tag{2}$$

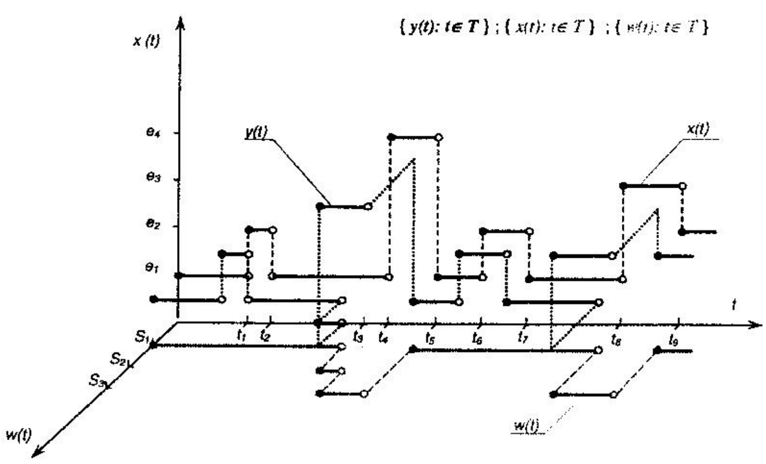

Fig. 1. An example of the realization of the two-dimension process $\{Y(t)\colon t \geq 0\}$, for instance of the ship power transmission system, also main engine (propulsion motor), propeller and so forth

Rys. 1. Egzemplifikacja realizacji procesu $\{Y(t)\colon t \geq 0\}$ na przykład układu napędowego, także silnika głównego, śruby napędowej itd.

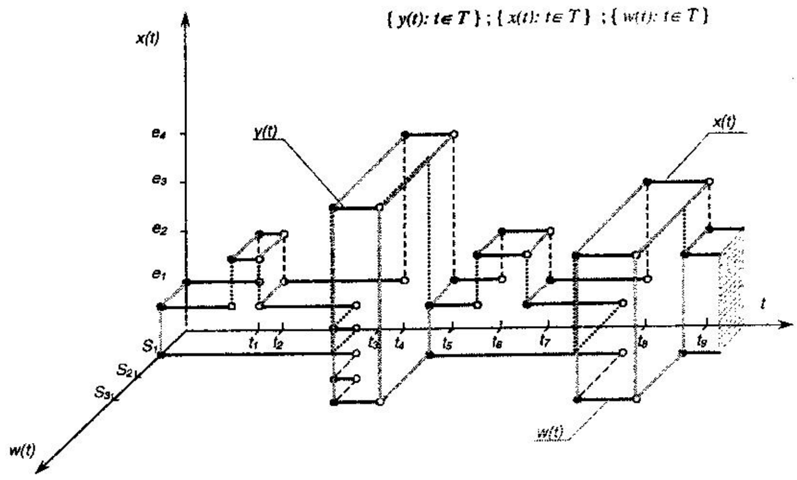

Fig. 2. Graphic examples of the processes $\{Y(t): t \geq 0\}$, $\{W(t): t \geq 0\}$ and $\{X(t): t \geq 0\}$
Rys. 2. Obrazowa egzemplifikacja procesów $\{Y(t): t \geq 0\}$, $\{W(t): t \geq 0\}$ *and* $\{X(t): t \geq 0\}$

Since the processes: $\{W(t): t \geq 0\}$ and $\{X(t): t \geq 0\}$ are interdependent it is not possible to find a combined distribution for the process $\{Y(t): t \geq 0\}$ on the basis of distributions of the processes: $\{W(t): t \geq 0\}$ and $\{X(t): t \geq 0\}$ presented above [5]. The distribution can be determined by forming a monovariate stochastic process $\{Y(t): t \geq 0\}$.

3. Semi-Markov process as a monovariate model of ship propulsion systems operating process

The combined consideration of the set of technical states of a given ship propulsion system $S = \{s_1, s_2, s_3\}$ and the set of its operating states $E = \{e_1, e_2, e_3, e_4\}$ permits forming the following set of states of the ship propulsion systems operating process [3, 4, 5]:

$$Z = \{z_1, z_2, z_3, z_4, z_5, z_6\} \tag{3}$$

The following: $z_1 = (s_1, e_2)$, $z_2 = (s_1, e_1)$, $z_3 = (s_2, e_1)$, $z_4 = (s_2, e_3)$, $z_5 = (s_3, e_3)$, $z_6 = (s_3, e_4)$ is an interpretation of the particular states.

The physical characteristics of the ship propulsion systems operating process imply a strictly defined form of a graph of process changes.

An exampleof a graph of changes of states of the operational process of the ship power transmission systems and their devices in the period between overhauls is shown in Fig. 3.

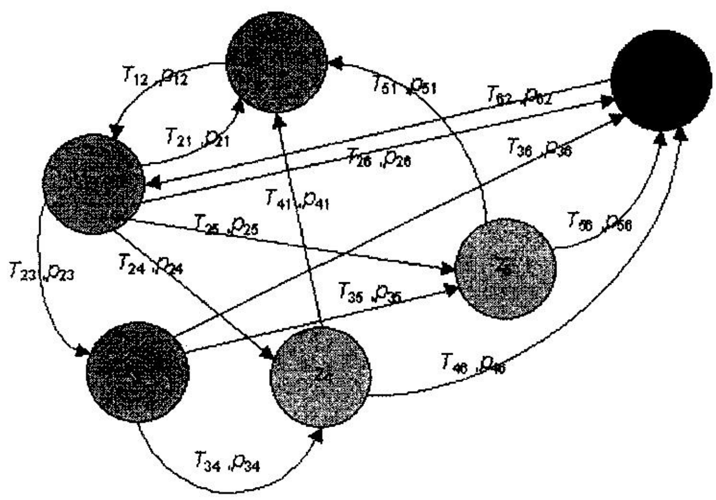

Fig. 3. Graph of changes of states of the operational process of the ship power transmission system, also the main engine (propulsion motor), propeller and so forth
Rys. 3. Graf zmian stanów układu napędowego, także silnika głównego, śruby napędowej itd.

The operating process under consideration $\{Y(t)\colon t \geq 0\}$ is a semi-Markov process – discrete in states and continuous in time. An example of the realization of the monovariate operating process $\{Y(t)\colon t \geq 0\}$ of a machine is shown in Fig. 4.

The semi-Markov process $\{Y(t)\colon t \geq 0\}$ is fully determined if the functional matrix is known:

$$Q(t) = \lfloor Q_{ij}(t) \rfloor \tag{4}$$

and when the initial distribution is given

$$P_i = P\{Y(0) = z_i\},\ i = \overline{1,6} \tag{5}$$

Functional matrix for the considered process has got the following form:

$$Q(t) = \begin{bmatrix} 0 & Q_{12}(t) & 0 & 0 & 0 & 0 \\ Q_{21}(t) & 0 & Q_{23}(t) & Q_{24}(t) & Q_{25}(t) & Q_{26}(t) \\ 0 & 0 & 0 & Q_{34}(t) & Q_{35}(t) & Q_{36}(t) \\ Q_{41}(t) & 0 & 0 & 0 & 0 & Q_{46}(t) \\ Q_{51}(t) & 0 & 0 & 0 & 0 & Q_{56}(t) \\ 0 & Q_{62}(t) & 0 & 0 & 0 & 0 \end{bmatrix} \tag{6}$$

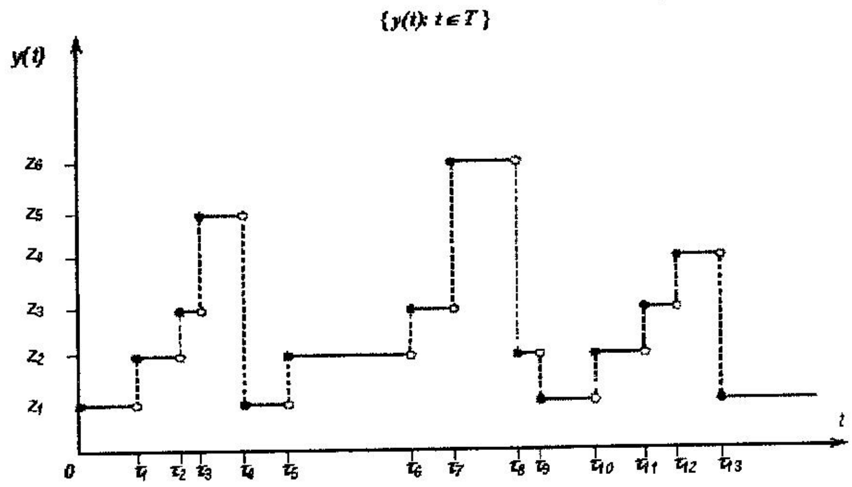

Fig. 4. An example of the process $\{Y(t): t \geq 0\}$ in the period between overhauls: $\tau_0 = 0, \tau_1, \ldots, \tau_{13}$ – moments in which changes in process values occur

Rys. 4. Egzemplifikacja procesu $\{Y(t): t \geq 0\}$ w okresie między obsługami profilaktycznymi: $\tau_0 = 0, \tau_1, \ldots, \tau_{13}$ – chwile, w których następują zmiany stanów procesu

Non-zero elements of the matrix Q can be determined on the basis of the following formula:

$$Q_{ij}(t) = P\{Y(\tau_{n+1}) = z_j,\ \tau_{n+1} - \tau_n < t \mid Y(\tau_n) = z_i\} = p_{ij}F_{ij}(t)$$

where:

$z_i, z_j \in Z(i, j = 0, 1, \ldots, 6; i \neq j)$;

p_{ij} – the probability of changing in one step of homogeneous Markov chain put in the process $\{Y(t): t \geq 0\}$;

$F_{ij}(t)$ – distribution function of the random variable T_{ij}, determining the duration time of the state z_i of the process $\{Y(t): t \geq 0\}$ on condition that z_j is the next state.

The probability p_{ij} is interpreted in the following way:

$$p_{ij} = P\{Y(\tau_{n+1}) = z_j \mid Y(\tau_n) = z_i\} = \lim_{t \to \infty} Q_{ij}(t) \tag{7}$$

In this situation, in order to solve the formulated problem, it is necessary to find a limiting distribution of the process $\{Y(t): t \geq 0\}$ of which the interpretation is:

$$P_j = \lim_{t \to \infty} P\{Y(t) = z_j\}, \quad j = \overline{1,6} \tag{8}$$

The initial distribution of the process $\{Y(t): t \geq 0\}$ is determined by the formula:

$$P_i = P\{Y(0) = z_i = \begin{cases} 1 & \text{dla} \quad i = 1 \\ 0 & \text{dla} \quad i = \overline{1,6} \end{cases} \tag{9}$$

The matrix elements $Q(t)$ are non-decreasing functions of the variable t, which determine the probability of changing the process $\{Y(t): t \geq 0\}$ from the state s_i into s_j (s_i, $s_j \in S$; $i, j = 1, 2, ..., 6$) in a time not longer than t. They are denoted as follows [5, 6]:

$$\lim_{t\to\infty} P_{ij}(t) = P_{ij} = P_j = \frac{\pi_j E(T_j)}{\sum_{j=1}^{6} \pi_j E(T_j)} \tag{10}$$

where $\pi_j = \lim_{t\to\infty} \frac{1}{n} \sum_{k=1}^{n} P\{Y(\tau_n) = z_j | Y(0) = z_i\}$, $[\pi_j; j = 0, 1, ..., 4]$ is a stationary distribution of the Markov chain $\{Y(\tau_n): n \in N\}$ put into the process $\{Y(t): t \geq 0\}$.

Using the formula (10) we can determine a limiting distribution of the process $\{Y(t): t \geq 0\}$ being the model of the ship Diesel engines operating process in the form of the following formulas:

$$P_1 = \frac{[p_{21} + p_{41}(p_{24} + p_{23}p_{34}) + p_{51}(p_{25} + p_{23}p_{35})]E(T_1)}{H}; \quad P_2 = \frac{E(T_2)}{H};$$

$$P_3 = \frac{p_{23}E(T_3)}{H}; \quad P_4 = \frac{(p_{24} + p_{23}p_{34})E(T_4)}{H}; \quad P_5 = \frac{(p_{25} + p_{23}p_{35})E(T_5)}{H};$$

$$P_6 = \frac{[1 - p_{21} - p_{41}(p_{24} + p_{23}p_{34}) - p_{51}(p_{25} + p_{23}p_{35})]E(T_6)}{H} \tag{11}$$

when

$$\begin{aligned} H = {} & [p_{21} + p_{41}(p_{24} + p_{23}p_{34}) + p_{51}(p_{25} + p_{23}p_{35})]E(T_1) + E(T_2) + \\ & + p_{23}E(T_3) + (p_{24} + p_{23}p_{34})E(T_4) + (p_{25} + p_{23}p_{35})E(T_5) + \\ & + [1 - p_{21} - p_{41}(p_{24} + p_{23}p_{34}) - p_{51}(p_{25} + p_{23}p_{35})]E(T_6) \end{aligned}$$

where:

p_{ij} – probability of changing the process $\{Y(t): t \geq 0\}$ from the state z_i to the state z_j (z_i, $z_j \in Z$; $i,j = 1, 2, ..., 6$: $i \neq j$);

$E(T_j)$ – expected value of random variable $T_j(j = 1, ..., 6)$ determining the duration time of the state $z_j \in Z(j = 1, ..., 6)$ of the process $\{Y(t): t \geq 0)$, regardless of the state from which the change takes place. Expected values $E(T)$ depend on the expected values $E(T_{ij})$ and probabilities p_{ij}, as follows:

$$E(T_j) = E(T_i) = \sum_j p_{ij} E(T_{ij}),\ i, j = \overline{1,6};\ i \neq j \qquad (12)$$

Particular probabilities $P_j(j = 1, 2, ..., 6)$ are interpreted as follows:

$$P_1 = \lim_{t \to \infty} P\{Y(t) = z_1\},\ P_2 = \lim_{t \to \infty} P\{Y(t) = z_2\},\ P_3 = \lim_{t \to \infty} P\{Y(t) = z_3\}$$

$$P_4 = \lim_{t \to \infty} P\{Y(t) = z_4\},\ P_5 = \lim_{t \to \infty} P\{Y(t) = z_5\},\ P_6 = \lim_{t \to \infty} P\{Y(t) = z_6\}$$

Because the states z_1 and z_2 of the process $\{Y(t): t \geq 0)$ exist only when the ship propulsion systems finds itself in full ability (s_1), the probability $P = P_1 + P_2$ can be considered as a measure of the ship propulsion systems reliability.

Receiving approximate values of probabilities $P_j(j = 1, 2, ..., 6)$ calls for an evaluation of p_{ij} and $E(T_j)$.

The evaluation of the probabilities p_{ij} and expected values $E(T_j)$ is possible after getting the realisation $y(t)$ of the process $\{Y(t): t \geq 0\}$ in the adequately long time interval of testing, so $t \in [0, t_b]$ where $t_b >> 0$. Now, it is possible to estimate the numbers n_{ij} $(i, j = 1, 2, ..., 6)$, which determine the amount of changes from the state s_i into the state s_j in an adequately long time.

The estimator of the greatest reliability of the transition probability p_{ij} is s-tatistics [5, 6].

$$\hat{P}_{ij} = \frac{N_{ij}}{\sum_j N_{ij}},\ i \neq j;\ i, j = 1,2,..,6 \qquad (13)$$

and its value $p_{ij} = \frac{n_{ij}}{\sum_j n_{ij}}$ is the estimation of transition probability p_{ij}.

From the mentioned $y(t)$ course, we can obtain realisations $T_j^{(m)}$, $m = 1, 2, \ldots, n_{ij}$ of random variables T_{ij}. Applying the point estimation allows to easily estimate $E(T_j)$ as a value of the arithmetic mean of realisations $t_j^{(m)}$.

4. Final conclusions

The rational, and above all optimum, control of ship propulsion systems operating process with the use of the model $\{Y(t): t \geq 0\}$ of this process is impossible without developing and employing a broadly-understood diagnostic system for the machines as well as using all the forms of the diagnostic activity (diagnosing, supervising, forecasting and originating). Proposals referring to such a diagnostic system have been included in the publication [3] and a concept for employment of the forms of diagnostic activity mentioned above has been discussed in the publications [3, 5].

References

1. Girtler J., Kuszmider S., Plewiński L.: *Wybrane zagadnienia eksploatacji statków morskich w aspekcie bezpieczeństwa żeglugi*. WSM, Szczecin 2003.
2. Girtler J., Kitowski Z., Kuriata A.: *Bezpieczeństwo okrętu na morzu. Ujęcie systemowe*. WKiŁ, Warszawa 1995.
3. Girtler J.: *Diagnostyka jako warunek sterowania eksploatacją okrętowych silników spalinowych*. Studia Nr 28. WSM, Szczecin 1997.
4. Girtler J.: On a Problem of Optimisation of Machines' Operating Process. Zagadnienia Eksploatacji Maszyn, z. 1(89), vol. 27. PWN, Warszawa 1992, s. 139-145.
5. Girtler J.: *Sterowanie procesem eksploatacji okrętowych silników spalinowych na podstawie diagnostycznego modelu decyzyjnego*. Zeszyty Naukowe WSMW (AMW) Nr 100A. AMW, Gdynia 1989.
6. Grabski F.: *Teoria semi-Markowskich procesów eksploatacji obiektów technicznych*. Zeszyty naukowe WSMW (AMW) Nr 75A. AMW, Gdynia 1982.
7. Howard R.A.: *Dynamic Probabilistic Systems. Semimarkov and Decision Processes*. Vol. II. Wiley Sons, New York, London, Sydney, Toronto, 1971.

Wpłynęło do redakcji w lutym 2004 r.

Recenzenci

dr hab. inż. Oleh Klyus, prof. AM
prof. dr hab. inż. Stefan Żmudzki

Adres Autora

prof. Jerzy Girtler, DSc, PhD, MSc, Eng.
Depertment of Ship Power Plant
Faculty of Ocean Engineering & Ship Technology
Gdansk University of Technology
11/12 G. Narutowicza Str.
e-mail: jgirtl@pg.gda.pl