JOURNAL OF THEORETICAL
AND APPLIED MECHANICS
**43**, 1, pp. 119-133, Warsaw 2005

# HYBRID METHOD OF EVALUATION OF SOUNDS RADIATED BY VIBRATING SURFACE ELEMENTS

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The paper provides the theoretical background of a hybrid method to be applied in engineering evaluation of sound radiation by vibrating surface elements. The intensity vector is obtained at a selected point in the acoustic medium surrounding the vibrating element. For convenience, certain assumptions are made for the purpose of far-field analysis, which makes the method allow for estimation of radiated acoustic pressure. The method is the verified experimentally. Vibrations of structural elements can be predicted theoretically using a finite element approach or from the measurement of vibrations of real structures.

*Key words:* hybrid method, structural vibrations, sound radiation

## 1. Introduction

The finite element method (FEM) is one of the most powerful and useful tools in engineering studies on dynamics of mechanical systems and can be well applied to the analysis of structural vibrations. Recently, the FEM concept was combined with spectral methods. It gives a new powerful technique to dynamical analysis of structures, especially for computational fluid dynamics (Karniadakis and Sherwin, 1999).

From the standpoint of acoustics, transverse vibrations of surface panels are the source of sound radiation. That it is really well confirmed by theoretical studies (Elliott and Nelson, 1997) and experimental programmes (Nizioł *et al.*, 1989). The analysis of coupled sound-structure interactions is now possible with the use of a finite element approach (e.g. the computer package *Ansys* (Łaczek, 1999)). However, engineering applications of FEM are not so widespread simply because most computers are not fast enough and they lack adequate computing power, particularly in the case of 3D problems and in the selection and identification of the type of required acoustic analysis (steady-state transition, near far-field). The concept of the acoustic intensity vector has been widely used in the analysis of acoustic fields for a very long tine, sometimes replacing the acoustic pressure. When the intensity vector is considered, we get information on both the amplitude of an expansion-compression wave and the direction of wave propagation.

This paper provides the theoretical background of a new, original hybrid method combining the advantages of these two approaches. The procedure of the method verification is outlined. The method is a modification of the earlier concepts by Mann *et al.* (1987) and Nakagawa *et al.* (1993a,b) which allow full engineering analysis of an acoustic field generated by vibrating surface elements. The major advantage of the method involves: lower computing power demands and available option for the selection of the type of acoustic field analysis (near or far field). Besides, the method is useful in the estimation of radiated acoustic pressure, assuming that velocity distributions for vibrating structural elements are found experimentally.

## 2. Fundamentals of the hybrid method

### 2.1. FEM in structural analysis

The finite element method (FEM) is an approximate method for solving the boundary problem formulated to describe behaviour of structural systems. A finite element (rod, panel, shell, elastic body) is defined on the basis of certain simplifying assumptions (hypotheses), simplified functionals are derived respectively and their minima are to be sought. This is a theoretical principle of the method stemming from variational mechanics. At that stage, the material type is taken into account through application of relevant constitutive equations and introduction of an adequate number of material constants. Within linear dynamics, formulas stem from the Hamilton principle (Kleiber, 1995)

$$\int\limits_{t_0}^{t_1} \delta(L - E_k)\; dt = \int\limits_{t_0}^{t_1} W(\delta u_i)\; dt \tag{2.1}$$

where $i, j = 1, 2, 3$ and

| | | |
|---|---|---|
| $\delta(\cdot)$ | – | symbol of variation |
| $t_0, t_1$ | – | time instants between which motion of a body is determined |
| $\delta L$ | – | virtual work produced by strain $\delta L = \int\limits_V \sum\limits_{i,j} \sigma_{ij}\delta\varepsilon_{ij}\, dV$ |
| $\delta E_k$ | – | variation of kinetic energy $\delta E_k = \int\limits_V \sum\limits_i \rho \frac{\partial u_i}{\partial t} \delta \frac{\partial u_i}{\partial t}\, dV$ |
| $W(\delta u_i)$ | – | virtual work of external loads $W(\delta u_i) = \int\limits_V \sum\limits_i b_i \delta u_i\, dV + \int\limits_S \sum\limits_i p_i \delta u_i\, dS$ |
| $u_i$ | – | component of displacement vector |
| $\sigma_{ij}$ | – | component of stress tensor |
| $\varepsilon_{ij}$ | – | component of strain tensor |
| $b_i$ | – | external (volumetric) load forces |
| $p_i$ | – | external (surface) load forces |
| $\rho$ | – | material density. |

When external loads acting upon a body are such that their virtual work $W(\delta u_i)$ can be expressed as a variation of a function $E_p$, called potential energy of the strain $W(\delta u_i) = -\delta E_p$, formula (2.1) can be rewritten into (2.2). The function $B = L - E_k + E_p$ is known as the Lagrange function

$$\delta \int\limits_{t_0}^{t_1} (L - E_k + E_p)\, dt = 0 \tag{2.2}$$

The problem is solved by minimisation of a relevant functional and an approximate solution is sought as a combination of baseline functions, known as shape functions, whose values are determined at specified points (i.e. nodes). At that stage, boundary conditions are formulated to represent the structure support. Node positions are associated with the selection of the type and manner of the division into finite elements such that the structure geometry be captured precisely enough. In dynamic problems, spatial discretisation of a structure has to be considered as well as time dependencies of nodal displacements. That can be achieved with the use of several methods, depending on the type of performed analysis (for instance the integration with a prescribed differentiation scheme for the analysis of transients). Theoretical backgrounds of FEM were provided by Zienkiewicz in his fundamental book (Zienkiewicz, 1972). FEM methods were first developed in Poland by a research group led by Szmelter from Warsaw (Szmelter *et al.*, 1979), who created an original finite element system WAT (Szmelter *et al.*, 1973). In the years to come, new applications of the FEM approach were investigated in other research centres, too: in Gdańsk (Kruszewski's group, see Kruszewski (1975), Kruszewski *et al.* (1984)) and in Kraków (Waszczyszyn and Cichoń, see Cichoń (1994), Waszczyszyn *et al.* (1990)).

It can be noted that the boundary element method is an alternative approach to solve differential equations in mechanics of deformable (elastic) bodies. Its theoretical background stems from Green-Gauss-Ostrogradsky's theorem whereby the theoretically formulated boundary condition is shifted from the whole domain to its boundary (Ciskowski and Brebbia, 1996; Burczyński, 1995). The method is particularly useful in acoustical studies.

### 2.2. Acoustic intensity vector

The notion of acoustic intensity is associated with the definition of intensity of any vector field understood as a prescribed domain in which each point at any time instant has an ascribed vector $\boldsymbol{K}(\boldsymbol{r},t)$ defined by a function of position and time, which is continuous and differentiable over the whole domain. The displacement field and the velocity of a deformable body are both vector fields. Another concept is introduced as well. It is the stream rate or the stream of the vector $Q$ – a scalar quantity associated with the specified surface area $S$, (often referred to as a vector stream by the surface). When we select an elementary surface $dS$, whose position and orientation is given by a vector $\boldsymbol{n}$ normal to it, the intensity of the vector field in the direction $K_n$ is expressed as $(2.3)_1$, and the derivative obtained accordingly is understood in boundary terms as in $(2.3)_2$ (Bukowski, 1959)

$$dQ = KdS_n = K_n\,dS \qquad\qquad \frac{dQ}{dS} = K_n = \lim_{\Delta S\to 0}\frac{\Delta Q}{\Delta S} \tag{2.3}$$

The intensity of a vector field at a point of the domain $K_n$ is equal to the projection of the vector $\boldsymbol{K}$ onto a line normal to the relevant surface element $dS$. Using the quantities applied to describe motion of deformable bodies within the linear range (stress tensor $\boldsymbol{\sigma}$, particle velocity $\boldsymbol{v}$) we introduce the vector of structural intensity $\boldsymbol{I}$, given by $(2.4)_1$, for any point within the acoustic space whose position is determined by the vector $\boldsymbol{r}$. It is readily apparent that in dynamic analysis all quantities present in $(2.4)_1$ are functions of time, which is explicitly implied by the definition. Hence, we obtain the instantaneous intensity vector $\boldsymbol{I}(\boldsymbol{r},t)$.

The physical interpretation of the intensity vector is based on its definition in acoustics, which was generalised to elastic media. Hence, the instantaneous intensity is interpreted as instantaneous power of a wave passing through a unit surface. In other words, the stream of the intensity vector over a prescribed surface expresses the power of a wave passing through this surface $(2.4)_2$

$$\boldsymbol{I}(\boldsymbol{r},t) = -\boldsymbol{\sigma}(\boldsymbol{r},t)\boldsymbol{v}(\boldsymbol{r},t) \qquad W(S,t) = \iint\limits_S \boldsymbol{I}(t)\boldsymbol{n}\, dS \tag{2.4}$$

In practical applications, we often resort to the acoustic intensity vector $\boldsymbol{I}$ averaged over the time of its realisation $T$ (2.5). This vector is often referred to as the intensity vector. In the case of monochromatic waves, the time $T$ is equal to the wave period

$$\boldsymbol{I} = \langle \boldsymbol{I}(t) \rangle_T = \frac{1}{T}\int\limits_0^T \boldsymbol{I}(t)\, dt \tag{2.5}$$

Assuming the acoustic medium to be an ideal fluid (a Newtonian one), constitutive equations for such media imply that the stress tensor has the form of $(2.6)_1$, where $p$ is acoustic pressure. A widely applied definition of the acoustic intensity vector $(2.6)_2$ in based on $(2.4)_1$ and $(2.6)_1$

$$\boldsymbol{\sigma} = \begin{bmatrix} -p & 0 & 0 \\ 0 & -p & 0 \\ 0 & 0 & -p \end{bmatrix} \qquad \boldsymbol{I}(\boldsymbol{r},t) = -p(\boldsymbol{r},t)\boldsymbol{v}(\boldsymbol{r},t) \tag{2.6}$$

Dynamic behaviour of mechanical systems is frequently analysed in the complex space. In such a case, the acoustic intensity vector might also be a complex quantity. Our considerations are restricted to monochromatic waves of the angular frequency $\omega$ and period $T$. Accordingly, the complex vector of instantaneous acoustic intensity can be obtained from $(2.7)_1$ (Mann *et al.*, 1987; Moorse and Ingard, 1968). On account of its complex form, the vector can be written as $(2.7)_2$ and has two components: the active and reactive intensity: $\boldsymbol{I}(\boldsymbol{r},t)$ and $\boldsymbol{Q}(\boldsymbol{r},t)$, respectively

$$\boldsymbol{I}_c(\boldsymbol{r},t) = \frac{1}{2}p(\boldsymbol{r},t)\boldsymbol{v}^*(\boldsymbol{r},t) \qquad \boldsymbol{I}_c(\boldsymbol{r},t) = \boldsymbol{I}(\boldsymbol{r},t) + \mathrm{i}\boldsymbol{Q}(\boldsymbol{r},t) \tag{2.7}$$

Averaging formula $(2.7)_1$ over the monochromatic wave period $T$ yields expression (2.8). Formulas that define $\boldsymbol{I}(r)$ and $\boldsymbol{Q}(r)$ can be found in Mann *et al.* (1987). Two important conclusions, vital for engineering applications, can be drawn:

- The averaged active component of the complex instantaneous intensity vector is the averaged intensity vector in the considered period for the analysed frequency.
- The averaged reactive component of the complex instantaneous intensity is zero, hence it fails to contribute to the period-averaged acoustic power flow.

In the light of the above conclusions, it is often claimed that the reactive component is of no importance from the standpoint of energy transfer, and the knowledge of the active component fully suffices for its quantitative description

$$\begin{aligned}\langle \boldsymbol{I}_c(\boldsymbol{r},t)\rangle_T &= \frac{1}{T}\int_0^T \boldsymbol{I}_c(\boldsymbol{r},t)\;dt = \frac{1}{T}\int_0^T \underbrace{2\boldsymbol{I}(\boldsymbol{r})\cos^2(\omega t-\varphi)}_{\boldsymbol{I}(\boldsymbol{r},t)}\;dt\;+\\ &+\mathrm{i}\frac{1}{T}\int_0^T \underbrace{\boldsymbol{Q}(r)\sin[2(\omega t-\varphi)]}_{\boldsymbol{Q}(\boldsymbol{r},t)}\;dt = \boldsymbol{I}(\boldsymbol{r})\end{aligned} \tag{2.8}$$

Basing on the definition of the complex intensity vector, we derive the concept of near and far acoustic fields which differ from any other geometrical definitions known from literature. The near acoustic field refers to the range close to the sound source where the reactive value of sound intensity is considerable in relation to the active component (Mann *et al.* 1987).

### 2.3. Determination of the acoustic intensity vector by analysis of structural vibrations – theoretical principles of the hybrid method

Analysis of an acoustic field generated by vibrating elements consists in finding the resultant intensity vector $\boldsymbol{I}$ at a selected point in the space. The method can be well applied to studies of vibrating surface elements. The whole vibrating structure is divided into subdomains. When the FEM approach is employed, the discretisation procedure is applied right from the moment the vibrating structure is divided into finite elements. When shell or plate elements are considered, each finite element naturally becomes a sub-domain.

Each vibrating sub-domain becomes a source of radiated sounds. For convenience, let us consider a monochromatic wave of a given frequency $\omega$ and the analysis is performed in the complex space. For the purpose of acoustic analysis, we assume that our case confines to a small radiating surface element. The smallness of the element is interpreted here in relation to the wavelength (associated with the wave frequency), in accordance with $(2.9)_1$, where

$r_0$ is the radius of the sub-domain or the greatest distance between the sub-domain centre and its boundary points (Kwiek, 1968; Moorse and Ingard, 1968; Śliwiński, 2001)

$$r_0 \ll \frac{\lambda}{4\pi} \qquad\qquad k = \frac{\omega}{c} = \frac{2\pi}{\lambda} \tag{2.9}$$

Our description utilises the Cartesian coordinate system $Oxyz$. Let the position of any control point $P(x, y, z)$ be given by the vector $\boldsymbol{R} = \overline{OP}$ and that of the central point in the $i$-th sub-domain $Q_i(x_i, y_i, z_i)$, i.e. by the vector $\boldsymbol{r}_i = \overline{Q_iP}$. The relationship between the two vectors is expressed by $(2.10)_1$, where $\boldsymbol{\rho}_i = \overline{OQ_i}$ denotes the vector of the central point position in the sub-domain, and equation $(2.10)_2$ is thus satisfied

$$\boldsymbol{r}_i = \boldsymbol{\rho}_i - \boldsymbol{R}_i \qquad\qquad r_i = \sqrt{(x - x_i)^2 + (y - y_i)^2 + (z - z_i)^2} \tag{2.10}$$

In this case, the acoustic pressure and components of particle velocity generated by the $i$-th sub-domain (defined by the position of its central point $Q(x_i, y_i, z_i)$ at a given point $P(x, y, z)$) are expressed by (2.11) (Kwiek, 1968), where: $\Delta S_i$ is the surface area of a sub-domain, $A_i$ – amplitude of sub-domain displacement. The geometry of the domain and sub-domain is schematically shown in Fig. .

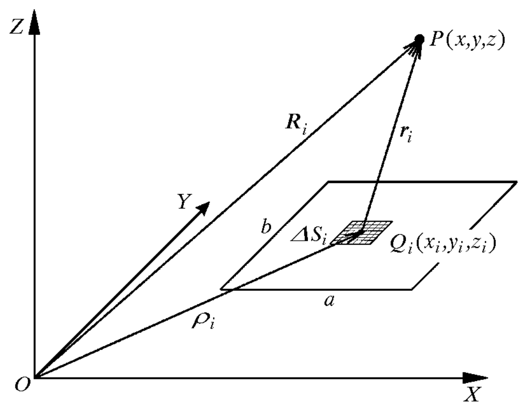


Fig. 1. Geometry of the sub-domain and the control point $P$

Formula $(2.11)_2$ can be written in the Cartesian coordinate system in a form of three scalar relationships to determine three components of the velocity vector $\boldsymbol{v}$, (2.12), which has the same direction as the vector $\boldsymbol{r}_i$

$$
\begin{aligned}
p_i &= p_i(\boldsymbol{r}_i, \boldsymbol{R}) = -\frac{1}{2\pi}\Delta S_i \omega^2 \rho \frac{A_i}{r_i} \mathrm{e}^{\mathrm{i}(\omega t - kr_i)} \\
\boldsymbol{v}_i &= \boldsymbol{v}_i(\boldsymbol{r}_i, \boldsymbol{R}) = \boldsymbol{v}_{r_i}(\boldsymbol{r}_i, \boldsymbol{R}) = A_i \Delta S_i \Big[-\frac{\omega^2}{2\pi c}\frac{1}{r_i} + \mathrm{i}\frac{\omega}{2\pi}\frac{1}{r_i^2}\Big] \mathrm{e}^e \mathrm{i}(\omega t - kr_i)\frac{\boldsymbol{r}_i}{r_i}
\end{aligned}
\tag{2.11}
$$

$$
\begin{aligned}
v_{i_x}(x_i, y_i, z_i, x, y, z) &= A_i \Delta S_i \Big[-\frac{\omega^2}{2\pi c}\frac{1}{r_i} + \mathrm{i}\frac{\omega}{2\pi}\frac{1}{r_i^2}\Big] \frac{x - x_i}{r_i} \mathrm{e}^e \mathrm{i}(\omega t - kr_i) \\
v_{i_y}(x_i, y_i, z_i, x, y, z) &= A_i \Delta S_i \Big[-\frac{\omega^2}{2\pi c}\frac{1}{r_i} + \mathrm{i}\frac{\omega}{2\pi}\frac{1}{r_i^2}\Big] \frac{y - y_i}{r_i} \mathrm{e}^e \mathrm{i}(\omega t - kr_i) \\
v_{i_z}(x_i, y_i, z_i, x, y, z) &= A_i \Delta S_i \Big[-\frac{\omega^2}{2\pi c}\frac{1}{r_i} + \mathrm{i}\frac{\omega}{2\pi}\frac{1}{r_i^2}\Big] \frac{z - z_i}{r_i} \mathrm{e}^e \mathrm{i}(\omega t - kr_i)
\end{aligned}
\tag{2.12}
$$

Another major consideration is the method of superposition of pressure and particle velocity contributions from sub-domains at the point $P$. The definition of the acoustic intensity vector, $(2.6)_2$, generalized by Mann *et al.* (1987) to systems of $N$ point sources, is given by formula (2.13). The assumption that a single element becomes a point source is acceptable as long as the surface area of a sub-domain (i.e. finite element) is small, and can be chosen arbitrarily (via grid density)

$$
\boldsymbol{I} = \frac{1}{2}\Big(\sum_{j=1}^{N} p_j\Big)\Big(\sum_{j=1}^{N} \boldsymbol{v}_j^*\Big)
\tag{2.13}
$$

After necessary transformations of $(2.11)_1$, (2.12) and (2.13), we get new concise formulas yielding components of real and imaginary parts of the acoustic intensity at a selected point. Coefficients $a_j$, $b_j$, $c_j$ and $d_j$ are given by formula (2.16)

$$
\begin{aligned}
\mathrm{Re}(I_x) &= \Big(\sum_{j=1}^{n} a_j\Big)\Big(\sum_{j=1}^{n} x_j b_j\Big) + \Big(\sum_{j=1}^{n} c_j\Big)\Big(\sum_{j=1}^{n} x_j d_j\Big) \\
\mathrm{Re}(I_y) &= \Big(\sum_{j=1}^{n} a_j\Big)\Big(\sum_{j=1}^{n} y_j b_j\Big) + \Big(\sum_{j=1}^{n} c_j\Big)\Big(\sum_{j=1}^{n} y_j d_j\Big) \\
\mathrm{Re}(I_z) &= \Big(\sum_{j=1}^{n} a_j\Big)\Big(\sum_{j=1}^{n} z_j b_j\Big) + \Big(\sum_{j=1}^{n} c_j\Big)\Big(\sum_{j=1}^{n} z_j d_j\Big)
\end{aligned}
\tag{2.14}
$$

$$\begin{aligned}
\mathrm{Im}(I_x) &= \Big(\sum_{j=1}^{n} a_j\Big)\Big(\sum_{j=1}^{n} x_j d_j\Big) - \Big(\sum_{j=1}^{n} b_j\Big)\Big(\sum_{j=1}^{n} x_j c_j\Big)\\
\mathrm{Im}(I_y) &= \Big(\sum_{j=1}^{n} a_j\Big)\Big(\sum_{j=1}^{n} y_j d_j\Big) - \Big(\sum_{j=1}^{n} b_j\Big)\Big(\sum_{j=1}^{n} y_j c_j\Big) \qquad (2.15)\\
\mathrm{Im}(I_z) &= \Big(\sum_{j=1}^{n} a_j\Big)\Big(\sum_{j=1}^{n} z_j d_j\Big) - \Big(\sum_{j=1}^{n} b_j\Big)\Big(\sum_{j=1}^{n} z_j c_j\Big)
\end{aligned}$$

$$\begin{aligned}
a_j &= \frac{A_j S_j}{r_j}\cos(kr_j) \qquad\qquad b_j = \frac{A_j S_j}{r_j}\sin(kr_j)\\
c_j &= \frac{A_j S_j}{r_j^2}\Big[k\cos(kr_j) - \frac{1}{r_j}\sin(kr_j)\Big] \qquad (2.16)\\
d_j &= \frac{A_j S_j}{r_j^2}\Big[k\sin(kr_j) + \frac{1}{r_j}\cos(kr_j)\Big]
\end{aligned}$$

Assuming velocities in individual sub-domains determined by structural analysis for steady states and transients using one of the available methods (for example FEM), the components of the resultant acoustic intensity vector are now found for the prescribed frequency. In steady-state analysis, the maximum (amplitude) acoustic intensity vector is obtained on the basis of the frequency response function, (2.5). In analysis of transients, the instantaneous acoustic intensity, $(2.7)_1$, is obtained and this formula enables us to determine whether the acoustic field in the selected point $P$ should be treated as a near or far field, in accordance with the definition provided in Subsection 2.2. This identification is based on a value of the imaginary-to-real part ratio.

One has to bear in mind that the geometry of a vibrating structure ought to be precisely chosen such that the assumption can be made that acoustic waves should not be absorbed or reflected from the surface.

The resultant acoustic intensity vector at the given point in space $P$ is an excellent parameter for the evaluation of human exposure in thus generated acoustic fields or for the evaluation of noise radiation by machines. Several standards specifying admissible noise levels utilise the concepts of acoustic pressure, acoustic power and intensity of an acoustic wave (e.g. Augustyńska *et al.*, 2000; Prascevic and Cvetkovic, 1997). When acoustic pressure is to be estimated at the point $P$, the acoustic wave at that point is assumed to be a plane wave, in accordance with (2.17). This formula, however, requires certain caution. The relationship between modelled zones of an acoustic wave radiated by a rigid 2D element: a spherical wave (near the source) and a plane wave (at a sufficiently great distance from the source $l > D^2/(4\lambda)$ (Fig. 2), where $D$ – source diameter) is only approximate (Śliwiński, 2001). Experimental tests performed by Weyna (1999) reveal that in the case of real radiating surfaces the near field range is considerable, while at some distance from the source the acoustic wave gets attenuated in the medium

$$I = \frac{p^2}{\rho c} \tag{2.17}$$

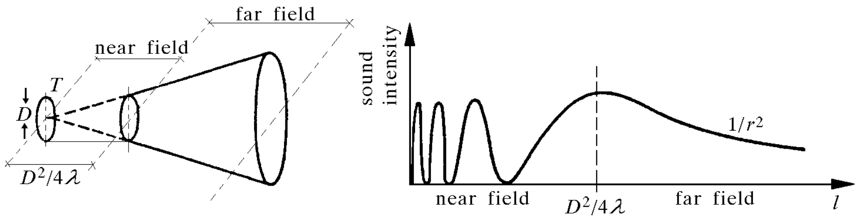


Fig. 2. Spherical and plane wave generated by a vibrating flat 2D element (left) and sound intensity in functiion of distance $r$ (right), Śliwiński (2001)

When structural analysis is performed for prescribed natural modes, the nodal lines of natural vibrations delineate sub-domains. In such a case, concise formulas are available to determine the acoustic intensity for individual modes. These formulas are provided by Nakagawa *et al.* (1993a,b) for characteristic modes of transverse vibrations of plates freely supported at edges. For the sake of simplicity, it is assumed that the analysed intensity vector component is normal to the surface of a vibrating plate, as it is directly associated with particle velocity in space. Besides, only the active component of the intensity vector is sought.

The formulas below should be treated only as a special case of the proposed hybrid method, where $u_z$ denotes the complex amplitude of vibrations in the $Oz$ direction for the given mode.

- Mode (1,1)

$$I_z = \frac{\omega^3 \rho u_z u_z^* S}{2\lambda} \frac{\sin(kr_i)}{kr_i} \tag{2.18}$$

- Mode $(1, N)$ or $(N, 1)$

$$I_z = \frac{\omega^3 \rho u_z u_z^* S}{2\lambda} \sum_{i=1}^{N} \frac{(-1)^{i-1} \sin(kr_i)}{kr_i} \tag{2.19}$$

- Mode $(M, N)$

$$I_z = \frac{\omega^3 \rho u_z u_z^* S}{2\lambda} \sum_{i=1}^{M} \frac{(-1)^{i-1}\sin(kr_i)}{kr_i} \sum_{j=1}^{N} \frac{(-1)^{j-1}\sin(kr_j)}{kr_j} \tag{2.20}$$

## 3. Experimental verification of the method

### 3.1. Verification procedure

The employed verification procedure consisted in comparing predicted and experimental values of parameters of acoustic fields produced by radiating surface elements. Theoretical values were obtained using the proposed hybrid method. As the author had no access to the required measurement facilities ( the anechoic chamber, reverberation room), the results obtained by Panuszka at AGH-UST and published in Cieślik and Panuszka (1993), Panuszka (1982a,b) had to be used instead. Conditions during the experiments were reproduced using the computer software *Ansys* for FEM analysis. Values of relevant acoustic parameters were derived from formulas applied in the acoustic intensity approach, with the use of FEM and the authors' own algorithm implemented on a PC.

### 3.2. Sound radiation from a round shaped plate

The first test was performed on a steel, homogeneous, round plate 0.487 m in diameter and 0.0009 m in thickness, fixed at the edges. Plate vibrations were induced by a rigid round piston placed at a small distance (5 mm) and parallel to the plate surface. Vibrations were transmitted via an air layer. In such an arrangement, the pressure distribution on the lower plate surface was roughly uniform. The amplitude of piston vibration acceleration was kept constant (14.7997 m/s$^2$). The first mode of plate vibrations was considered: 28.3 Hz (experiment), 35.6 (predicted value), 36.5 (FEM calculation). The level of acoustic pressure at a point on the line normal to the plate surface and passing through its center at the distance 3.02 m from the surface was taken as the reference parameter. An assumption was made, following Panuszka (1982a), that the acoustic field at that point was far, and hence the acoustic pressure level could be obtained from (2.17). The acoustic pressure level found experimentally was 80 dB, Panuszka (1982a). The pressure level obtained using the hybrid method was 80.7 dB. The reference level of the acoustic pressure was assumed to be $p_0 = 2.0 \cdot 10^{-5}$ Pa.

### 3.3. Sound radiation from a round shaped rigid piston

The second test was aimed to investigate the sound radiation from a vibrating, rigid and round piston 0.07 m in diameter. Vibrations were excited directly by a shaker and the amplitude was constant 0.05 m/s throughout the analysed frequency range. Monochromatic vibrations of the frequency 1150 Hz were considered in experiments and calculations. The level of the acoustic vector component $I_z$ at a point on the line normal to the piston surface and passing through its centre at the distance 0.6 m from the surface was taken as the reference parameter. The level of sound intensity obtained experimentally was 95 dB (Cieślik and Panuszka, 1993), and in the hybrid analysis 93 dB. The reference value of the sound intensity was $I_0 = 1.0 \cdot 10^{-12}\,\mathrm{W/m^2}$.

Some more comparisons between results of experimental analysis (Cieślik and Panuszka, 1993), results obtained by theoretical formulas (Wyrzykowski, 1972) and results obtained from the hybrid method for the control points lying on the same line but different distances from the piston are discussed by the author in Kozień (2004).

## 4. Conclusions

The FEM approach is now widespread in structural analysis of complex mechanical systems and in engineering applications. However, full analysis of coupled structural-acoustic interactions, though possible, is subject to certain limitations, chiefly due to dramatic increase of the number of dofs of a model, and hence the size of relevant matrices. As a result, calculations become time-consuming, and it is a major drawback in optimisation and vibration control tasks, which prove difficult or even impossible to handle using FEM only. Application of the FEM approach to analysis of structural vibrations, followed by the intensity method applied to studies of acoustic fields, is possible and often brings good results, which is confirmed by the experimental verification. The accuracy of thus combined analysis is sufficiently high for engineering applications. The original formulation of the acoustic intensity method, put forward and verified by the author, allows the FEM discretisation procedure to be further utilised in the analysis of acoustic fields.

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**Hybrydowa metoda oceny promieniowanego dźwięku przez drgające elementy powierzchniowe**

Streszczenie

W pracy przedstawiono podstawy teoretyczne metody hybrydowej służącej inżynierskiemu oszacowaniu promieniowania akustycznego przez drgające powwierzchniowe elementy strukturalne. Metoda umożliwia wyznaczenie wektora natężenia akustycznego w wybranym punkcie ośrodka otaczającego drgający element. Przy pewnych założeniach upraszczczających (pole dalekie) możliwa jest estymacja promieniowanaego ciśnienia akustycznego. Metoda została zweryfikowana doświadczalnie. Drgania elementu strukturalnego mogą być wyznaczone na drodze teoretycznej metodą elementów skończonych bądź pochodzić z pomiarów drgań rzeczywistch struktur.

*Manuscript received March 9, 2004; accepted for print November 8, 2004*