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NONPARAMETRIC APPROACH TO IMPROVEMENT OF QUALITY OF MODAL PARAMETERS ESTIMATION

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In the paper, issues concerning curve smoothing methods such as the locally weighted scatter plot smooth method (LOESS), moving average method (MA), Savitzky-Golay filter method (SG) are discussed. Results of the ERA analysis for measured noisy frequency response functions smoothed by curve smoothing methods as well as frequency response functions with no additional noise introduced and smoothed by the curve smoothing methods are presented. As reference values, natural frequencies and modal damping factors corresponding with poles estimated by the use of the ERA method for the frequency response functions measured in a harmonic test are assumed.

Key words: noise reduction, locally weighted scatter plot smooth method, moving average, Savitzky-Golay filter

List of the most important symbols and abbreviations used in the paper

Curve Fitting Toolbox – a set of functions for Matlab 6.5 (Release 13)
LOESS – locally weighted scatter plot smooth method
MA – moving average method
MAC – modal assurance criterion, modal model quality coefficient used for comparing sets of mode shapes
n. – noise
SG – Savitzky-Golay filter method
VIOMA (Virtual In-Operation Modal Analysis Toolbox) – a set of Matlab

functions designed for classical and in-operation modal analysis created at the Department of Robotics and Machine Dynamics of AGH Technical University, Cracow (Uhl *et al.*, 2000).

1. Introduction

Nowadays, changes in parameters of mechanical structures modal models are widely used for diagnosing the condition of such structures (Uhl, 2003). There are several problems with direct application of modal models to structure diagnostics. The main one is related to accuracy of estimation of modal parameters. Noise reduction of measurement characteristics on the basis of which the structure modal models are estimated results in increase in the accuracy of estimated modal parameters and, what fallows, in increase in aptness and credibility of drawn conclusions.

One of the practical applications of curve smoothing algorithms is emphasizing the measurement data features by noise reduction (Chaudhuri and Marron, 2000; Beauchamp, 1978). As the curve smoothing, a local approximation based on an assumed number of adjacent measured system responses is understood (Cleveland and Loader, 1996).

In case of smoothing a real data of a changeable character, assumption of a constant value of the smoothing parameter λ (e.g. length of a smoothing band) does not result in optimal solutions (Lee, 2004).

Curve areas of high concentration of local extremes require application of small values of the parameter λ, while remaining areas – adequately higher values of λ (Fig. 1).

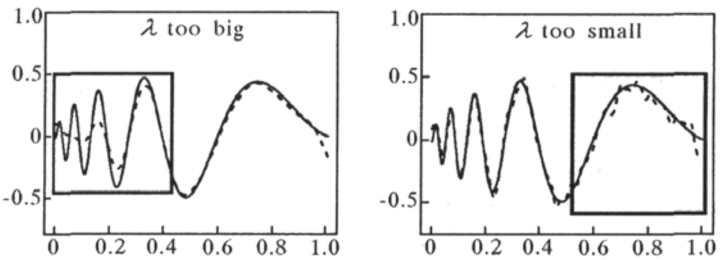

Fig. 1. Estimates (– –) of a real characteristic (——) obtained with the use of the curve smoothing method for a constant value of the parameter λ

In the literature, there are proposed some methods for analysing properties of a given characteristic on the basis of a set of curves smoothed with different values of the smoothing parameter. Two of them are shortly discussed below.

Lee in his work (Lee, 2004) proposed a smoothing method where for each point x_i of a curve of interest $f(x_i)$ a set of smoothing splines $f_\lambda(x_i)$ is created. From the set of smoothing splines an estimate $f_\lambda(x_i)$ minimising the equation

$$R_\lambda(x_i) = E\{f(x_i) - f_\lambda(x_i)\}^2$$

is chosen.

Chaudhuri and Marron (2000), on the contrary to classical methods consisting in looking for the optimal λ, assumed simultaneous considering of smoothing parameters belonging to a given band. Such a procedure is based on the fact that at each level of smoothing different information is available. A set of smoothed curves (Fig. 2a) arranged according to a growing value of the smoothing parameter λ can be represented by the so called *scale space surface* (Fig. 2b) that models properties of data seen at different levels of smoothing. The smoothing parameter is treated as a parameter of curve scaling.

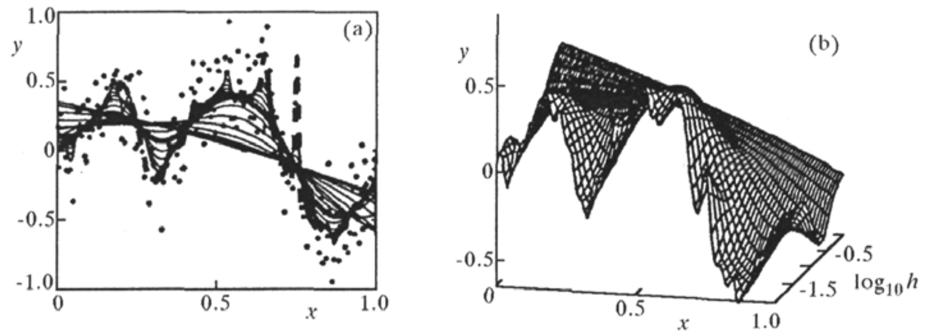

Fig. 2. (a) Set of smoothing curves of different values of the smoothing parameter λ, (b) scale space surface

With increase in the level of smoothing, the extremes of the scale space surface should disappear monotonically. This assumption, known also as the Lindeberg *casuality* assumption, guarantees that the extremes visible on the scale space surface are physical (not computational).

The methods presented above can provide more information about data of interest than the methods for a single characteristic, but they are time-consuming and computationally expensive.

In this paper, because of the large number of considered characteristics and their invariable character in the whole analysed frequency band, the curve smoothing methods with a constant smoothing parameter for a single characteristic will be used for data analysis.

Among methods of nonparametric curve smoothing, two main groups can be distinguished: filtration methods (Ruffin and King [8]; MathWorks Inc., 2002) and methods of local regression (Cleveland and Loader, 1996). These methods are discussed in the following part of this paper.

2. Filtration methods

Main applications of filters in signal processing are data averaging, noise detection, bandwidth limiting and improving signal-to-noise ratio. Signals can be filtered at an arbitrary stage of the analysis, but usually they are filtered at the stage of preliminary processing.

2.1. Moving average method

The moving average algorithm (MA) (MathWorks Inc., 2002), (The Scientists and Engineers Guide to Digital Signal Processing) consists in data smoothing by replacing consecutive measurement points by an average of points neighboring within an assumed smoothing band. In such a case, the smoothing process can be treated as equivalent to the use of a low-pass filter of a response described by the equation

$$y_s(i) = \frac{1}{2n+1}[y(i+n) + y(i+n-1) + y(i-n)] \tag{2.1}$$

where: $y_s(i)$ is a smoothed value of the ith measurement point, n – number of points neighbouring with a considered point from both sides, $(2n+1)$ – smoothing band length (filter bandwidth; Chaudhuri and Marron, 2000; Cleveland and Loader, 1996).

While considering filter equation (2.1), it can be stated that a middle point from the smoothing band of an odd number of elements is smoothed. Points from the band edges reamain unchanged. The odd number $2n+1$ is called the filter bandwidth. The wider is the filter band, the better is the smoothing effect. Increase in the filter bandwidth or multiple method application results in an increase in the signal-to-noise ratio. As the main method disadvantage, the loss of signal information in case of resonant peaks narrow in comparison with the filter bandwidth is treated.

2.2. Savitzky-Golay filter method

The Savitzky-Golay filter method, known also as a polynomial digital smoothing filter or a least-squares smoothing filter, is a generalization of the moving average algorithm. By the use of the least squares method, a set of consecutive measurement points is fitted to a polynomial of a certain order (Ruffin and King [8]). The value of the central point of a fitted-to-polynomial curve is assumed as a new smoothed point. The highest is the polynomial order, the better *reconstructed* is the height and width of the narrow resonant peaks.

Savitzky and Golay revealed that it is possible to formulate a set of integrals $(A_{-n}, A_{-(n-1)}, \ldots, A_{n-1}, A_n)$ that can be treated as weights in a smoothing process. The application of these weights is equivalent to fitting data to a polynomial and makes it possible to formulate an effective and fast algorithm. A point smoothed by the use of the Savitzky-Golay algorithm $(y_k)_s$ is described by the equation

$$(y_k)_s = \frac{\sum_{i=-n}^{n} A_i y_{k+i}}{\sum_{i=-n}^{n} A_i} \tag{2.2}$$

With respect to an assumed filter bandwidth and a polynomial order, various sets of weights can be used. The method is used for noise reduction of broad frequency band signals for which, contrary to the moving average method, it preserves high-frequency signal components (Ruffin and King [8]).

2.3. Local regression method

The local regression smoothing method LOESS (or locally weighted scatter plot smooth) consists in establishing regression weights for each point from an assumed smoothing band (Shipley and Hunt, 1996) according to the formula

$$w_i = \left(1 - \left|\frac{x - x_i}{d(x)}\right|^3\right)^3 \tag{2.3}$$

where: x is a value corresponding with the response y which is going to be smoothed; x_i – nearest neighbour of x (according to defined neighbourhood); $d(x)$ – distance between x and its furthest neighbour.

Weight values have the following characteristic features (Shipley and Hunt, 1996):

- the point that is going to be smoothed has the highest weight and the greatest influence on the smoothing process,
- points from the outside of an assumed smoothing band have zero weights and no influence on the smoothing process.

Smoothed response values are determined as a result of a weighted linear or square regression. If the right-side and left-side neighbourhoods of the x value contain the same number of points, then the weight function is symmetrical. In case of the local regression smoothing method, contrary to the moving average method, the length of the smoothing band is fixed.

3. Application of curve smoothing methods to noise reduction of the frequency response function

Curve smoothing methods were applied to noise reduction of frequency response functions measured by the use of a harmonic test on a real object – SW3 helicopter propeller blade (Ligęza *et al.*, 2001), shown schematically in Fig. 3. Twelve measurement points were chosen. The structure was excited by a harmonic force applied to the point 1 in direction opposite to the Oz axis of the assumed coordinate system (Fig. 3).

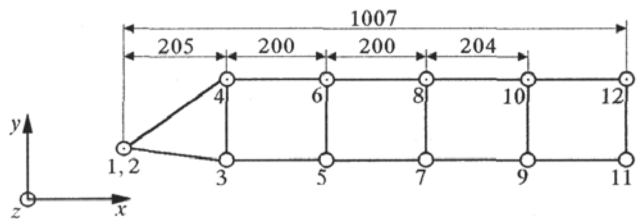

Fig. 3. Scheme of the SW3 helicopter propeller blade

System responses were measured in each assumed measurement point by the use of one-axial piezoelectric accelerometers. A four-channel Siglab analyzer was used for registration of measured frequency response functions and estimation of transfer functions necessary to carry out the modal analysis. A scheme of the test stand is presented in Fig. 4.

The analysis was carried out by the use of the ERA method implemented in the VIOMA toolbox (Uhl *et al.*, 2000). By the use of the stabilising diagrams method a model order and appropriate system of poles were chosen. Two cases were considered: measured frequency response functions burdened additionally with random noise of an amplitude of 10% measured characteristics amplitudes smoothed by the curve smoothing methods as well as frequency response functions obtained directly from harmonic test measurements (Ligęza *et al.*, 2001) smoothed by the curve smoothing methods. As reference values (ideal values), values of natural frequencies and modal damping factors

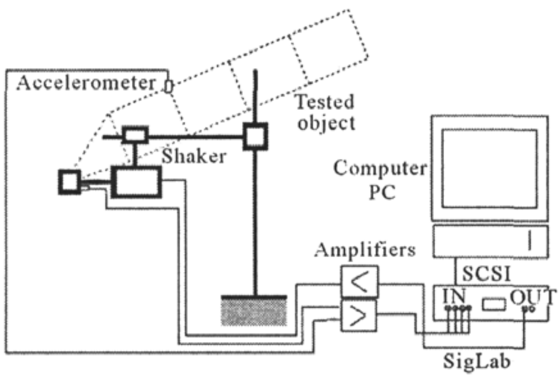

Fig. 4. Scheme of the test stand

corresponding with the poles estimated by the use of the ERA method for the frequency response functions measured by the use of the harmonic test were assumed. The assumption of such reference values is based on the fact that a harmonic test is the most accurate measurement method used in the modal analysis that allows one to obtain measurement characteristics of a small signal-to-noise ratio (Uhl, 1997).

3.1. Analysis of data measured on a real structure with additional noise

For the measured frequency response functions (marked as 'n. 0%') and measured frequency response functions burdened with an additional random 10% noise (marked as n. 10%), system poles were estimated by the use of the ERA method. Obtained values of natural frequencies and modal damping factors are gathered in Table 1.

Table 1. Results of estimation of modal model parameters

No.	n. 0% (ERA)		n. 10% (ERA)	
	f [Hz]	ζ [%]	f [Hz]	ζ [%]
1.	78.754	0.43	78.676	0.49
2.	95.033	3.90	–	–
3.	127.988	8.13	–	–
4.	219.587	1.20	220.084	1.10
5.	233.576	0.64	232.813	0.83
6.	280.535	1.89	–	–
7.	285.215	2.21	–	–

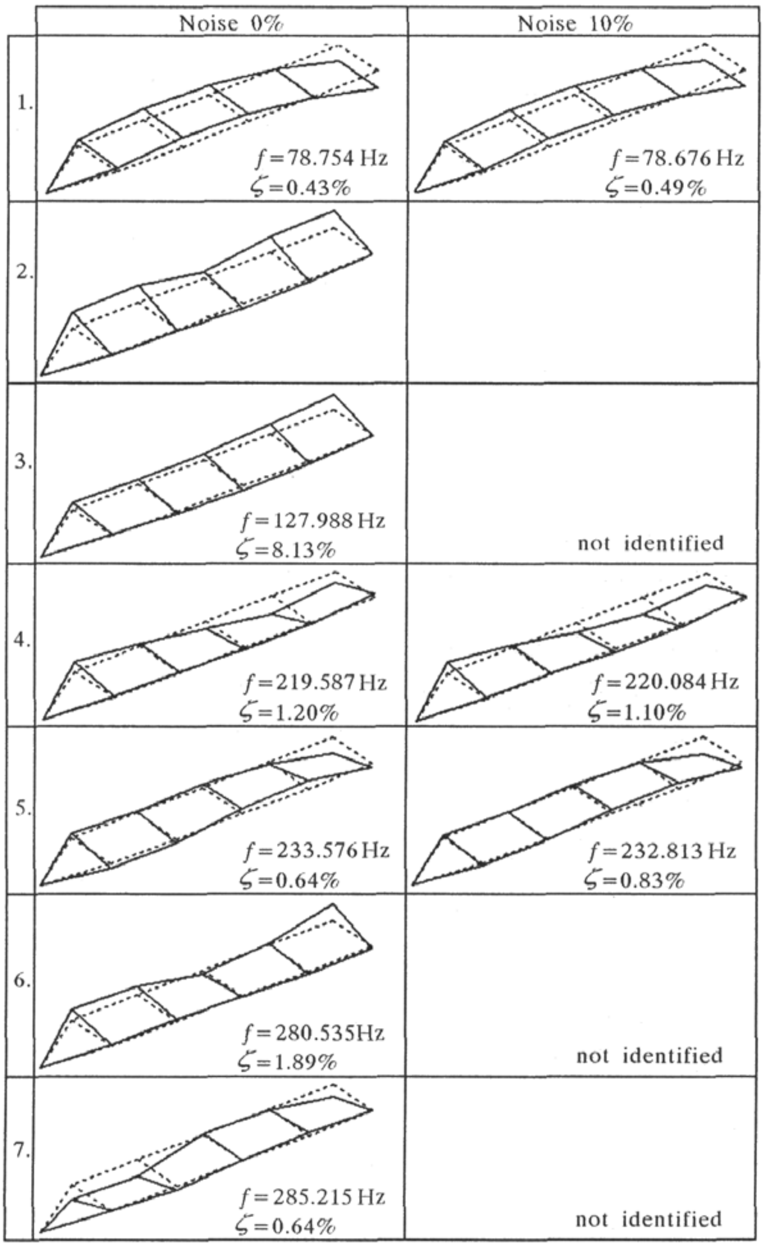

Fig. 5. On the left: mode shapes of the SW3 helicopter propeller blade for the data without noise (solid line). On the right: mode shapes of the SW3 helicopter propeller blade for the data with random 10% noise (solid line, 2) and the data without noise marked (solid line, 1)

In Fig. 5, mode shapes corresponding with the consecutive system poles estimated by the use of the ERA method for n. 0% and n. 10% (the numbering is consistent with the poles numbering from Table 1) are presented.

The introduced noise resulted in decrease in the number of estimated poles and corresponding mode shapes – only 3 out of 7 poles and mode shapes were identified.

Frequency response functions burdened with 10% random noise were smoothed by the use of the moving average (MA), locally weighted scatter plot smooth (LOESS) and Savitzky-Golay (SG) methods implemented in the Curve Fitting Toolbox [7]. For the smoothed characteristics, ERA analysis was carried out.

For the measurement data unsmoothed and smoothed by the MA method, stabilizing diagrams determined by the ERA method are shown in Fig. 6.

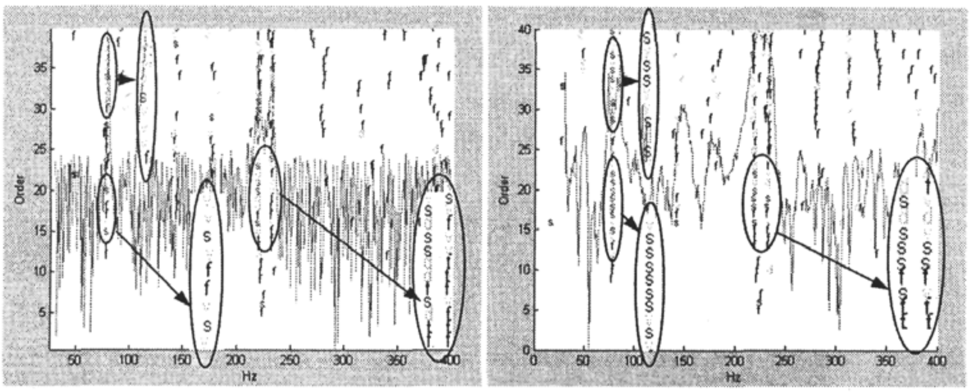

Fig. 6. Stabilizing diagrams obtained by the ERA method for measurement data unsmoothed (on the left) and smoothed by the MA method (on the right)

The application of the MA method to smoothing of frequency response functions resulted in increase in number of stable poles in areas marked in Fig. 6, which improved quality of stabilizing diagrams.

In Fig. 7, stabilizing diagrams obtained by the use of the ERA method for the data unsmoothed and smoothed by the LOESS method are shown.

For the measured frequency response functions smoothed by the use of the LOESS method, an increase in the number of estimated stable poles (with respect to diagrams obtained for the unsmoothed data) was observed (Fig. 7).

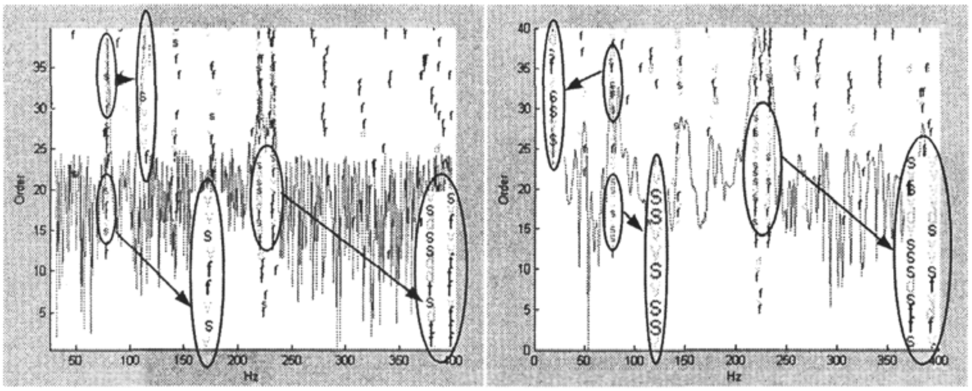

Fig. 7. Stabilizing diagrams obtained by the ERA method for data unsmoothed (on the left) and smoothed by the LOESS method (on the right)

Stabilizing diagrams determined by the use of the ERA method for the frequency response functions (measured in a harmonic test) unsmoothed and smoothed by the use of the SG method are presented in Fig. 8.

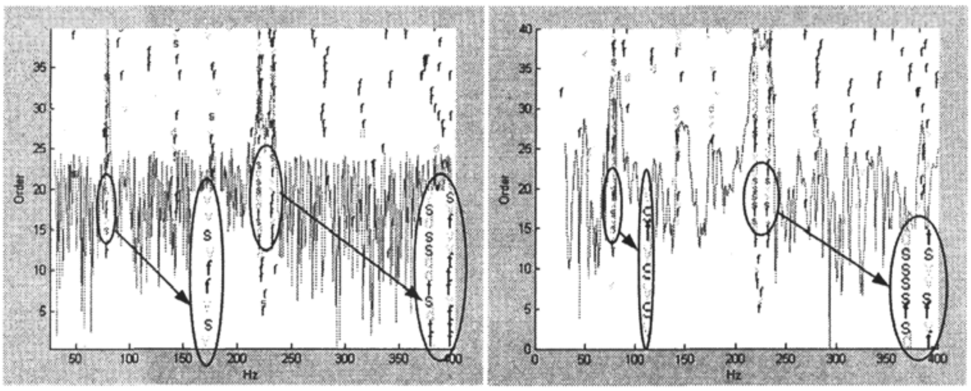

Fig. 8. Stabilizing diagrams determined by the ERA method for data unsmoothed (on the left) and smoothed by the SG method (on the right)

For the measured frequency response functions smoothed by the use of the SG method more stable poles (than in the case of diagrams obtained for the unsmoothed data) were estimated (Fig. 8).

Natural frequencies and modal damping factors corresponding with the poles estimated by the use of the ERA method for noisy frequency response functions smoothed by the MA, LOESS and SG methods are shown in Table 2.

Table 2. Natural frequencies and modal damping factors corresponding with the poles of noisy frequency response functions smoothed by the MA, LOESS and SG methods

No.	noise 0% (ERA)		noise 10% (ERA)		noise 10% MA, (ERA)		noise 10% LOESS, (ERA)		noise 10% SG, (ERA)	
	f [Hz]	ζ [%]	f [Hz]	ζ [%]	f [Hz]	ζ [%]	f [Hz]	ζ [%]	f [Hz]	ζ [%]
1.	78.754	0.43	78.676	0.49	78.948	1.78	78.788	0.75	78.787	0.68
2.	95.033	3.90	–	–	–	–	–	–	–	–
3.	127.988	8.13	–	–	–	–	–	–	–	–
4.	219.587	1.20	220.084	1.10	219.461	1.69	219.839	1.30	219.912	1.19
5.	233.576	0.64	232.813	0.83	232.529	1.10	232.839	0.67	232.634	0.50
6.	280.535	1.89	–	–	–	–	–	–	–	–
7.	285.215	2.21	–	–	–	–	–	–	–	–

In Table 3, values of percentage relative errors of estimated natural frequencies e_f and modal damping factors e_ζ for smoothed characteristics defined according to the following formula are gathered

$$e_f = \left| \frac{f_{0\%}^{ERA} - f_{10\% smoothed}^{ERA}}{f_{0\%}^{ERA}} \right| \cdot 100\% \qquad e_\zeta = \left| \frac{\zeta_{0\%}^{ERA} - \zeta_{10\% smoothed}^{ERA}}{\zeta_{0\%}^{ERA}} \right| \cdot 100\% \tag{3.1}$$

where $f_{0\%}^{ERA}$, $\zeta_{0\%}^{ERA}$ are natural frequency and modal damping factor estimated by the use of the ERA method for the measured data with no additional noise; $f_{10\% smoothed}^{ERA}$, $\zeta_{10\% smoothed}^{ERA}$ – natural frequency and modal damping factor estimated by the use of the ERA method for the measured data with additional 10% random noise smoothed by the curve smoothing methods.

Table 3. Values of percentage relative errors of natural frequencies e_f and modal damping factors e_ζ estimated by the ERA method for smoothed characteristics

No.	noise 0% (ERA)		noise 10% (ERA)		noise 10% MA, (ERA)		noise 10% LOESS, (ERA)		noise 10% SG, (ERA)	
	f [Hz]	ζ [%]	e_f [%]	e_ζ [%]	e_f [%]	e_ζ [%]	e_f [%]	e_ζ [%]	e_f [%]	e_ζ [%]
1.	78.754	0.43	0.099	13.95	0.246	232.55	0.0432	74.4	0.042	58.14
2.	95.033	3.90	–	–	–	–	–	–	–	–
3.	127.988	8.13	–	–	–	–	–	–	–	–
4.	219.587	1.20	0.226	8.33	0.057	40.83	0.115	8.33	0.148	0.83
5.	233.576	0.64	0.327	29.69	0.448	71.87	0.312	4.69	0.403	21.87
6.	280.535	1.89	–	–	–	–	–	–	–	–
7.	285.215	2.21	–	–	–	–	–	–	–	–

For all the considered curve smoothing methods, the percentage relative errors of estimated natural frequencies are small ($e_f \in \langle 0.042\%, 0.448\% \rangle$), while the percentage relative errors of estimated modal damping factors are highest for modal damping factors corresponding with a low frequency band. The smallest percentage relative errors of estimated modal damping factors were observed for characteristics smoothed by the use of the LOESS and SG methods.

Table 4 contains MAC values (Heylen *et al.*, 1994) for mode shapes corresponding with the considered system poles shown in Table 2.

Table 4. MAC coefficients for mode shapes corresponding with considered system poles shown in Table 2

No.	noise 10% (ERA), MAC	noise 10%, MA (ERA), MAC	noise 10%, LOESS (ERA), MAC	noise 10%, SG (ERA), MAC
1.	0.292	0.977	0.991	0.983
2.	–	–	–	–
3.	–	–	–	–
4.	0.012	0.901	0.887	0.974
5.	0.098	0.817	0.823	0.865
6.	–	–	–	–
7.	–	–	–	–

In Fig. 9, the obtained MAC values are presented in a graphical way.

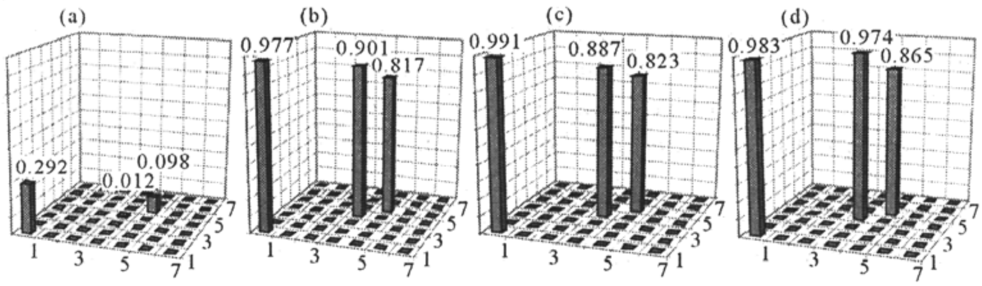

Fig. 9. Comparison of mode shapes obtained by the ERA method for the data with no noise and (a) burdened with random 10% noise, burdened with random 10% noise and smoothed by the (b) MA method, (c) LOESS method, (d) SG method

The MAC values for mode shapes estimated on the basis of noisy unsmoothed characteristics are very low $\mathrm{MAC} \in \langle 0.098, 0.292 \rangle$. For smoothed noisy characteristics, the MAC values are high for all considered curve smoothing methods $\mathrm{MAC} \in \langle 0.817, 0.991 \rangle$).

In the discussed case, the application of the LOESS method to frequency response functions smoothing proved to give better results – lower values of the relative errors e_f, e_ζ than those obtained by other considered methods.

3.2. Analysis of data measured on a real object by the use of a harmonic test

Frequency response functions measured on the SW3 helicopter propeller blade with the use of a harmonic test were subjected to smoothing by the use of the moving average (MA) locally weighted scatter plot smooth (LOESS) and Savitzky-Golay (SG) methods. It was assumed that data measured by the use of the harmonic test (that is proved to be the most accurate measurement method) can be treated as noiseless.

For the measurement data unsmoothed and smoothed by the MA method, stabilizing diagrams found from the ERA method are presented in Fig. 10.

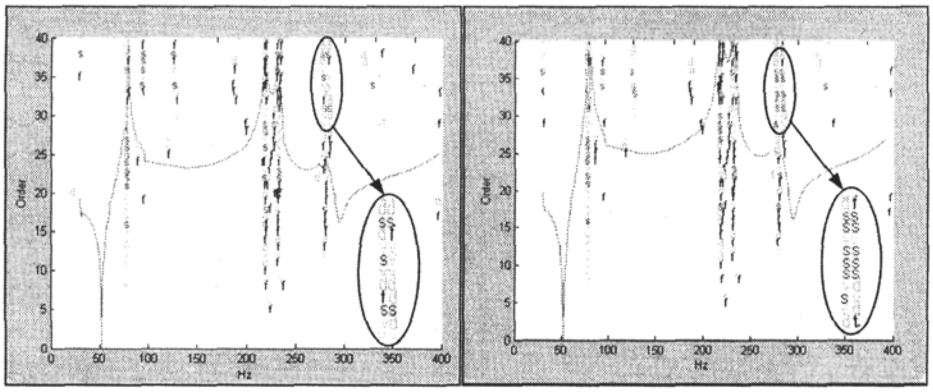

Fig. 10. Stabilizing diagrams obtained by the ERA method for the measurement data unsmoothed (on the left) and smoothed by the MA method (on the right)

The application of the MA method to measured frequency response functions smoothing resulted in increase in the number of identified stable poles, which improved the quality of stabilizing diagrams (Fig. 10).

In Fig. 11, stabilizing diagrams determined by the use of the ERA method for the measured data unsmoothed and smoothed by the use of the LOESS method are shown.

For the data smoothed by the use of the LOESS method, more stable poles (than in the case of stabilizing diagrams determined for the unsmoothed data) were estimated (Fig. 11).

Stabilizing diagrams estimated by the use of the ERA method for frequency response functions unsmoothed and smoothed by the use of the SG method are presented in Fig. 12.

The application of the SG method to frequency response functions with no noise added did not result in quality improvement of stabilizing diagrams.

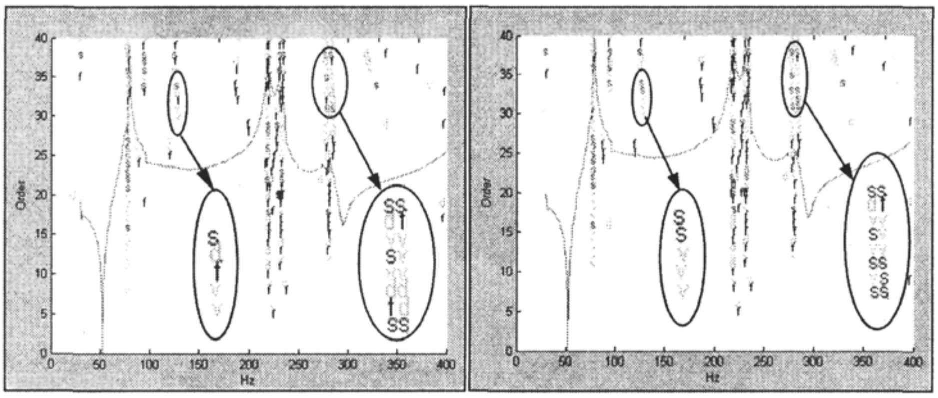

Fig. 11. Stabilizing diagrams obtained by the ERA method for the measurement data unsmoothed (on the left) and smoothed by the LOESS method (on the right)

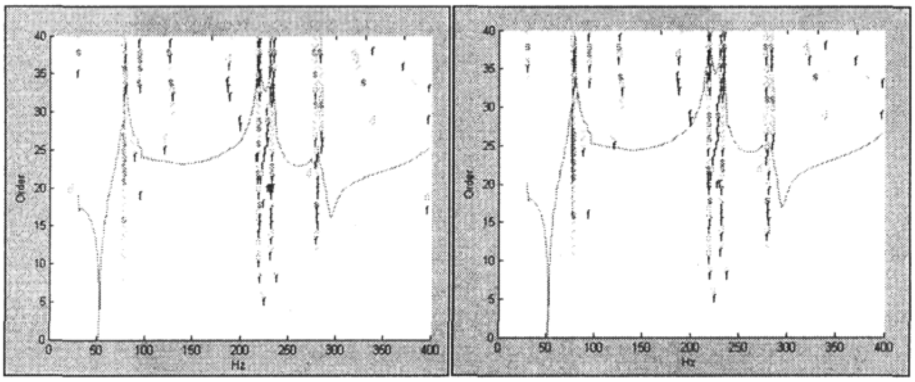

Fig. 12. Stabilizing diagrams obtained by the ERA method for the measurement data unsmoothed (on the left) and smoothed by the SG method (on the right)

Table 5. Natural frequencies and modal damping factors estimated for the frequency response functions smoothed by the MA, LOESS and SG methods

No.	noise 0% (ERA)		noise 0% MA, (ERA)		noise 0% LOESS, (ERA)		noise 0% SG, (ERA)	
	f [Hz]	ζ [%]	f [Hz]	ζ [%]	f [Hz]	ζ [%]	f [Hz]	ζ [%]
1.	78.754	0.43	78.634	0.84	78.751	0.43	78.753	0.43
2.	95.033	3.90	94.613	4.78	95.099	3.91	95.073	3.92
3.	127.988	8.13	128.029	8.63	128.077	8.10	128.027	8.12
4.	219.587	1.20	219.057	1.21	219.529	1.13	219.174	1.07
5.	233.576	0.64	233.809	0.78	233.583	0.64	233.574	0.64
6.	280.535	1.89	280.589	1.98	278.875	1.36	279.385	1.28
7.	285.215	2.21	285.658	2.29	285.192	2.21	285.169	2.21

On the basis of the presented above stabilizing diagrams determined by the use of the ERA method, natural frequencies and modal damping factors corresponding with the consecutive poles were estimated (Table 5).

In Table 6, percentage relative errors of estimated natural frequencies e_f and modal damping factors e_ζ for smoothed characteristics are gathered.

Table 6. Values of percentage relative errors of estimated natural frequencies e_f and modal damping coefficients e_ζ for smoothed characteristics

No.	noise 0% (ERA)		noise 0% MA, (ERA)		noise 0% LOESS, (ERA)		noise 0% SG, (ERA)	
	f [Hz]	ζ [%]	e_f [Hz]	e_ζ [%]	e_f [Hz]	e_ζ [%]	e_f [Hz]	e_ζ [%]
1.	78.754	0.43	0.152	95.35	0.004	0.00	0.001	0.51
2.	95.033	3.90	0.442	22.56	0.069	0.26	0.042	0.51
3.	127.988	8.13	0.032	6.15	0.069	0.37	0.030	0.12
4.	219.587	1.20	0.241	0.83	0.026	5.83	0.188	10.83
5.	233.576	0.64	0.099	21.87	0.003	0.00	0.000	0.00
6.	280.535	1.89	0.019	4.76	0.591	28.04	0.409	32.27
7.	285.215	2.21	0.155	3.62	0.008	0.00	0.016	0.00

For all considered methods of curve smoothing, the percentage relative errors of frequency estimation are small $e_f \in \langle 0\%, 0.591\% \rangle$. The lowest values of percentage relative errors of modal damping factor estimation were observed for characteristics smoothed by the use of the LOESS method.

The obtained values of relative errors e_f and e_ζ can be treated as a measure of accuracy of tested methods of the curve smoothing.

Table 7 contains MAC values for the mode shapes corresponding with the poles of the considered system described in Table 5.

Table 7. MAC values for mode shapes corresponding with poles gathered in Table 5

No.	noise 0%, MA (ERA), MAC	noise 0%, LOESS (ERA), MAC	noise 0%, SG (ERA), MAC
1.	0.999	1.000	1.000
2.	0.958	0.999	0.999
3.	0.999	1.000	0.999
4.	0.991	0.996	0.762
5.	0.999	1.000	1.000
6.	1.000	0.330	0.472
7.	0.997	1.000	0.999

In Fig. 13, the obtained MAC values are presented in a graphical way.

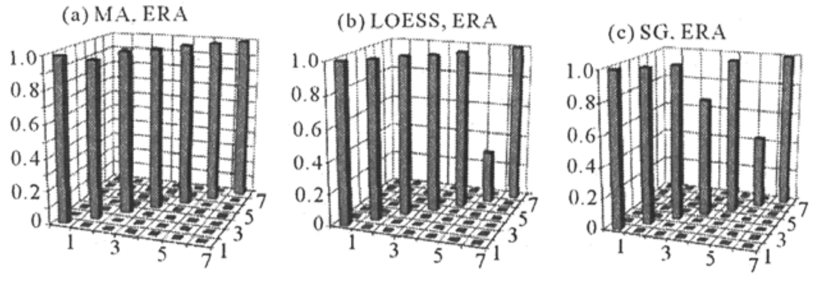

Fig. 13. Comparison of mode shapes obtained by the ERA method for frequency response functions unsmoothed and smoothed by: (a) MA, (b) LOESS, (c) SG methods

MAC values for the mode shapes estimated on the basis of the measured characteristics smoothed by the use of the MA, LOESS and SG methods are very high MAC $\in \langle 0.958, 1\rangle$ with the exception of MAC values for the 6th mode shape estimated by the use of the LOESS and SG methods $MAC_{\mathrm{LOESS}\,6.6} = 0.33$, $MAC_{\mathrm{SG}\,6.6} = 0.452$. Low values of MAC coefficients can result from the assumption of too long (with respect to the width of the resonant area) smoothing band.

Also in this case, the LOESS method performance proved to be better than the performance of other considered methods.

4. Conclusions and final remarks

Application of the curve smoothing methods to noise reduction resulted in obtaining stabilizing diagrams of better quality than those in case of unsmoothed data: pole lines contain more stable poles. For all the considered curve smoothing methods, the percentage relative errors of estimated natural frequencies are low, while the percentage relative errors of modal damping coefficients are highest for the poles of low modal damping levels.

In most cases, MAC values for the data with noise reduced by the use of the curve smoothing methods are much better than for the unsmoothed data MAC > 0.9. The filter bandwidth has significant influence on the obtained results. Assumption of big value results in the estimation of mode shapes differing form the mode shapes of the system with no noise introduced.

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Zastosowanie nieparametrycznych metod wygładzania krzywych do poprawiania jakości estymacji parametrów modalnych

Streszczenie

W pracy omówiono zagadnienia dotyczące możliwości zastosowania nieparametrycznych metod wygładzania krzywych: metody MA (ang. *moving average*), LOESS (ang. *locally weighted scatter plot smooth*) oraz SG (ang. *Savitzky-Golay filtering*) do poprawiania jakości estymowanych modeli modalnych. Dla danych bez szumu oraz danych zaszumionych wygładzonych przy użyciu nieparametrycznych metod wygładzania krzywych przeprowadzono estymację parametrów modalnych metodą ERA zaimplementowaną w przyborniku VIOMA. Wyznaczono błędy estymacji współczynnika tłumienia modalnego oraz częstości drgań własnych dla poszczególnych metod przyjmując jako wartości odniesienia współczynniki tłumienia modalnego oraz częstości drgań własnych estymowane metodą ERA dla charakterystyk niezaszumionych.

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