AN AVERAGED ISOTROPIC MODEL OF NONSTATIONARY HEAT TRANSFER IN ANISOTROPIC HEXAGONAL-TYPE CONDUCTORS

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A new macroscopic model for non-stationary heat transfer processes in a periodic hexagonal-type anisotropic rigid conductor is formulated. The main aim of this contribution is to show that the macroscopic properties of such a conductor are transversally isotropic. The tolerance averaging technique as a tool of macroscopic modelling is taken into account.

Key words: macroscopic modelling, heat conduction, periodic conductors

1. Introduction

The problem of examination of isotropic macroscopic models for materials having micro-periodic structures is not new in the recent literature. A continuum isotropic model for a certain class of frames was formulated by Woinowski-Krieger (1970). This model was generalized in a series of papers and summarized by Woźniak (1970) to the case of hexagonal lattices with rigid joints. An alternative approach to this problem was proposed by Lewiński (1984-1988). The anisotropic heat transfer effective modulus for chessboard structures was determined by Jikov *et al.* (1994). Within the framework of dynamic problems, a continuum isotropic model for hexagonal frames was analyzed by Cielecka *et al.* (2000). An isotropic averaged model of dynamic problems for periodically ribbed plates was formulated by Woźniak and Wierzbicki (2000b), for linear elastic composites by Nagórko and Wągrowska (2002), and for grids by Cielecka and Jędrysiak (2003). Averaged isotropic properties of linear elastic chessboard-type dynamic models were investigated by Augustowska and Wierzbicki (2000), Wierzbicki (2002).
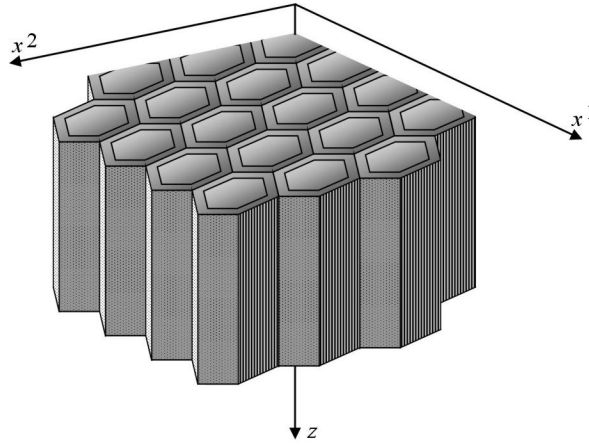
Fig. 1. Fragment of hexagonal-type periodic conductor

In this paper, a new averaged model for non-stationary heat transfer processes in a hexagonal-type rigid conductor is proposed. A fragment of such conductor is shown in Fig. 1. The main result of this contribution is to show that, under a certain condition, the overall properties of the considered conductor are transversally isotropic even if its constituents are anisotropic. The tolerance averaging technique as a tool of macroscopic modelling is taken into account. The general form of the averaged equations, which can be found in Woźniak and Wierzbicki (2000a), will be specified in order to describe the hexagonal-type periodic rigid conductor, whose material and geometrical properties are invariant for rotation by $2\pi/3$ with respect to the center of an arbitrary hexagonal cell. All considerations will be carried out within the framework of the parabolic heat transfer equations for nonstationary problems. The scope of the paper is restricted to the formulation of 3D-nonstationary heat transfer model equations; some applications of these equations will be presented in a separate paper.

Throughout the paper, we use superscripts A, B which run over $1, \ldots, N$, superscripts a, b which run over $1, \ldots, n$ and subscripts r, s which run over $1, 2, 3$. Summation convention with respect to these indices holds. Tensor product related to vectors $\boldsymbol{p}$, $\boldsymbol{q}$ is denoted by $\boldsymbol{p} \otimes \boldsymbol{q}$.

2. Preliminaries

In order to make the paper self-consistent, we outline in this section some concepts which were explained in Woźniak and Wierzbicki (2000a).

Throughout the paper, we shall use the concept of the tolerance system. The definition of the tolerance system is based on the concept of the tolerance introduced by Zeeman (1965), and from the formal viewpoint it coincides with a pair $T = (F, \varepsilon(\cdot))$, where F will be identified in this paper with a set of functions defined for every $\boldsymbol{x} = (x_1, x_2) \in R^2$ endowed with pertinent unit measures, and $\varepsilon(\cdot)$ is a mapping which assigns to every $f(\cdot) \in F$ a positive real number ε_f which has the same unit measure as f. The positive number ε_f is called the tolerance parameter and is treated as a certain admissible accuracy related to computations of f. The concept of the tolerance parameter coincides with that introduced by Fichera (1992) and called *an upper bound of negligibles*. For every $f(\cdot) \in F$ and $\boldsymbol{x}', \boldsymbol{x}'' \in R^2$ we shall write $f(\boldsymbol{x}') \cong f(\boldsymbol{x}'')$ if and only if $|f(\boldsymbol{x}') - f(\boldsymbol{x}'')| < \varepsilon_f$, and say that the values of $f(\cdot)$ at $\boldsymbol{x}'$ and $\boldsymbol{x}''$ we shall not discern. It can be seen that every tolerance is a certain symmetric and reflexive but not transitive binary relation. For the sake of simplicity, every tolerance relation will be denoted by the same symbol $\sim$.

Now, following Woźniak and Wierzbicki (2000a), we shall recall the concept of the slowly-varying function. To this end, denote by Δ a representative hexagonal periodicity cell on the Ox_1x_2-plane. A continuous function $\psi \in F$ will be called *slowly varying* (with respect to a certain tolerance system T and a periodicity cell Δ) and written as $\psi(\cdot) \in SL_\Delta(T)$ if and only if for every $\boldsymbol{x}', \boldsymbol{x}'' \in R^2$, such that $\boldsymbol{x}' - \boldsymbol{x}' \in \Delta$, condition $|\psi(\boldsymbol{x}') - \psi(\boldsymbol{x}'')| < \varepsilon_\psi$ holds; it will be assumed that this condition should be satisfied for all derivatives of ψ. Let Oz be an axis normal to the periodicity Ox_1x_2-plane. In the direction of the z-axis, the conductor will be treated as homogeneous. At the same time every plane $z = \text{const}$ is assumed to be a plane of the material symmetry. The symbol ∂ stands for a partial derivative with respect to the z-coordinate and $\overline{\nabla}$ is a gradient with respect to $\boldsymbol{x} = (x_1, x_2)$. For an arbitrary integrable function $f(\cdot)$ defined in R^2, we shall introduce the averaging operator

$$\langle f \rangle(\boldsymbol{x}) = \frac{1}{area\Delta} \int\limits_{\Delta(\boldsymbol{x})} f(\boldsymbol{y}) \, d\boldsymbol{y}$$

where $\Delta(\boldsymbol{x}) = \boldsymbol{x} + \Delta$ for $\boldsymbol{x} \in R^2$. If f depends exclusively on z and is Δ-periodic function then the above definition determinates a constant which will be denoted by $\langle f \rangle$.

In order to describe the heat transfer in the hexagonal-type rigid conductor we shall assume the heat conductivity Δ-periodic tensor field $\boldsymbol{A}(\cdot)$ in the form

$$\boldsymbol{A}(\boldsymbol{x}) = \begin{bmatrix} \overline{\boldsymbol{A}} & \boldsymbol{0} \\ \boldsymbol{0} & A_{33} \end{bmatrix} \qquad \boldsymbol{x} \in R^2 \qquad\qquad (2.1)$$

where $A_{33} = \text{const}$. Moreover, according to the periodicity of the hexagonal structure under consideration, we assume that this field, together with the specific heat field $c(\cdot)$, are both Δ-periodic fields and that they attain constant values in every constituent of the conductor. The problem of heat transfer in a honeycomb-type periodic conductor is described by the parabolic equation

$$\overline{\nabla} \cdot \overline{\boldsymbol{q}}(\boldsymbol{x}, z, t) + \partial q_3(\boldsymbol{x}, z, t) - c(\boldsymbol{x}, z)\dot{\theta}(\boldsymbol{x}, z, t) = f(\boldsymbol{x}, z, t) \qquad (2.2)$$

where the total flow $\boldsymbol{q} = (\overline{\boldsymbol{q}}, q_3)$ is determined by constitutive equations

$$\overline{\boldsymbol{q}}(\boldsymbol{x}, z, t) = -\overline{\mathbf{A}}(\boldsymbol{x}) \cdot \overline{\nabla}\theta(\boldsymbol{x}, z, t)$$

$$q_3 = -A_{33}(\boldsymbol{x})\partial\theta(\boldsymbol{x}, z, t) \tag{2.3}$$

and where $\theta(\cdot, z, t)$, $z \in R$, $t \in (t^0, t^1)$ is a temperature field. The temperature field will be represented here in a characteristic for the tolerance averaging technique description form

$$\theta(\boldsymbol{y}, z, t) = \theta^o(\boldsymbol{y}, z, t) + g^A(\boldsymbol{y})W^A(\boldsymbol{x}, z, t) \qquad \boldsymbol{y} \in (\boldsymbol{x}) \qquad (2.4)$$

where $\theta^o(\cdot, z, t) = \langle c \rangle^{-1}\langle c\theta \rangle(\cdot, z, t)$ is an averaged temperature field and $W^A(\cdot, z, t)$, $A = 1, \ldots, N$, represent the fluctuation of the temperature field caused by nonhomogeneous periodic structure of the conductor and that is why will be called *fluctuation variables*. Moreover, $g^A(\cdot)$, $A = 1, \ldots, N$, are postulated *a priori* Δ-periodic functions, usually called shape functions and satisfying conditions $\langle g^A \rangle = 0$, $\langle cg^A \rangle = 0$ and $\langle \mathbf{A}g^A \rangle = 0$. Fields $\theta^o(\cdot, z, t)$ and $W^A(\cdot, z, t)$ are assumed to be slowly varying functions for every $(z, t) \in R \times (t^0, t^1)$ and represent new basic unknowns. For these unknowns, the following system of equations

$$\overline{\nabla} \cdot \langle \overline{\mathbf{A}} \rangle \cdot \overline{\nabla}\theta^0 + \langle A_{33} \rangle \partial^2\theta - \langle c \rangle \dot{\theta}^0 + \overline{\nabla} \cdot \langle \overline{\mathbf{A}} \cdot \overline{\nabla}g^A \rangle W^A = \langle f \rangle$$

$$\langle cg^A g^B \rangle \dot{W}^B + \langle \overline{\nabla}g^A \cdot \overline{\mathbf{A}} \cdot \overline{\nabla}g^B \rangle W^B - \langle A_{33}g^A g^B \rangle \partial^2 W^B + \tag{2.5}$$
$$+ \langle \overline{\nabla}g^A \cdot \overline{\mathbf{A}} \rangle \cdot \overline{\nabla}\theta^0 = -\langle fg^A \rangle$$

has been formulated in Woźniak and Wierzbicki (2000a). Equations (2.5) have constant coefficients. Let us observe that the shape functions depend on the period length l and satisfy conditions $g^A(\cdot) \in O(l)$, $l\nabla g^A(\cdot) \in O(1)$, where $O(\cdot)$ is the known ordering symbol. The coefficients $\langle \mathbf{A}g^A g^B \rangle$ and $\langle cg^A g^B \rangle$ in Eqs (2.5) are of the order l^2 and, hence, model equations (2.4) describe the effect of the microstructure size on the averaged properties of the conductor.

It is worth noting that the known homogenized models of a periodic solid are not able to describe the above effect. The choice of shape functions g^A, $A = 1, \ldots, N$, depends on the finite element discretization of the periodicity cell by using a procedure similar to that given in Żmijewski (1987).

A model of the heat transfer in a hexagonal-type rigid conductor, given by Eqs (2.5), is the starting point for the subsequent considerations. For detailed discussion of model equations (2.4) the reader is referred to Woźniak and Wierzbicki (2000a,b).

3. Modelling of hexagonal-type conductors

In order tu discuss isotropic properties of model equations (2.5) we shall restrict considerations to the case of hexagonal-type composites satisfying the following two assumptions.

Assumption 1. *The material structure of the conductor is invariant over rotation by $2\pi/3$ with respect to the center of an arbitrary hexagonal periodicity cell.*

Since we deal with a Δ-periodic conductor, the above assumption can be written as

$$\overline{\mathbf{A}}(\mathbf{Q}\boldsymbol{y}) = \mathbf{Q}\overline{\mathbf{A}}(\boldsymbol{y})\mathbf{Q}^\top \qquad c(\mathbf{Q}\boldsymbol{y}) = c(\boldsymbol{y}) \qquad \boldsymbol{y} \in \Delta \qquad (3.1)$$

where $\mathbf{Q}$ represents rotation by $2\pi/3$ related to the center of the representative periodicity cell Δ. In the subsequent considerations, we shall use the following representation of the conductivity tensor $\mathbf{A}(\boldsymbol{y}, z)$ at $(\boldsymbol{y}, z) \in \Delta(\boldsymbol{x}) \times R$

$$\mathbf{A}(\boldsymbol{y}, z) = a^{(1)}(\boldsymbol{y})\boldsymbol{d}^1(\boldsymbol{y}) \otimes \boldsymbol{d}^1(\boldsymbol{y}) + a^{(2)}(\boldsymbol{y})\boldsymbol{d}^2(\boldsymbol{y}) \otimes \boldsymbol{d}^2(\boldsymbol{y}) + A_{33}\boldsymbol{e}^3 \otimes \boldsymbol{e}^3 \quad (3.2)$$

in which $\boldsymbol{d}^1(\boldsymbol{y})$, $\boldsymbol{d}^2(\boldsymbol{y})$ denote a pair of unit vectors determining the principal directions of the conductivity tensor at a point $(\boldsymbol{y}, z) \in R^3$. The scalars $a^{(1)}(\boldsymbol{y})$, $a^{(2)}(\boldsymbol{y})$ denote conductivity moduli related to these directions, respectively, and $\boldsymbol{e}^3$ is the $0x_3$-axis unit vector. Bearing in mind Assumption 1 we conclude that the fields $a^{(1)}(\cdot)$, $a^{(2)}(\cdot)$ are invariant over rotation $\mathbf{Q}$, i.e. formulae

$$a^{(1)}(\mathbf{Q}\boldsymbol{y}) = a^{(1)}(\boldsymbol{y}) \qquad a^{(2)}(\mathbf{Q}\boldsymbol{y}) = a^{(2)}(\boldsymbol{y}) \qquad \boldsymbol{y} \in \Delta \qquad (3.3)$$

describe a transformation procedure over rotation $\mathbf{Q}$ for these fields in the representative periodicity cell. Bearing in mind (3.2) and (3.3), we conclude that

$$\mathbf{A} + \mathbf{Q}\mathbf{A}\mathbf{Q}^\top + \mathbf{Q}^\top\mathbf{A}\mathbf{Q} = \frac{3}{2}\mathbf{1}\,\mathrm{tr}\,\mathbf{A} \qquad (3.4)$$

Similarly, bearing in mind Assumption 1 we conclude that the unit vector fields $\boldsymbol{d}^1(\cdot)$, $\boldsymbol{d}^2(\cdot)$ are invariant over rotation $\mathbf{Q}$, i.e. formulae

$$\boldsymbol{d}^1(\mathbf{Q}\boldsymbol{y}) = \mathbf{Q}\boldsymbol{d}^1(\boldsymbol{y}) \qquad\qquad \boldsymbol{d}^2(\mathbf{Q}\boldsymbol{y}) = \mathbf{Q}\boldsymbol{d}^2(\boldsymbol{y}) \qquad \boldsymbol{y} \in \Delta \qquad (3.5)$$

describe a transformation procedure over the rotation $\mathbf{Q}$ for these fields in the periodicity cell.

Passing to the formulation of the second assumption, we shall denote by G the set of all shape functions taken into account in every special problem analyzed in the framework of model equations (2.5). Since the set G has to conform with the material structure of the conductor, we shall also assume that

Assumption 2. *The set G is invariant over rotation by $2\pi/3$ with respect to the center of an arbitrary hexagonal periodicity cell.*

Having used the denotations $G_\mathbf{Q} \equiv \{g_\mathbf{Q}(\cdot) : \ g(\cdot) \in G\}$, where $g_\mathbf{Q}(\boldsymbol{y}) \equiv g(\mathbf{Q}\boldsymbol{y})$ for $\boldsymbol{y} \in \Delta$, this condition can be written as

$$G_\mathbf{Q} \subset G \qquad (3.6)$$

At the same time, let us recall the transformation formula (over rotation $\mathbf{Q}$) for shape function gradients. Namely, bearing in mind the above stated definition of the symbol $g_\mathbf{Q}$, we obtain

$$\overline{\nabla}g_\mathbf{Q}(\boldsymbol{y}) = \mathbf{Q}\overline{\nabla}g(\mathbf{Q}^\top\boldsymbol{y}) \qquad\qquad \boldsymbol{y} \in \Delta \qquad (3.7)$$

for every $g(\cdot) \in G$.

Now, we shall outline the approach leading from model equations (2.5) to the isotropic averaged model equations. This approach is implied by the assumptions formulated above and will be divided into five steps. To this end, we shall introduce a new enumerations of shape functions and fluctuation variables.

Step 1. *Introduction of new enumeration of the shape functions*

Let us observe that the natural consequence of Assumption 2 is that the set G of all shape functions can be represented the disjoint sum $G = G_1 \cup G_2 \cup \ldots \cup G_n$ of classes G_a, $a = 1, \ldots, n$. Every class G_a consists exclusively of one or three elements. Indeed, every shape function $g \in G$ uniquely determines two other shape functions $g_{\mathbf{Q}}$ and $g_{\mathbf{Q}^\top}$, defined by $g_{\mathbf{Q}}(\boldsymbol{y}) = g(\mathbf{Q}\boldsymbol{y})$ and $g_{\mathbf{Q}^\top}(\boldsymbol{y}) = g(\mathbf{Q}^\top \boldsymbol{y})$ for every $\boldsymbol{y} \in \Delta$. The class of shape functions determinined by the shape function g will be denoted by $G(g)$. Hence, $g, g_{\mathbf{Q}}, g_{\mathbf{Q}^\top} \in G(g)$, and $G(g)$ includes no other shape functions. It must be emphasized that in many cases, in which a certain shape function $g \in G$ is invariant over the $2\pi/3$ rotation $\mathbf{Q}$ (i.e. $g_{\mathbf{Q}} = g_{\mathbf{Q}^\top} = g$), the set $G(g)$ consists of exclusively one element g and then $G(g) = \{g\}$. Denoting by n_3 the number of three-element classes, we conclude that $N = n + 2n_3$. Hence, it is possible to introduce a new enumeration of the shape functions. Namely, in every class G_a, shape functions will be denoted by $g^a{}_1$, $g^a{}_2$, $g^a{}_3$ and related with each other by formulas $g^a{}_2 = (g^a{}_1)_{\mathbf{Q}}$, $g^a{}_3 = (g^a{}_2)_{\mathbf{Q}}$. It must be emphasized that in the case of one element class G_a, we have $g^a{}_1 = g^a{}_2 = g^a{}_3$ and we admiss a situation in which one shape function posseses three different indices. At the same time, formula (2.4), for every $z \in R$, $t \in (t^0, t^1)$, will be rewritten in the form

$$\theta(\boldsymbol{x}, z, t) = \theta^o(\boldsymbol{x}, z, t) + g^a{}_1(\boldsymbol{y})W^a{}_1(\boldsymbol{x}, z, t) + g^a{}_2(\boldsymbol{y})W^a{}_2(\boldsymbol{x}, z, t) + \tag{3.8}$$

$$+ g^a{}_3(\boldsymbol{y})W^a{}_3(\boldsymbol{x}, z, t) \qquad \boldsymbol{y} \in \Delta(\boldsymbol{x})$$

where the fluctuation variable field related to the shape functions $g^a{}_1$, $g^a{}_2$, $g^a{}_3$ will be denoted by $W^a{}_1$, $W^a{}_2$, $W^a{}_3$, respectively.

Step 2. *Introduction of new fluctuation variables*

Let us introduce in every cell Δ a local coordinate system uniquely determined by the unit vectors $\boldsymbol{e}^1$, $\boldsymbol{e}^2$ related to the $0\,x_1$- and $0x_2$-axes, respectively. Moreover, let $\boldsymbol{t}^1 = \boldsymbol{e}^1$, $\boldsymbol{t}^2 = -\frac{1}{2}\boldsymbol{e}^1 + \frac{\sqrt{3}}{2}\boldsymbol{e}^2$, $\boldsymbol{t}^3 = -\frac{1}{2}\boldsymbol{e}^1 - \frac{\sqrt{3}}{2}\boldsymbol{e}^2$ and $\widetilde{t}^1 = \boldsymbol{\epsilon}\boldsymbol{t}^1$, $\widetilde{t}^2 = \boldsymbol{\epsilon}\boldsymbol{t}^2$, $\widetilde{t}^3 = \boldsymbol{\epsilon}\boldsymbol{t}^3$, where $\boldsymbol{\epsilon}$ denote the Ricci-type tensor. The set of just introduced vectors is shown in Fig. 2.

Now we can define new variables

$$U^a = W^a{}_1 + W^a{}_2 + W^a{}_3$$
$$\tag{3.9}$$
$$\boldsymbol{V}^a = \boldsymbol{t}^1 W^a{}_1 + \boldsymbol{t}^2 W^a{}_2 + \boldsymbol{t}^2 W^a{}_3 \qquad a = 1, \ldots, n$$
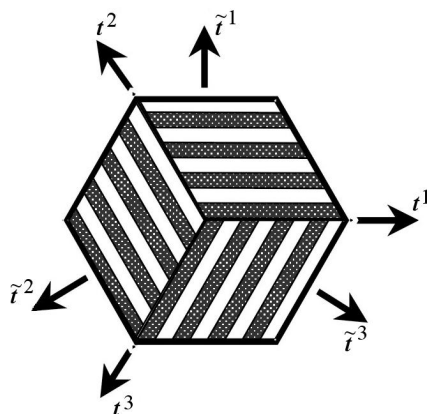
Fig. 2. Geometrical and material structure of hexagonal-type periodicity cell invariant over rotation by $2\pi/3$

strictly related to introduced above formulated two invariant assumptions and to the geometry of the hexagonal cell described by the vectors $\boldsymbol{t}^1$, $\boldsymbol{t}^2$, $\boldsymbol{t}^3$. It can be proved that relation (3.9) is an invertible transformation, and the formula

$$W^a{}_1 = \frac{1}{3}U^a + \boldsymbol{t}^1\boldsymbol{V}^a \qquad\qquad W^a{}_2 = \frac{1}{3}U^a + \boldsymbol{t}^2\boldsymbol{V}^a$$

$$W^a{}_3 = \frac{1}{3}U^a + \boldsymbol{t}^3\boldsymbol{V}^a \qquad\qquad a = 1,\dots,n \tag{3.10}$$

determines new fluctuation variable fields $U^a(\cdot)$, $\boldsymbol{V}^a(\cdot)$ by the old fluctuation variable fields $W^a{}_1(\cdot)$, $W^a{}_2(\cdot)$, $W^a{}_3(\cdot)$. Now we are to introduce the just defined new fluctuation variable U^a, $\boldsymbol{V}^a$ into model equations (2.5). To this end we define new coefficients

$$\mathbf{A}_2^{ab} = \langle\overline{\nabla}g^a{}_r\mathbf{A}(\overline{\nabla}g^b{}_s)^\top\rangle\boldsymbol{t}^r\otimes\boldsymbol{t}^s \qquad\qquad \mathbf{A}_3^{ab} = \langle A_{33}g^a{}_rg^b{}_s\rangle\boldsymbol{t}^r\otimes\boldsymbol{t}^s$$

$$[\mathbf{B}^a] = \langle\mathbf{A}\cdot\overline{\nabla}g^a{}_r\rangle\otimes\boldsymbol{t}^r \qquad\qquad\qquad [\mathbf{B}^a]^\top = \boldsymbol{t}^r\otimes\langle\mathbf{A}\cdot\overline{\nabla}g^a{}_r\rangle$$

$$\mathbf{C}_2^{ab} = \langle cg^a{}_rg^b{}_s\rangle\boldsymbol{t}^r\otimes\boldsymbol{t}^s$$

$$\overline{a}_2^{ab} = \langle\overline{\nabla}(g^a{}_1 + g^a{}_2 + g^a{}_3)\cdot\mathbf{A}\cdot\overline{\nabla}(g^b{}_1 + g^b{}_2 + g^b{}_3)\rangle \tag{3.11}$$

$$\overline{a}_3^{ab} = \langle A_{33}(g^a{}_1 + g^a{}_2 + g^a{}_3)(g^b{}_1 + g^b{}_2 + g^b{}_3)\rangle$$

$$\overline{c}_2^{ab} = \langle c(g^a{}_1 + g^a{}_2 + g^a{}_3)(g^b{}_1 + g^b{}_2 + g^b{}_3)\rangle$$

Moreover, let

$$[f^a] = \langle f(g^a{}_1 + g^a{}_2 + g^a{}_3)\rangle \qquad\qquad \boldsymbol{f}^a = \langle g^a{}_rf\rangle\boldsymbol{t}^r \tag{3.12}$$

After the formal introducing of (3.10) to model equations (2.5), bearing in mind (3.11), (3.12) and

$$\langle \overline{\mathbf{A}} \cdot \overline{\nabla}(g^a{}_1 + g^a{}_2 + g^a{}_3) \rangle = 0$$
$$\langle \overline{\nabla}(g^a{}_1 + g^a{}_2 + g^a{}_3) \cdot \overline{\mathbf{A}} \cdot \overline{\nabla}g^a{}_1 \rangle \boldsymbol{t}^1 + \langle \overline{\nabla}(g^a{}_1 + g^a{}_2 + g^a{}_3) \cdot \overline{\mathbf{A}} \cdot \overline{\nabla}g^a{}_2 \rangle \boldsymbol{t}^2 +$$
$$+\langle \overline{\nabla}(g^a{}_1 + g^a{}_2 + g^a{}_3) \cdot \overline{\mathbf{A}} \cdot \overline{\nabla}g^a{}_3 \rangle \boldsymbol{t}^3 = 0$$

we conclude the following alternative form of model equations

$$\overline{\nabla} \cdot \langle \mathbf{A} \rangle \cdot \overline{\nabla}\theta^o + [\mathbf{B}^a] : \overline{\nabla}\boldsymbol{V}^a + \langle A_{33} \rangle \partial^2 \theta^o - \langle c \rangle \dot{\theta}^o = \langle f \rangle$$
$$\overline{c}_2^{ab}\dot{U}^b + \overline{a}_2^{ab}U^b + \overline{a}_3^{ab}\partial^2 U^b = [f^a] \tag{3.13}$$
$$\mathbf{C}_2^{ab}\dot{\boldsymbol{V}}^b + \mathbf{A}_2^{ab}\boldsymbol{V}^b + [\mathbf{B}^a]^\top\overline{\nabla}\theta^o + \mathbf{A}_3^{ab}\partial^2 \boldsymbol{V}^{ab} = -\boldsymbol{f}^a$$

In the subsequent analysis, term $[\mathbf{B}^a] : \overline{\nabla}\boldsymbol{V}^a$ placed in the first from Eqs (3.13) will be called *the fluctuation term*.

Step 3. *Investigation of isotropic coefficients and their isotropic representations*

Now we are going to show that the coefficients $\mathbf{A}_2^{ab}$, $\mathbf{C}_2^{ab}$ and $\mathbf{A}_3^{ab}$ are isotropic. Indeed, formulas (3.1) and (3.7) yield

$$\langle \overline{\nabla}g^a{}_1 \mathbf{A}\overline{\nabla}g^b{}_1 \rangle = \langle \overline{\nabla}g^a{}_2 \mathbf{A}(\overline{\nabla}g^b{}_2)^\top \rangle = \langle \overline{\nabla}g^a{}_3 \mathbf{A}(\overline{\nabla}g^b{}_3)^\top \rangle$$
$$\langle \overline{\nabla}g^a{}_1 \mathbf{A}\overline{\nabla}g^b{}_2 \rangle = \langle \overline{\nabla}g^a{}_2 \mathbf{A}(\overline{\nabla}g^b{}_3)^\top \rangle = \langle \overline{\nabla}g^a{}_3 \mathbf{A}(\overline{\nabla}g^b{}_1)^\top \rangle$$

Hence, we denote

$$\widehat{a}_2^{ab} = \frac{2}{9}\langle \overline{\nabla}g^a{}_1 \mathbf{A}(\overline{\nabla}g^b{}_1)^\top + \overline{\nabla}g^a{}_2 \mathbf{A}(\overline{\nabla}g^b{}_2)^\top + \overline{\nabla}g^a{}_3 \mathbf{A}(\overline{\nabla}g^b{}_3)^\top \rangle$$
$$\tag{3.14}$$
$$\widetilde{a}_2^{ab} = \frac{2}{9}\Big[\langle \overline{\nabla}g^a{}_1 \mathbf{A}(\overline{\nabla}g^b{}_2)^\top + \overline{\nabla}g^a{}_2 \mathbf{A}(\overline{\nabla}g^b{}_3)^\top + \overline{\nabla}g^a{}_3 \mathbf{A}(\overline{\nabla}g^b{}_1)^\top \rangle -$$
$$-\langle \overline{\nabla}g^b{}_1 \mathbf{A}(\overline{\nabla}g^a{}_2)^\top + \overline{\nabla}g^b{}_2 \mathbf{A}(\overline{\nabla}g^a{}_3)^\top + \overline{\nabla}g^b{}_3 \mathbf{A}(\overline{\nabla}g^a{}_1)^\top \rangle\Big]$$

Moreover, cf. Woźniak and Wierzbicki (1999)

$$\boldsymbol{t}^1 \otimes \boldsymbol{t}^1 + \boldsymbol{t}^2 \otimes \boldsymbol{t}^2 + \boldsymbol{t}^3 \otimes \boldsymbol{t}^3 = \boldsymbol{\epsilon}(\boldsymbol{t}^1 \otimes \boldsymbol{t}^2 + \boldsymbol{t}^2 \otimes \boldsymbol{t}^3 + \boldsymbol{t}^3 \otimes \boldsymbol{t}^1) = \frac{3}{2}\mathbf{1} \tag{3.15}$$

Bearing in mind (3.14) and (3.15), we arrive at the following form of the coefficient $\mathbf{A}_2^{ab}$

$$\mathbf{A}_2^{ab} = \widehat{a}_2^{ab}\mathbf{1} + \widetilde{a}_2^{ab}\boldsymbol{\epsilon}^\top \tag{3.16}$$

On the similar way it can be proved that

$$\mathbf{A}_3^{ab} = \widehat{a}_3^{ab}\mathbf{1} + \widetilde{a}_3^{ab}\boldsymbol{\epsilon}^\top \qquad\qquad \mathbf{C}_2^{ab} = \widehat{c}_2^{ab}\mathbf{1} + \widetilde{c}_2^{ab}\boldsymbol{\epsilon}^\top \qquad (3.17)$$

where

$$\widehat{a}_3^{ab} = \frac{2}{9}\langle A_{33}(g^a{}_1 g^b{}_1 + g^a{}_2 g^b{}_2 + g^a{}_3 g^b{}_3)\rangle$$

$$\widehat{c}_2^{ab} = \frac{2}{9}\langle c(g^a{}_1 g^b{}_1 + g^a{}_2 g^b{}_2 + g^a{}_3 g^b{}_3)\rangle$$

$$\qquad\qquad(3.18)$$

$$\widetilde{a}_3^{ab} = \frac{2}{9}\Big[\langle A_{33}(g^a{}_1 g^b{}_2 + g^a{}_2 g^b{}_3 + g^a{}_3 g^b{}_1)\rangle - \langle A_{33}(g^b{}_1 g^a{}_2 + g^b{}_2 g^a{}_3 + g^b{}_3 g^a{}_1)\rangle\Big]$$

$$\widetilde{c}_2^{ab} = \frac{2}{9}\Big[\langle c(g^a{}_1 g^b{}_2 + g^a{}_2 g^b{}_3 + g^a{}_3 g^b{}_1)\rangle - \langle c(g^b{}_1 g^a{}_2 + g^b{}_2 g^a{}_3 + g^b{}_3 g^a{}_1)\rangle\Big]$$

Analogously, we conclude that the averaged conductivity tensor $\langle \mathbf{A}\rangle$ is transversally isotropic. Indeed, denoting

$$a = \frac{1}{2}\langle\,\mathrm{tr}\,\overline{\mathbf{A}}\rangle \qquad (3.19)$$

one can prove that

$$\langle\overline{\mathbf{A}}\rangle = a\mathbf{1} \qquad (3.20)$$

It must be emphasized that the representations of coefficients $\langle\overline{\mathbf{A}}\rangle$, $\mathbf{A}_2^{ab}$, $\mathbf{A}_3^{ab}$ and $\mathbf{C}_2^{ab}$, given by Eqs (3.16), (3.17), (3.20) are isotropic.

Step 4. *Investigation of the isotropic representation of fluctuation term* $[\mathbf{B}^a]$: $\overline{\nabla}\boldsymbol{V}^a$

In order to investigate the isotropic form of the fluctuation term $[\mathbf{B}^a]$: $\overline{\nabla}\boldsymbol{V}^a$ we introduce the following vector field

$$\delta\boldsymbol{V}^a \equiv \overline{\nabla}\boldsymbol{V}^a + \mathbf{Q}\overline{\nabla}\boldsymbol{V}^a\mathbf{Q}^\top + \mathbf{Q}^\top\overline{\nabla}\boldsymbol{V}^a\mathbf{Q} \qquad (3.21)$$

It is easy to verify that

$$\delta\boldsymbol{V}^a = \mathbf{Q}\cdot\delta\boldsymbol{V}^a\cdot\mathbf{Q}^\top \qquad (3.22)$$

and hence

$$[\mathbf{B}^a] : \overline{\nabla}\boldsymbol{V}^a = [\mathbf{B}^a] : \delta\boldsymbol{V}^a = \mathbf{Q}[\mathbf{B}^a]\mathbf{Q}^\top : \delta\boldsymbol{V}^a = \mathbf{Q}^\top[\mathbf{B}^a]\mathbf{Q} : \delta\boldsymbol{V}^a \qquad (3.23)$$

From the above representations of the fluctuation term, bearing in mind (3.4) and the representation of $[\mathbf{B}^a]$ given by the third formula from (3.11), we conclude that

$$[\mathbf{B}^a] : \overline{\nabla} \boldsymbol{V}^a = \frac{1}{3}\Big([\mathbf{B}^a] + \mathbf{Q}[\mathbf{B}^a]\mathbf{Q}^\top + \mathbf{Q}^\top[\mathbf{B}^a]\mathbf{Q}\Big) : \delta \boldsymbol{V}^a =$$

$$= \frac{1}{2}\Big(\langle \overline{\nabla} g^a{}_1 \operatorname{tr}\overline{\mathbf{A}}\rangle \otimes \boldsymbol{t}^1 : \delta \boldsymbol{V}^a + \langle \overline{\nabla} g^a{}_2 \operatorname{tr}\overline{\mathbf{A}}\rangle \otimes \boldsymbol{t}^2 : \delta \boldsymbol{V}^a + \langle \overline{\nabla} g^a{}_3 \operatorname{tr}\overline{\mathbf{A}}\rangle \otimes \boldsymbol{t}^3 : \delta \boldsymbol{V}^a\Big)$$

(3.24)

Now, we shall apply the decompositions of the shape function gradients $\overline{\nabla} g^a{}_1$, $\overline{\nabla} g^a{}_2$, $\overline{\nabla} g^a{}_3$ with respect to the three vector bases $(\boldsymbol{t}^1, \widetilde{\boldsymbol{t}}^1)$, $(\boldsymbol{t}^2, \widetilde{\boldsymbol{t}}^2)$, $(\boldsymbol{t}^3, \widetilde{\boldsymbol{t}}^3)$, respectively. To this end, we shall apply the following formulas

$$\langle \overline{\mathbf{A}} \cdot \overline{\nabla} g^a{}_1 \rangle \cdot \boldsymbol{t}^1 = \langle \overline{\mathbf{A}} \cdot \overline{\nabla} g^a{}_2 \rangle \cdot \boldsymbol{t}^2 = \langle \overline{\mathbf{A}} \cdot \overline{\nabla} g^a{}_3 \rangle \cdot \boldsymbol{t}^3$$

$$\langle \overline{\mathbf{A}} \cdot \overline{\nabla} g^a{}_1 \rangle \cdot \widetilde{\boldsymbol{t}}^2 = \langle \overline{\mathbf{A}} \cdot \overline{\nabla} g^a{}_2 \rangle \cdot \widetilde{\boldsymbol{t}}^3 = \langle \overline{\mathbf{A}} \cdot \overline{\nabla} g^a{}_3 \rangle \cdot \widetilde{\boldsymbol{t}}^1$$

(3.25)

which have to hold for every $a = 1, \ldots, n$. Moreover, cf. Woźniak and Wierzbicki (2000b), we recall that

$$\boldsymbol{t}^1 \otimes \boldsymbol{t}^1 + \boldsymbol{t}^2 \otimes \boldsymbol{t}^2 + \boldsymbol{t}^3 \otimes \boldsymbol{t}^3 = \widetilde{\boldsymbol{t}}^1 \otimes \widetilde{\boldsymbol{t}}^1 + \widetilde{\boldsymbol{t}}^2 \otimes \widetilde{\boldsymbol{t}}^2 + \widetilde{\boldsymbol{t}}^3 \otimes \widetilde{\boldsymbol{t}}^3 = \frac{3}{2}\mathbf{1} \qquad (3.26)$$

Bearing in mind formulas (3.18) and denoting

$$[\widehat{b}^a] = \frac{3}{4}\langle \overline{\nabla} g^a{}_r \cdot \boldsymbol{t}^r \operatorname{tr}\overline{\mathbf{A}}\rangle \qquad\qquad [\widetilde{b}^a] = \frac{3}{4}\langle \overline{\nabla} g^a{}_r \cdot \widetilde{\boldsymbol{t}}^r \operatorname{tr}\overline{\mathbf{A}}\rangle \qquad (3.27)$$

we arrive at the final isotropic form of the fluctuation term

$$[\mathbf{B}^a] : \overline{\nabla} \boldsymbol{V}^a = ([\widehat{b}^a]\mathbf{1} + [\widetilde{b}^a]\boldsymbol{\epsilon}) : \overline{\nabla} \boldsymbol{V}^a \qquad (3.28)$$

being final result of Step 4.

Step 5. *Formulation of governing equations*

Bearing in mind the results obtained in the framework of Steps 1-4, we can rewrite model equations (2.5) in an equivalent isotropic form

$$a\overline{\nabla}^2 \theta^o + [\widehat{b}^a]\overline{\nabla} \cdot \boldsymbol{V}^a + [\widetilde{b}^a]\overline{\nabla} \cdot (\boldsymbol{\epsilon} \cdot \boldsymbol{V}^a) + \langle A_{33}\rangle \partial^2 \theta^o - \langle c\rangle \dot{\theta}^o = \langle f\rangle$$

$$\overline{c}_2^{ab}\dot{U}^b + \overline{a}_2^{ab}U^b + \overline{a}_3^{ab}U^b{}_{,33} = [f^a]$$

$$(\widehat{c}_2^{ab}\mathbf{1} + \widetilde{c}_2^{ab}\boldsymbol{\epsilon}^\top) \cdot \dot{\boldsymbol{V}}^a + (\widehat{a}_2^{ab}\mathbf{1} + \widetilde{a}_2^{ab}\boldsymbol{\epsilon}^\top) \cdot \boldsymbol{V}^a + ([\widehat{b}^a]\mathbf{1} + [\widetilde{b}^a]\boldsymbol{\epsilon}^\top) \cdot \overline{\nabla}\theta^o +$$

$$+(\widehat{a}_3^{ab}\mathbf{1} + \widetilde{a}_3^{ab}\boldsymbol{\epsilon}^\top) \cdot \partial^2 \boldsymbol{V}^a = -\boldsymbol{f}^a$$

(3.29)

Moreover, if we assume that the periodicity cell of the hexagonal type composite has threefold symmetry axes, then model equations (3.29) reduce to the form

$$
\begin{aligned}
&a\overline{\nabla}^2\theta^o + [\widehat{b}^a]\overline{\nabla}\cdot\boldsymbol{V}^a + \langle A_{33}\rangle\partial^2\theta^o - \langle c\rangle\dot{\theta}^o = \langle f\rangle \\
&\overline{c}_2^{ab}\dot{U}^b + \overline{a}_2^{ab}U^b + \overline{a}_3^{ab}U^b{},_{33} = [f^a] \\
&\widehat{c}_2^{ab}\dot{\boldsymbol{V}}^a + \widehat{a}_2^{ab}\boldsymbol{V}^a + [\widehat{b}^a]\overline{\nabla}\theta^o + \widehat{a}_3^{ab}\partial^2\boldsymbol{V}^a = -\boldsymbol{f}^a
\end{aligned}
\tag{3.30}
$$

in which the first and third from equations (3.30) have exclusively scalar coefficients in contrast to $(3.29)_{1,3}$.

Model equations (3.29) represent an averaged model of the parabolic heat transfer in hexagonal-type rigid conductors and are equvallent to equations (2.5). The basic unknowns are: the averaged temperature field θ^o and new fluctuation variables $\boldsymbol{V}^a$, U^a, $a = 1,\dots,n$. These new fluctuation variables are interrelated with the fluctuation variables W^A, $A = 1,\dots,N$, by means of formula (3.9). The inverse relation is given by (3.10). Thus, we conclude that the initial boundary value problems discussed by Woźniak and Wierzbicki (2000a), and related to Eqs (2.5) can also be applied to the problems related to above obtained model equations (3.29). It has to be emphasized that equations (3.29) have a physical sense only if basic unknowns θ^o, $\boldsymbol{v}^a$, U^a are slowly varying functions of $\boldsymbol{x} \equiv (x_1, x_2) \in R^2$.

The characteristic features of Eqs (3.29) can be listed as follows.

- Model equations (3.29) are transversally isotropic, and hence we have proved the main thesis of this contribution that the averaged heat transfer response of hexagonal-type rigid conductors can be transversally isotropic.

- It has to be underlined that the components of the periodic conductor under consideration, in the general case, can be anisotropic in the plane $z = \text{const}$. We have to recall that every such plane is a material symmetry plane for every component.

- Obtained equations (3.29), in a special case reduced to (3.30), concern hexagonal-type conductors for which every cell has a threefold axis of symmetry in every plane $z = const$. A similar result for elastodynamics was obtained by Nagórko and Wągrowska (2002).

In the next section, a few special cases of the obtained model equations will be examined.

4. Special cases

As the first special case, we shall discuss heat propagation in the periodicity plane. Hence we shall assume that $\theta^o = \theta^o(x_1, x_2, t)$, $U^a = U^a(x_1, x_2, t)$ and $V^a = V^a(x_1, x_2, t)$. In this case, model equations (3.29) yield

$$a\overline{\nabla}^2\theta^o + [\widehat{b}^a]\overline{\nabla}\cdot V^a + [\widetilde{b}^a]\overline{\nabla}\cdot(\epsilon\cdot V^a) - \langle c\rangle\dot{\theta}^o = \langle f\rangle$$

$$\overline{c}_2^{ab}\dot{U}^b + \overline{a}_2^{ab}U^b = [f^a] \tag{4.1}$$

$$(\widehat{c}_2^{ab}\mathbf{1} + \widetilde{c}_2^{ab}\epsilon^\top)\cdot\dot{V}^b + (\widehat{a}_2^{ab}\mathbf{1} + \widetilde{a}_2^{ab}\epsilon^\top)\cdot V^b + ([\widehat{b}^a]\mathbf{1} + [\widetilde{b}^a]\epsilon^\top)\cdot\overline{\nabla}\theta^o = -f^a$$

It can be observed that the coefficient $(\widehat{c}_2^{ab}\mathbf{1} + \widetilde{c}_2^{ab}\epsilon^\top)$ in Eq. (4.1) is of the order l^2, and hence represents the effect of the periodicity cell size on the macroscopic heat transfer behaviour. Applying the formal limit passage $l \to 0$, we arrive at algebraic equations for the fluctuation variables V^a

$$(\widehat{a}_2^{ab}\mathbf{1} + \widetilde{a}_2^{ab}\epsilon^\top)\cdot V^b + ([\widehat{b}^a]\mathbf{1} + [\widetilde{b}^a]\epsilon^\top)\cdot\overline{\nabla}\theta^o = \mathbf{0} \tag{4.2}$$

It can be shown that this system has a unique solution for V^a given by

$$V^a = -\mathbf{M}^{ab}([\widehat{b}^b]\mathbf{1} + [\widetilde{b}^b]\epsilon^\top)\cdot\overline{\nabla}\theta^o \tag{4.3}$$

where $\mathbf{M}^{ab}$ is defined by

$$\mathbf{M}^{ab}\cdot(\widehat{a}_2^{ac}\mathbf{1} + \widetilde{a}_2^{ac}\epsilon^\top) = \delta^{ac}\mathbf{1} \tag{4.4}$$

Thus, after substituting the right-hand sides of formula (4.3) into the first from equations (4.1), we obtain a single equation for the averaged temperature given by

$$\mathbf{A}^{eff} : \overline{\nabla}\otimes\overline{\nabla}\theta^o - \langle c\rangle\dot{\theta}^o = \langle f\rangle \tag{4.5}$$

where

$$\mathbf{A}^{eff} = a\mathbf{1} - [\widehat{b}^a][\widehat{b}^b]\mathbf{M}^{ab} + [\widehat{b}^a][\widetilde{b}^b](\mathbf{M}^{ab}\cdot\epsilon^\top) - [\widetilde{b}^a][\widehat{b}^b](\epsilon\cdot\mathbf{M}^{ab}) - [\widetilde{b}^a][\widetilde{b}^b](\epsilon\cdot\mathbf{M}^{ab}\cdot\epsilon^\top) \tag{4.6}$$

If the periodicity cell of a hexagonal type composite has the threefold symmetry axis, i.e. if the model equations has a special form of (3.30), then equation (4.5) for the averaged temperature reduces to the well known form of the parabolic heat transfer equation

$$a^{eff}\overline{\nabla}^2\theta^o - \langle c\rangle\dot{\theta}^o = \langle f\rangle \tag{4.7}$$

in which

$$a^{eff} = a - [\widehat{b}^a][\widehat{b}^b]M^{ab} \tag{4.8}$$

and M^{ab} is defined by

$$M^{ab} \cdot \widehat{a}_2^{ac} = \delta^{ac} \tag{4.9}$$

where δ^{ac} denotes the Kronecker delta symbol.

In the second special case, let us consider heat propagation in the direction normal to the periodicity plane, i.e. $U^a = U^a(z,t)$, $\boldsymbol{V}^a = \boldsymbol{V}^a(z,t)$, $\theta^o = \theta^o(z,t)$. In this case, we arrive at $[\widehat{b}^a] = [\widetilde{b}^a] = 0$ and governing equations (3.29) reduce to the form

$$\langle A_{33}\rangle \partial^2 \theta^o - \langle c\rangle \dot{\theta}^o = \langle f\rangle$$
$$\overline{c}_2^{ab}\dot{U}^b + \overline{a}_2^{ab}U^b + \overline{a}_3^{ab}U^b,_{33} = [f^a] \tag{4.10}$$
$$(\widehat{c}_2^{ab}\mathbf{1} + \widetilde{c}_2^{ab}\boldsymbol{\epsilon}^\top) \cdot \dot{\boldsymbol{V}}^a + (\widehat{a}_2^{ab}\mathbf{1} + \widetilde{a}_2^{ab}\boldsymbol{\epsilon}^\top) \cdot \boldsymbol{V}^a + (\widehat{a}_3^{ab}\mathbf{1} + \widetilde{a}_3^{ab}\boldsymbol{\epsilon}^\top) \cdot \partial^2 \boldsymbol{V}^a = -\boldsymbol{f}^a$$

It can be observed that above equations are separated, i.e. every equation describes independent evolution of exclusively one basic unknown.

In the third special case, let us assume that the set G of all shape functions consists of only one shape function which will be denoted here by g, i.e. $G = \{g\}$. Since in this case $n = 1$, the superscript a attain the number $a = 1$ and it can be omitted. Moreover, we arrive at $[\widehat{b}] = [\widetilde{b}] = 0$, $\widehat{a}_3 = \widetilde{a}_3 = \widehat{c}_2 = \widetilde{c}_2 = 0$ and, sinilarly to the previous case of heat propagation in the direction normal to the periodicity plane, model equations (3.29) reduce to the form

$$a\overline{\nabla}^2\theta^o + \langle A_{33}\rangle \partial^2\theta^o - \langle c\rangle \dot{\theta}^o = \langle f\rangle$$
$$\overline{c}_2^a\dot{U}^a + \overline{a}_2^aU^a + \overline{a}_3^aU^a,_{33} = [f^a] \tag{4.11}$$
$$\widehat{c}_2^{ab}\dot{\boldsymbol{V}}^a + \widehat{a}_2^{ab}\boldsymbol{V}^a + \widehat{a}_3^{ab}\partial^2\boldsymbol{V}^a = -\boldsymbol{f}^a$$

in which every equation contains only one variable.

5. Conclusions and summary of new results

At the end of this paper let us summarize the main new results and informations on the heat transfer in 3D-hexagonal-type conductors.

- A new macroscopic model for heat transfer in 3D rigid conductors with a transversally honeycomb-type periodic structure is formulated. The main characteristic feature of this model is that the governing equations are transverally isotropic. Hence, the considered conductors have transversally isotropic properties on the macroscopic level.

- It is shown that the isotropic behaviour of the conductor takes place on the macroscopic level provided that the material properties of the hexagonal cell are invariant over rotation by $2\pi/3$ with respect to the center of the cell. In a special case, this situation occurs if there exist threefold axes of the cell material symmetry, which is a stronger condition.

- The obtained isotropic model also describes situations in which a hexagonal-type conductor is made of anisotropic constituents. It means that the isotropic properties of the constituents are not a necessary condition for the isotropic behaviour of the conductor on the macroscopic level.

The scope of this paper is restricted to the formulation of model equations. The application of these equations for finding a solution to some initial boundary-value problems will be given in a separate paper.

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Makroskopowe modelowanie problemów niestacjonarnego przepływu ciepła w przewodnikach heksagonalnych

Streszczenie

Celem pracy jest sformułowanie makroskopowego modelu niestacjonarnego przepływu ciepła w lokalnie anizotropowych heksagonalnych przewodnikach prostych. Jako narzędzie modelowania zastosowano technikę tolerancyjnego uśredniania. Sformułowano warunki wystarczające na to, by otrzymane równania były izotropowe. Zasadniczym rezultatem pracy jest wykazanie, że własności makroskopowe lokalnie anizotropowych przewodników mogą być transwersalnie izotropowe.