

## RELIABILITY BASED OPTIMIZATION OF STEEL FRAMES UNDER SEISMIC LOADING CONDITIONS USING EVOLUTIONARY COMPUTATION

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Earthquake-resistant design of structures using probabilistic analysis and performance-based design criteria is an emerging field of structural engineering. These new analysis and design methodologies are aimed at improving the existing practice and design codes for better prediction of the structural performance. In this paper, a robust and efficient methodology is presented for performing reliability-based structural optimum design of steel frames under seismic loading. The optimization part is realised with evolution strategies, while the reliability analysis is carried out with the Monte Carlo simulation method incorporating the latin hypercube sampling technique for the reduction of the sample size. The probability of failure of the frame structures, in terms of interstorey drift limits, is determined via the multi-modal response spectrum analysis.

*Key words:* structural optimization, reliability analysis, Monte Carlo simulation, evolutionary computation, multi-modal response spectrum analysis

### 1. Introduction

The inherent probabilistic nature of geometry, material properties and loading conditions involved in structural analysis is an important factor that

influences structural safety. Reliability analysis leads to safety measures that a design engineer has to take into account due to the aforementioned uncertainties. The modern conceptual approach towards seismic structural design follows the so-called Performance-Based Earthquake Engineering (PBEE) ([12], [2], [27], Fajfar and Krawinkler, 1997). The most important ingredient of PBEE is the structural reliability: straightforward consideration of all uncertainties and variabilities that arise in structural design, construction and serviceability in order to calculate the level of confidence about the structure ability to meet the desired performance goals.

Within this probabilistic framework, the seismic hazard is typically expressed in terms of occurrence of earthquakes having a certain (or bigger) intensity over a specific time period, which is normally 50 years. The structural performance in PBEE is measured as the probability that damages caused by a certain seismic hazard level are kept under a specified level. For example, one PBEE goal would be to calculate the probability to have collapse prevention if the earthquake has 2% probability of exceedance in 50 years period. Obviously, according to PBEE methodology, one can obtain all *levels of confidence* for various combinations of the structural capacity and seismic demand levels. In recent years a number of publications have appeared dealing with the performance and reliability-based optimum structural design (Alimoradi, 2003; Beck *et al.*, 1997; Collins *et al.*, 1996; Ganzerli *et al.*, 2000; Hasan and Grierson, 2002; Wen, 2000).

Due to the uncertain nature of earthquake loading, the structural design is often based on the design response spectrum of the region of interest and on some simplified assumptions of the structural behaviour under earthquakes. In the case of direct consideration of the earthquake loading, the optimization of structural systems requires multiple solution of dynamic equations of motion which can be orders of magnitude more computationally intensive than a case of static loading. In the present study the reliability-based sizing optimization of multi-story steel frames under seismic loading is investigated. The objective function is the weight of a structure while constraints are both deterministic (stress and displacement limitations imposed by the design codes) as well as probabilistic (limitation on the overall probability failure of the structure). Randomness of excitation due to ground motion and material properties are taken into consideration in the reliability analysis using Monte Carlo simulation (MCS). The probability of failure of frame structures, in terms of interstorey drift limits, is determined via multi-modal response spectrum analysis. The optimization part is solved using the Evolution Strategies (ES) method, which in most cases is more robust and present better global behaviour than

mathematical programming methods (Papadrakakis *et al.*, 1999; Lagaros *et al.*, 2002).

## 2. Structural reliability analysis

In the design of structural systems, limiting uncertainties and increasing safety is an important issue to be considered. The structural reliability, which is defined as the probability that a system meets some specified demands for a specified time period under specified environmental conditions, is used as a probabilistic measure to evaluate the reliability of structural systems. The performance function of a structural system must be determined to describe the system behaviour and to identify the relationship between the basic parameters in the system. It should be noted that in the earthquake loading environment the uncertainties related to seismic demand and structure capacity are strongly coupled.

The probability of failure  $p_f$  can be determined using the time invariant reliability analysis procedure with the following relationship

$$p_f = p[R < S] = \int_{-\infty}^{\infty} F_R(t) f_S(t) dt = 1 - \int_{-\infty}^{\infty} F_s(t) f_R(t) dt \quad (2.1)$$

where  $R$  denotes the structure bearing capacity and  $S$  the external loads. The randomness of  $R$  and  $S$  can be described by known probability density functions  $f_R(t)$  and  $f_S(t)$ , with  $F_R(t) = p[R < t]$ ,  $F_S(t) = p[S < t]$  being the cumulative probability density functions of  $R$  and  $S$ , respectively.

Most often, the limit state function is defined as  $G(R, S) = S - R$  and the probability of structural failure is given by

$$p_f = p[G(R, S) \geq 0] = \int_{G \geq 0} f_R(R) f_S(S) dR dS \quad (2.2)$$

It is practically impossible to evaluate  $p_f$  analytically for complex and/or large-scale structures, especially in the case of dynamic Reliability-Based Optimization (RBO) problems that are considered in the present study. In such cases, the integral of Eq. (2.2) can be calculated only approximately using either simulation methods, such as the Monte Carlo Simulation (MCS), or approximation methods like the first order reliability method (FORM) and the

second order reliability method (SORM), or response surface methods (RSM) (Gasser and Schueller, 1997; Huh and Haldar, 2000; Gupta and Manohar, 2004). Despite its high computational cost, MCS is considered as an efficient method and is commonly used for the evaluation of the probability of failure in computational mechanics, either for comparison with other methods or as a standalone reliability analysis tool.

### 2.1. Monte Carlo simulation

In reliability analysis, the MCS method is often employed when the analytical solution is not attainable and the failure domain can not be expressed or approximated by an analytical form. This is mainly the case in problems of complex nature with a large number of basic variables where all other reliability analysis methods are not applicable. Expressing the limit state function as  $G(\mathbf{x}) < 0$ , where  $\mathbf{x} = [x_1, x_2, \dots, x_M]^T$  is the vector of the random variables, Eq. (2.2) can be written as

$$p_f = \int_{G(\mathbf{x}) \geq 0} f_x(\mathbf{x}) d\mathbf{x} \quad (2.3)$$

where  $f_x(\mathbf{x})$  denotes the joint probability of failure for all random variables. Since MCS is based on the theory of large numbers ( $N_\infty$ ) an unbiased estimator of the probability of failure is given by

$$p_f = \frac{1}{N_\infty} \sum_{j=1}^{N_\infty} I(\mathbf{x}_j) \quad (2.4)$$

in which  $I(\mathbf{x}_j)$  is an indicator for considering *successful* or *unsuccessful* simulations, defined as

$$I(\mathbf{x}_j) = \begin{cases} 1 & \text{if } G(\mathbf{x}_j) \geq 0 \\ 0 & \text{if } G(\mathbf{x}_j) < 0 \end{cases} \quad (2.5)$$

thus in every violation a successful simulation is encountered and the failure counter is increased by 1.

It is important, while using simulation methods in the structural reliability, to efficiently and accurately evaluate the probability of failure for a given performance function. In order to estimate  $p_f$ , an adequate number of  $N_{sim}$  independent random samples is produced using a specific, usually uniform, probability density function of the vector  $\mathbf{x}$ . The value of the failure function

is computed for each random sample  $\mathbf{x}_j$  and the Monte Carlo estimation of  $p_f$  is given in terms of the sample mean by

$$p_f \cong \frac{N_H}{N_{sim}} \quad (2.6)$$

where  $N_H$  is the number of successful simulations.

## 2.2. Latin hypercube sampling

Although the mathematical formulation of MCS is relatively simple and has the ability of handling practically every possible case, regardless of its complexity, the computational effort involved in conventional MCS is excessive. For this reason a lot of sampling techniques, also called variance reduction techniques, have been developed in order to improve the computational efficiency of the method by minimizing the sample size and reducing the statistical error that is inherent in MCS. Among them are the importance sampling, adaptive sampling technique, stratified sampling, latin hypercube sampling, antithetic variate technique, conditional expectation technique (Kamal and Ayyub, 2000). Latin Hypercube Sampling (LHS) is generally recognized as one of the most efficient size reduction techniques (McKay *et al.*, 1979; Stein, 1987). The basis of LHS is full stratification of the sampled distribution with a random selection inside each stratum. In consequence, sample values are randomly shuffled among different variables. Apart from the standard LHS there are also improved LHS schemes, which combine LHS with descriptive sampling methods (Ziha, 1995) or adaptive importance sampling (Olsson *et al.*, 2003), in order to further increase the efficiency of this sampling procedure.

In the LHS method, the range of probable values for each random variable is divided into  $M$  non-overlapping segments of equal probability of occurrence. Thus, the whole parameter space, consisting of  $N$  parameters, is partitioned into  $M^N$  cells. For example, for the case of 3 parameters and 5 segments, the parameter space is divided into  $5^3$  cells. Then, the random sample generation is performed, by choosing  $M$  cells from the  $M^N$  space with respect to the density of each interval, and the cell number of each random sample is calculated. The cell number indicates the segment number the sample belongs to with respect to each of the parameters. For example, cell number (3,2,1) indicates that the sample lies in the segments 3, 2 and 1 with respect to the first, second and third parameter, respectively. In LHS, the sampling is realized independently, whereas, the matching of random samples is performed either randomly or in a restricted manner. All necessary random samples are produced and they are accepted only if they do not agree with any previous combination of the

segment numbers. The advantage of the LHS approach is that random samples are generated from all ranges of possible values, thus giving a more thorough insight into the tails of the probability distributions.

### 3. Structural design under seismic loading

The equations of equilibrium for a finite element system in motion can be written in the usual form

$$\mathbf{M}(\mathbf{s}_i)\ddot{\mathbf{u}}_t + \mathbf{C}(\mathbf{s}_i)\dot{\mathbf{u}}_t + \mathbf{K}(\mathbf{s}_i)\mathbf{u}_t = \mathbf{R}_t \quad (3.1)$$

where  $\mathbf{M}(\mathbf{s}_i)$ ,  $\mathbf{C}(\mathbf{s}_i)$ , and  $\mathbf{K}(\mathbf{s}_i)$  are the mass, damping and stiffness matrices for the  $i$ th design vector  $\mathbf{s}_i$ ;  $\mathbf{R}_t$  is the external load vector, while  $\mathbf{u}_t$ ,  $\dot{\mathbf{u}}_t$  and  $\ddot{\mathbf{u}}_t$  are the displacement, velocity, and acceleration vectors of the finite element assemblage, respectively. The design approach based on the multi-modal response spectrum analysis, which is based on the mode superposition approach, will be considered in the following section.

#### 3.1. Multi-Modal Response Spectrum analysis

The Multi-Modal Response Spectrum (MMRS) analysis is based on simplification of the mode superposition approach in order to avoid time history analyses which are required by both, the direct integration and mode superposition approaches. In the case of the multi-modal response spectrum analysis Eq. (3.1) is modified according to the modal superposition approach in the following form

$$\bar{m}^j(\mathbf{s}_i)\ddot{u}_t + \bar{c}^j(\mathbf{s}_i)\dot{u}_t + \bar{k}^j(\mathbf{s}_i)u_t = \bar{r}_t \quad (3.2)$$

where

$$\begin{aligned} \bar{m}_i^j &= (\boldsymbol{\phi}_i^j)^\top \mathbf{M}_i \boldsymbol{\phi}_i^j & \bar{c}_i^j &= (\boldsymbol{\phi}_i^j)^\top \mathbf{C}_i \boldsymbol{\phi}_i^j \\ \bar{k}_i^j &= (\boldsymbol{\phi}_i^j)^\top \mathbf{K}_i \boldsymbol{\phi}_i^j & \bar{r}_t &= (\boldsymbol{\phi}_i^j)^\top \mathbf{R}_t \end{aligned} \quad (3.3)$$

are the generalized values of the corresponding matrices and the loading vector, while  $\boldsymbol{\Phi}_i$  is the eigenmode shape matrix. For simplicity, the matrices  $\bar{m}^j(\mathbf{s}_i)$ ,  $\bar{c}^j(\mathbf{s}_i)$ ,  $\bar{k}^j(\mathbf{s}_i)$  are denoted by  $\bar{m}_i^j$ ,  $\bar{c}_i^j$ ,  $\bar{k}_i^j$ , respectively. According to the modal superposition approach, the system of  $N$  simultaneous differential equations ( $j = 1, 2, \dots, N$ ), which are coupled with the off-diagonal terms in the mass, damping and stiffness matrices, is transformed to a set of  $N$  independent normal-coordinate equations. The dynamic response can therefore be obtained

by solving separately for the response of each normal (modal) coordinate and by superposing the response in the original coordinates.

In the MMRS analysis, a number of different formulas have been proposed to obtain reasonable estimates of the maximum response based on the spectral values without performing time history analyses for a considerable number of transformed dynamic equations. The simplest and the most popular one is the Square Root of Sum of Squares (SRSS) of modal responses. According to this estimate the maximum total displacement for a degree of freedom is approximated by

$$u_{i,max} = \sqrt{(u_i^1)^2 + (u_i^2)^2 + \dots + (u_i^N)^2} \quad (3.4)$$

where the subscript  $i$  denotes the design vector;  $u_i^j$  corresponds to the maximum displacement of the  $j$ th transformed dynamic equation over the complete time period. The use of Eq. (3.4) permits this type of *dynamic* analysis by knowing only the maximum modal displacement  $u_i^j$ .

The MMRS analysis method is summarized in the following steps, where the subscript  $i$  refers to the  $\mathbf{s}_i$  design vector

- Obtain first  $m < N$  eigenfrequencies and the corresponding eigenmode shape matrices, which are classified in the following order  $(\omega_i^1, \omega_i^2, \dots, \omega_i^m)$  and  $\Phi_i = [\phi_i^1, \phi_i^2, \dots, \phi_i^m]$ , respectively;  $\omega_i^j$ ,  $\phi_i^j$  are the  $j$ th eigenfrequency and eigenmode, respectively.  $N$  is the total number of modes, while  $m$  – the number of important modes considered. The number of important modes is specified by the condition that the sum of modal masses considered must be equal or greater than 90% of the total participating mass of the system.
- Calculate the modal masses, according to the following equation

$$\bar{m}_i^j = (\phi_i^j)^\top \mathbf{M}_i \phi_i^j \quad (3.5)$$

calculate the coefficients  $L_i^j$ , according to the following equation

$$L_i^j = (\phi_i^j)^\top \mathbf{M}_i \mathbf{r} \quad (3.6)$$

where  $\mathbf{r}$  is the influence vector, which represents the displacements of the vibrating masses resulting from static application of the unit ground displacement along the direction of the seismic excitation.

- Calculate the modal participation factors  $\Gamma_i^j$ , according to

$$\Gamma_i^j = \frac{L_i^j}{\bar{m}_i^j} \quad (3.7)$$

- Calculate the effective modal mass for each design vector and for each eigenmode, by the following equation

$$m_{eff,i}^j = \frac{L_i^{j2}}{m_i^j} \quad (3.8)$$

- Calculate the spectral accelerations  $R_d(T_j^i)$  for each period of the  $m$  modes considered. For this step, the knowledge of the design response spectrum is necessary.
- Finally, obtain the modal displacements according to the relations

$$(SD^i)_j = \frac{R_d(T_j^i)}{\omega_j^{i2}} = \frac{R_d(T_j^i)T_j^{i2}}{4\pi^2} \quad (3.9)$$

$$\mathbf{u}_i^j = \Gamma_i^j \phi_i^j (SD^i)_j$$

The total maximum displacement is obtained by superimposing the maximum modal displacements using the SRSS rule of Eq. (3.4).

### 3.2. Load combinations

In the Eurocode earthquake, the loading is taken as a random action, therefore it must be considered for the structural design with the following loading combination [7]

$$S_d = \sum G_{kj} \text{ " + " } E_d \text{ " + " } \sum \psi_{2i} Q_{ki} \quad (3.10)$$

where " + " implies "to be combined with",  $\sum$  implies "the combined effect of",  $G_{kj}$  denotes the characteristic value of the permanent action  $j$ ,  $E_d$  is the design value of the seismic action, and  $Q_{ki}$  refers to the characteristic value of the variable action  $i$ , while  $\psi_{2i}$  is the combination coefficient for the quasi permanent value of the variable action  $i$ , here taken as 0.30. Design code checks are implemented in the optimization algorithm as constraints. Each structural member should be checked for actions that correspond to the most severe load combination obtained from Eq. (3.10) and the persistent load combination

$$S_d = 1.35 \sum G_{kj} \text{ " + " } 1.50 \sum Q_{ki} \quad (3.11)$$

It should be pointed out that the seismic action is obtained from the elastic response spectrum reduced by the behaviour factor  $q$ . This is done because the structure is expected to absorb the energy through inelastic deformation. The maximum values of the  $q$ -factor are suggested by design codes and vary according to the material and type of the structural system. For the framed steel structures considered in this study  $q = 4.0$ .



### 3.3. Probabilistic definition of Seismic Response Spectra

The most common approach for the definition of the seismic input is the use of the design code response spectrum. This is a general approach which is easy to implement. However, if higher precision is required the use of spectra derived from natural earthquake records is more appropriate. Since a significant dispersion on the structural response due to the use of different natural records has been observed, these spectra must be scaled to the same desired earthquake intensity. The most commonly applied scaling procedure is based on the peak ground acceleration (PGA).

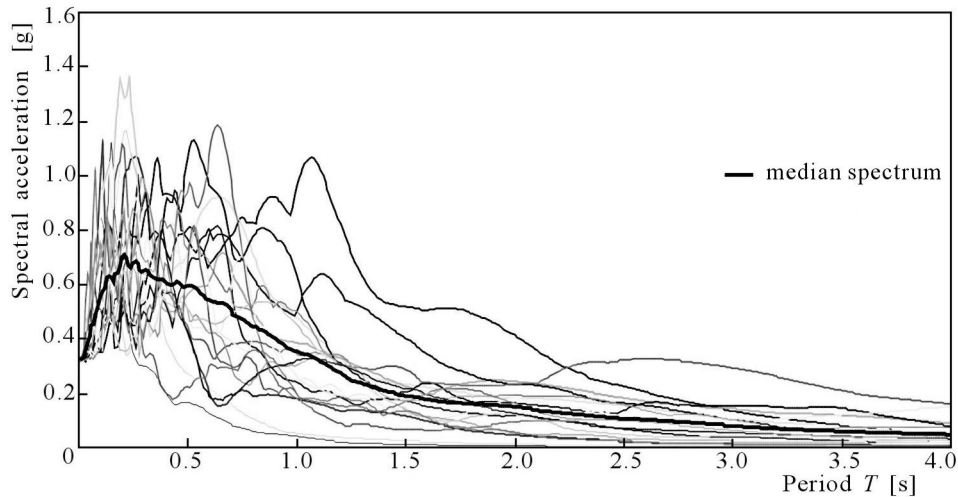


Fig. 1. Natural record response spectra and their median

In this study, a set of twenty natural accelerograms, shown in Table 1, is used. It can be seen that each record corresponds to different earthquake magnitudes and soil properties. The records of this set correspond to a wide range of PGA and peak acceleration over peak displacement ratio ( $a/v$ ) values. The latter parameter is considered to describe the damage potential of the earthquake more reliably than PGA. The records are scaled, to the same PGA according to Eurocode 8 in order to ensure compatibility between the records. The response spectrum for each scaled record are shown in Figure 1. It has been observed that the response spectra follow the lognormal distribution (Chintanapakdee and Chopra, 2003). Therefore, the median spectrum  $\hat{x}$ , also

**Table 1.** List of natural accelerograms

	Earthe name (Date)	Site \ Soil conditions	Orien- tation	$M_S$	PGA [g]	PGV [cm/s]	$a/v$ [s]
1	Victoria Mexico (06.09.80)	Cerro Prieto \ Alluvium	45	6.40	0.62	31.57	19.30
2	Kobe (16.01.95)	Kobe \ Rock	0	6.95	0.82	81.30	9.91
3	Imperial Valley (19.05.40)	El Centro Array \ CWB: D, USGS: C	180	7.20	0.31	29.80	10.32
4	Duzce (12.11.99)	Bolu \ CWB: D, USGS: C	90	7.30	0.82	62.10	12.99
5	San Fernando (09.02.71)	Pacoima dam \ Rock	164	6.61	1.22	112.49	10.69
6	Gazli (17.05.76)	Karakyr \ CWB: A	90	7.30	0.72	71.56	9.83
7	Friuli (06.05.76)	Bercis \ CWB: B	0	6.50	0.03	1.33	21.17
8	Aigion (17.05.90)	OTE building \ Stiff soil	90	4.64	0.20	9.76	20.00
9	Central California (25.04.54)	Hollister City Hall \ CWB: D, USGS: C	271	–	0.05	3.90	12.77
10	Alkyonides (24.02.81)	Korinthos OTE building \ Soft soil	90	6.69	0.31	22.70	13.34
11	Northridge (17.01.94)	Jensen filter Plant \ CWB: D, USGS: C	292	6.70	0.59	99.10	5.86
12	Athens (07.09.99)	Sepolia (Metro Station) \ Unknown	0	5.60	0.24	17.89	13.32
13	Cape Mendocino (25.04.92)	Petrolia \ CWB: D, USGS: C	90	7.10	0.66	89.72	7.24
14	Erzihan, Turkey (13.03.92)	Erzikan East-East Comp \ CWB: D, USGS: S	270	6.90	0.49	64.28	7.56
15	Kalamata (13.09.86)	Kalamata, Prefecture \ Stiff soil	0	5.75	0.21	32.90	6.41
16	Iran (16.09.78)	Tabas \ CWB: S	0	7.40	0.85	121.40	6.89
17	Loma Prieta <sup>1</sup> (18.10.89)	Hollister Diff Array \ CWB: D	255	7.10	0.28	35.60	7.69
18	Loma Prieta <sup>2</sup> (18.10.89)	Coyote Lke dam \ CWB: D	285	7.10	0.48	39.70	11.95
19	Mammoth Lakes (27.05.80)	McGee Creek \ CWB: D	0	5.00	0.33	8.55	37.29
20	Irpinia, Italy (23.11.80)	Sturno \ Unknown	270	6.50	0.36	52.70	6.66

$M_S$ : surface moment magnitude

shown in Fig. 1, and the standard deviation are calculated from the above set of spectra using the following expressions

$$\hat{x} = \exp\left\{\frac{1}{n} \sum_{i=1}^n \ln[R_{d,i}(T)]\right\}$$

$$\delta = \sqrt{\frac{1}{n-1} \sum_{i=1}^n [\ln R_{d,i}(T) - \ln \hat{x}]^2}$$
(3.12)

where  $R_{d,i}(T)$  is the response spectrum value for a period equal to  $T$  of the  $i$ th record ( $i = 1, \dots, n$ , where  $n = 20$  in this study). For a given period value, the acceleration  $R_d$  is obtained as a random variable following the log-normal distribution whose mean value is equal to  $\hat{x}$  and standard deviation is equal to  $\delta$ .

#### 4. Evolutionary computation in structural optimization

The two most widely used optimization algorithms belonging to a class of the evolutionary computation that imitates the nature by using biological methodologies are the Genetic Algorithms (GA) and Evolution Strategies (ES). The ES method was proposed for parameter optimization problems in the seventies (Schwefel, 1981). In this work, ES are used as the optimization tool for addressing RBO problems under earthquake loading. Both GA and ES imitate biological evolution in the nature and have three characteristics that make them differ from mathematical optimization algorithms

- (i) in the place of usual deterministic operators they use randomised operators
- (ii) instead of a single design point they work simultaneously with a population of design points
- (iii) they can handle continuous, discrete and mixed optimization problems.

The second characteristic allows for a natural implementation of ES in parallel computing environments (Papadrakakis *et al.*, 1999).

In studies by Papadrakakis *et al.* (1999), Lagaros *et al.* (2002) it was found that probabilistic search methods are computationally more efficient than mathematical programming methods, even though more optimization steps were required in order to reach the optimum. In the former case, the optimization

steps were computationally less expensive than in the latter case since there was no need for gradient information. This property of the probabilistic search methods is of greater importance in the case of RBO problems since the calculation of derivatives of reliability constraints is very time-consuming. Furthermore, the probabilistic methodologies can be considered, due to their random search, as global optimization methods because they are capable of finding the global optimum, whereas the mathematical programming algorithms may be trapped in local optima.

The ES optimization procedure initiates with a set of parent vectors, and if any of these parent vectors gives an infeasible design then it is modified until it becomes feasible. Subsequently, the offspring design vectors are generated and checked if they are in the feasible region. According to  $(\mu + \lambda)$  selection scheme in every generation, the values of the objective function of the parent and the offspring vectors are compared and the worst vectors are rejected, while the remaining ones are considered to be the parent vectors of the new generation. This procedure is repeated until the chosen termination criterion is satisfied. The ES algorithm for structural optimization applications under seismic loading can be stated as follows

1. **Selection step** – selection of  $\mathbf{s}_i$  ( $i = 1, 2, \dots, \mu$ ) parent design vectors
2. **Analysis step** – solve  $\mathbf{M}(\mathbf{s}_i)\ddot{\mathbf{u}}_t + \mathbf{C}(\mathbf{s}_i)\dot{\mathbf{u}}_t + \mathbf{K}(\mathbf{s}_i)\mathbf{u}_t = \mathbf{R}_t$  ( $i = 1, 2, \dots, \mu$ )
3. **Constraints check** – all parent become feasible
4. **Offspring generation** – generate  $\mathbf{s}_j$  ( $j = 1, 2, \dots, \lambda$ ) offspring design vectors
5. **Analysis step** – solve  $\mathbf{M}(\mathbf{s}_j)\ddot{\mathbf{u}}_t + \mathbf{C}(\mathbf{s}_j)\dot{\mathbf{u}}_t + \mathbf{K}(\mathbf{s}_j)\mathbf{u}_t = \mathbf{R}_t$  ( $j = 1, 2, \dots, \lambda$ )
6. **Constraints check** – if satisfied continue, else go to Step 4
7. **Selection step** – selection of the next generation parent design vectors
8. **Convergence check** – If satisfied stop, else go to Step 4

## 5. Reliability-based structural optimization under earthquake loading

In deterministic sizing optimization problems the main goal is to minimize the weight of the structure under certain deterministic behavioral constraints usually on stresses and displacements. In the RBO design, additional probabilistic constraints are imposed in order to take into account various random

parameters. So far, many articles have been devoted to the RBO design research field and efficient methods have been presented (Alimoradi, 2003; Beck *et al.*, 1997; Collins *et al.*, 1996; Ganzerli *et al.*, 2000; Gasser and Schueller, 1997; Hasan and Grierson, 2002; Tsompanakis and Papadrakakis, 2004; Papadrakakis and Lagaros, 2002; Wen, 2000).

### 5.1. Performance based earthquake engineering

Over the last ten years various design codes and guidelines ([12], [2], [27], Fajfar and Krawinkler, 1997) have introduced performance-based engineering concepts to the evaluation or improvement of the existing structures and the analysis and design of new ones. The main objective of this effort is to increase the safety of old and new buildings against natural hazards and in the case of earthquakes to make them having a predictable and reliable seismic performance. In other words, the structures should be able to resist earthquakes in a quantifiable manner and hold possible damages desired within levels.

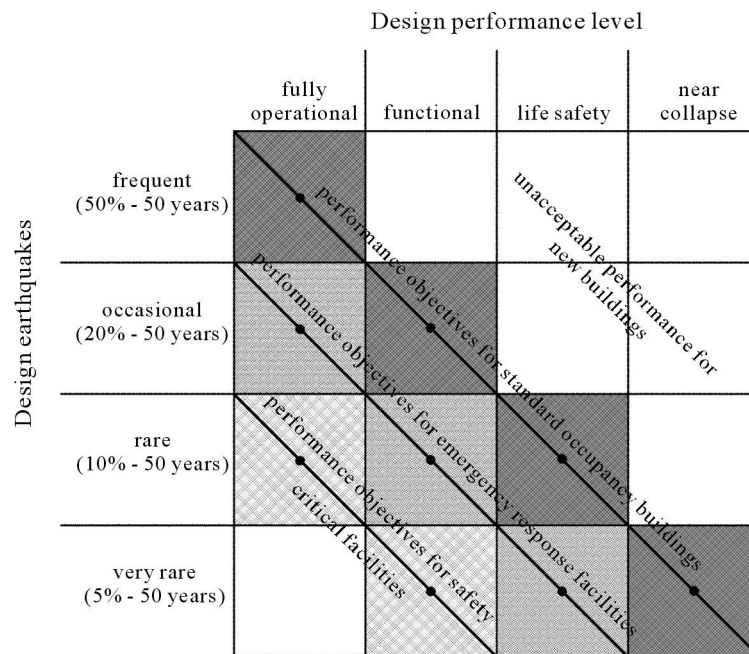


Fig. 2. Performance objectives [27]

A typical limit-state based design according to the modern codes concept can be viewed as a two-level approach: serviceability (damage control) and ul-

timate strength (life safety) limit-states. On the other hand, the Performance-Based Earthquake Engineering (PBEE) is a multi-level design approach where various levels of the structural performance are encountered ([12], [2], [27], Fajfar and Krawinkler, 1997): operational, immediate occupancy, life-safety, collapse-prevention as it is shown in Fig. 2. In other words, taking into account all the important uncertainties related to seismic demand and structural capacity, PBEE design criteria try to define certain levels of structural performance for various levels of seismic hazard. In Table 2, the description of the performance levels is shown, and the relation of these levels with the inter-storey drift limits is depicted in Table 3, whereas the various seismic risk levels are presented in Table 4. The structural performance can be measured either in terms of stresses, or displacements. Since the latter approach provides a better indicator of damages it is usually preferred, especially in terms of drift limits.

**Table 2.** Structural performance levels

Performance level		Description
NEHRP Guidelines	Vision 2000	
Operational	Fully functional	No significant damage has occurred to structural and non-structural components. The building is suitable for normal intended occupancy and use.
Immediate occupancy	Operational	No significant damage has occurred to the structure, which retains nearly all of its pre-earthquake strength and stiffness. Non-structural components are secure and most would function, if utilities available. The building may be used for intended purpose, albeit in an impaired mode.
Life safety	Life safe	Significant damage to structural elements, with substantial reduction in stiffness, however, margin remains against collapse. Non-structural elements are secured but may not function. Occupancy may be prevented until repairs can be instituted.
Collapse prevention	Near collapse	Substantial structural and non-structural damage. Structural strength and stiffness substantially degraded. Little margin against collapse. Some falling debris hazards may have occurred.

**Table 3.** Performance levels, corresponding damage state and drift limits

Performance level	Damage state	Drift limits
Fully operational Immediate occupancy	No damage	< 0.2%
Operational Damage control Moderate	Repairable	< 0.5%
Life safe – Damage state	Irreparable	< 1.5%
Near collapse Limited safety Hazard reduced	Severe	< 2.5%
Collapse	Collapse	> 2.5%

**Table 4.** Earthquake hazard levels

Earthquake frequency	Return period [years]	Probability of exceedance
Frequent	72	50% in 50 years
Occasional	225	20% in 50 years
Rare	475	10% in 50 years
Very rare	970	5% in 50 years
Extremely rare	2475	2% in 50 years

In the aforementioned guidelines, the use of various types of analysis methods is suggested: linear static, non-linear static, linear dynamic, non-linear dynamic, etc. A commonly used approach is the non-linear static, or push-over analysis method. According to this method the ground motion is *projected* to a diagram where the base shear seismic force vs lateral drift is plotted. In many cases (Freeman, 1998; Fajfar, 1999), the Capacity Spectrum method is used, where the push-over curve is plotted in a diagram having  $S_a - S_d$  (i.e. spectral acceleration and spectral displacement) axes. In this plot, the response spectra of the seismic excitation, for different hazard levels, are drawn in order to determine if the basic relation: *structural capacity* > *seismic demand*, is true or not. The response spectrum (in terms of  $S_a - S_d$ ) of the seismic excitation is produced by transforming the peak ground acceleration (PGA) which is given by Probabilistic Seismic Hazard Analysis (PSHA).

PSHA quantifies the probability of exceeding a certain level of seismic excitation at a specific site. Apart from its probabilistic considerations, it consists of two main parts: specification of the source model of the earthquake

as well as the modeling of the ground motion. Usually, the description of the source models is related to magnitude, location and rate of occurrence (annual or periodical). The ground motion model is given by the so-called attenuation relationships, where the PGA (or the  $\ln(\text{PGA})$  since PGA is considered to follow logarithmic distribution and  $\ln(\text{PGA})$  normal or Gaussian distribution) is given with regard to the magnitude and distance. It is also given in terms of local conditions which sometimes can amplify the ground motion. Once these two basic aspects are specified, probabilistic calculations can be applied in order to calculate the probability of exceedance of a certain  $\ln(\text{PGA})$  value in annual or periodical terms and produce *seismic hazard curves* for various scenarios of earthquake excitations (Field, 2003; [28]).

## 5.2. Formulation of RBO problems

In the present study the reliability-based sizing optimization of multi-storey framed structures under earthquake loading is investigated. In deterministic sizing optimization problems the aim is to minimize the weight of a structure under certain deterministic behavioral constraints usually imposed on stresses and displacements. In reliability-based optimal design additional probabilistic constraints are imposed in order to take into account various random parameters and to ensure that the probability of failure of the structure is within acceptable limits. The probabilistic constraints enforce the condition that the probability of failure of the system is smaller than a certain value. In this work, the overall probability of failure of the structure, as a result of multi-modal response spectrum analysis, is taken as the global reliability constraint. The failure is detected when the maximum interstorey drift exceeds the threshold value, here considered as 4% of the storey height. Due to engineering practice demands, the members are divided into groups having the same design variables. This linking of the elements results in a trade-off between the use of more material and the need of symmetry and uniformity of structures due to practical considerations. Furthermore, it has to be taken into account that, due to manufacturing limitations, the design variables are not continuous but discrete since the cross-sections belong to a certain set.

A discrete RBO problem can be formulated in the following form:

$$\begin{aligned}
 \min \quad & F(\mathbf{s}) \\
 \text{subject to} \quad & g_j(\mathbf{s}) \leq 0 & j = 1, \dots, m \\
 & s_i \in R^d & i = 1, \dots, n \\
 & p_f(d_r > d_{al}) \leq p_a
 \end{aligned} \tag{5.1}$$



where  $F(\mathbf{s})$  is the objective function,  $\mathbf{s}$  is the vector of design variables which can take values only from the given discrete set  $R^d$ ,  $g_j(\mathbf{s})$  are deterministic constraints and  $p_f$  is the probability of failure of the structure, i.e. the probability that the interstorey drift  $d_r$  exceeds the allowable value  $d_{al}$  for various structural performance levels. Most frequently, deterministic constraints of a structure are member stresses and nodal displacements or the inter-storey drifts. For rigid frames with  $W$ -shape cross sections, as in this study, the design constraints were taken from the design requirements specified by Eurocodes 3 [6] and 8 (Olsson *et al.*, 2003).

Two safety limit states have been considered in this work: ultimate and serviceability limit states. In the ultimate limit state the following equations should be verified. For beams the capacity design against shear requires that the following condition is satisfied

$$\frac{V_{G,Sd} + V_{M,Sd}}{V_{pl,Rd}} \leq \frac{1}{2} \quad (5.2)$$

where  $V_{G,Sd}$  is the shear force due to non seismic actions and  $V_{M,Sd}$  is the shear force due to the application of resisting moments with opposite signs at the extremities of the beam. Moreover, the applied moment should be less than  $M_{pl,Rd}$  while the axial load should be less than the 15% of  $N_{pl,Rd}$ .

For columns subjected to bending with the presence of an axial load the following formula should be satisfied

$$\frac{N_{sd}}{\chi_{min} N_{pl,Rd}} + \frac{\kappa_y M_{sd}}{M_{pl,Rd}} \leq 1 \quad (5.3)$$

where  $\chi_{min}$  factor is taken equal to 0.7 and  $\kappa_y$  equal to 1. Moreover, the shear capacity should be two times greater than the applied shear force. The plastic capacities for each member section are determined from the expressions

$$M_{pl,Rd} = \frac{W_{pl} f_y}{\gamma_{M0}} \quad N_{pl,Rd} = \frac{A f_y}{\gamma_{M1}} \quad V_{pl,Rd} = \frac{1.04 h t_w f_y}{\sqrt{3} \gamma_{M0}} \quad (5.4)$$

where  $\gamma_{M0}$  and  $\gamma_{M1}$  are considered equal to 1.10. The second order effects are not considered since the following condition is assumed to be fulfilled in all storeys

$$\frac{P_{tot} d_r}{V_{tot} h} \leq 0.10 \quad (5.5)$$

where  $P_{tot}$  is the total gravity load at the storey considered,  $d_r$  is the inter-storey drift,  $V_{tot}$  is the total seismic shear and  $h$  is the storey height.

For the serviceability limit state the interstorey drift should be limited to

$$\frac{d_r}{\nu} \leq 0.006h \quad (5.6)$$

where  $\nu$  is a reduction factor for the serviceability limit state (taken equal to 2.5 for the test example considered in this study). Different drift limits are adopted for the probabilistic and deterministic constraints, since the failure is supposed to take place for considerably higher deformations. Furthermore, the strength ratio of the column to beam is calculated and also a check of whether the sections chosen are of class 1, as EC3 suggests, is carried out. The later check is necessary in order ensure that the members have the capacity to develop their full plastic moment and rotational ductility, while the former is necessary in order to have a design consistent with the strong column-weak beam design philosophy.

The proposed reliability-based sizing optimization methodology proceeds with the following steps

1. At the outset of the optimization procedure the geometry, boundaries and loads of the structure under investigation are defined.
2. The constraints are defined in order to formulate the optimization problem as in Eq. (5.1).
3. The optimization phase is carried out with evolution strategies where feasible designs should be generated at each generation. The feasibility of the designs is checked for each design vector with respect to both deterministic and probabilistic constraints of the problem.
4. The satisfaction of the deterministic constraints is monitored through a MMRS analysis of the structure.
5. The satisfaction of the probabilistic constraints is realized with the reliability analysis of the structure using the MCS technique in order to evaluate its probability of failure.

If the convergence criteria for the optimization algorithm are satisfied then the optimum solution has been found and the process is terminated, else the whole process is repeated from step 3 with a new generation of the design vectors.

## 6. Numerical results

One test example has been considered in the present study in order to illustrate the efficiency of the proposed methodology for reliability-based sizing

optimization problems under earthquake loading. This test example is a four-bay, three-storey moment resisting the plane frame shown in Fig. 3. The frame has been previously studied in Gupta and Krawinkler (2000) where a detailed description of the structure is given. The frame consists of rigid moment connections and fixed supports. Each bay has a span of 9.15 m (30 ft), while each storey is 3.96 m (13 ft) high. The permanent action considered is equal to  $5 \text{ kN/m}^2$  while the variable action is equal to  $2 \text{ kN/m}^2$ , both distributed along the beams. The frame is considered to be a part of a 3D structure where each frame is 4.5 m (15 ft) apart. The median spectrum used for the determination of the base shear corresponds to a peak ground acceleration of 0.32 g. The structural members are divided into five groups, as shown in Fig. 3, corresponding to five design variables of a discrete structural optimization problem. The cross-sections are *W*-shape beam and column sections available from manuals of the American Institute of Steel Construction. The objective function is the weight of the structure that is to be minimized.

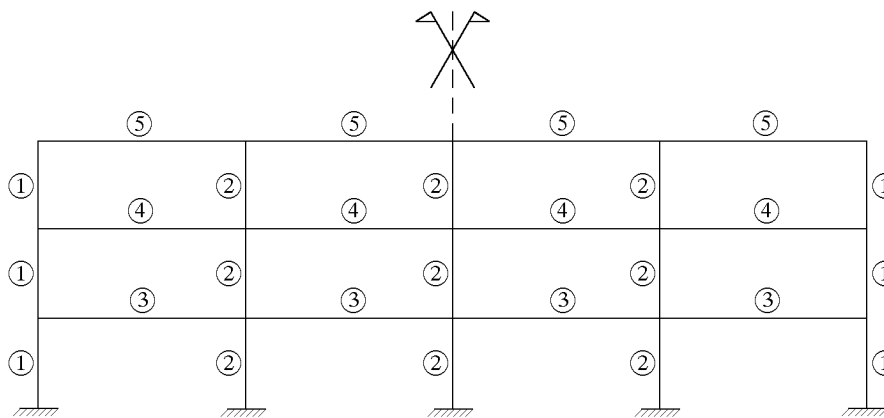


Fig. 3. Test example – geometry and member grouping

The deterministic constraints are those discussed in Section 5. The probabilistic constraint is imposed on the probability of structural collapse which is set equal to  $p_f = 0.001$ . The probability of failure caused by uncertainties related to seismic loads and material properties of the structure is estimated using both MCS and LHS techniques. The earthquake ground motion parameter, as described in Eq. (5.2), yield stress and elastic modulus are considered to be random variables. The type of probability density functions, mean values, and variances of the random parameters are shown in Table 5. The seismic action follows a log-normal probability density function, while the rest of the random

variables follow a normal probability density function. For more details on the probabilistic formulations of uncertainties the reader is referred to [19].

**Table 5.** Characteristics of the random variables

Random variable	Probability density function	Mean value	Standard deviation
$E$	$N$	$2.1 \cdot 10^6$ MPa	$0.10E$
$\sigma_y$	$N$	235 MPa	$0.10\sigma_y$
Seismic load	$\log N$	$\hat{x}, (3.12)_1$	$\delta, (3.12)_2$

For this test case the  $(\mu + \lambda)$ -ES approach is used with  $\mu = \lambda = 5$ , while a sample size of 5000 and 1000 simulations is taken for MCS and LHS techniques, respectively. Table 6 depicts the performance of the optimization procedure for this test case. As it can be seen, the probability of failure corresponding to the optimum computed by the deterministic optimization procedure is much larger than the specified value of  $10^{-3}$ . In this example, the increase in the optimum weight achieved, when the probabilistic constraints are considered, is approximately 26% compared to the deterministic one, as it can be observed in Table 6.

**Table 6.** Performance of the methods

Optimization procedure	ES generations	$p_f$	Optimum volume [m <sup>3</sup> ]
DBO	157	$9.32 \cdot 10^{-2}$	15.95
RBO-MCS (5000 siml.)	65	$0.08 \cdot 10^{-2}$	21.44
RBO-LHS (1000 siml.)	72	$0.10 \cdot 10^{-2}$	21.30

## 7. Concluding remarks

In most cases, the optimum design of structures is based on deterministic parameters and is focused on the satisfaction of associated deterministic constraints. Since there are many random factors that affect the design, manufacturing and performance of a structure during its lifetime, the deterministic optimum is not indeed a *safe* optimum. In order to find the *real* optimum the designer has to take into account all necessary random parameters, and via reliability analysis of the structure to determine its optimum design taking into account the desired level of probability of the structural failure. Only after

forming and solving this RBO problem, even with an additional cost in weight and computing time, a *global* and realistic optimum structural design can be found.

The aim of the proposed RBO procedure is twofold. To increase the safety margins of structures optimized under various uncertainties, while at the same time minimizing their weight, and to reduce substantially the required computational effort. The solution to realistic RBO problems in structural mechanics is an extremely computationally intensive task. As it can be observed from numerical results, the computational cost of the solution to realistic RBO problems can be order(s) of magnitude larger than the corresponding cost of the deterministic optimization. Due to the size and complexity of RBO problems, a stochastic optimization method, such as ES, appears to be the most suitable choice.

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### **Niezawodnościowo zorientowana optymalizacja ram stalowych poddanych obciążeniom sejsmicznym za pomocą ewolucyjnej techniki obliczeniowej**

#### Streszczenie

Konstruowanie budowli o zwiększonej odporności na trzęsienia ziemi poprzez wykorzystanie rachunku prawdopodobieństwa i kryteriów eksploatacyjnych jest nowa

rozwijająca się dziedzina inżynierii konstrukcji. Nowa metodologia i procedury postępowania celują w udoskonalanie istniejących programów i pakietów obliczeniowych przewidujących zachowanie się danej budowli w zadanych warunkach. W pracy zaprezentowano sztywną i wydajną metodologię optymalizacji zorientowaną na niezawodność ram stalowych poddanych obciążeniom sejsmicznym. Optymalizację oparto na obliczeniach ewolucyjnych, natomiast analizę niezawodności zrealizowano metodą Monte-Carlo, w której do redukcji wymiarowości zagadnienia wykorzystano technikę LHS (*Latin Hypercube Sampling*). Prawdopodobieństwo uszkodzenia konstrukcji, w sensie przekroczenia granicznych przemieszczeń międzykondygnacyjnych, obliczono za pomocą wielomodalnej analizy widma odpowiedzi konstrukcji.

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