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BUCKLING OF I-CORE SANDWICH PANELS[1]

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Necessity of optimisation of ship hull structural mass calls for application of innovative materials and structural components. One option is based on using structural components with internal structure. The considered sandwich panels are composed of two plates stiffened by vertical ribs (I-core) or ribs of different shape (V-core). Such panels are applied as the ship hull structural components, replacing the conventional stiffened panels. They are subject to typical loadings acting in the ship hull; tension, compression and lateral loading. Analysis of stability of sandwich panels subject to compressive loading is presented in the paper. Stabilities of conventional and innovative ship panels were compared. Influence of the filling foam was also investigated.

Key words: I-core sandwich panels, buckling

1. Introduction

Innovative structural components such as sandwich panels have been recently applied in shipbuilding. The sandwich panels have proven to have many advantages over traditional plates; low weight, modular prefabrication, decrease of labour demand etc. The panels are used in production of walls, decks, bulkheads, staircases and deckhouses on the ships. One of the first examples of application, according to the Lloyd's Register publication (2000) was a vehicle deck section made of the Sandwich Plate System (SPS), assembled into the RoPax vessel. Presently after a year in service, the ship has not experienced

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any problems concerning the strength or degradation of the innovative structure. Since the sandwich panels are quite a new application in shipbuilding, the knowledge concerning the behaviour of this type of structure is still insufficient. They are even not referred to by the rules of classification societies.

Analysis of sandwich panels without core and with different cores has been a subject investigated and presented by many authors. The local buckling of sandwich panels made up of hybrid laminated faces and flexible core is investigated by Aiello and Ombres (1997). Stability analysis of sandwich panels with a flexible core is presented by Frostig (1998). The analysis uses high-order theory and determines the bifurcation loads and local and overall buckling modes of panels. Wrinkling analysis of sandwich panels containing holes is presented by Hadi and Matthews (2001). Razi *et al.* (1999) used cylindrical holes to model the sandwich panels with damages. They present an analytical method to determine the stress distribution in panels with arbitrarily located damage. Most sheet faces of sandwich panels are modelled using two-dimensional plate and cores cosisting of three-dimensional solid elements. The face plates were modelled with a nine-node isoparametric elements based on the Mindlin plate theory of bending and vibration analysis presented by Lee and Fan (1996). Philippe *et al.* (1999) developed a new model of sandwich structure referred to as a tri-particle model. The tri-particle solution was compared with the exact Pagano solution and the results were obtained by using the Mindlin-Reissner plate theory. The core material is considered to be isotropic for cellular cores or orthotropic for honeycomb. Review of the analytic solutions for bending and buckling of flat rectangular orthotropic plates is presented by Bao *et al.* (1997). The experimental research of double skin composite elements under lateral and axial loads was carried out by Oduyemi and Wright (1989), Wright *et al.* (1991).

Laser-welded panels, known as I-core panels, produced by Meyer-Werft shipyard in Papenburg were presented by Kozak (2002). He described the tests of sandwich panels developed under the European Union Project "SANDWICH". The purpose of experimental tests of I-core and V-core sandwich panels is to define the strength properties of such innovative structures. Naar *et al.* (2002) in their paper analysed the strength of various types of double bottom structures. Among other types compared are conventional ship and steel sandwich structures. Behaviour of fibre-reinforced plastic deck of bridge structures is described by Qiao *et al.* (2000). To simplify the analysis of the bridge deck, an equivalent orthotropic plate was used instead of an exact model of the actual deck geometry.

In the present paper stability of the I-core panel under compressive loading using the finite elements method is analyzed. The considered I-core plate is a laser-welded steel sandwich panel produced by the Meyer-Werft shipyard. The panel is composed of two thin face plates joined by ribs.

2. Theoretical background

A mathematical model is considered for the structure composed of the plate and the filling foam – Fig. 1.

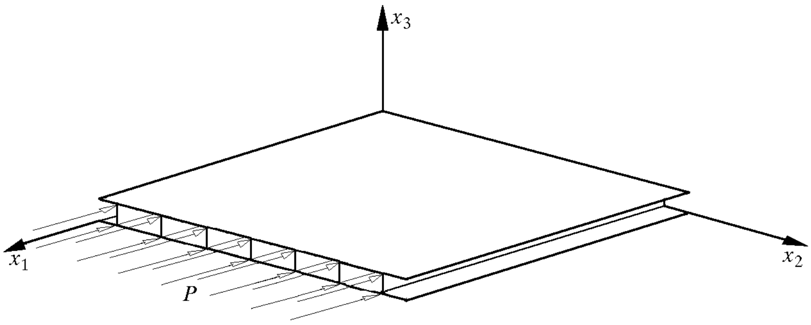


Fig. 1. Model of compressed I-core panel

The following assumptions are made: (i) materials of the plate and foam are linear elastic and (ii) strain components are small. Assuming the compressive stresses to act in the direction of X_1 axis of the global coordinate system, the principle of virtual work is

$$\int\limits_{V_p} \sigma_{ij}^p \delta\varepsilon_{ij}\ dV + \int\limits_{V_f} \sigma_{ij}^f \delta\varepsilon_{ij}\ dV = P_p \int\limits_{l} \delta\varDelta\ dx + P_s \int\limits_{l} \delta\varDelta\ dx \qquad (2.1)$$

where $[\sigma_{ij}^p]$ is the stress tensor for the plate, $[\sigma_{ij}^f]$ – stress tensor for the foam, $[\delta\varepsilon_{ij}]$ – variation of the strain tensor, P_p and P_f are compressive forces acting in the plate and foam, respectively, $\delta\varDelta\, dx$ – variation of the change of elementary length. Expressions on the left-hand side of Eq. (2.1) are virtual works of stresses in the plate and foam, respectively, and the expressions on the right-hand side are virtual works of compressive loading.

Ordering the components of the stress and strain tensors in the form typical for the finite element method, Eq. (2.1) can be rewritten as

$$\int\limits_{V_p} \sigma_i^p \delta\varepsilon_i \, dV + \int\limits_{V_f} \sigma_i^f \delta\varepsilon_i \, dV = P_p \int\limits_l \delta\Delta \, dx + P_f \int\limits_l \delta\Delta \, dx \tag{2.2}$$

The compressive forces are given by

$$P_p = \int\limits_{A_p} \sigma_1^p \, dA \qquad\qquad P_f = \int\limits_{A_f} \sigma_1^f \, dA \tag{2.3}$$

Material properties are defined by the constitutive matrices $[C_{ij}^p]$, $[C_{ij}^f]$ used for the definition of stress-strain relationships for the plate and foam

$$\sigma_i^p = C_{ij}^p \varepsilon_j \qquad\qquad \sigma_i^f = C_{ij}^f \varepsilon_j \tag{2.4}$$

It is usually assumed in the analysis of the linearised buckling that the compressive stresses are constant over the cross-sectional area of a homogenous material. Since in the present approach we have a combination of two different materials, the stresses are assumed to be proportional to the Young moduli of materials

$$\sigma_1^p = E_p \varepsilon_1 \qquad\qquad \sigma_1^f = E_f \varepsilon_1 \tag{2.5}$$

with the strain being the same for both materials. The change of the elementary length is expressed in the linearised analysis as a function of derivatives of the out-of-plane displacements

$$\varDelta dx = \frac{1}{2}(U_{2,X_1}^2 + U_{3,X_1}^2) \tag{2.6}$$

and the variation of the elementary length change is

$$\delta\varDelta dx = (U_{2,X_1}\delta U_{2,X_1} + U_{3,X_1}\delta U_{3,X_1}) \tag{2.7}$$

where index $(\cdot)_{,X_1}$ denotes differentiation with respect to X_1. Employing Eqs (2.3)-(2.7), Eq. (2.2) becomes

$$\begin{aligned} &\int\limits_{V_p} C_{ij}\varepsilon_j \delta\varepsilon_i \, dV + \int\limits_{V_f} C_{ij}^f \varepsilon_j \delta\varepsilon_i \, dV = \\ &= \int\limits_l \Big[\int\limits_{A_p} E^p \varepsilon_1 (U_{2,X_1}\delta U_{2,X_1} + U_{3,X_1}\delta U_{3,X_1}) \, dA \Big] \, dx + \\ &+ \int\limits_l \Big[\int\limits_{A_f} E^f \varepsilon_1 (U_{2,X_1}\delta U_{2,X_1} + U_{3,X_1}\delta U_{3,X_1}) \, dA \Big] \, dx \end{aligned} \tag{2.8}$$

Displacements U_2 and U_3 given in the global local coordinate system are related to the displacements $\{u_j\}$ in the local coordinate system via the transformation matrix $[T_{ij}]$

$$U_2 = T_{j2}u_j \qquad U_3 = T_{j3}u_j \tag{2.9}$$

Consequently,

$$\begin{aligned} U_{2,X_1} &= T_{j2}u_{j,X_1} & \delta U_{2,X_1} &= T_{k2}\delta u_{k,X_1} \\ U_{3,X_1} &= T_{j3}u_{j,X_1} & \delta U_{3,X_1} &= T_{k3}\delta u_{k,X_1} \end{aligned} \tag{2.10}$$

Employing the chain rule for differentiation

$$u_{j,X_1} = \frac{\partial u_j}{\partial X_1} = \frac{\partial u_j}{\partial x_m}\frac{\partial x_m}{\partial X_1} = T_{m1}u_{j,x_m} \tag{2.11}$$

Eq. (2.8) becomes

$$\begin{aligned} &\int\limits_{V_p} C^p_{ij}\varepsilon_j\delta\varepsilon_i\;dV + \int\limits_{V_s} C^s_{ij}\varepsilon_j\delta\varepsilon_i\;dV = \\ &= \varepsilon_1\int\limits_{V_p} E^p(T_{j2}T_{m1}u_{j,x_m}T_{k2}T_{n1}\delta u_{k,x_n} + T_{j3}T_{m1}u_{j,x_m}T_{k3}T_{n1}\delta u_{k,x_n})dV + \\ &+\varepsilon_1\int\limits_{V_s} E^s(T_{j2}T_{m1}u_{j,x_m}T_{k2}T_{n1}\delta u_{k,x_n} + T_{j3}T_{m1}u_{j,x_m}T_{k3}T_{n1}\delta u_{k,x_n})\;dV \end{aligned} \tag{2.12}$$

Two types of finite elements will be employed in the finite element modelling: plate elements for the plating and solid elements modelling the foam. Deformations of the plating are consistent with Kirchoff-Love plate theory, while three-dimensional stress and strain is assumed for the foam. Displacement field for the plate is approximated using shape functions of the four-noded rectangular element

$$u_i = N^p_{ij}d_j \tag{2.13}$$

and for the foam eight-noded hexahedral element

$$u_i = N^s_{ij}d_j \tag{2.14}$$

where $\{d_j\}$ is the nodal displacement vector and $[N^p_{ij}]$, $[N^s_{ij}]$ are matrices of the shape functions for plate and solid elements, respectively. Strains for plates and solids are obtained using the matrices of derivatives of the shape functions

$$\varepsilon_j = B^p_{jq}d_q \qquad \varepsilon_j = B^s_{jq}d_q \tag{2.15}$$

Employing the finite element formulations, Eqs. (2.13)-(2.15), Eq. (2.12) is rewritten

$$\begin{aligned}\int\limits_{V_p} C^p_{ij}B^p_{jp}d_pB^p_{iq}\delta d_q\;dV + \int\limits_{V_p} C^s_{ij}B^s_{jp}d_pB^s_{iq}\delta d_q\;dV =\\ = \varepsilon_1\int\limits_{V_p} E^p(T_{j2}T_{m1}N^p_{jp,x_m}d_pT_{k2}T_{n1}N^p_{kq,x_n}\delta d_q +\\ +T_{j3}T_{m1}N^p_{jp,x_m}d_pT_{k3}T_{n1}N^p_{kq,x_n}\delta d_q)\;dV +\\ +\varepsilon_1\int\limits_{V_s} E^s(T_{j2}T_{m1}N^s_{jp,x_m}d_pT_{k2}T_{n1}N^s_{kq,x_n}\delta d_q +\\ +T_{j3}T_{m1}N^s_{jp,x_m}d_pT_{k3}T_{n1}N^s_{kq,x_n}\delta d_q)\;dV\end{aligned} \tag{2.16}$$

Since the above equation is true for arbitrary variation of displacements, it follows

$$\begin{aligned}\Big[\int\limits_{V_p} C^p_{ij}B^p_{jp}B^p_{iq}\;dV + \int\limits_{V_s} C^s_{ij}B^p_{jp}B^p_{iq}\;dV\Big]\;d_p =\\ = \varepsilon_1\Big[E^p(T_{j2}T_{m1}T_{k2}T_{n1} + T_{j3}T_{m1}T_{k3}T_{n1})\int\limits_{V_p} N^p_{jp,x_m}N^p_{kq,x_n}\;dV +\\ +E^s(T_{j2}T_{m1}T_{k2}T_{n1} + T_{j3}T_{m1}T_{k3}T_{n1})\int\limits_{V_s} N^s_{jp,x_m}N^s_{kq,x_n}\;dV\Big]\;d_p\end{aligned} \tag{2.17}$$

which can be written in the form

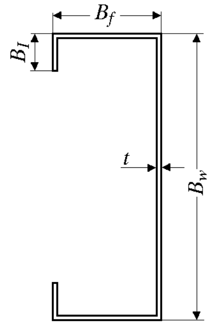
$$[K^p_{qp} + K^s_{qp} - (K^{Gp}_{qp} + K^{Gs}_{qp})]d_p = 0 \tag{2.18}$$

where $[K^p_{qp}]$, $[K^s_{qp}]$ are stiffness matrices for the plate and solid, respectively, and K^{Gp}_{qp}, K^{Gs}_{qp} are geometrical matrices. Equation (2.18) is a typical formulation of an eigenvalue problem for linearised buckling, with strain ε_1 being the searched value instead of the stress as in the standard formulation. The problem in the present approach is solved by the subspace iteration method, originally developed by Bathe (1996).

3. Numerical examples

3.1. Reference example

Cold-formed steel lipped channels investigated numerically by Dubina and Goina (1997) were taken as reference cases. Three beams with sections presented in Fig. 2 subject to axial compression were analysed using ANSYS. The beams were modelled with four-noded shell elements. Pinned support was assumed at the beam ends. In the present analysis, the pinned support was realised by the diaphragms situated at the ends of the beams having thickness 20 mm, considerably larger than the thickness of the profiles which is less than 1.5 mm.



Model	B_l [mm]	B_f [mm]	B_w [mm]	t [mm]	L [mm]
L36P0280	12.6	37.1	97.2	1.48	279.9
L36P0815	12.7	37.0	97.4	1.48	814.6
L36P1315	12.4	36.9	97.1	1.47	1316.4

Fig. 2. Section of investigated profiles

Buckling modes obtained for the present analysis are presented in Fig. 3.

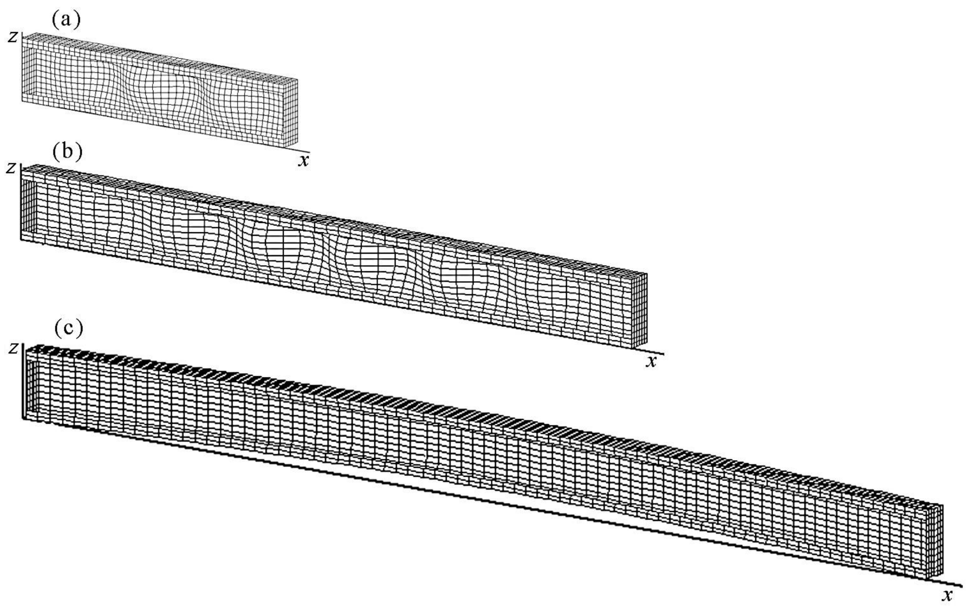


Fig. 3. Buckling mode of L36P0280 (a), L36P0815 (b) and L36P1315 (c)

The reference values and results of the present analysis are given in Table 1. Comparing the results we note that the reference cases were analysed in a geometrically non-linear range. The results thus refer to the conjugation of the local and global buckling modes. In the present analysis it is the local buckling mode which was found to be the first buckling mode for models L36P0280 and L36P0815 while the global buckling mode is the first one for model L36P1315.

Table 1. Comparison of results

Model	Critical force [kN]	
	Reference example	Presented formulation
L36P0280	75	66.9
L36P0815	68	63.6
L36P1315	40	51.1

3.2. Comparison of conventional and I-core ship panels

An essential idea of application of the innovative I-core panels in structural ship design is to replace the conventional structures composed of plating, stiffeners and girders. Due to increased stiffness and strength of I-core panels under lateral loading, the stiffeners can be eliminated from the structural design to simplify assembling of the structures – Fig. 4. A method of selection of the scantlings of the I-core panel equivalent to the conventional ship panel was described by Pyszko (2002), who considered requirements concerning minimal thickness, section modulus and stability according to the Rules of Polish Register of Shipping.

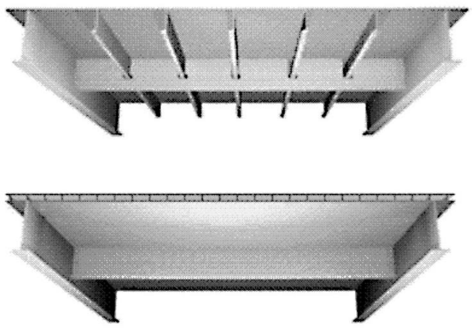


Fig. 4. Conventional ship panel and I-core panel

Comparative analysis was performed for the two panels: conventional panel of size 2400×2400 mm stiffened with three angles $100 \times 50 \times 8$ and equivalent I-core panel of the same size and scantlings facing thickness 2 mm, rib thickness 3 mm, distance between facings 50 mm. Structural design with application of the I-core panel is more advantageous as its mass is 235.9 kg what should be compared with the mass of the conventional panel – 451.2 kg. Critical stresses are also in favour of the innovative panel – 266.7 MPa – against 207.4 MPa for the conventional panel. The buckling modes of both panels is presented in Fig. 5 and Fig. 6

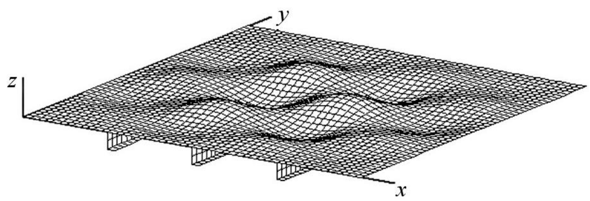


Fig. 5. Buckling mode of conventional ship panel

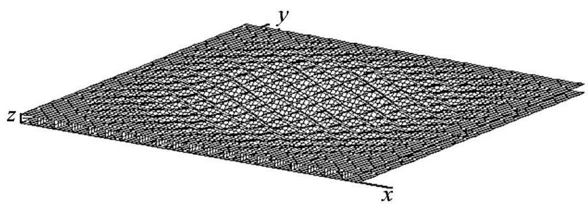


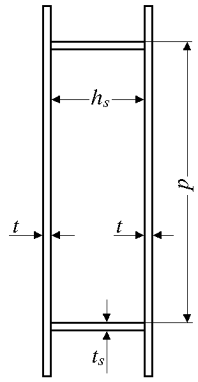
Fig. 6. Buckling mode of equivalent I-core panel

3.3. Buckling of I-core and V-core panels

The presented examples are I-core and V-core panels which were taken from the catalogue of the panel series. The I-core panels (Fig. 7) of size 600×600 mm were analysed assuming that the edges were clamped.

The panels were compressed in the direction in accordance with the position of the ribs. Buckling modes of the analysed I-core panels are shown in Fig. 8.

Another innovative structural design is a V-core panel, where the ribs are not situated vertically but at a certain angle with respect to the facings. The models of such a panel taken for investigation are presented in Fig. 9. A typical local buckling mode of the analysed V-core panels is shown in Fig. 10.



Type	t [mm]	t_s [mm]	h_s [mm]	p [mm]	σ_{cr} [MPa]
2/3/40	2	3	40	120	304
3/4/40	3	4	40	120	655
4/5/50	4	5	50	120	1122
5/6/50	5	6	50	120	1722
6/7/60	6	7	60	120	2429
7/8/70	7	8	70	120	3227
8/9/80	8	9	80	120	4112
9/9/90	9	9	90	120	4815
10/10/100	10	10	100	120	5696

Fig. 7. Scantlings of analysed I-core panels

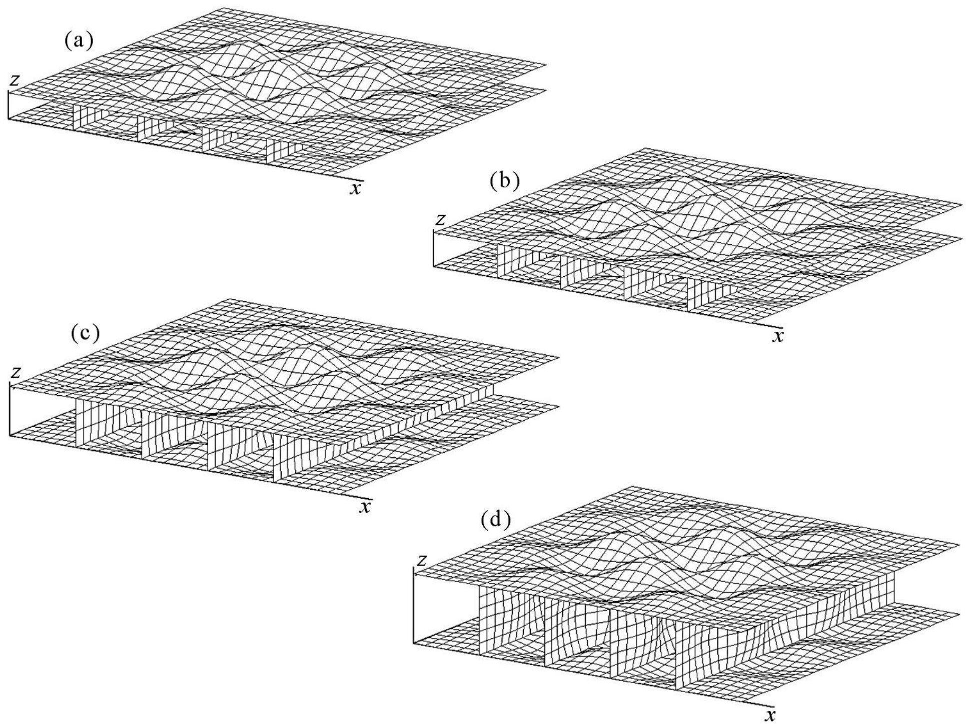
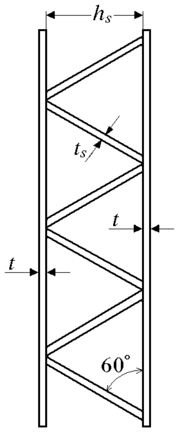


Fig. 8. Buckling mode of panel 2/3/40 (a), 5/6/50 (b), 7/8/70 (c), 10/10/100 (d)



t [mm]	t_s [mm]	h_s [mm]	σ_{cr} [MPa]
0.8	1	80	73
0.8	1	100	64
2	3	80	492
2	3	100	315
4	5	80	1800
4	5	100	1154

Fig. 9. Dimensions of analysed V-core panels

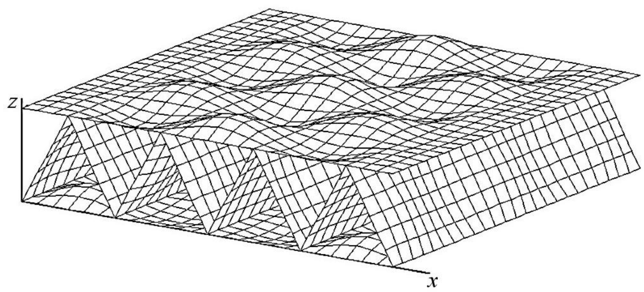


Fig. 10. Typical local buckling mode of V-core panel

3.4. Buckling of I-core panels filled with foam

I-core panels are also offered in a variant in which the structure is filled with foam. Models of such structures were built using plate elements for modelling facings and ribs and eight-noded solid elements for modelling the foam. Two types of isotropic foams were applied: foam with Young modulus 20 MPa denoted *foam* 1 and 100 MPa denoted *foam* 2. Poisson's ratio in both cases is equal to 0.3. Buckling mode of I-core panel filled with foam is shown in Fig. 11. The comparison of critical stresses of I-core panels with and without foam is given in Table 2.

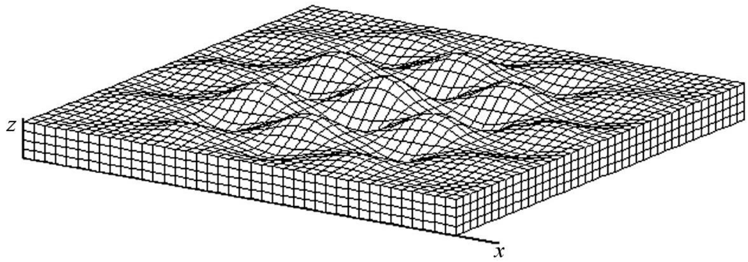


Fig. 11. Buckling mode of I-core panel with foam

Table 2. Critical stresses of analysed I-core panels with foam

Dimensions of the I-core panel				Panel without foam	Panel with foam 1	Panel with foam 2
t [mm]	t_s [mm]	h_s [mm]	p [mm]	σ_{cr} [MPa]	σ_{cr} [MPa]	σ_{cr} [MPa]
2	3	40	120	304	433	781
3	4	40	120	655	796	1063
4	5	50	120	1122	1305	1558
7	8	70	120	3227	3365	3668

Strengthening effect of the foam can be observed. Even the application of the foam with small elastic modulus significantly increases the buckling stress. The stabilizing effect of the foam with larger elastic modulus is large and increases the buckling stress for the panels with the thinnest elements almost twice.

4. Conclusions

A method of investigation of linearised buckling for structures with ribs modelled by plate elements was presented. The method was implemented in the finite element code. Examples of the analysis of buckling of the I-core and V-core panels subject to compressive loading with a possibility to detect both the overall and local buckling modes were given. Advantageous aspects of the application of the I-core panels as compared to the conventional structural design in terms of their stability were indicated. The stabilizing effect of the foam filling the I-core panels was also proven.

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Wyboczenie paneli typu I-core

Streszczenie

Optymalizacja masy kadłuba okrętowego wymaga zastosowania innowacyjnych materiałów i elementów konstrukcyjnych. Możliwe jest wykorzystanie elementów konstrukcyjnych ze strukturą wewnętrzną. Analizowane w pracy panele składają się z dwóch płyt usztywnionych żebrami pionowymi (I-core) lub żebrami innego kształtu (V-core). Panele takie stosowane są jako elementy konstrukcyjne kadłuba okrętowego zastępując konwencjonalne usztywnione panele. Poddane są one obciążeniu występującemu w kadłubie statku: rozciąganiu, ściskaniu i obciążeniu poprzecznemu. W pracy przedstawiono analizę stateczności sandwiczowych paneli typu I-core i V-core poddanych ściskaniu. Porównano stateczność konwencjonalnych i innowacyjnych paneli okrętowych. Zbadano także wpływ piany wypełniającej panele typu I-core.

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