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PLANE CONTACT PROBLEMS WITH PARTIAL SLIP FOR ROUGH HALF-SPACE

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Plane contact problems with the partial slip in the contact area are considered in the paper. To make the problems more realistic, the deformation of roughness of the contacting boundary is involved. The Shtayerman model of roughness is generalized on the case of tangential problems. The problems are treated by the boundary integral method. Examples of the contact of a flat rigid punch and a rigid cylinder with an elastic half-space involving boundary imperfections are studied. The effects of roughness parameters on the distribution of normal and shearing tractions as well as on the stick-slip transition are investigated.

Key words: Cattaneo-Mindlin problem; boundary roughness; integral equation

1. Introduction

When two elastic bodies are normally pressed against each other and, subsequently, shifted by a monotonically increasing shearing force in the tangential direction, slip zones develop in the mutual contact area. This kind of a contact problem is referred to as the Cattaneo-Mindlin problem, see Cattaneo (1938), Mindlin (1949). The practical importance of this problem is very great, its results are applied to the investigation of the fretting in the contact zone. The bibliography on the Cattaneo-Mindlin problem is wide. It can be found in well-known monographs on contact mechanics, Johnson (1985), Hills *et al.* (1993). Some generalizations of this problem were done by Jäger (1997), Ciavarella (1998).

Known solutions to partial slip tangential contact problems were obtained on the assumption that the contacting surfaces are ideally smooth. But real boundaries of real bodies are not perfectly smooth, they include roughness which has an influence on the contact.

There are many approaches to the modelling of boundary roughness. Our approach, which is presented in Section 2, is based on the Shtayerman (1949) assumption, that the boundary roughness causes additional deformation under the punch. We extend this assumption on that relevant to tangential contact problems. On the base of the model proposed, the normal and tangential contact problems are considered in the next Sections. We will study uncoupled problems postulating that the normal traction has no effect on the tangential displacements and the shearing traction on the normal displacements. The problems are assumed to be plane and steady-state. One contacting body is considered as a rigid punch while for the second body the Hertz assumptions are applied, and it is considered as an elastic half-space. Two main contact geometries are studied: the flat punch and the cylinder approximated by a parabola. To solve the partial slip contact problem, Cattaneo's superposition for the shearing traction is used and integral equations for a corrective traction are derived. Contrary to the case of ideal contact boundaries, the solution to these integral equations can not be obtained analytically, and a numerical technique has to be applied.

2. Model of boundary roughness

The boundary roughness acts like a thin compliant layer on the surface of a body, Johnson (1985). As a result, additional deformation takes place under the contact of rough bodies. This assumption was first used by Shtayerman (1949), who proposed a model of boundary imperfections postulating that the normal displacement of the rough boundary subjected to the normal load $p(x)$ consists of two parts

$$v(x) = v^e(x) + v^r(x) \tag{2.1}$$

where $v^e(x)$ is the displacement due to the elastic deformation of the body and $v^r(x)$ are additional local displacements due to the roughness deformation.

The first part of the displacements can be found as a solution to the elasticity equations. To describe the additional displacement, Shtayerman used the relation

$$v^r(x) = \alpha p(x) \tag{2.2}$$

where the constant α is called the roughness parameter. The Shtayerman model of the boundary roughness states an analogy with the well-known Winkler (1867) assumption, and can be successfully applied to the solution to normal contact problems for rough bodies. But this model is not useable in the tangential contact problems because it neglects the shearing traction and displacements.

In the solution to the tangential contact problems, we will use a similar idea, presenting the normal displacements in form (2.1) and tangential ones as

$$u(x) = u^e(x) + u^r(x) \tag{2.3}$$

where $u^e(x)$ is a displacement due to elastic deformation of the body and

$$u^r(x) = \beta q(x) \tag{2.4}$$

states the tangential displacement due to the deformation of the boundary roughness subjected to the action of the shearing traction $q(x)$. The constant β will be called the roughness parameter. Thus the proposed model is characterized by two roughness parameters α and β. Equations (2.2), (2.4) present the simplest model of the boundary roughness. Another model of the roughness known as the Greenwood-Williamson model (see Greenwood and Williamson, 1966) has a broader application range, and is widely used for the investigation of the normal contact of rough bodies. However, it is not easy to generalize the Greenwood-Williamson model, i.e. extend it onto the case of tangential problems.

Analysing formulae (2.2) and (2.4) it is easy to observe an analogy between the proposed model of the boundary roughness and the simplified model of the elastic foundation used by Kalker (1973) for the investigation of rolling contact. The proposed model treats the boundary roughness as a set of independent springs or as a "wire brush". The problem of determination of the roughness parameters is discussed in Appendix.

Considering the body as an elastic half-space and taking the solutions $u^e(x)$ and $v^e(x)$ in the well-known forms, Johnson (1985), the total displacements of the rough boundary of the half- space can be presented as

$$\begin{aligned} v(x) &= \alpha p(x) + \frac{2(1-\nu^2)}{\pi E} \int\limits_{-a}^{a} p(\xi) \ln|\xi - x|\, d\xi + \\ &+ \frac{(1-2\nu)(1+\nu)}{2E} \int\limits_{-a}^{a} q(\xi) \operatorname{sgn}(x-\xi)\, d\xi \\ u(x) &= \beta q(x) + \frac{2(1-\nu^2)}{\pi E} \int\limits_{-a}^{a} q(\xi) \ln|\xi - x|\, d\xi - \\ &- \frac{(1-2\nu)(1+\nu)}{2E} \int\limits_{-a}^{a} p(\xi) \operatorname{sgn}(x-\xi)\, d\xi \end{aligned} \tag{2.5}$$

where ν, E are Poisson's ratio and Young's modulus of the half-space, respectively, a is the half-width of the contact area.

As was stated in Introduction, in the further analysis we will consider an uncoupled problem in which the tangential traction has no effect on the normal displacements and the normal pressure on the tangential displacements. This situation takes place if $\nu = 0.5$ (assumed here) or when the mechanical properties of contacting bodies are identical. It is important to notice that the effect of the coupling between the tangential and normal problems is not great also in the general case of material properties, see Johnson (1985), and thus can be neglected.

3. Normal contact problems

Let us assume that the rigid punch is pressed symmetrically by the normal load P against the rough boundary of the elastic half-space. The punch geometry is described by the function $h(x)$. Satisfying the boundary condition by making use of expression $(2.5)_1$

$$v(x) = \delta_y - h(x) \qquad\qquad x \in (-a, a) \tag{3.1}$$

where $\delta_y = \text{const}$ describes the normal approach of the contacting bodies, we arrive at the following integral equation

$$\alpha p(x) + \frac{2(1-\nu^2)}{\pi E} \int\limits_{-a}^{a} p(\xi) \ln|\xi - x|\, d\xi = \delta_y - h(x) \qquad\qquad x \in (-a, a) \tag{3.2}$$

This equation with the equilibrium condition

$$\int_{-a}^{a} p(x)\,dx = P \tag{3.3}$$

determines a system of integral equations of the normal contact problem. We will consider two types of the punch geometry.

3.1. Flat punch

In this case. the function $h(x) = 0$ and after introducing dimensionless variables, i.e. contact pressure and parameters

$$\begin{aligned} s &= \frac{x}{a} & \eta &= \frac{\xi}{a} & p^*(s) &= \frac{ap(x)}{P} \\ \alpha^* &= \frac{\alpha E}{a(1-\nu^2)} & \delta_y^* &= \frac{\delta_y E}{1-\nu^2} \end{aligned} \tag{3.4}$$

the system of integral equations (3.2), (3.3) can be transformed to the form

$$\begin{aligned} &\alpha^* p^*(s) + \frac{2}{\pi}\int_{-1}^{1} p^*(\eta)\ln|\eta - s|\,d\eta = \delta_y^* \qquad s \in (-1,1) \\ &\int_{-1}^{1} p^*(s)\,ds = 1 \end{aligned} \tag{3.5}$$

Equations (3.5) are then solved numerically for different values of the dimensionless roughness parameter α^*. The effects of boundary roughness on the contact pressure distribution is presented in Fig. 1a by dotted curves. For $\alpha^* = 0$, we obtain the well-known solution for the smooth half-space, Johnson (1985)

$$p^*(s) = \frac{1}{\pi\sqrt{1-s^2}} \tag{3.6}$$

which is unbounded for $s \to \pm 1$. But if $\alpha^* > 0$, the contact pressure no longer tends to the infinity at the punch edges. This result is due to the boundary roughness and was first obtained by Shtayerman (1949).

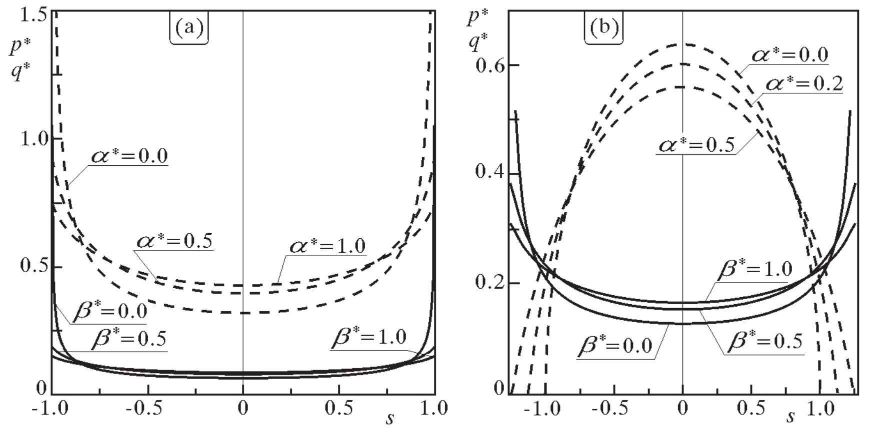

Fig. 1.

3.2. Cylindrical punch

Assuming the punch geometry in the form

$$h(x) = \frac{x^2}{2R} \tag{3.7}$$

and introducing dimensionless parameters

$$\begin{aligned} s &= \frac{x}{a_H} \qquad & \eta &= \frac{\xi}{a_H} \qquad & p^*(s) &= \frac{a_H p(x)}{P} \\ a^* &= \frac{a}{a_H} \qquad & \alpha^* &= \frac{\alpha E}{a_H(1-\nu^2)} \qquad & \delta_y^* &= \frac{\delta_y E}{1-\nu^2} \end{aligned} \tag{3.8}$$

we obtain the dimensionless form of integral equations (3.2), (3.3)

$$\begin{aligned} &\alpha^* p^*(s) + \frac{2}{\pi}\int\limits_{-a^*}^{a^*} p^*(\eta)\ln|\eta - s|\,d\eta = \delta_y^* - \frac{2}{\pi}\frac{P_H}{P}s^2 \qquad s \in (-a^*, a^*) \\ &\int\limits_{-a^*}^{a^*} p^*(s)\,ds = 1 \end{aligned} \tag{3.9}$$

Here, a_H and P_H are the contact size and normal load in the Hertz problem, respectively, see Johnson (1985)

$$a_H^2 = \frac{4(1-\nu^2)RP_H}{\pi E} \tag{3.10}$$

In the numerical analysis, the normal load is equal to that in the Hertz problem, i.e. $P_H/P = 1$, and the unknown contact size a^* is determined iteratively from the physical condition

$$p(\pm a^*) = 0 \tag{3.11}$$

The distribution of the contact pressure in the present case is shown in Fig. 1b by dotted curves for three values of the roughness parameter α^*. For $\alpha^* = 0$ the classical solution, Johnson (1985)

$$p^*(s) = \frac{2}{\pi}\sqrt{1-s^2} \tag{3.12}$$

is obtained. We observe that the contact area is bigger, and the maximum value of the contact pressure is lower in the presence of boundary imperfections.

4. Complete stick contact problems

Let us now assume that the bodies are in contact as was stated in Section 3 and, subsequently, the tangential load Q is applied. First, we will consider fully adhesive contact described by the condition

$$u(x) = \delta_x \qquad\qquad x \in (-a, a) \tag{4.1}$$

where $\delta_x = \text{const}$ is the tangential component of the rigid motion of contacting bodies.

Satisfying this condition, using formula $(2.5)_2$, we obtain an integral equation for the shearing traction

$$\beta q(x) + \frac{2(1-\nu^2)}{\pi E}\int\limits_{-a}^{a} q(\xi)\ln|\xi - x|\, d\xi = \delta_x \qquad\qquad x \in (-a, a) \tag{4.2}$$

which has to be considered together with the equilibrium condition

$$\int\limits_{-a}^{a} q(x)\, dx = Q \tag{4.3}$$

Integral equations (4.2), (4.3) have been solved numerically. The effect of the dimensionless roughness parameter $\beta^* = \beta E/[a_H(1-\nu^2)]$ on the distribution of the dimensionless tangential traction $q^*(s) = a_H q(x)/P$ is presented in

Fig. 1 by solid curves in the cases of the flat punch and the parabolic cylinder, respectively. These results were obtained for $Q^* = Q/P = 0.2$, and the contact area in the case of the cylindrical punch was equal to that in the normal contact problem for $\alpha^* = 0.5$.

The presented results need more comments. Integral equations (4.2), (4.3) have a structure like equations (3.2), (3.3) of the normal contact of the rigid flat punch. Thus, the tangential traction $q(x)$ is unbounded at the edge of the contact area if $\beta^* = 0$, and is limited when $\beta^* > 0$. Note that this behaviour is independent of the punch geometry.

Considering the ratio $q^*(a^*)/p^*(a^*)$ in the case of the rigid cylinder (Fig. 1b) we can state that this value is always equal to infinity. It means that in order to satisfy the complete stick condition over the whole contact area, we must apply the infinite friction force at the contact zone edges, which is physically impossible. So, some slip under the punch near the points $s = \pm a^*$ is inevitable in the case of the parabolic geometry, and the partial slip contact problem has to be solved. Identical behaviour takes place for this geometry in the complete stick contact problem for the ideally smooth boundary, Johnson (1985).

A significantly different situation is observed in the case of the flat punch. In the classical case, when the boundary is ideal $(\alpha = \beta = 0)$, the shearing traction is

$$q^*(s) = Q^* p^*(s) \qquad\qquad s \in (-1, 1) \tag{4.4}$$

where the normal pressure has the form of (3.7). It means that the stick occurs everywhere when $Q^* \leqslant f$ (f is the friction coefficient), and if $Q^* > f$, the punch slides over the half-space. Thus, no partial slip solution exists on the classical assumptions. Let us note here, that Ciavarella *et al.* (1998) obtained the partial slip solution for the flat punch assuming that the punch edges were slightly rounded.

To study possible slip near the flat punch edges, let us examine the ratio $q^*(1)/p^*(1)$ for different values of the roughness parameters. Figure 2 presents this ratio versus the dimensionless load Q^* for some values of the parameter α^* (straight solid lines; $\beta^* = 0.5$ is fixed) and for some values of β^* (straight dotted lines; $\alpha^* = 0.5$ is fixed). Drawing a horizontal line $f = \text{const}$ (for example $f = 0.4$), we can conclude that no partial slip solution exists if $\alpha^* < \beta^* = 0.5$. If $\alpha^* > \beta^* = 0.5$ there are regions in which the stick conditions

$$q^*(1) < f p^*(1) \qquad\qquad 0 < Q^* < f \tag{4.5}$$

are not satisfied, see Fig. 2. Thus, some slip must occur for these values of input parameters. For example, the bold line on the horizontal axis in Fig. 2

indicates the range of the load Q^* in which the slip at the punch edges is inevitable. This range is $Q^* \in (Q_0, f = 0.4)$, where the value of Q_0 can be read from Fig. 2 as $Q_0 \approx 0.32$ for fixed $\alpha^* = 1.0$, $\beta^* = 0.5$. For another roughness $\alpha^* = 1.0$, $\beta^* = 0.0$ the value is smaller $Q_0 \approx 0.08$.

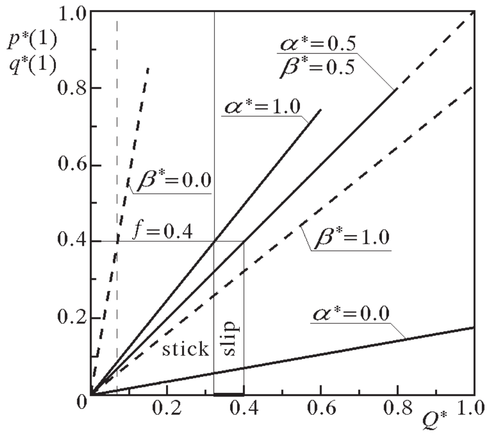

Fig. 2.

The calculation performed for other sets α^* and β^* confirms the general property of the contact of the flat punch: if $\alpha^* < \beta^*$ there is no partial slip solution, and this kind of solution is possible when $\alpha^* > \beta^*$. This property can also be proved using an analogy between the normal and tangential problem for identical elastic half-spaces discovered by Jäger (1997) and Ciavarella (1998).

Thus, if the tangential load Q^* is monotonically increasing from zero to the value Q_0, the contact is fully adhesive; if $Q_0 < Q^* < f$, some slip takes place near the punch edges, and the partial slip contact problem has to be considered; and, finally, if $Q^* > f$, the punch slides over the half-space.

5. Partial slip contact problems

The previous section shows that, even if $Q < fP$, the regions of slip near the contact area edges exist for all values of input parameters in the case of parabolic geometry and for some values of the roughness parameters in the case of the flat punch. From the symmetry of the problem, the stick zone can be defined as $(-c, c)$, where the size $c < a$ must be found.

The boundary conditions in the stick zone are, Johnson (1985)

$$\begin{aligned} &u(x) = \delta_x && |x| \leqslant c \\ &|q(x)| < f|p(x)| && |x| \leqslant c \end{aligned} \tag{5.1}$$

In the slip zones, the normal and shearing tractions are connected by the relationship

$$|q(x)| = f|p(x)| \qquad c < |x| \leqslant a \tag{5.2}$$

In addition, the direction of $q(x)$ in the slip zones must be opposite to the direction of micro- sliding, i.e.

$$\operatorname{sgn} q(x) = -\operatorname{sgn} s_x(x) \qquad c < |x| \leqslant a \tag{5.3}$$

where

$$s_x(x) = u(x) - \delta_x \tag{5.4}$$

stands for the value of relative tangential displacements.

As was stated above, the problems are considered to be uncoupled. Thus, the normal pressure $p(x)$ is already known from Section 3. To seek the shearing traction in the partial slip contact problems, we will use the idea of superposition presented by Cattaneo (1938)

$$q(x) = \begin{cases} fp(x) & \text{for} \quad c < |x| \leqslant a \\ fp(x) + q_0(x) & \text{for} \quad |x| \leqslant c \end{cases} \tag{5.5}$$

where $q_0(x)$ is an unknown corrective shearing traction defined in the stick zone.

Substituting this presentation into $(2.5)_2$ and taking into account integral equation (3.2), after some transformations, we can satisfy boundary condition $(5.1)_1$, which leads to the integral equation written in the stick zone for the unknown $q_0(x)$ $(|x| \leqslant c)$

$$\beta q_0(x) + \frac{2(1-\nu^2)}{\pi E} \int\limits_{-c}^{c} q_0(\xi) \ln|\xi - x| \, d\xi = \delta_x - f[\delta_y - h(x) + (\beta - \alpha)p(x)] \tag{5.6}$$

Equilibrium condition (4.3) with the help of expression (5.5) reads

$$\int\limits_{-c}^{c} q_0(x) \, dx = Q - fP \tag{5.7}$$

The size c of the stick zone has to be found from the condition

$$q_0(\pm c) = 0 \tag{5.8}$$

which provides continuous distribution of the shearing tractions under the punch.

The system of equations (5.6)-(5.8) can be transformed to a dimensionless form

$$\begin{aligned}
&\beta^* q_0^*(s) + \frac{2}{\pi}\int\limits_{-c^*}^{c^*} q_0^*(\eta)\ln|\eta - s|\,d\eta = \delta_x^* - f[\delta_y^* - h^*(s) + (\beta^* - \alpha^*)p^*(s)] \\
&\hspace{20em} |s| \leqslant c^* \\
&\int\limits_{-c^*}^{c^*} q_0^*(s)\,ds = Q^* - f \\
&q_0^*(\pm c^*) = 0
\end{aligned} \tag{5.9}$$

where $q_0^*(s) = a_H q_0(x)/P$, $c^* = c/a_H$ (a_H is defined by (3.10) in the case of the parabolic cylinder or $a_H = a =$ half-width of the flat punch). Other dimensionless quantities have been defined above.

Equations (5.9) have been solved numerically. The input parameters were: f – friction coefficient, Q^* – dimensionless tangential load, and α^*, β^* – dimensionless roughness parameters.

Let us first discuss the results for the contact of a rigid cylinder with the half-space. Figure 3a presents the effects of the parameter α^* on the total shearing traction $q^*(s)$ (solid curves) and on the corrective traction $q_0^*(s)$ (dotted curves) for $\beta^* = 0.0$, $f = 0.2$ and $Q^* = 0.1$. Typical distributions of the shearing traction are shown in Fig. 3b,c for some values of the roughness parameter β^* ($\alpha^* = 0.2$, $f = 0.2$ and $Q^* = 0.1$) and the tangential load Q^* ($\alpha^* = 0.2$, $\beta^* = 0.2$ and $f = 0.2$ and $Q^* = 0.1$). Generally speaking, the boundary imperfections cause a decrease in the shear traction. The effect of the roughness on the stick zone size is also important.

It is easy to check that the distributions of shearing traction presented in Fig. 3 satisfy boundary condition $(5.1)_2$, (5.2). To make sure that we have obtained the correct solution to the partial slip contact problem, condition (5.3) has to be checked. Substituting expresions $(2.5)_2$, (5.5) into formula (5.4),

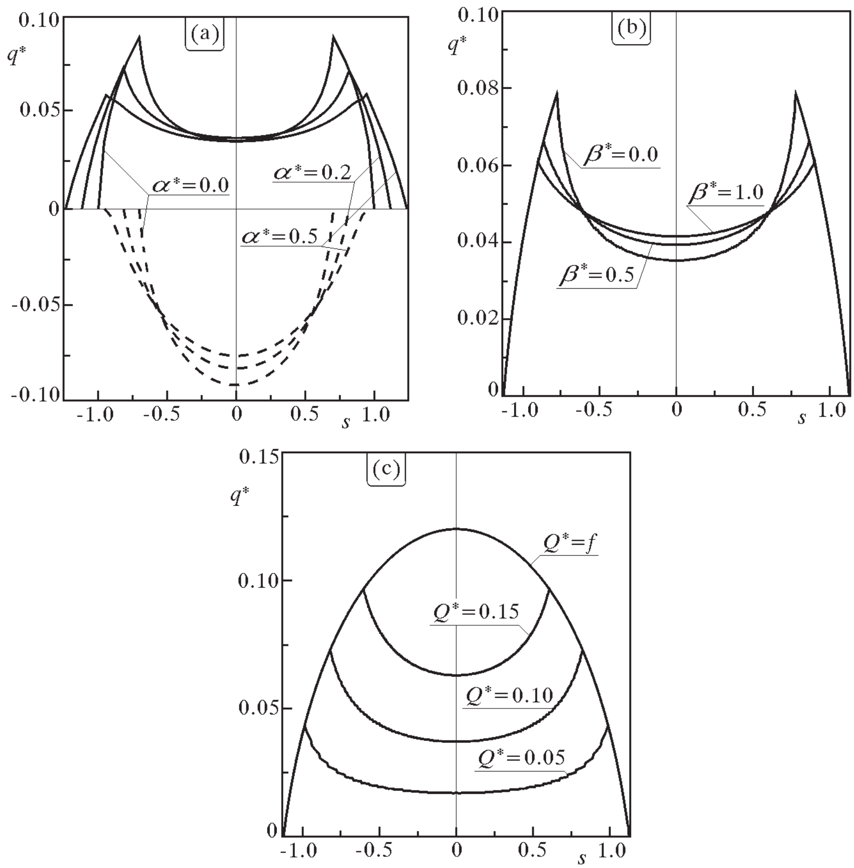

Fig. 3.

after some transformations, we arrive at the dimensionless form of the relative tangential displacements in the slip zones

$$
\begin{aligned}
s_x^*(s) = \frac{Es_x(x)}{(1-\nu^2)P} &= -\delta_x^* + f[\delta_y^* - h^*(s) + (\beta^* - \alpha^*)p^*(s)] + \\
&+ \frac{2}{\pi}\int_{-c^*}^{c^*} q_0^*(\eta)\ln|\eta - s|\,d\eta \qquad c^* \leqslant |s| \leqslant a^*
\end{aligned}
\tag{5.10}
$$

where $q_0^*(s)$ is the solution to equations (5.9).

The distribution of relative tangential displacements in the slip zone is presented in Fig. 4 for some values of the parameter β^* ($\alpha^* = 0.2$, $f = 0.2$

and $Q^* = 0.1$) and the load Q^* ($\alpha^* = 0.2$, $\beta^* = 0.2$ and $f = 0.2$ and $Q^* = 0.1$). Since, these displacements are negative, boundary condition (5.3) is satisfied.

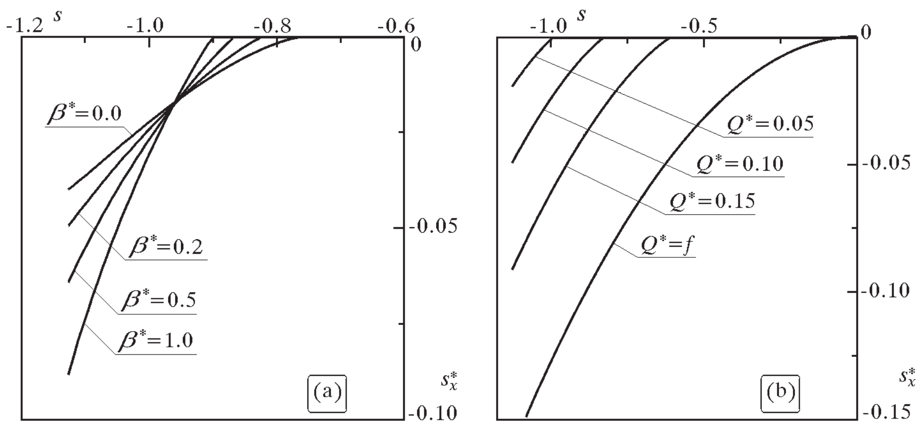

Fig. 4.

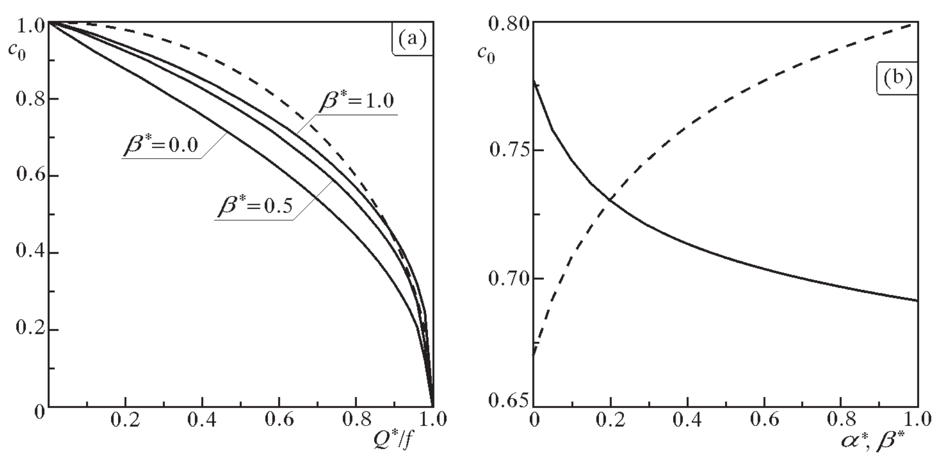

Fig. 5.

The effects of the roughness parameter β^* on the relative half-width of the stick zone $c_0 = c/a = c^*/a^*$ as a function of the ratio Q^*/f is presented in Fig. 5a for $\alpha^* = 0.5$ and $f = 0.2$. For comparison, the classical result, Cattaneo (1938)

$$c_0 = \sqrt{\frac{1 - Q^*}{f}} \tag{5.11}$$

is presented by the dotted line. Figure 5b shows the value of c_0 as a function of the parameter α^* (solid curve, $\beta^* = 0.2$) and of the parameter β^* (dotted curve, $\alpha^* = 0.2$) for Q^*. A general tendency is that the stick zone size increases with the parameter β^* growth, and decreases with the growth of α^*.

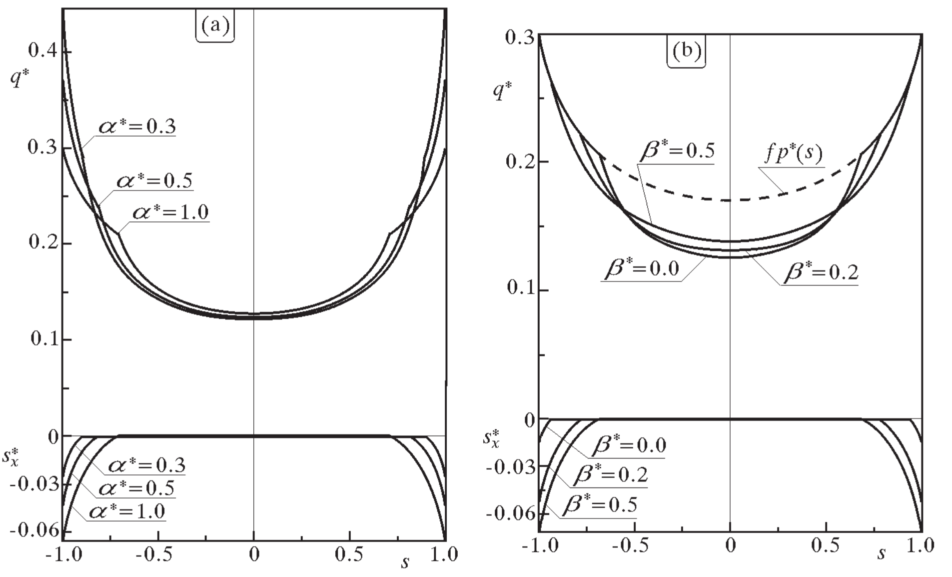

Fig. 6.

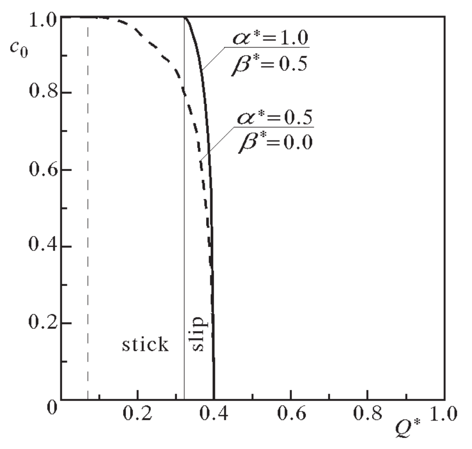

Fig. 7.

This tendency is also observed in the case of the rigid flat punch. The effects of the roughness parameters α^*, β^* on the total shearing traction under the flat punch and on the relative tangential displacements in the slip zones for $f = 0.4$ and $Q^* = 0.35$ are shown in Fig. 6a ($\beta^* = 0.1$ is fixed) and in Fig. 6b ($\alpha^* = 1.0$ is fixed).

The dependence between the stick zone size c_0 and the shearing load Q^* is presented in Fig. 7 for two cases of the roughness: $\alpha^* = 0.5$, $\beta^* = 0.0$ (dotted curve) and $\alpha^* = 1.0$, $\beta^* = 0.5$ (solid curve) and for fixed $f = 0.4$. The stick zone size is equal to the punch width for $Q^* < Q_0$, and quickly decreases to zero if Q^* approaches f.

6. Conclusions

A new formulation of the tangential partial slip contact problem was presented in the paper. Additional displacements in the contact zone due to boundary roughness deformation were taken into account. Two geometries of the punch profiles were considered: a parabolic cylinder and a flat punch. The problems were reduced to boundary integral equations which were solved numerically. The obtained result allowed one to draw the following conclusions.

- Boundary imperfections have great effect on solutions to partial slip contact problems.

- In the case of a parabolic geometry, these effects are quantitative: the contact zone and the stick zone are bigger, and the shearing traction is lower in the presence of roughness. However, the general behaviour is similar to the classical one.

- In the case of the flat punch, new behaviour is observed. Contrary to the classical formulation, which gives no partial slip solution for the flat geometry, the proposed formulation provides partial slip for some values of the roughness parameters and tangential load.

The main difficulty with the application of the obtained results is that the known experimental tests of rough boundaries do not provide data for the roughness parameters α and β. Some evaluation of the roughness parameters is given in Appendix.

A. Appendix

We propose an approach for the determination of the roughness parameters from the comparison of the solution obtained here with the known one. For example, the solution to the normal contact problem for a rough half-space presented in Section 3 can be compared with the solution obtained using the Greenwood-Williamson model of roughness, see Greenwood and Williamson (1966). This solution, in the plane case, was presented by Lo (1969). Figure 8 shows (solid line) the distribution of the normalized normal pressure $p^*(x)$ (see (3.8)) obtained in the framework of the Greenwood-Williamson model for two dimensionless parameters

$$\delta = \frac{8}{3}\frac{\sqrt{\gamma R}\,\eta\sigma}{\pi\sqrt{\pi}} = \frac{1}{2} \qquad\qquad \widetilde{P} = \frac{2P(1-\nu^2)}{\pi\sigma E} = \frac{1}{10} \tag{A.1}$$

where γ is the radius of the tip of asperities distributed with the density η, σ is the standard deviation of the Gaussian distribution of asperity heights. Note that δ plays the role of the roughness parameter in the Greenwood-Williamson model.

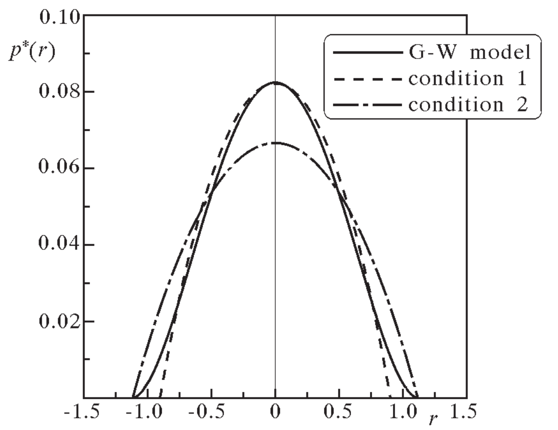

Fig. 8.

Solving now the normal contact problem for the rough half-space and the parabolic punch as has been stated in Section 3, we can guess the roughness parameter α^* from one of the two conditions: the Greenwood-Williamson and Shtayerman models produce the same maximum of the contact pressure (Condition 1), or both models produce the same value of the contact size (Condition 2). We have found, respectively, $\alpha^* \approx 0.18$ and $\alpha^* \approx 0.37$. The corresponding solutions are presented in Fig. 8 by dotted lines. We can conclude that, for the parameters $\delta = 0.5$, $\widetilde{P} = 0.1$ from the Greenwood-Williamson model of roughness, the value $\alpha^* \approx 0.275$ of the roughness parameter describing the Stayerman model corresponds with the good accuracy. In a similar way, we can find the parameter α^* for another sets of δ and $\widetilde{P}$, and state a relation between both models of boundary roughness in the case of the normal problem. It is impossible however, to perform a similar procedure for the tangential problem and find a relation for another roughness parameter γ because there is no model analogous to the Greenwood-Williamson one in the case of the tangential problem.

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Płaskie zagadnienia kontaktowe dla półprzestrzeni chropowatej z uwzględnieniem częściowego poślizgu

Streszczenie

Praca dotyczy zagadnień kontaktowych uwzględniających powstawanie poślizgów pomiędzy powierzchniami styku. Dodatkowo zakłada się, że te powierzchnie są chropowate. Rozważa się dwie podstawowe geometrie stempla: stempel o płaskiej podstawie oraz stempel walcowy. Do rozwiązania zagadnień kontaktowych stosuje się metodę równań całkowych. Ujawniono wpływ chropowatości na rozwiązania zagadnień kontaktowych.

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