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MODELLING OF VIBRATIONS OF MACHINES MODELS BY USE OF THE HYBRID BOND GRAPHS

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In the paper the problem of the modelling of machine systems by making use of the hybrid bond graphs method in a matrix representation and in terms of differential equations has been formulated. The presented method of the dynamic analysis of mechatronical machines models on the basis of the hybrid network graphs constitutes a very efficient algorithm. Such a method consists in the modelling of the given mechatronical model in terms of the hybrid bond graphs with a neural net and the Mason signal flow as subgraphs. Using the impedance method frequency characteristics and natural frequencies for the vibrating model of a machine are analysed on an example of a railway vehicle. In the paper, the sensitivity model and its dynamic characteristics are formulated and examined with the help of the hybrid bond graphs method.

Key words: modelling, bond graphs, hybrid bond graphs method, neural network

1. Introduction

In the paper, the problem of the analysis of dynamic machines models in terms of the network graphs, in particularl by the use of the bond graphs method (Ford and Fulkerson, 1969; Korzan, 1978; Wojnarowski and Nowak, 1989; Wojnarowski, 1985) in a matrix representation has been formulated. The topological way of the analysis of the dynamics of mechatronical machines models on the basis of the network is characterised by algorithms of great efficiency (Bukowski, 1959; Robichaud *et al.*, 1968). In our paper, mathematical principies of the hybrid bond graphs method for the modelling of a mechatronic system is presented. Such a method gives possibility to formulate a general mechatronical model of machine systems by coupling the bond grahs and the neural net with the Mason flow graphs, which yields a single universal graph model. In the numerical algorithm the mathematical model of the machine dynamic characteristics in a matrix representation is formulated on the basis of the *global stiffness matrix* and the *global flexibility matrix* of the system. Using the impedance method, frequency characteristics and natural frequencies for the dynamic model of a machine in terms of the network graphs are analysed. The goal of the analysis in this paper consists in the formulation of the sensitivity model of the dynamic characteristics of the machine by applying the hybrid bond graphs method. Such a method gives one the possibility to elaborate the complex and the unit bond graph model of the mechatronical model of the machine with regard to the sensitivity parameters of the system. In the paper, the unit hybrid bond graph on an example of the dynamic model of a railway vehicle with the driving system and the electrical motor is presented (Bukowski, 1959; Robichaud *et al.*, 1968; Wojnarowski, 1999).

By *the hybrid bond graphs* (HBG) we name the bond graph (BG) that includes one or more *subgraphs* (S) in terms of the Mason signal flow graph (M) or in terms of *the neural network graph* (N), connected with the BG by the use of the branches with analogy elements (integrator, summator, invertor).

For the modelling of a mechatronical system with a neural network subsystem represented by the hybrid bond graphs, we introduce *the logic element* (Fig. 1a) by *the logister element* named. For the modelling of nonlinear functions (dependencies between the input and the output signals) the *activator element* is introduced (Fig. 2).

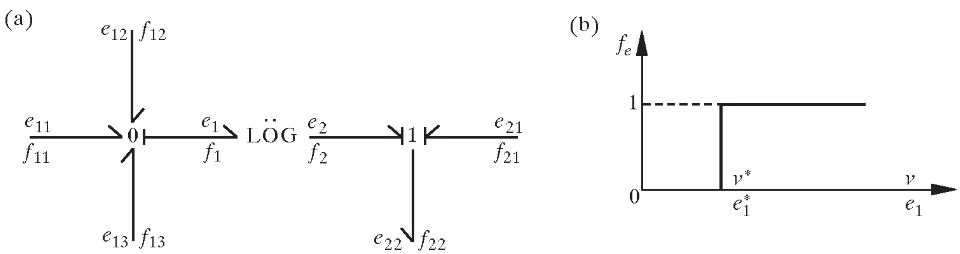

Fig. 1. Bond graph of the logister element (a) and its characteristic (b)

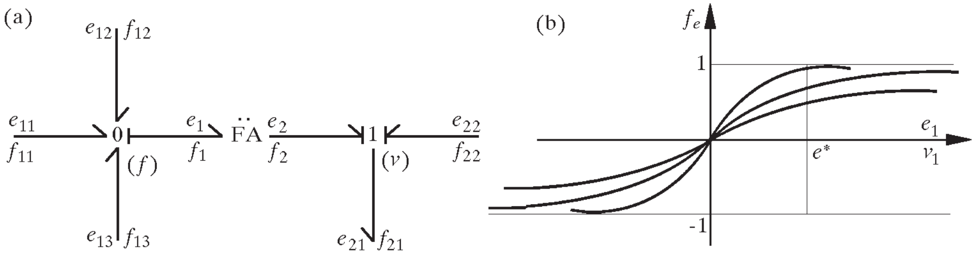

Fig. 2. Bond graph of the activator element (a) and sigmoidal function (b)

2. Modelling of the driving system

We assume the dynamic model of a driving system with a flexible coupling and steady-current electric motor (Fig. 3a). The block scheme of the driving system in the form of a model of the hybrid bond graph with a subsystem in terms of the neural network graph is presented in Fig. 3b.

The detailed model of the neural network for the modelling of the electric driving system with the steady-current motor in aspect of the identification process of its parameters is illustrated in Fig. 3c. The both graphs, i.e. the bond graph and the neural network by the Mason flow signals are connected with summators, potentiometers and invertors elements. Then we consider the sensitivity model of the driving system with regard to the spring of the coupling and the resistance of the electric motor in terms of the hybrid bond graph, as shown in Fig. 6. There, the integrator element is applied for calculation of the input signal from the bond graph to the sensitivity subgraph.

3. Models of hybrid bond graphs of the railway vehicle

By making use of the hybrid bond graphs method, sensitivity models of mechatronical and dynamical systems can be constructed in the network graphs representation. For example, we consider a dynamic model of the railway vehicle powered with an electrical driving system in which the effect of the stiffnesses of its elements is examined.

The global dynamic model of the vehicle consists of the following subsystems: the driving system, the four wheelset assembly, the car body of the vehicle and the road with rails. The global dynamic model of the railway vehicle with the driving system and a steady-current motor is presented in Fig. 5.

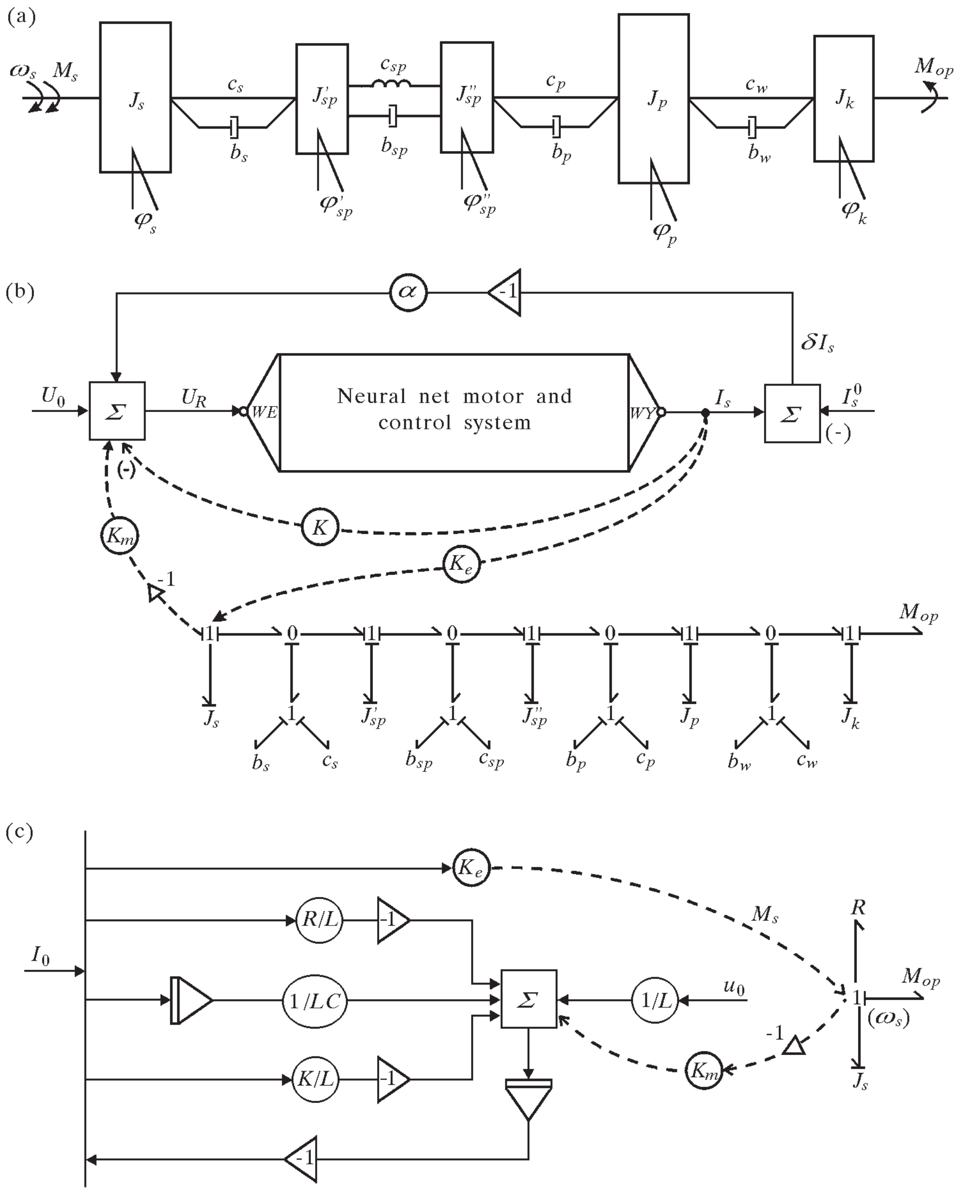

Fig. 3. Model of the driving system (a), hybrid bond graph (b), and neural network (c)

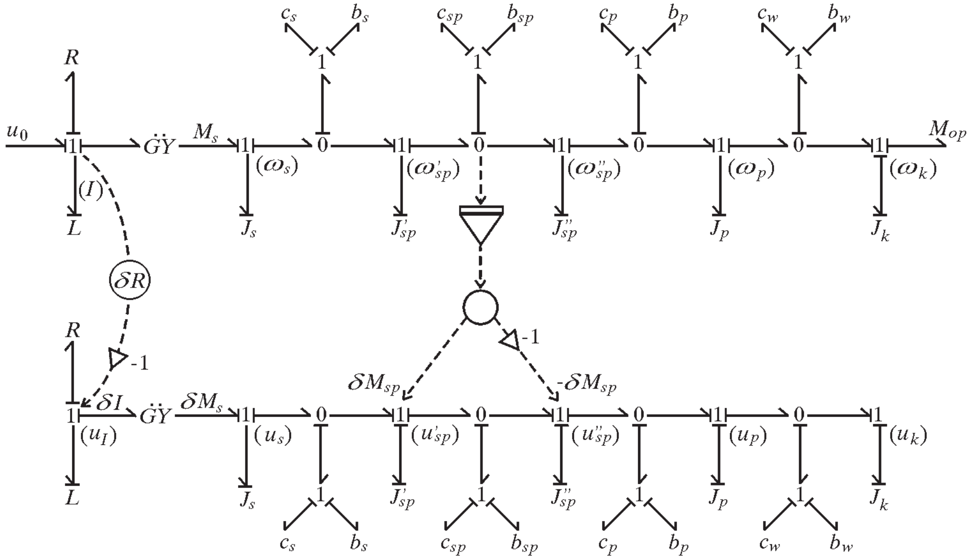

Fig. 4. Sensitivity hybrid bond graph of the electric driving system

In this model a horizontal distribution of the loads is given. Motion of the vehicle is expressed in terms of the hybrid bond graph, as shown in Fig. 6.

On the basis of the formulated physical and mathematical models of the railway vehicle a numerical procedure was prepared for particular calculations. For the assumed vibraiting model and for the input harmonic forces with parameters $Z_{0i} = Z_0 = 0.01\,\mathrm{m}$, and $\omega \in \langle 0, 600\rangle$ rd/s in the range of frequencies $f \in \langle 0, 100\rangle$ Hz, one may calculate the natural frequencies and characteristics of the system. As a result of the analysis of the fully loaded the following natural frequencies of the system were determined

$$
\begin{array}{lll}
f_1 = 1.57\ \mathrm{Hz} & f_2 = 3.55\ \mathrm{Hz} & f_3 = 4.65\ \mathrm{Hz} \\
f_4 = 6.07\ \mathrm{Hz} & f_5 = 7.88\ \mathrm{Hz} & f_6 = 10.4\ \mathrm{Hz}
\end{array}
$$

The natural frequency f_1 corresponds to the torsional vibration mode, whereas the frequency f_2 describes the vertical vibrations about the axis Oz. In Fig. 7a the amplitude-frequency characteristics of the car body of the vehicle in a logarithmic scale for the shock absorbers parameters $c_a = 200$, 500, 800, 1100 kN/m are presented (corresponding curve number are 1, 2, 3, 4). The diagrams of the acceleration frequencies for the wheelset are shown in Fig. 7b. On the basis of the calculated natural frequencies, and for the wheel radius $r = 0.6\,\mathrm{m}$ and the transmission ratio of the driving system $p = 2$, the critical

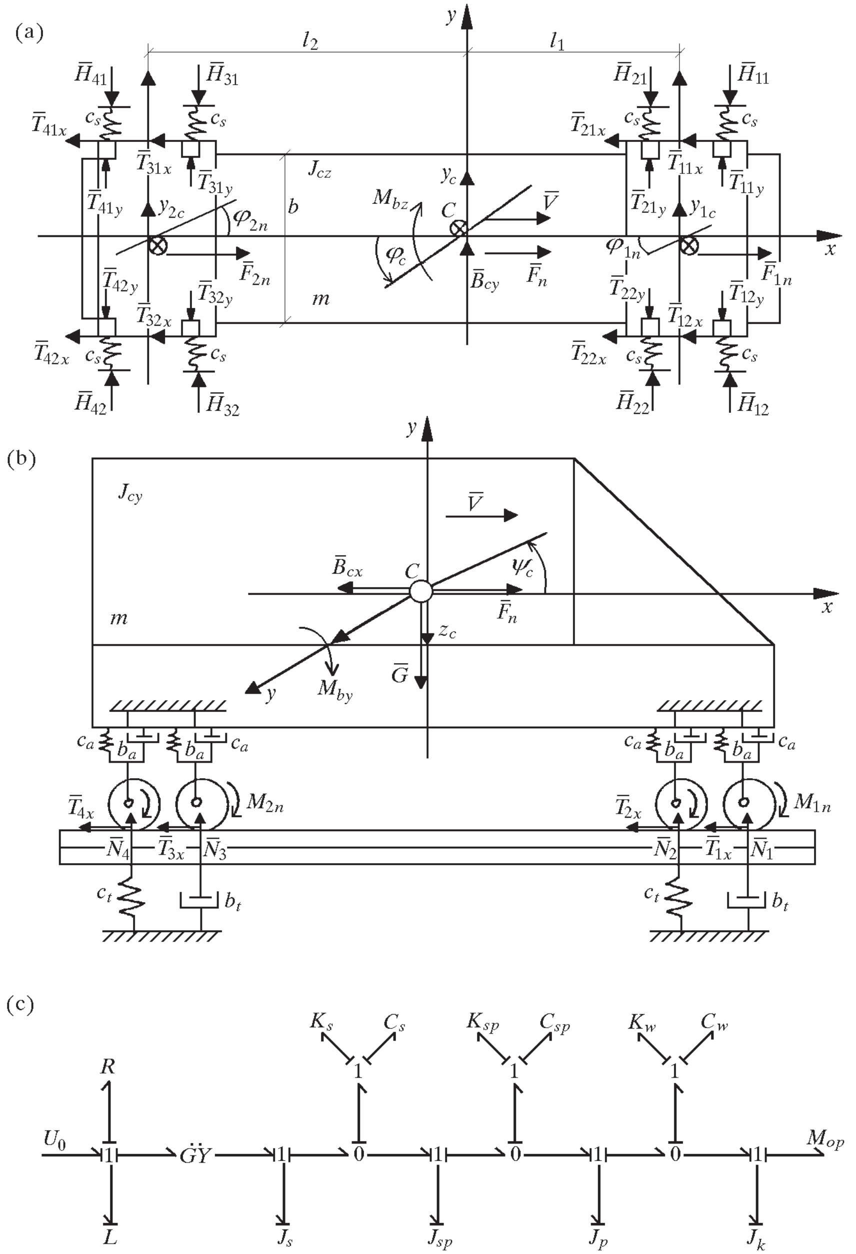

Fig. 5. Dynamic model of the railway vehicle with the electric driving system

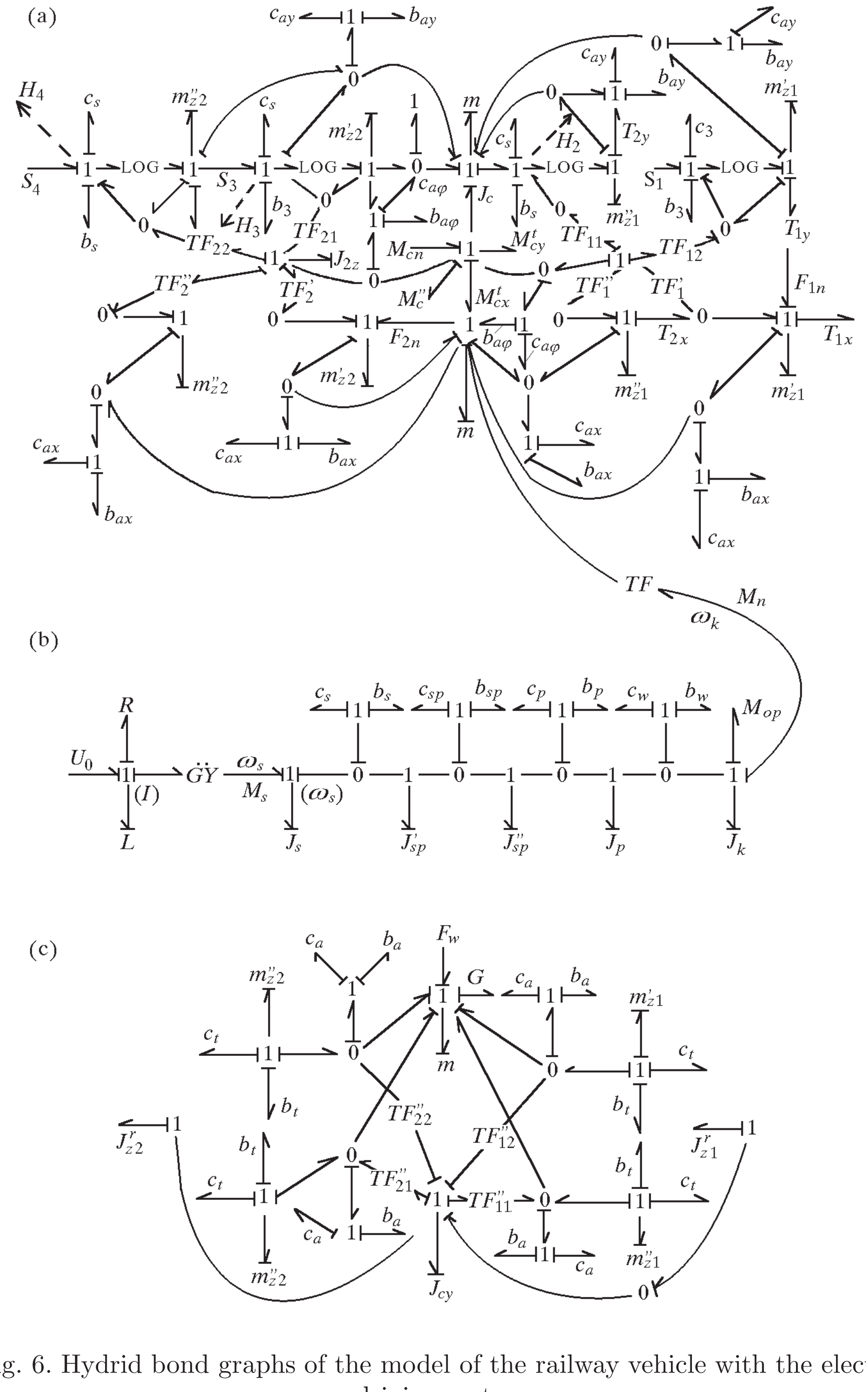

Fig. 6. Hydrid bond graphs of the model of the railway vehicle with the electric driving system

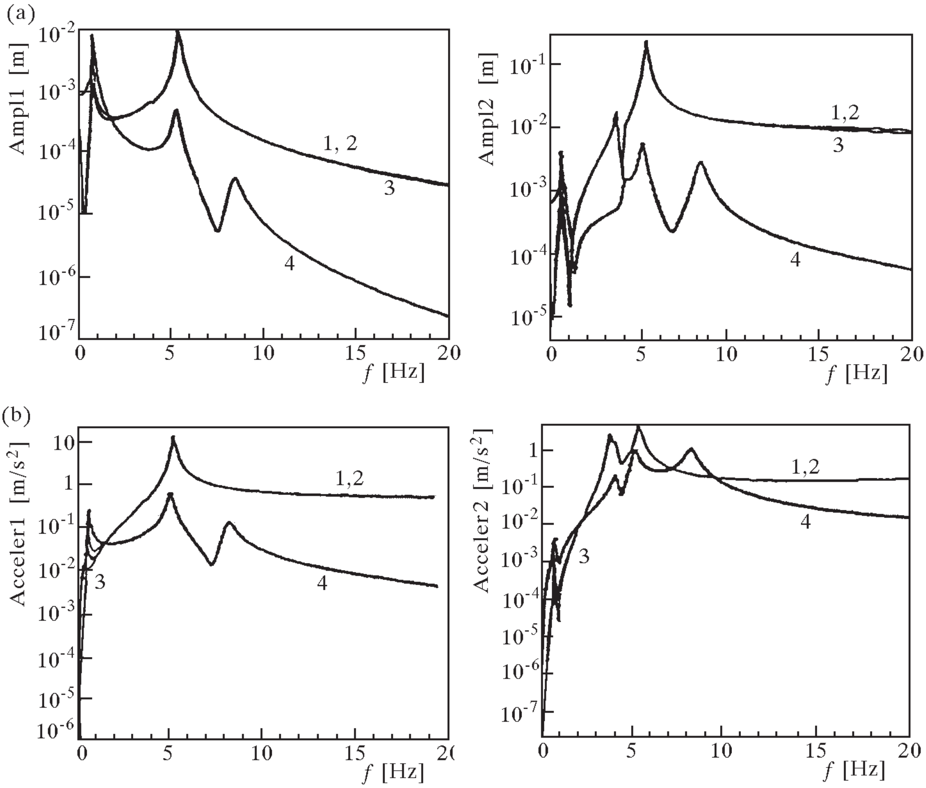

Fig. 7. Frequency characteristics (a), and acceleration frequency characteristics (b)

drive velocities of the fully loaded railway vehicle were determined as in the following

$$v_1 = 33.75 \text{ km/h} \qquad v_2 = 56.9 \text{ km/h} \qquad v_3 = 80.6 \text{ km/h}$$
$$v_4 = 90.65 \text{ km/h} \qquad v_5 = 152.1 \text{ km/h}$$

4. Modelling of the flows in hydraulical systems by use of the network graphs

In the paper the problem of the analysis of dynamic network flows by making use of the graphs method (Bukowski, 1959; Wojnarowski and Nowak, 1989; Kaczmarek and Nowak, 2000, 2001) in a matrix representation has been formulated. The classisal model of the flows, which incorporates the

Ford-Fulkerson algorithm, was formulated for the stationary case (Ford and Fulkerson, 1969; Korzan, 1978). In our paper, the model of the dynamic flows is defined by flow intensities in function of the potential variables in the vertices of the network graph at network branches. The elementary capacities of the branches are expressed in terms of the differential operator $s = d/dt$. Such a method gives one the possibility to describe viscous dampings and inertial resistancies in the network graph with side branches. The dynamical flows are realized by impulse and harmonic input flows. The mathematical model of the dynamic network flows in a matrix representation is formulated on the basis of the global matrix capacity of the system. Using the impedance method, the frequency characteristics and the natural frequencies for the resonance flows in the potential network system are analysed. The potential network is isomorphous with different physical systems, in particular with mechanical vibrating systems, electric and hydraulic ones.

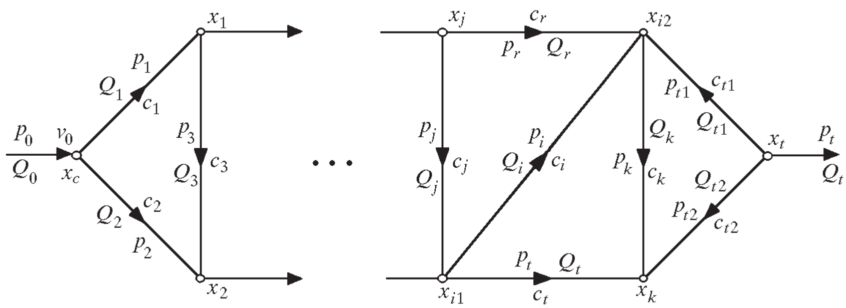

Fig. 8. Model of the dynamic flow network

We assume a model of the network with only one input and output node (Fig. 8). For each vertex x_i, $i = 1, 2, ..., n$, the potential variable q_i (general variable) is defined. For each branch u_j, $j = 1, 2, ..., N$ with the capacity c_j, the flow variable Q_j is given in terms of the formula

$$Q_j = c_j \Delta q_j = c_j(q_{j1} - q_{j2}) \tag{4.1}$$

In the network graph, we define a potential function of the flows

$$V = \frac{1}{2}\sum_{j=1}^{N} c_j|\Delta q_j|^2 = \frac{1}{2}\sum_{j=1}^{N} c_j(q_{j1} - q_{j2})^2 \tag{4.2}$$

Using the Lagrange method, the column matrix of the network flows is determined

$$\boldsymbol{Q} = (\mathbf{ACA}^\top)\boldsymbol{q} = \mathbf{Z}\boldsymbol{q} \tag{4.3}$$

where

$$\mathbf{Z} = \mathbf{A}\mathbf{C}\mathbf{A}^\top \tag{4.4}$$

$\mathbf{C}$ is the *global capacity matrix of the network*, $\mathbf{Z}$ is the *global stiffness matrix* of the flow system, and $\mathbf{A}$ denotes the structural matrix of the network graph.

Using formula (4.3), the matrix equation for the equilibrium of the network flows is

$$\mathbf{Z}\boldsymbol{q} = \boldsymbol{Q}_0 \tag{4.5}$$

where $\boldsymbol{Q}_0$ is the input flow in the network. We consider the potential network with:

— viscosity dampings (for the first order edges)

$$Q_i(t) = c_i q_i + b_i \dot{q}_i \quad \Leftrightarrow \quad Q_i(s) = z_i q_i = (c_i + b_i s) q_i \tag{4.6}$$

— inertial resistances (for the second order edges)

$$B_i(t) = a_i \dot{v}_i \;\Leftrightarrow\; B_i(s) = a_i s^2 q_i \quad \text{or} \quad B_i(j\omega) = -a_i \omega^2 q_i \tag{4.7}$$

where a_i is the inertia capacity coefficient.

In the input vertex of the network, we consider the harmonic input flow signal

$$Q_s = Q_0 + Q_1 \sin(\omega t) \qquad\qquad Q_1 \ll Q_0 \tag{4.8}$$

The introduced notions of the first and second order branches define the *dynamical potential network*. The corresponding equations of the model are as in the following

$$\begin{aligned} \mathbf{M}\ddot{\boldsymbol{q}}_s + \mathbf{B}_s\dot{\boldsymbol{q}} + \mathbf{C}_s\boldsymbol{q} &= \boldsymbol{Q}_{0s}(t) \\ \mathbf{B}_m\dot{\boldsymbol{q}} + \mathbf{C}_m\boldsymbol{q} &= \boldsymbol{Q}_{om}(t) \end{aligned} \tag{4.9}$$

with the initial conditions:

$$\boldsymbol{q}_0(0) = \boldsymbol{q}_0 \qquad\qquad \dot{\boldsymbol{q}}_0(0) = \boldsymbol{v}_{s0}$$

where $\boldsymbol{q} = [q_1, q_2, ..., q_5]^\top$ is the column matrix of the general coordinates.

In *the free dynamical network* without the input flow, the dynamic flows in the branches should be shown in terms of the natural modes, i.e. expressed by a series of harmonic signals with natural frequencies ω_k and phase shifts φ_k

$$Q_i^{(k)}(t) = \sum_{i=}^{s} Q_{ik} \sin(\omega_k t + \varphi_k) \tag{4.10}$$

Equations (4.9) are the second order differential equations for the sth degree of freedom of the flow system, and for the mth $(m = n - s)$ degree of freedom in the case of the first order differential equations. Reducing equations (4.9) eith respect to the general coordinates, we obtain the second order matrix equation expressed by such coordinates

$$\mathbf{M}_s\ddot{\boldsymbol{q}}_s + \mathbf{B}_{sm}\dot{\boldsymbol{q}}_s + \mathbf{C}_{sm}\boldsymbol{q}_s = \boldsymbol{Q}_{sm}(t) \tag{4.11}$$

where

$$\begin{aligned} &\mathbf{B}_{sm} = \mathbf{B}_{1s} - \mathbf{B}_{1m}\mathbf{B}_{ms} \qquad\qquad \mathbf{C}_{sm} = \mathbf{C}_{1s} - \mathbf{C}_{1m}\mathbf{B}_{ms} \\ &\boldsymbol{Q}_{sm}(t) = \boldsymbol{Q}_{0s}(t) - \mathbf{C}_{1m}\mathbf{C}_{2m}^{-1}\boldsymbol{Q}_{0m}(t) - \mathbf{B}_{2m}\mathbf{C}_{2m}^{-1}\dot{\boldsymbol{Q}}_{0m}(t) \end{aligned} \tag{4.12}$$

For the harmonic input signals, we can determine the frequency characteristics of the flow system

$$\boldsymbol{X}_s(j\omega) = \mathbf{Y}_s(j\omega)\boldsymbol{Q}^0_{sm}(j\omega) \tag{4.13}$$

where $\mathbf{Y}_s(j\omega) = \mathbf{Z}_s(j\omega)^{-1}$ is the *global flexibility matrix of the flow system.*

Next, we define the frequency characteristics of the flow intensities for the network branches

$$\boldsymbol{Q}(j\omega) = [\mathbf{A}^\top\mathbf{Z}(j\omega)]\boldsymbol{X}(j\omega) \qquad\qquad Q^0_i(j\omega) = |\boldsymbol{Q}_i(j\omega)| \tag{4.14}$$

The absolute values of the complex flows are determined on the basis of actual frequency characteristics of the network system.

For each potential network, we can construct an isomorphous physical system with a similar structure. For example, for the potential network given in Fig. 9a with second order branches, one can propose a series of isomorphous systems, in particular the *vibrating mechanical system* that consists of the base elements: masses, viscous dampings, springs (Fig. 9b), and *the electric system* (Fig. 9c).

5. Analysis of dynamical flows in the network model of a hydropneumatical vibroisolation system

The object of the investigations is a hydropneumatical vibroisolation system of the operator's seat, whose model is shown in Fig. 10a. In the scheme the stiffness of the hydropneumatical element is denoted by c_s. The servo-motor is

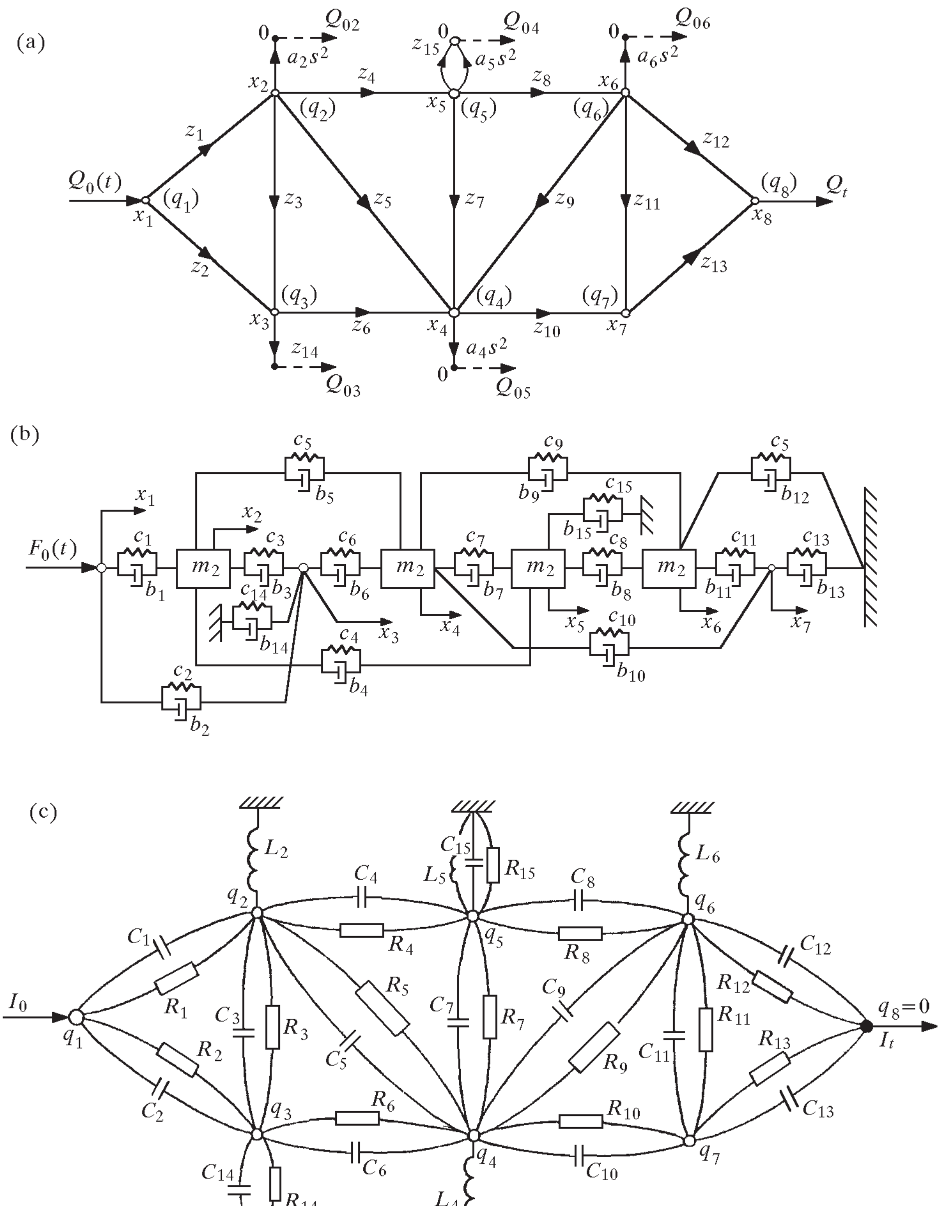

Fig. 9. Potential network graph (a), isomorphous mechanical system (b), isomorphous electric system (c)

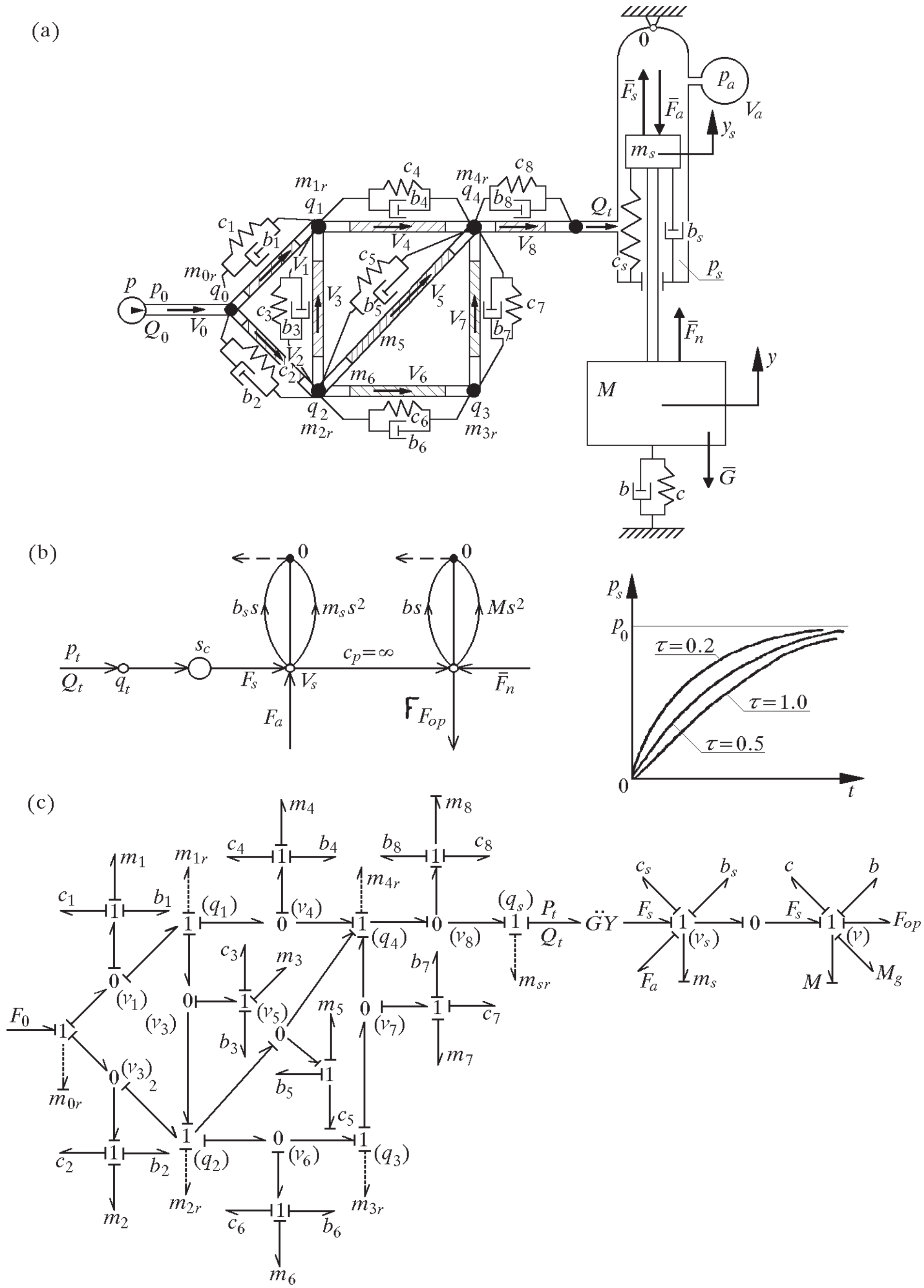

Fig. 10. Model of the vibroisolation system (a), the network (b), and the bond graph (c)

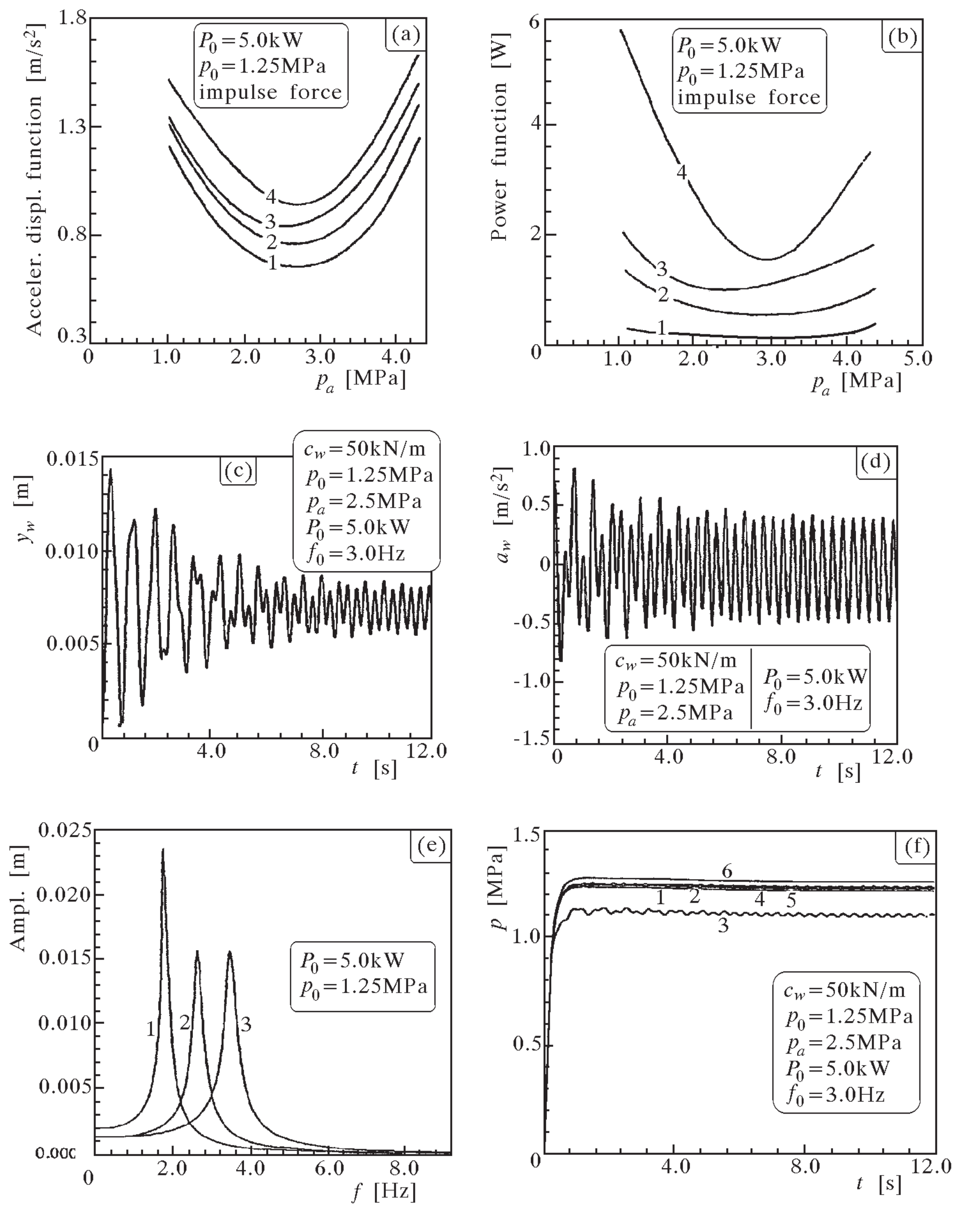

Fig. 11. Results of calculations for the vibroisolation system of the operator's seat

moved by the force F_s, generated in the hydraulic pump system. The network and bond graphs of the hydraulical system in terms of the potential is formulated, see Fig. 10b,c. As a result, the influence of the accumulator pressure p_a on the displacement-acceleration criterial function (Fig. 11a) and on the power criterial function (Fig. 11b) has been found. The minimum of the criterial functions for the pressure $p_a = 2.8\,\mathrm{MPa}$ has been determined. In such a case, the displacement (Fig. 11c) and the acceleration (Fig. 11d) of the operator's seat have been calculated for a harmonic force with the frequency $f_0 = 3.0\,\mathrm{Hz}$ acting on the mass m_1.

The frequency characteristics of the vibroisolation system are shown in Fig. 11e. The pressure function in the cylinder of the servo-motor is given in Fig. 11f.

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Modelowanie drgań modeli maszyn metodą hybrydowych grafów wiązań

Streszczenie

W pracy sformułowano zagadnienie modelowania modeli dynamicznych układów maszyn z zastosowaniem metody hybrydowych grafów wiązań w reprezentacji macierzowej i w postaci układu równań różniczkowych ruchu. Opracowana metoda hybrydowych grafów wiązań stanowi efektywny sposób analizy układów mechatronicznych maszyn. Metoda polega na sformułowaniu globalnego grafu wiązań z uwzględnieniem grafu przepływu sygnałów Masona lub sieci neuronowej jako podgrafów. Stosując metodę impedancji wyznaczono charakterystyki amplitudowo-częstotliwościowe oraz częstości własne na przykładzie modelu dynamicznego pojazdu szynowego. W pracy opracowano także modele wrażliwości ruchu maszyn i ich charakterystyk dynamicznych przy zastosowaniu metody hybrydowych grafów wiązań.

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