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OPTIMUM DESIGN OF COMPOSITE CHANNEL CROSS-SECTION COLUMNS UNDER COMPRESSION

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Thin-walled open cross-section bars are effective structural members (with respect to overall stability). But the weight efficiency of bars with flat walls is constrained by their low stiffness in local buckling. The use of composites and multi-layer materials is one of perspective ways to overcome this shortage. The minimum-weight problem for such laminated bars has not been studied until now. In the paper a solution to the structural optimization problem for three-layer channel cross-section columns under compression is presented. The optimization problem is formulated as a nonlinear programming problem. Basic constraints are conditions of overall and local stability. In addition, strength, wrinkling and geometrical constraints (on minimal thickness of skin layers and maximal total thickness) have been accounted for. For the given material properties the optimal dimensions of the cross-section and the optimal thickness ratio (for the steel and composite layers) are determined. The three-layer members are shown to be much more efficient in comparison with columns made of homogeneous materials. The sensitivity of optimal projects to changes in the material properties (in particular, to changes of the modulus of elasticity in tension and in shear) is also studied.

Key words: optimization, thin-walled structures, channel, composite

1. Introduction

The minimum weight design problems for thin-walled bars-beams under compression or bending with respect to their stability were investigated in a few papers already in the 60-70-s on the basis of various theoretical models. A review of the literature concerning the optimal design and some new works in this field can be found, in Adali (1995), Birger and Panovko (1968), Walker *et al.* (1996), Życzkowski and Gajewski (1983), and in proceedings of recent international conferences (Zaraś *et al.*, 2001; Gutkowski and Mróz, 1997).

Thin-walled bars of open cross-section are effective structural members (with respect to overall stability), however the weight efficiency of bars with flat walls is constrained by their low stiffness in local buckling. The use of composites and multi-layer members is one of perspective ways to overcome this shortage. Over the past years an interest was exhibited to new types of structural elements, in particular, to metal-composite-metal members which were developed for applications in manufacturing of parts for cars, train and other machines (Ashafi *et al.*, 2000). But the minimum-weight problem for such laminated bars with respect to their stability and strength has not been studied systematically until now.

In the paper a solution of the structural optimization problem for composite and three-layer channel cross-section columns (metal-composite-metal members) under compression is presented. The solution is based on the classical plate theory and the linear stability theory and employs two program packages:

- WARST package for calculation of critical stresses of compressed and bent plate assemblages composed of an arbitrary number of orthotropic layers (see Kołakowski and Kowal-Michalska, 1999)

- LMRG package for solution of the general nonlinear programming problem by the linearized method of reduced gradient (Manevich, 1979).

Results of the solution are verified by analyzing the obtained optimal channels using ANSYS 5.7 with account of shear deformation.

2. Formulation of the problem and method of solution

A simply supported thin-walled member of channel cross-section loaded centrally by the compressive force P is considered (Fig. 1). Two cases of manufactured channels are studied:

case I – the member is made of a composite material (Fig. 1a)

case II – the member is constructed from two steel skins and a composite layer between them (Fig. 1b).

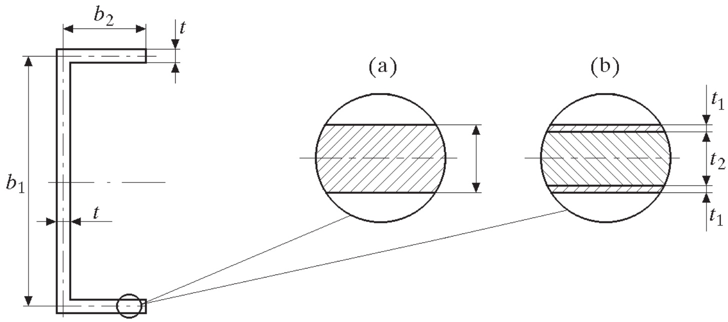

Fig. 1. Two cases of manufactured channels: composite material (a); steel+composite+steel (b)

The length L and the material properties of all layers are considered to be given, and the optimal dimensions of the cross-section and the optimal thickness ratios (for the steel and composite layers) are to be determined. The composite is considered as a structurally orthotropic material, and the classical plate theory is employed in the buckling analysis.

The optimization problem is formulated as a nonlinear programming problem. For a given value of the weight per unit length Q the cross-section geometrical parameters for which the critical load is maximal are sought. The bending mode in the symmetry plane, flexural-torsional modes and local modes (Fig. 2) are taken into account, and the minimal value of critical stresses for these three types of buckling modes determine the critical load. In addition, the strength, wrinkling (local buckling of external layers) and geometrical constraints were accounted for.

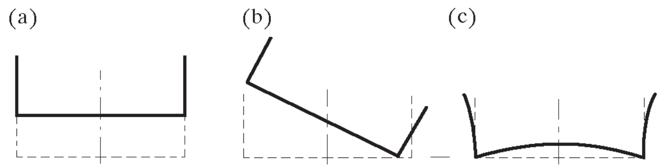

Fig. 2. Considered buckling mode: (a) overall flexural mode; (b) flexural-torsional mode; (c) local mode

Critical stresses for all buckling modes were calculated by program WARST which was worked out for thin-walled girders built of plates with closed or open cross-sections. The plate elements under consideration can be multi-layer walls made of orthotropic materials. The classical laminated plate theory (Jones, 1975) is used in the theoretical analysis, the effect of shear deformation through the thickness of the laminate is neglected. The materials they are made of are subjected to Hooke's law.

In order to account for all modes of global and local buckling, a plate model of thin-walled structures is employed. Sinusoidal deflection along the length is assumed and the amplitude changing in the transverse direction is determined by numerical integration. During the integration of the equations, Godunov's orthogonalization method is employed. The most important advantage of this method is that it enables us to describe the complete range of behaviour of thin-walled structures from all global (flexural, flexural-torsional, lateral, distortional buckling and their combinations) to the local stability (Kołakowski and Kowal-Michalska, 1999). The column global buckling occurs at one sinusoid half-wave along the column length, whereas the local buckling takes place at the number of half-waves $m > 1$ with $b_k \ll \ell$.

Due to the symmetry or antisymmetry conditions only a half of the profile was considered. For the overall bending mode (in the symmetry plane, see Fig. 2a) at the point $x = 0$ (center of the wall) the symmetry condition was imposed, and the number of halfwaves m was assumed equal to 1. For flexural-torsional modes (Fig. 2b) the antisymmetry condition and $m = 1$ were assumed. For local modes (Fig. 2c) the symmetry condition was imposed but there was assumed that $m > 1$ (for more details see Kołakowski and Kowal-Michalska, 1999; Kołakowski *et al.*, 1999).

Critical stresses for wrinkling of the external layer (skin) were calculated according to (Birger and Panovko, 1968)

$$\sigma_w = \begin{cases} 0.91\sqrt[3]{\dfrac{E_1E_2G_2}{1-\nu_1^2}} & \text{when} \quad \dfrac{t_2}{2t_1} > 0.4\sqrt[3]{\dfrac{E_1E_2}{(1-\nu_1^2)G_2^2}} \\[2ex] 0.58\sqrt{\dfrac{2E_1E_2t_1}{(1-\nu_1^2)t_2}} & \text{when} \quad \dfrac{t_2}{2t_1} < 0.4\sqrt[3]{\dfrac{E_1E_2}{(1-\nu_1^2)G_2^2}} \end{cases} \tag{2.1}$$

where E_1 is the elasticity modulus of the skin, ν_1 Poisson's ratio of the skin, E_2, G_2 are the effective elasticity modulus and shear modulus of the core, respectively.

The strength constraint was taken in the form $\sigma \leqslant \sigma_y$, where σ_y is the yield stress.

Geometrical constraints were imposed on the minimal thickness of skin layers and the maximal total thickness. The thickness constraints had the form: $t_1 \geqslant t_{1,\min}$ and $t = 2t_1 + t_2 \leqslant t_{\max}$.

The nonlinear programming problem was solved by the linearized method of reduced gradient (Manevich, 1979). This method, which uses the idea of change of the independent variables set in the vicinity of the allowable region boundary by means of linear operations with sensitivity matrices, effectively overcomes familiar difficulties arising at solving nonlinear programming problems (in particular, connected with zigzag-type motion in the boundary vicinity). As a rule, 20-30 iterations were required to achieve the optimum with a relative error of order 0.001.

For generality of the analysis, nondimensional parameters of the load P^*, weight Q^* and stress σ^* were used

$$P^* = \frac{P \cdot 10^6}{EL^2} \qquad Q^* = \frac{Q \cdot 10^3}{\rho g L^2} \qquad \sigma^* = \frac{\sigma \cdot 10^3}{E} \tag{2.2}$$

where E stands for E_x in case I (modulus of the orthotropic composite in the longitudinal direction) and for E_1 – in case II (modulus of the isotropic external layer), ρ is the density (in the case of a three-layered channel – the density of the external layer). The following *effective* parameters of load and weight were used for comparison of the optimal members made of various materials

$$P^*_{eff} = P^* \frac{E}{E_0} \qquad Q^*_{eff} = Q^* \frac{\rho_{av}}{\rho_0} \tag{2.3}$$

where E_0, ρ_0 are the elasticity modulus and density of an isotropic material which the given composite material is compared with, ρ_{av} is the average density for the three-layered channel.

Two equivalent formulations of the optimization problem are possible (in the given nondimensional parameters):

- for a given Q^* two nondimensional cross-section parameters $(b_1/L, b_2/b_1)$ are determined yielding the maximum of P^* (minimal value of critical loads for all possible buckling modes)

- for a given P^* the optimal geometrical parameters are determined yielding the minimum of Q^*.

In this work, the first approach was chosen, as a rule, which is more convenient for the sensitivity analysis.

All optimal nondimensional parameters of the compressed member are defined by specifying the single nondimensional parameter Q^* (for given properties of the material); when the length L and modulus E_x (or E_1) are given, and the dimensions of optimal cross-sections are to be determined.

For testing the algorithm, calculations of optimal members of the isotropic material were carried out. Solution to this problem was obtained in Manevich and Raksha (2000) with using the Vlasov theory for overall critical stresses (flexural and torsional-flexural modes) and the analytical solution of the buckling problem for local modes (by conjugation of analytical solutions for all constituent plates). Results of both solutions – Manevich and Raksha (2000) and obtained with using the WARST package – coincide for the isotropic material.

3. Results of the solution

3.1. Composite channel

Two composites with the following characteristics are considered:

- composite A:

$$\frac{E_y}{E_x} = 0.0796 \qquad \frac{G}{E_x} = 0.04316 \qquad \nu_{xy} = 0.3$$

$$E_x = 1.393 \cdot 10^{11} \text{ Pa} \qquad \rho = 1.56 \frac{\text{g}}{\text{cm}^3}$$

- composite B (6-layer graphite/epoxy composite):

$$\frac{E_y}{E_x} = 0.1417 \qquad \frac{G}{E_x} = 0.05157 \qquad \nu_{xy} = 0.3$$

$$E_x = 0.37385 \cdot 10^{11} \text{ Pa} \qquad \rho = 2.2 \frac{\text{g}}{\text{cm}^3}$$

E_x, E_y, G are, correspondingly, the modulus of elasticity in the longitudinal and transverse directions and the modulus of shear, ρ is the density ($E_x\nu_{yx} = E_y\nu_{xy}$).

Comparison of the optimal cross-sections for these materials and for the isotropic material enables us to estimate the influence of ratios E_y/E_x, G/E_x, and also E_x/E_0 on optimal parameters and (with account of ρ/ρ_0) their comparative efficiency.

Values of Q^* were chosen in the range $(0, ..., 1.2)$ with the step 0.1 (in this range the critical stresses for optimal members from usual materials do not exceed the yield stress).

The active constraints are found to be the buckling constraints on the torsional-flexural mode and one (or two) local mode(s); note that the critical stresses for the overall flexural buckling were higher by 15-25% (similarly to the optimal channels of the isotropic material (Manevich and Raksha, 2000)).

In Table 1 there are compared cross-section dimensions, critical loads and critical stresses for thin-walled members with the length $L = 1\,\mathrm{m}$, obtained for three cases:

a) isotropic material – steel with Young's modulus $E_0 = 2 \cdot 10^{11}\,\mathrm{Pa}$ and density $\rho_0 = 7.8\,\mathrm{g/cm^3}$, for $Q^* = 0.2$

b) composites A and B at the same Q^* value (i.e. with the same cross-section area)

c) composite A at $Q^* = 0.865$ and composite B at $Q^* = 0.71$, i.e. at the same weight (for the effective weight parameter Q^*_{eff} according to (2.2), equal to 0.2).

Table 1. Comparison of the optimal cross-section of thin-walled channels made of steel and composites (length $L = 1\,\mathrm{m}$)

Material	Q^*	h [mm]	b_1 [mm]	b_2 [mm]	P^*	P [kN]	σ [MPa]
Steel	0.2	1.4	78.2	34.1	0.190	38	189.8
Composite A	0.2	1.82	58.35	25.76	0.099	13.8	68.7
Composite A	0.865	4.8	96.3	42.1	1.099	153	177
Composite B	0.2	1.75	61.1	27.0	0.107	4.004	20.02
Composite B	0.71	4.03	94.0	41.1	0.864	32.3	45.51

The optimal profiles of the isotropic member and channels of composites with the same area ($Q^* = 0.2$) for $L = 1\,\mathrm{m}$ are shown in Fig. 3, and the optimal profiles of the same weight ($Q^*_{eff} = 0.2$) are compared in Fig. 4.

As in this analysis we used the classical plate model based on Kirchhoff's hypothesis, it is interesting to test the results obtained by comparison with more precise models accounting for the shear deformation (which can have significant effect on buckling loads (Whitney, 1987)). Calculations of the critical stresses of the optimal channels by the FEM with using ANSYS 5.7 were carried out. The shell finite elements (SHELL 91) were employed. On the loaded edges forces were applied at the gravity center, and boundary conditions of the simple support were satisfied.

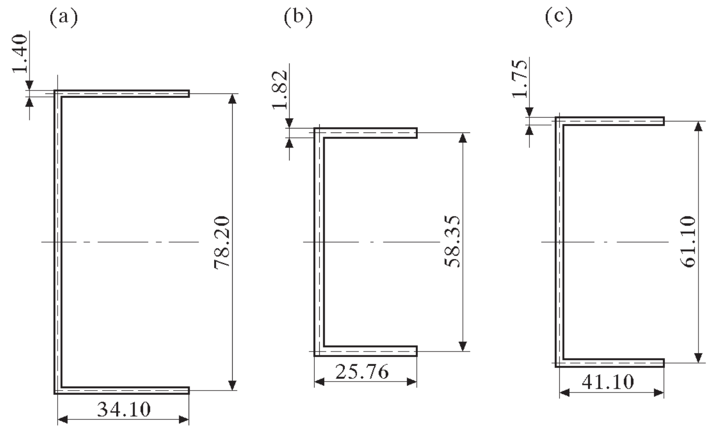

Fig. 3. Optimal profiles of channels with the same area $(Q^* = 0.2)$ for $L = 1\,\mathrm{m}$: (a) steel; (b) composite A; (c) composite B

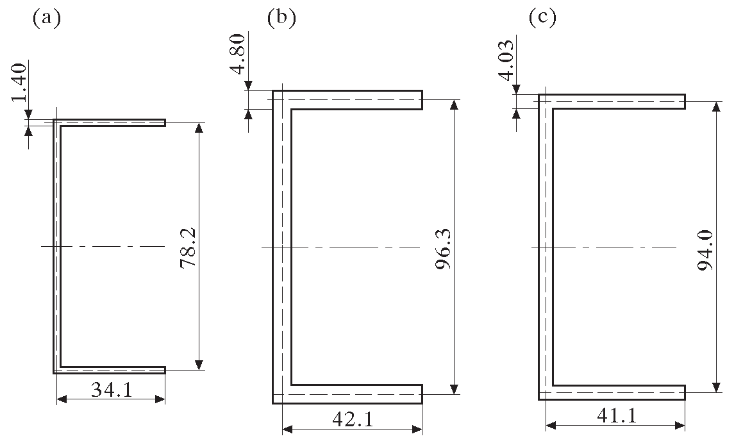

Fig. 4. Optimal profiles of channels with the same weight $(Q^* = 0.2)$ for $L = 1\,\mathrm{m}$: (a) steel; (b) composite A; (c) composite B

Table 2. Comparison of the critical loads for two optimal channels obtained in the present analysis and by ANSYS 5.7

		Present analysis		ANSYS 5.7	
Optimal variants of channel	Q^*	Local buckling [kN]	Overall (Euler) buckling [kN]	Local buckling [kN]	Overall (Euler) buckling [kN]
Composite A	0.865	153 $(m = 4)$	180.3	145.9 $(m = 4)$	181.7
Composite B	0.71	32.3	39.2	30.5	39.5

Typical results of such comparison are presented in Table 2, where the values of critical loads for the local mode and the overall flexural (Euler) mode, obtained in the present analysis and by using ANSYS 5.7 are given for two optimal channels from Table 1.

We see that the critical loads in both models are close to those in local and overall modes (for the local modes ANSYS 5.7 gives a little lesser value, for the Euler overall modes – a slightly greater value). So, we can conclude that, at least for the adopted material properties, our analysis is valid.

The optimal configurations of channels of the isotropic material and the composite with the same area are essentially different. The composite channels have a larger thickness, but lesser width of the web and flanges. If to compare channels of the same weight then all dimensions of the composite channel become larger than those of the isotropic channel – the thickness gets larger by several times.

The comparative efficiency of the composite channels is determined, first of all, by E_x/E_0 and ρ/ρ_0 ratios. The optimal member of composite A, for which the modulus E_x lower than that of steel only by 30%, and density – almost by 4 times, has the critical force 4 times larger than that of the optimal steel channel of the same weight. At the same time the channel of composite B, for which the modulus E_x equals about $0.19E_0$, turns out to be less efficient than the isotropic steel channel.

It has been revealed in Manevich and Raksha (2000) that optimal channels of an isotropic material have a practically constant flange width to a web width the ratio b_2/b_1 (in the whole range of Q^* considered) equal to $b_2/b_1 = 0.42$-0.43. For the optimal composite channels this ratio it also almost constant, and practically the same ($b_2/b_1 = 0.43$-0.44).

Other nondimensional parameters depend upon the parameter Q^*. In Fig. 5 cross-sectional nondimensional parameters are given versus Q^* for composite A and for the isotropic material. Note that these curves do not depend on the value E_x and are determined only by the ratios E_y/E_x, G/E_x and therefore are close to composites A and B (but differ from the corresponding dependencies for the isotropic material).

In Fig. 6 curves for the parameter t/L versus Q^* for the isotropic and composite materials are presented. Values of the nondimensional load parameter P^* and critical stresses σ^* versus Q^* are given in Fig. 7 and Fig. 8.

The presented optimal nondimensional parameters enable us to determine all cross-sectional dimensions when L and E_x are given.

Let us estimate the influence of the elasticity modulus in two directions and shear modulus on the critical load of optimal composite channels. Nondi-

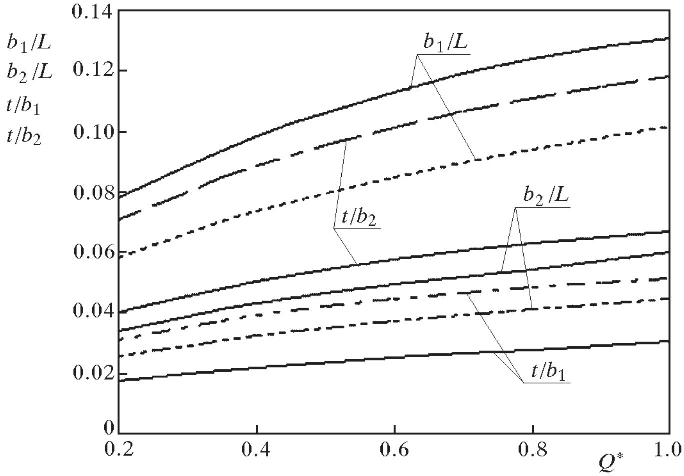

Fig. 5. Cross-sectional nondimensional parameters b_1/L, b_2/L, t/b_1, t/b_2 versus Q^* for composite A (broken line) and for the isotropic material (full line)

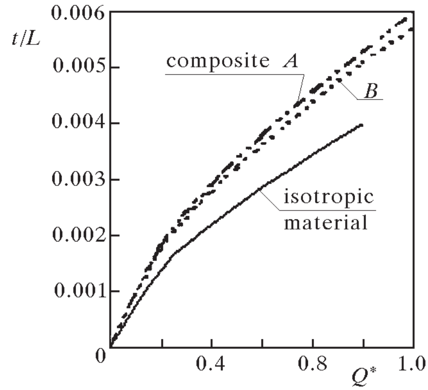

Fig. 6. Parameter t/L versus Q^* for the isotropic and composite materials

mensional load parameter (2.2) depends upon two parameters of the material: $\eta_1 = E_y/E_x$, $\eta_2 = G/E_x$. Calculations of the sensitivity of the critical load to changes in these parameters for the optimal channel of composite A at $Q^* = 0.865$ (i.e., $Q^*_{eff} = 0.2$) were carried out. Calculations were performed in two variants:

- cross-section remained invariable;
- cross-section dimensions were changed so that the cross-section became the optimal for the composite with changed properties (i.e. at simultaneous optimization of the channel with the same area).

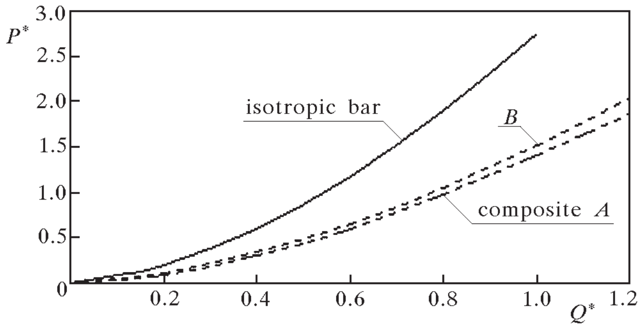

Fig. 7. Nondimensional load parameter P^* versus Q^* for the isotropic and composite materials

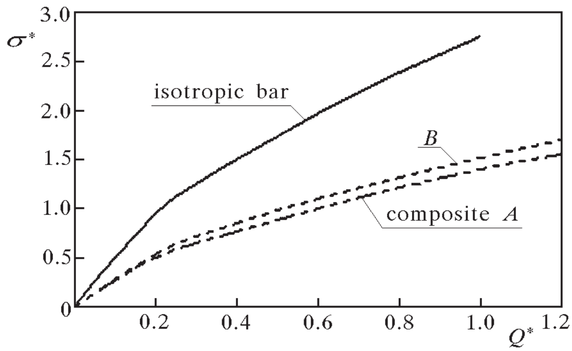

Fig. 8. Nondimensional critical stresses σ^* versus Q^* for the isotropic and composite materials

In the first case the sensitivity coefficients were calculated separately for those buckling modes that govern the carrying capacity – the overall mode (torsional-flexural) and the local mode with minimal critical stresses. In the second case the sensitivity coefficients referred simultaneously to all critical modes. For more generality, the relative sensitivity coefficients, i.e. coefficients k_1 and k_2 in the relations

$$\frac{\delta P}{P} = k_1 \frac{\delta \eta_1}{\eta_1} \qquad\qquad \frac{\delta P}{P} = k_2 \frac{\delta \eta_2}{\eta_2}$$

were calculated.

These coefficients are presented in Table 3. In the case of an invariable cross-section the influence of the parameters η_1 and η_2 on the overall stability is found to be lesser than on the local buckling (especially for the parameter η_1). So, when η_1 is increased by 10% the local critical load increases by 4.4%, but the overall critical load – only by 0.58%; for the parameter η_2

these values equal 4.4% and 1.7%, respectively. This is clearly since the overall buckling is governed mainly by the modulus E_x.

Table 3. Sensitivity coefficients for the critical load with respect to parameters η_1 and η_2

Coefficient	Invariable cross-section		Optimized cross-section
	overall buckling	local buckling	
k_1	0.058	0.44	0.086
k_2	0.17	0.44	0.214

If the cross-section is optimized simultaneously with the varying parameters η_1 and η_2 the sensitivity coefficients lie between their values for the overall and local modes obtained for the fixed cross-section. The parameter η_2 has more influence on the critical load than η_1: when η_1 is increased by 10% the critical load increases by 0.86%, when η_2 is increased by 5% the load increases by 2.14%.

3.2. Three-layered channel

Calculations were carried out for the following core material: 6-layer graphite-epoxy material with the parameters $E_y/E_x = 0.0796$, $G/E_x = 0.04316$, $E_x = 1.393 \cdot 10^{11}$ Pa, $\rho = 1.8\,\mathrm{g/cm^3}$ (material A, Section 3.1). Values of the weight parameter Q^* were considered in the interval (0,...,0.35) for which the use of the linear stability theory for elastic materials can be justified.

The computations showed that the strength- and/or geometrical constraints are of principal importance for the optimization problem. In distinction from the isotropic thin-walled bar, the solution to the weight optimization problem with respect only to buckling constraints for usual elastic materials has no sense (it results in unrealistic thin external layers and very high critical stresses considerably exceeding the yield stress for usual materials). Only when the geometrical and/or strength constraints are imposed we come to realistic optimal projects. We considered the following values of the relative minimal skin thickness: $t^*_{\min} = t_{1,\min} \cdot 10^3/L = 0.1$; 0.15; 0.2; 0.25; 0.3 and dimensionless yield stress values (for the skin): $\sigma^*_y = \sigma_y \cdot 10^3/E_1$ in the range (1.0; 2.2).

Optimal bars with constraint on minimal thickness of external layers

Consider the results of the solution obtained with the only geometrical constraint on the minimal thickness of skin layers $t^*_{\min} = 0.2$ mm. This constraint is always active, so for all these optimal projects $t_1/L = t_{\min}/L = 0.0002$. In Table 4 the optimal nondimensional cross-section parameters and critical values of the load and stresses for several values of the weight parameter Q^* are presented.

Table 4. Optimal nondimensional parameters of the 3-layered channel ($t^*_{\min} = t_{1,\min} \cdot 10^3/L = 0.2$)

	Geometrical parameters			Critical load and stress values		
Q^*	b_1/L	b_2/L	h/L	P^*	σ^*_{av}	σ^*_1
0.10	0.0375	0.0296	0.00357	0.237	0.687	0.940
0.15	0.0468	0.0341	0.00492	0.5	0.884	1.226
0.20	0.0540	0.0376	0.00614	0.832	1.049	1.464
0.25	0.0602	0.0404	0.00686	1.222	1.193	1.673
	0.1170	0.0512	0.00409	1.289	1.434	1.975
0.30	0.1246	0.0544	0.00483	1.792	1.591	2.204

For $Q^* = 0.25$ there exist two local optima, both presented in Table 4. These local optima have essentially different cross-section dimensions – *narrow* and *wide* web and flanges, and correspondingly *large* and *small* wall thickness, but the values of the critical loads are rather close (two local optima also exist for other Q^* values, but for lower Q^* the first optimum is global, for the larger ones – the second, and transition from the first local optima to the second occurs approximately at $Q^* = 0.25$). Note that at the first local optimum three-layered channels have equal critical loads for the flexural mode (in the symmetry plane) and for the flexural-torsional mode. At the second local optimum the bars have equal critical loads for the flexural-torsional mode and one of local modes (critical stresses for the flexural mode are found to be larger by up to 20%).

In the last three columns of Table 4 the values of the load parameter P^*, nondimensional average (over the cross-section) stress σ^*_{av} and the critical stress in the external (steel) layer σ^*_1 are given. We see that with an increasing Q^* the critical load P^* quickly raises as well as the critical stress in the external layer, and this stress becomes larger than the yield stress for usual steels.

In Table 5 cross-section dimensions, critical loads for the optimal three-layered channel of the length $L = 1\,\mathrm{m}$ for $Q^* = 0.2$ and the optimal single-layered (steel, composite A) channel of the same weight are compared.

Table 5. Cross-section dimensions and critical loads for the optimal single- and three-layered channel; $Q^* = 0.2$, length $L = 1\,\mathrm{m}$

Type of bar	h [mm]	b_1 [mm]	b_2 [mm]	P^*	P [kN]
1-layered, steel	1.4	78.2	34.1	0.19	38
1-layered, composite	4.8	96.3	42.1	1.099	153
3-layered, steel+composite	6.14	54.0	37.6	0.832	166.4

The optimal configurations of laminated channel columns essentially differ from those made of a homogeneous material. Big advantage of the optimal three-layered channel is due to high characteristics of the composite layer, i.e. low density and high elasticity modulus E_x.

In order to test the results obtained on the basis of the Kirchhoff hypothesis, calculations were carried out with using ANSYS 5.7. Spatial (3D) elements were employed for the composite (core) and shell elements – for the steel (external) layers. For the optimal 3-layered channel, presented in the last line of Table 5, the critical force in ANSYS 5.7 was 167.3 kN, i.e. the difference was less than 0.4%.

Influence of constraint on minimal thickness upon optimal bars

Let us consider now results of the solution for various values of $t^*_{\min}$. The characteristics of the optimal channels for the load parameter $Q^* = 0.2$ at various values of $t^*_{\min}$ are presented in Table 6.

Table 6. Optimal channels for the load parameter $Q^* = 0.2$ at various values of $t^*_{\min} = t_{1,\min} \cdot 10^3/L$

$t^*_{\min}$	Geometrical parameters			Critical load and stress values		
	b_1/L	b_2/L	h/L	P^*	σ^*_{av}	σ^*_1
0.10	0.0606	0.0370	0.00663	0.867	0.971	1.376
0.15	0.0572	0.0374	0.00637	0.856	1.017	1.431
0.20	0.0540	0.0376	0.00614	0.832	1.049	1.464
0.25	0.0511	0.0376	0.00592	0.799	1.069	1.481
0.30	0.1052	0.0467	0.00263	0.667	1.275	1.665

An increase in the minimal thickness of the external (skin) layer results in perceptible changes in the geometrical parameters $(b_1/L, h/L)$ and reduction of the critical load. So, we can draw the conclusion that the weight efficiency of the three-layered channel largely depends upon the constraint on the minimal skin thickness.

Influence of strength constraint upon optimal parameters

Calculations of optimal channels with constraints on the ultimate stress were carried out. We assumed that $Q^* = 0.3$, $t^*_{\min} = 0.2$ and varied the values $\sigma^*_y = \sigma_y \cdot 10^3/E_1$ in a certain range. The results of the calculations are presented in Table 7.

Table 7. Optimal channels for $Q^* = 0.3$, $t^*_{\min} = 0.2$ at various values of $\sigma^*_y = \sigma_y \cdot 10^3/E_1$

σ^*_y	Geometrical parameters			Critical load and stress values		
	b_1/L	b_2/L	h/L	P^*	σ^*_{av}	σ^*_1
2.2	0.1246	0.0544	0.00485	1.792	1.591	2.2
2.0	0.1116	0.0487	0.00558	1.675	1.436	2.0
1.8	0.0520	0.0416	0.00949	1.639	1.277	1.8
1.6	0.0381	0.0387	0.01140	1.488	1.131	1.6
1.4	0.0313	0.0360	0.01290	1.319	0.988	1.4
1.2	0.0264	0.0330	0.01460	1.143	0.846	1.2
1.0	0.0220	0.0298	0.01680	0.964	0.704	1.0

The strength constraint becomes active when $\sigma^*_y < 2.20$ (for given Q^* and $t^*_{\min}$). At $\sigma^*_y \approx 1.9$ transition from one local optimum to the other occurs, with a snap in the optimal parameters. The strength constraint strongly affects the optimal projects, and it should be taken into account. The same regards the geometrical constraints.

Summarizing, we can conclude that three-layered members under compression may be much more efficient in comparison with columns made of homogeneous materials.

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Optymalne projektowanie kompozytowych słupów o przekroju otwartym poddanych ściskaniu

Streszczenie

Cienkościenne słupy o otwartym przekroju poprzecznym są efektywnymi elementami konstrukcjami (z uwzględnieniem stateczności globalnej). Bardziej wydajne wagowo słupy o ścianach płaskich mają niższą sztywnością na wyboczenie lokalne. Użycie kompozytów i materiałów wielowarstwowych jest jedną z perspektywicznych dróg podniesienia ich obciążalności. Problem minimalnej wagi takich laminatowych słupów nie był do chwili obecnej jeszcze rozpatrywany. W prezentowanej pracy przedstawiono rozwiązanie zagadnienia optymalizacji konstrukcji trzywarstwowego ceownika poddanego ściskaniu. Zagadnienie optymalizacji sformułowano jako problem programowania nieliniowego. Podstawowymi ograniczeniami są warunki globalnej i lokalnej stateczności. Ponadto uwzględniono wytrzymałość, pomarszczenie i geometryczne ograniczenia (na minimalną grubość okładzin oraz maksymalną całkowitą grubość). Dla zadanych stałych materiałowych zostały określone optymalne stosunki wymiarów przekroju i grubości (dla stalowej i kompozytowych warstw). Trzywarstwowa konstrukcja jest bardziej wydajna na ściskanie niż słup z materiału homogenicznego. Analizowano także czułość optymalnej zmiennej projektowej na własności materiałowe (w szczególności zmiana modułów sprężystości na rozciąganie i ścinanie).

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