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ACTIVE AND PASSIVE DAMPING OF VIBRATIONS BY TORSIONAL DAMPER DURING STEADY STATE MOTION OF A POWER TRANSMISSION SYSTEM

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This paper presents a theoretical study of the process of damping of nonlinear vibrations in a two-mass model of a mechanical system with a torsion damper. The steady-state motion of the system subject to harmonic excitation is considered on the assumption of a uniform frequency and constant amplitude of the forcing torque. Simultaneous structural friction phenomena (passive damping) and the piezoelectric effect (active damping) are been considered as well. The problem is considered on the assumption of a uniform unit pressure distribution between the contacting surfaces of the friction discs and plunger. The aim of the analysis is to asses the influence of geometric parameters, external load, unit pressure and electric parameters on the resonance curves of the steady-state vibrations. The equations of motion of the examined system are solved by means of the Van der Pol method.

Key words: active damping, vibrations, torsional damper, structural friction

1. Introduction

Facing the complexity of problems concerned with structural friction in mechanical systems, including transmission systems with clutches, simplifications of the friction model are being assumed. The elastic strain of discs and plunger as well as the influence of the piezoelectric effect on damping of vibrations in such systems are taken into account. The author based his considerations on previously derived physical formulations of a frictional torsion damper. In the design process of a power transmission system and selection of existing models of dampers, basic standard calculation methods are applied. It is essential, however, to take into account the natural source of vibration damping by means of the structural friction and piezoelectric effect. Development in new materials have made it possible to create the so-called "intelligent" materials (electrorheological fluids FL, ML; alloys with shape memory, piezoelectric polymers). These materials change their properties under the influence of magnetic and electric field, temperature or mechanical stress.

The basic electromechanical properties of the piezoelectric material – lead zirconate titanate (PZT) are presented in the paper in terms of their capacity to damp torsional vibrations in the considered mechanical system. Piezoceramics exhibit natural shear effect, which is three times stronger than the longitudinal one. This phenomenon can be successfully utilized in torsional systems for vibration control, as piezoceramics constitute perfect elements for actuator applications. Active control is achieved through closed loop with proportional and velocity feedback (proportional-plus-derivative controller).

Meng-Kao Yeh and Chih-Yuan Chin (1994) proved the applicability of piezoelectric sensors for measuring torsional vibration of shafts. Introductory theoretical studies of shafts vibration induced by piezoelements based on PZT ceramics were presented by Kurnik and Przybyłowicz (1995), Przybyłowicz (1995), Tzou (1991). Experimental investigations dealing with an actively controlled torsional system were carried out by Chia-Chi Sung *et al.* (1994).

2. Equations of motion

We assume a two-mass model of a mechanical system which contains a frictional torsion damper, as shown in Fig. 1. Structural friction occurs between the cooperating surfaces of discs 2 and plunger 1. Discs 2 are pressed down to plunger 1 by means of springs 3. The shaft is equipped with two piezoelectric elements: actuator 4 and sensor 5. The actuator is posed by a ring-shaped element of considerable thickness, thus, its moment of inertia must be taken into account. The sensor may be made of PZT or with PVDF (piezoelectric foil), yet it must be thin enough to be neglected in the total inertia balance of the system. The actuator and the sensor are electronically coupled with proportional and velocity feedback ruling the performance of the thus arranged control system. Therefore, the equations of motion of the considered system can be written down as follows

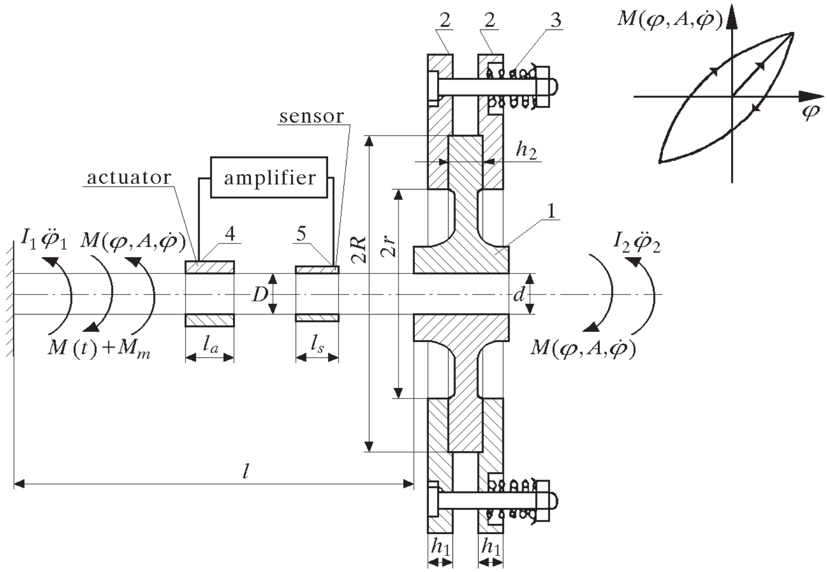

Fig. 1. Physical model of the considered friction torsional damper

$$
\begin{aligned}
&I_1\ddot{\varphi}_1 + M(\varphi, A, \dot{\varphi}) = M_m + M(t) + M_a \\
&I_2\ddot{\varphi}_2 - M(\varphi, A, \dot{\varphi}) = 0
\end{aligned}
\tag{2.1}
$$

where

φ_1, φ_2	–	angular displacements of the active and passive damper
φ, A	–	relative angular displacement of the discs and plunger and its amplitude, respectively
I_1, I_2	–	reduced moments of inertia of the movable parts of the engine, actuator and plunger and of the discs in the damper
$M(t) + M_m$	–	variable engine torque described by a constant average value M_m and a discrete torque $M(t)$ in the form of harmonic excitation with a uniform frequency and constant amplitude as in Giergiel (1990), Grudziński *et al.* (1992), Osiński (1986), Osiński and Kosior (1976), Skup (1987, 1991, 2002), Szadkowski and Morford (1992), Zagrodzki (1994)
M_a	–	torque generated by the actuator.

$M(\varphi, A, \dot{\varphi})$ – damper torque in a cycle represented by a structural hysteresis loop (Fig. 1) dependent on the relative displacement, amplitude and its velocity signum.

Therefore

$$M(t) = M_0 \cos \omega t \tag{2.2}$$

where

M_0	–	excitation amplitude of the forcing torque
ω	–	angular velocity of the excitation torque
t	–	time.

According to the studies presented in works by Skup (1976, 2002), the relationship between the damper torque and the relative angular displacement is as follows

$$\begin{aligned} M(\varphi, A, \dot{\varphi}) &= \frac{k}{\kappa_1} \operatorname{sgn} \dot{\varphi} \Big[2\sqrt{1 + \kappa_1(\varphi - A) \operatorname{sgn} \dot{\varphi}} - 1 - \sqrt{1 - 2\kappa_1 A \operatorname{sgn} \dot{\varphi}} \Big] \\ M_a &= \kappa_2 \operatorname{sgn} \dot{\varphi} \Big(1 - \sqrt{1 - 2\kappa_1 \varphi_s \operatorname{sgn} \dot{\varphi}} \Big) \end{aligned} \tag{2.3}$$

where

$$\begin{array}{lll} k = \dfrac{GI_0}{l} & I_0 = \dfrac{\pi d^4}{32} & \kappa_1 = \dfrac{G^2 I_0^2}{2l^2 k_z \pi^2 p \mu R^5} \\ k_z = \dfrac{k_1 k_2}{k_1 + k_2} & k_1 = Gh_1 & k_2 = Gh_2 \\ \kappa_2 = \dfrac{\pi G_a G_s d_{15}^a d_{15}^s l_s d(D^3 - d^3) k_p k}{12 G l_a I_0 \kappa_1 \varepsilon_0 \varepsilon_s} & & \end{array} \tag{2.4}$$

The following denote: k – stiffness of the elastic shaft of the length l and diameter d, κ_1 – nondimensional parameter, k_1, k_2, h_1, h_2 – discs and plunger stiffness and their thickness, μ – friction coefficient, p – pressure per unit area, R – external radius of the discs, G – shear modulus, I_0 – cross-sectional moment of inertia of the shaft, G_a – shear modulus of PZT material (for actuator), l_a – width of the actuator, d_{15}^a – coupling constant of the actuator, k_p – gain factor introduced by the electronic circuit, d, D – inner and outer actuator diameter, ε_0, ε_s – absolute and relative dielectric permittivity of the sensor, G_s – shear modulus of the sensor material, l_s – width of the sensor, d_{15}^s – electromechanical coupling constant of the sensor.

3. Solution to the equation of motion

After introducing the relative angular displacement

$$\varphi = \varphi_1 - \varphi_2 \tag{3.1}$$

and the reduced moment of inertia

$$I_z = \frac{I_1 I_2}{I_1 + I_2} \tag{3.2}$$

we can transform equations of motion (2.1) into the following form

$$\ddot{\varphi} + \frac{M(\varphi, A, \dot{\varphi})}{I_z} = \frac{1}{I_1}[M(t) + M_m + M_a] \tag{3.3}$$

Let the solution to equation (3.3) be approximated by

$$\varphi = A\cos z \qquad\qquad z = \omega t + \varphi_0 \tag{3.4}$$

where: z – forcing phase, φ_0 – initial forcing phase, A, φ_0 – are slowly varying functions of time t. Then

$$\dot{\varphi} = \dot{A}\cos z - A\dot{\varphi}_0 \sin z - A\omega \sin z \tag{3.5}$$

By analogy to Lagrange's method of variation of a parameter, it is permissible to set

$$\dot{A}\cos z - A\dot{\varphi}_0 \sin z = 0 \tag{3.6}$$

Thus

$$\ddot{\varphi} = -\dot{A}\omega \sin z - A\omega^2 \cos z - A\omega\dot{\varphi}_0 \cos z \tag{3.7}$$

Substituting equation (3.7) into the equation of motion (3.3), using formula $(3.4)_2$, gives

$$-\dot{A}\omega \sin z - A\omega^2 \cos z - A\omega\dot{\varphi}_0 \cos z + \frac{M(\varphi, A, \dot{\varphi})}{I_z} = \frac{M_m + M_a}{I_1} + \frac{M_0}{I_1}\cos(z - \varphi_0) \tag{3.8}$$

By multiplying equation (3.6) by $\omega\cos z$, equation (3.8) by $\sin z$ and substracting the sides while using formula $(3.4)_2$, we obtain

$$\begin{aligned} -\dot{A}\omega - A\omega^2 \sin z \cos z + \frac{M(\varphi, A, \dot{\varphi})}{I_z}\sin z &= \\ = \frac{M_m + M_a}{I_1}\sin z + \frac{M_0}{I_1}\sin z \cos(z - \varphi_0) \end{aligned} \tag{3.9}$$

Since A and φ_0 are slowly varying parameters in equation (3.3), equation (3.9) takes, after integrating over the interval $(0, 2\pi)$, the following form

$$-2\pi\dot{A}\omega + \frac{1}{I_z}\int\limits_0^{2\pi} M(\varphi, A, \dot{\varphi})\sin z\,dz = \int\limits_0^{2\pi}\frac{M_m + M_a}{I_1}\sin z\,dz + \frac{M_0\pi\sin\varphi_0}{I_1} \tag{3.10}$$

Multiplying equation (3.6) by $\omega\sin z$, Eq (3.8) by $\cos z$, adding the sides, using formula $(3.4)_2$ and averaging over one cycle of z, gives

$$\begin{aligned}&-2\pi A\dot{\varphi}_0\omega - \pi A\omega^2 + \frac{1}{I_z}\int\limits_0^{2\pi} M(\varphi, A, \dot{\varphi})\cos z\,dz = \\ &= \int\limits_0^{2\pi}\frac{M_m + M_a}{I_1}\cos z\,dz + \frac{M_0\pi\cos\varphi_0}{I_1}\end{aligned} \tag{3.11}$$

Steady-state equations (3.10) and (3.11) can be obtained when $\dot{A} = \dot{\varphi}_0 = 0$, therefore these equations are reduced to the form

$$\begin{aligned}\sin\varphi_0 &= \frac{1}{\pi\beta M_0}\int\limits_0^{2\pi} M(\varphi, A, \dot{\varphi})\sin z\,dz - \frac{1}{\pi M_0}\int\limits_0^{2\pi}(M_m + M_a)\sin z\,dz \\ I_z\omega^2 + \frac{\beta M_0}{A}\cos\varphi_0 &= \frac{1}{\pi A}\int\limits_0^{2\pi} M(\varphi, A, \dot{\varphi})\cos z\,dz - \frac{\beta}{\pi A}\int\limits_0^{2\pi}(M_m + M_a)\cos z\,dz\end{aligned} \tag{3.12}$$

In accordance with the Ritz method, the integral with equation (3.3) as the integrand must be equal zero. Therefore

$$\int\limits_0^{2\pi}\Big[\ddot{\varphi} + \frac{M(\varphi, A, \dot{\varphi})}{I_z} - \frac{M_m}{I_1} - \frac{M_m + M_a}{I_1} - \frac{M_0\cos(z - \varphi_0)}{I_1}\Big]\partial\varphi\,dt = 0 \tag{3.13}$$

where the variation $\partial\varphi$ is equal

$$\partial\varphi = \partial A\cos z + \partial\varphi_0 A\sin z \tag{3.14}$$

Substituting equation (3.4) and (3.14) into (3.13) and basing on the linear independence of the variations ∂A and $\partial\varphi_0$, we can obtain two independent equations for the displacement amplitude A and phase shift angle φ_0. We obtain a result which is identical with equations (3.12).

While integrating Eqs (3.12) there is a discontinuity of $M(\varphi, A, \dot{\varphi})$ and M_a encounterred for $\dot{\varphi} = 0$. To avoid this, we confine our considerations to a single half-period (motion between two stops).

Thus, the integration interval (from 0 to 2π) of the right-hand terms of the above equations is divided into two sub-intervals, from 0 to π for negative $d\varphi/dt$ and from π to 2π for positive $d\varphi/dt$. This is, for instance, the procedure adopted by Giergiel (1990), Osiński (1986), Osiński and Kosior (1976), Skup (1991, 2002).

Therefore, substitution of formulas (2.3) into equations (3.12), and subsequent integration gives after some transformations

$$
\begin{aligned}
&\sin\varphi_0 = \frac{1}{\pi\beta M_0}\Big(\int\limits_0^{\pi} M(\varphi, A, \dot{\varphi}) \sin z \; dz\Big|_{\operatorname{sgn}\dot{\varphi}<0} + \\
&+ \int\limits_{\pi}^{2\pi} M(\varphi, A, \dot{\varphi}) \sin z \; dz\Big|_{\operatorname{sgn}\dot{\varphi}>0}\Big) + \\
&- \frac{1}{\pi M_0}\Big(\int\limits_0^{\pi} (M_m + M_a) \sin z \; dz\Big|_{\operatorname{sgn}\dot{\varphi}<0} + \int\limits_{\pi}^{2\pi} (M_m + M_a) \sin z \; dz\Big|_{\operatorname{sgn}\dot{\varphi}>0}\Big) = \\
&= \frac{2k}{\pi M_0}\Big\{\frac{1}{\beta\kappa_1}\Big[\sqrt{x} + \sqrt{y} + 2 + \frac{2}{3\kappa_1 A}(\sqrt{y^3} - \sqrt{x^3})\Big] - \kappa_2(\sqrt{x} + \sqrt{y} - 2)\Big\} \\
&x = 1 + 2\kappa_1 A \qquad\qquad y = 1 - 2\kappa_1 A
\end{aligned}
\tag{3.15}
$$

$$
\begin{aligned}
&I_z\omega^2 + \frac{\beta M_0}{A}\cos\varphi_0 = \frac{1}{\pi A}\Big(\int\limits_0^{\pi} M(\varphi, A, \dot{\varphi}) \cos z \; dz\Big|_{\operatorname{sgn}\dot{\varphi}<0} + \\
&+ \int\limits_{\pi}^{2\pi} M(\varphi, A, \dot{\varphi}) \cos z \; dz\Big|_{\operatorname{sgn}\dot{\varphi}>0}\Big) + \\
&- \frac{\beta}{\pi A}\Big(\int\limits_0^{\pi} (M_m + M_a) \cos z \; dz\Big|_{\operatorname{sgn}\dot{\varphi}<0} + \int\limits_{\pi}^{2\pi} (M_m + M_a) \cos z \; dz\Big|_{\operatorname{sgn}\dot{\varphi}>0}\Big) = \\
&= k\Big(1 + \frac{15}{32}\kappa_1^2 A^2\Big)
\end{aligned}
$$

Introducing the notations:

a	–	dimensionless vibration amplitude, $a = A/\varphi_{st}$
φ_{st}	–	static displacement in the form of the relative angular displacement of the damper plates, $\varphi_{st} = M_0/k$

γ – dimensionless frequency, $\gamma = \omega/\omega_0$
ω_0 – frequency of the free vibration of the system, $\omega_0 = \sqrt{k/I_z}$

we find

$$\begin{aligned}
&\sin\varphi_0 = \frac{2k}{\pi M_0}\Big\{\frac{1}{\beta\kappa_1}\Big[\eta_1 + 2 + \frac{2}{3a\psi}(\sqrt{y_1^3} - \sqrt{x_1^3})\Big] - \kappa_2(\eta_1 - 2)\Big\} \\
&\gamma^2 + \frac{\beta}{a}\cos\varphi_0 = 1 + \frac{15}{32}a^2\psi^2
\end{aligned} \tag{3.16}$$

where

$$\begin{aligned}
&\psi = \frac{M_0\kappa_1}{k} \qquad \kappa_1 A = a\psi \qquad x_1 = 1 + 2a\psi \\
&y_1 = 1 - 2a\psi \qquad \eta_1 = \sqrt{x_1} + \sqrt{y_1}
\end{aligned} \tag{3.17}$$

Finally, in basic equations (3.16) we have the relations for the tangent of the phase angle φ_0 and the dimensionless frequency γ as functions of the external load, geometric and electric parameters, friction forces and the dimensionless amplitude a. Therefore

$$\begin{aligned}
&\tan\varphi_0 = \frac{2k\beta\Big\{\frac{1}{\beta\kappa_1}\Big[\eta_1 + 2 + \frac{2}{3a\psi}(\sqrt{y_1^3} - \sqrt{x_1^3})\Big] - \kappa_2(\eta_1 - 2)\Big\}}{\pi M_0 a\Big(1 + \frac{15}{32}a^2\psi^2 - \gamma^2\Big)} \\
&\gamma = \sqrt{1 + \frac{15}{32}a^2\psi^2 \mp \frac{\beta}{a}\sqrt{1 - \sin^2\varphi_0}}
\end{aligned} \tag{3.18}$$

4. Numerical results

The following data has been taken for numerical calculations:
$h_1 = 0.004\,\mathrm{m}$, $h_2 = 0.010\,\mathrm{m}$, $R = 0.070\,\mathrm{m}$, $D = 0.070\,\mathrm{m}$, $M_0 = 20\,\mathrm{Nm}$, $\mu = 0.25$, $d = 0.054\,\mathrm{m}$, $I_1 = 0.560\,\mathrm{kgm}^2$, $I_2 = 0.04\,\mathrm{kgm}^2$, $l = 0.65\,\mathrm{m}$, $p = 0.8\cdot 10^5\,\mathrm{N/m}^2$, $l_a = 0.04\,\mathrm{m}$, $l_c = 0.04\,\mathrm{m}$, $G = 8.2\cdot 10^{10}\,\mathrm{N/m}^2$, $G_a = 6.3\cdot 10^9\,\mathrm{N/m}^2$, $G_s = 2\cdot 10^9\,\mathrm{N/m}^2$, $d_{15}^a = 5.6\cdot 10^{-10}\,\mathrm{m/V}$ – for PZT (PIC 255), $d_{15}^s = 0.23\cdot 10^{-10}\,\mathrm{m/V}$ – for PVDF, $\varepsilon_s = 12$, $\varepsilon_0 = 0.088\cdot 10^{-10}\,\mathrm{F/m}$, $k_p = 1.0$, $a = 0.25$.

On the basis of the results of numerical analysis it has been found that all resonance curves start from zero, pass through a resonance and tend again asymptotically to zero in the superresonance range (Fig. 2 to Fig. 4). They also become more steep in that range. In Fig. 2a we can see that the growing amplitude M_0 leads to a decrease in the dimensionless amplitudes a. It obviously results from the fact that increment in M_0 entails enlargement of the slip zone, thus the amount of the dissipated energy grows.

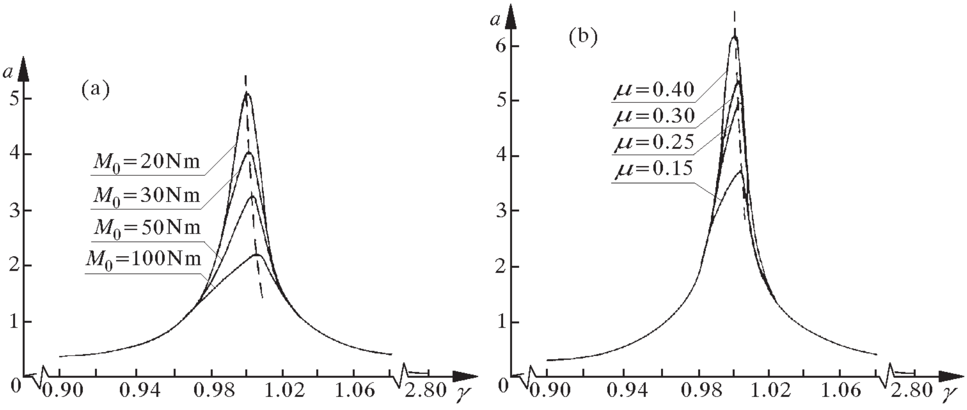

Fig. 2. Resonant curves for various values: (a) of the excitation torque amplitude M_0, (b) of the friction coefficient μ

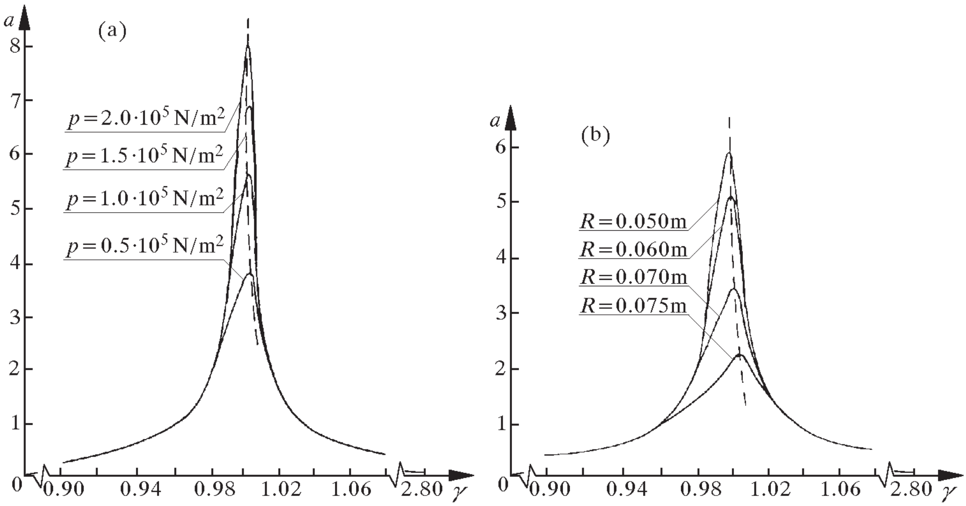

Fig. 3. Resonant curves for various values: (a) of the unit pressure p, (b) of the external radius R

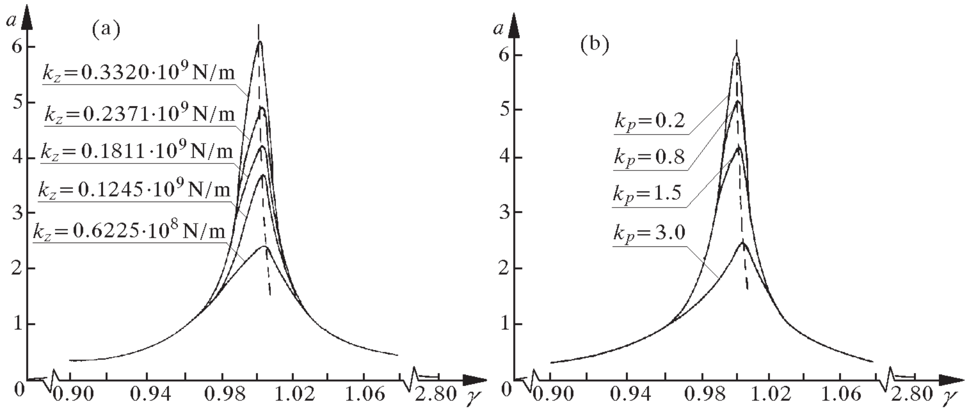

Fig. 4. Resonant curves for various values: (a) of the discs and plunger equivalent rigidity k_z, (b) of the gain factor k_p

An extreme damping intensity can be observed for a particular value of the friction forces $q = p\mu$ (the largest zone of the slip between the discs and plunger surfaces). Figures 2a, 3a disclose a clear effect of the changing of values of μ and p on the nondimensional resonant amplitudes a (all the other parameters remain fixed). The response curves for the two-degree-of-freedom power transmission system reduced to a one-degree-of-freedom non-linear hysteretic system are reveal a typical "soft" resonance (Fig. 2 to Fig. 4). For the excitation frequency ω close to the natural frequency of vibrations ω_0, the nondimensional amplitudes a assume big values. As the presented graphs (Fig. 2 to Fig. 4) and numerical calculations show, the most dangerous frequency range for the real values of the parameters of the damper is $0.98 < \gamma < 1.02$.

The effect of damping is also best for a suitable value of the external radius R and a reduced discs rigidity k_z because of the zone of the biggest slide of the discs in the plunger. The graphs in Fig. 3b and Fig. 4a can serve as an illustration of this conclusion as they show that with the same geometric and electric parameters as well as loading and friction coefficient but varied radius R and reduced rigidity k_z, their influence on the resonance amplitude a is clearly visible. The greater amplifier gain k_p is the more visible the effect (Fig. 4b).

The phase shift angle φ_0 is a measure of the vibration damping in a mechanical system. If it increases, then the energy dissipation also increases and, consequently, so does the damping. The dependence of the phase shift angle φ_0 upon the dimensionless frequency γ for different values of the external radius R is represented by diagrams in Fig. 5a and, for different values of the friction coefficient μ, in Fig. 5b. A clear influence of the external radius R and friction coefficient μ on the angle φ_0 can be observed there. The energy dissipation is the largest for big values of R (Fig. 5a) and μ (Fig. 5b) (big slide zone).

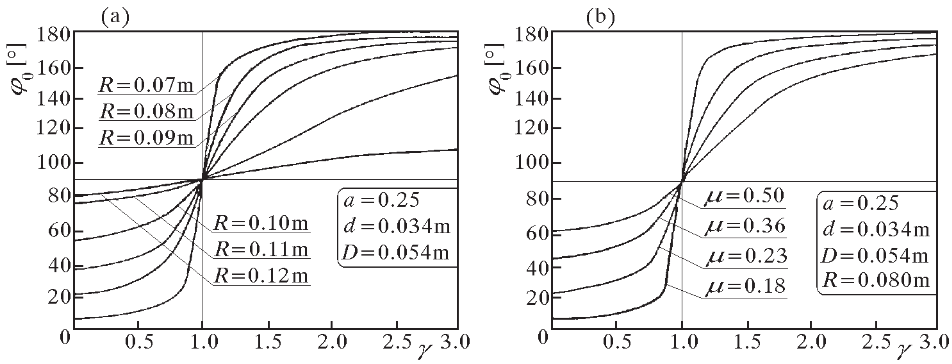

Fig. 5. Pphase displacement angle φ_0 as a function of the dimensionless frequency γ for various values: (a) of the external radius R, (b) of the friction coefficient μ

5. Concluding remarks

Structural friction between contacting surfaces of the discs and plunger, causes increased performance of the examined system as for as vibration damping is concerned. Further development of the damping level is achieved by making use of piezoelectric materials controlled by an electronic circuit. Properties of piezoelectric materials can be utilized in torsional systems for vibration control as piezoceramics constitute perfect elements for actuator applications.

Active control by piezoelectric elements and structural friction proves to be a powerful tool in reducing the vibration amplitude of torsional systems. Application of the electronic damping gives excellent results, especially in systems with velocity feedback – even for different points of actuator application and spatial sensor/actuator dislocation. This is highly important since real technical conditions may not always allow arbitrary piezoelements application. The paper emphasizes some aspects related to the active control under more realistic conditions.

Admittelly, it should be said that the vibration damping by friction dampers is considerably influenced by the following factors: forcing amplitude, stiffness of the discs and plunger, unit pressure, friction coefficient and gain. The examined system has "soft" frequency characteristic and attenuation diagram.

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Aktywne i pasywne tłumienie drgań poprzez tłłumik drgań skrętnych podczas ruchu ustalonego układu napędowego

Streszczenie

Praca dotyczy teoretycznego badania tłumienia drgań nieliniowych układu mechanicznego o dwóch stopniach swobody zawierającego tłumik drgań skrętnych. Rozważany jest ruch ustalony układu przy wymuszeniu harmonicznym z jednostajną częstością o stałej amplitudzie momentu wymuszającego. Uwzględniono tłumienie drgań jednocześnie poprzez zjawisko tarcia konstrukcyjnego (tłumienie pasywne) i elementy piezoelektryczne (tłumienie aktywne). Zagadnienie rozpatrywane jest przy założeniu równomiernego rozkładu nacisków jednostkowych występujących pomiędzy współpracującymi powierzchniami tarcz ciernych i bezwładnika. Zbadano wpływ parametrów geometrycznych układu, nacisku jednostkowego, obciążenia zewnętrznego oraz parametrów elektrycznych na krzywe rezonansowe drgań ustalonych. Równania ruchu badanego układu mechanicznego rozwiązano metodą Van der Pola.

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