LOCAL AND GLOBAL INSTABILITY AND VIBRATION OF OVERBRACED EULER'S COLUMN

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The theoretical and experimental research on the stability and free vibration of a geometrically non-linear Euler's column is presented in the paper. The influence of the asymmetry of the flexural stiffness distribution of the rod in an individual column on the value of the critical force and existence of local and global instability regions is determined. The column consists of three planar members. Two outside members have the same flexural stiffness while the third central one has different flexural stiffness. The members are jointed in such a way that their deflection at the joint points equals zero and the deflection angles are identical.

Key words: elastic column, divergence instability, natural frequency

1. Introduction

Geometric research on non-linear columns subjected to Euler's load (with a longitudinal force with the fixed-point of force application and a fixed direction) was presented by Godley and Chilver (1970), Tomski (1985), Przybylski and Tomski (1992), Tomski and Kukla (1992), Przybylski *et al.* (1996), Tomski *et al.* (1997).

The considered systems (in the bending plane) consisted of three beams with asymmetrical distribution of the flexural rigidity was investigated by Godley and Chilver (1970), Przybylski and Tomski (1992), Tomski and Kukla (1992), Przybylski *et al.* (1996) or co-axially mounted rod and pipe by Tomski (1985), Tomski *et al.* (1997). Those three beams or rod and pipe were connected in such a way that both displacements and deflection angles were identical at the end of the columns.

The following problems were presented in the above mentioned above papers:

- buckling of a pre-stressed column (theoretical and experimental research)

- pre-buckling behaviour of a cantilever beam (without pre-stressing) in the range from linear to bending line (Tomski, 1985) (theoretical research)

- natural vibration and stability of the pre-stressed hinged-clamped column (Przybylski and Tomski, 1992; Tomski and Kukla, 1992) (theoretical research) and Przybylski *et al.* (1996) (theoretical and experimental research)

- free vibration and stability of a column with a member developing rotational inertia (Tomski *et al.*, 1997) (theoretical and experimental research).

The small parameter perturbation method (Nayfeh, 1973) and the Ritz method (Tomski and Kukla, 1992) were applied to solve natural vibration problems by Przybylski and Tomski (1992), Przybylski *et al.* (1996), Tomski *et al.* (1997).

A pre-stressed geometrically non-linear system (externally unloaded) was tested (theoretically and experimentally) by Tomski *et al.* (1994). It was a cantilever beam with a mass at the unloaded end. The geometrically non-linear system was also considered by Tomski and Kukla (1990), but it was constructed in a different way than in the above quoted papers. The system consists of a column with hinged-clamped ends and an axially located spring (at one end). The column is loaded eccentrically by a longitudinal force.

The natural vibration frequency depends on the pre-stress of the system, which is the feature of geometrically non-linear systems.

The system, constructed from three beams with identical bending line, is considered by Kerr (1976). Such a system is geometrically linear. It was stated, on the basis of theoretical and experimental research, that the pre-stress does not influence the natural vibration frequency – what is obvious in the case of geometrically linear systems.

The theoretical research, concerning the stability of a geometrically non-linear column (overbraced frame), is conducted in this paper. The influence of the asymmetry of the flexural stiffness distribution of the individual column rods on the critical force value and the existence of local and global instability regions was determined.

The experimental research on the course of the load – natural vibration frequency curve in the case of local and global stability loss in the plane is carried out. The influence of pre-stress on the stability of the considered system in the range of local and global stability loss is also analysed.

2. Column classification

Three systems $(A, B$ and $C)$, presented in Fig. 1, are examined in the paper. Both the flexural rigidity $E_i J_i$ and the axial rigidity $E_i A_i$ are identical for all systems. Additionally, taking into account the symmetry of the considered system in the plane perpendicular to the vibration plane, the following equations are assumed

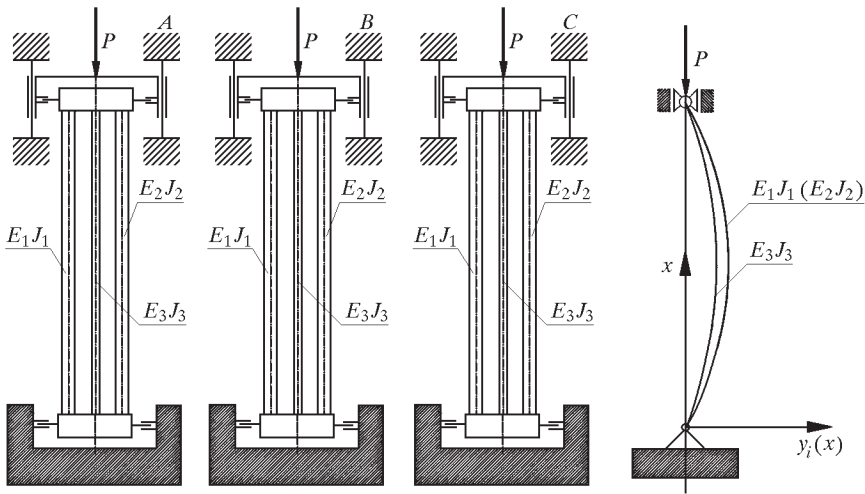$$E_1 J_1 = E_2 J_2 \qquad E_1 A_1 = E_2 A_2 \qquad (2.1)$$

Fig. 1. Physical schemes of the considered columns

The characteristic of columns is presented as follows:

A – geometrically non-linear column (overbraced planar frame). The system consists of three rods with the symmetrical flexural stiffness distribution $E_1 J_1$, $E_2 J_2$, $E_3 J_3$ and the compressive stiffness distributions $E_1 A_1$, $E_2 A_2$, $E_3 A_3$, respectively. The longitudinal displacements at the point

of the hinged clamping of all rods are identical. Taking into account Hooke's law and the influence of the pre-stress, the distributions of forces in the rods are as follows

$$S_1 = P\frac{E_1 A_1}{\sum\limits_{i=1}^{3} E_i A_i} + \frac{S_0}{2} \qquad\qquad S_2 = P\frac{E_2 A_2}{\sum\limits_{i=1}^{3} E_i A_i} + \frac{S_0}{2}$$

$$S_3 = P\frac{E_3 A_3}{\sum\limits_{i=1}^{3} E_i A_i} - S_0 \tag{2.2}$$

The pre-stressing of the geometrically non-linear column induces an additional internal force. This force causes that:

- rods with $E_1 J_1$, $E_2 J_2$ flexural stiffness are totally compressed with the force S_0
- rod with $E_3 J_3$ flexural stiffness is stretched with the force S_0.

Because of the pre-stress, suitable denotations of the geometrically non-linear column (A) are introduced in the further part of the paper:

- AUC – column loaded by an external compressing force without the pre-stress
- APC – column pre-stressed and loaded by an external compressing force.

B – linear column consisting of two rods with $E_1 J_1$, $E_2 J_2$ flexural stiffness and with $E_1 A_1$, $E_2 A_2$ compression (without the third member). The rod stiffness of column B is the same as for rods with index 1, 2 of column A at the assumed asymmetry of the stiffness of the geometrically non-linear column.

C – linear column, which consists of three rods like column A, but rod 3 does not transmit the longitudinal force (movable mounting), so

$$S_1 = \frac{P}{2} \qquad\qquad S_2 = \frac{P}{2} \qquad\qquad S_3 = 0 \tag{2.3}$$

3. Solution to the boundary value problem

Equations of motion of the considered geometrically non-linear column, after separation of variables of the $W_i(x,t)$ function with respect to time and

displacement given by

$$W_i(x,t) = y_i(x)\sin(\omega t) \tag{3.1}$$

take the following form

$$E_i J_i y_i^{IV}(x) + S_i y_i''(x) - \rho_i A_i \omega^2 y_i(x) = 0 \qquad \sum_{i=1}^{3} S_i = P \tag{3.2}$$

The boundary conditions in the hinge-mounted points are expressed as

$$y_1(x)\Big|_{\substack{x=0 \\ x=l}} = y_2(x)\Big|_{\substack{x=0 \\ x=l}} = y_3(x)\Big|_{\substack{x=0 \\ x=l}} = 0$$

$$y_1'(x)\Big|_{\substack{x=0 \\ x=l}} = y_2'(x)\Big|_{\substack{x=0 \\ x=l}} = y_3'(x)\Big|_{\substack{x=0 \\ x=l}} = 0 \tag{3.3}$$

$$\sum_{i=1}^{3} E_i J_i y_i''(x)\Big|_{\substack{x=0 \\ x=l}} = 0$$

General solution to equation (3.2) gives

$$y_i(x) = C_{1i}\cosh(\alpha_i x) + C_{2i}\sinh(\alpha_i x) + C_{3i}\cos(\beta_i x) + C_{4i}\sin(\beta_i x) \tag{3.4}$$

where C_{ji} are integration constants $(j = 1, 2, 3, 4)$ and

$$\alpha_i^2 = -\frac{1}{2}k_i^2 + \sqrt{\frac{1}{4}k_i^2 + \Omega_i^2} \qquad \beta_i^2 = \frac{1}{2}k_i^2 + \sqrt{\frac{1}{4}k_i^2 + \Omega_i^2} \tag{3.5}$$

where

$$\Omega_i^2 = \frac{\rho_i A_i \omega^2}{E_i J_i} \qquad k_i = \sqrt{\frac{S_i}{E_i J_i}} \tag{3.6}$$

$i = 1, 2, 3$ for column A and C, while for column B $i = 1, 2$.

By inserting solutions (3.4) into boundary equations (3.3) a transcendental equation for eigenvalues of the considered geometrically non-linear column is obtained (AUC, APC).

4. Buckling load. Stability of columns

The transcendental equations for the critical load value, obtained on the basis of boundary conditions (3.3) with taking into account the identical longitudinal displacements of all rods (1,2) of the considered systems, are as follows:

— A (AUC, APC) column

$$-2k_I^2 k_{II}^2 \mu(\cos k_I \cos k_{II} - 1) + k_I k_{II}(k_I^2 + k_{II}^2 \mu^2) \sin k_I \sin k_{II} + $$

$$+2(k_I^2 + k_{II}^2 \mu)[k_I(\cos k_{II} - 1)\sin k_I + k_{II}\mu(\cos k_I - 1)\sin k_{II}] = 0 \tag{4.1}$$

— B column

$$\sin k_I = 0 \qquad\qquad P_{cr} = \frac{\pi^2(E_1 J_1 + E_2 J_2)}{l^2} \tag{4.2}$$

— C column

$$4\mu(6\mu - k_I^2) - 8\mu(3\mu + k_I^2)\cos k_I + k_I[k_I^2 - 12(\mu - 1)\mu]\sin k_I = 0 \tag{4.3}$$

where according to (2.3) and (3.6), equations for k_I, k_{II} can be written as

$$k_I^2 = \frac{1}{2}l^2(k_1^2 + k_2^2) \qquad\qquad k_{II}^2 = \frac{S_3 l^2}{E_3 J_3} = k_3^2 l^2 \tag{4.4}$$

The flexural stiffness of columns A and C is assumed constant

$$\sum_{i=1}^{3} E_i J_i = idem \tag{4.5}$$

The coefficient of the flexural stiffness distribution μ is defined in the form

$$\mu = \frac{E_3 J_3}{E_1 J_1 + E_2 J_2} \tag{4.6}$$

The following relationship is derived from equations (4.5) and (4.6)

$$\mu = \frac{idem}{E_1 J_1 + E_2 J_2} - 1 \tag{4.7}$$

$k_I \neq 0$ for the considered system and additionally $k_{II} \neq 0$, $\mu \neq 0$ for column A.

4.1. Unprestressed column (AUC)

For comparison of columns A, B and C in relation to the critical force P_c, the critical load parameter λ takes form

$$\lambda = \frac{P_{cr} l^2}{\sum\limits_{i=1}^{3} E_i J_i} \tag{4.8}$$

The stability loss of the AUC and B column (A and B curves, Fig. 2) for a given coefficient of the asymmetry of the flexural stiffness distribution (4.7) in the $\mu = (0,1)$ range is determined on the basis of equations (4.1)-(4.3). Curve B (Fig. 2) represents the course of the critical parameter of the load in accordance with equations (4.2) and (4.8). The rod rigidity of column B is the same as for the rods with index 1,2 of the AUC column at the assumed asymmetry of the flexural stiffness of the geometrically non-linear column. Column B is comparable with column A, so the parameter of the critical load (λ_A, λ_B) is compared to the coefficient of the asymmetry of the flexural stiffness distribution of the column A. Column C (Fig. 1 and Fig. 2) losses its stability in a global way at every considered range of the coefficient μ.
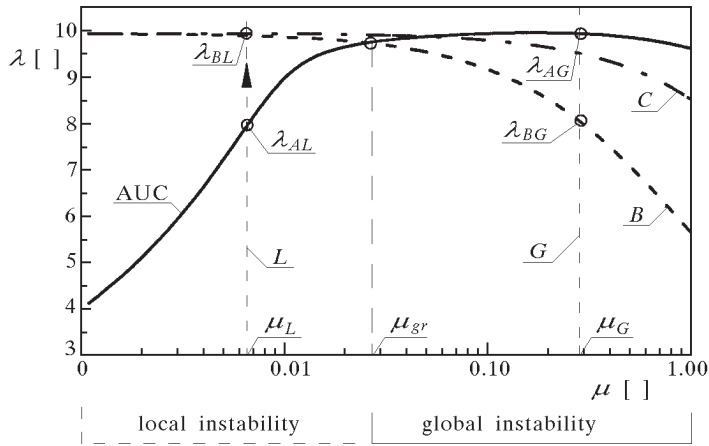

Fig. 2. The value of the critical load parameter λ in relation to the asymmetry coefficient of the flexural stiffness distribution μ

The problem of local and global stability loss can be explained by taking into consideration the values of the critical load parameter for the AUC and B columns. Two values of the asymmetry coefficient of the flexural stiffness distribution μ_L (where $\mu_L \in (0, \mu_{gr})$) and μ_G (where $\mu_G \in (\mu_{gr}, 1)$ were pointed by broken lines in Fig. 2. The local loss of stability $(\lambda_{AL} < \lambda_{BL})$ occurs on the L line, because if the rod with $E_3 J_3$ stiffness is removed from the system then a sudden increase in the critical force takes place (transition from (μ_L, λ_{AL}) point to (μ_L, λ_{BL}) point – Fig. 2). Such a phenomenon takes place for every μ_L. The system will lose its global stability $(\lambda_{AG} > \lambda_{BG})$ for optional μ_G value (G line).

4.2. Columns with prestress and external force (APC)

The influence of prestressing rate on the value of the critical load parameter for a geometrically non-linear Euler's column is determined. This column is characterised by local and global loss of stability, which occurs respectively at $\mu = \mu_L$ and $\mu = \mu_G$ (Fig. 2).
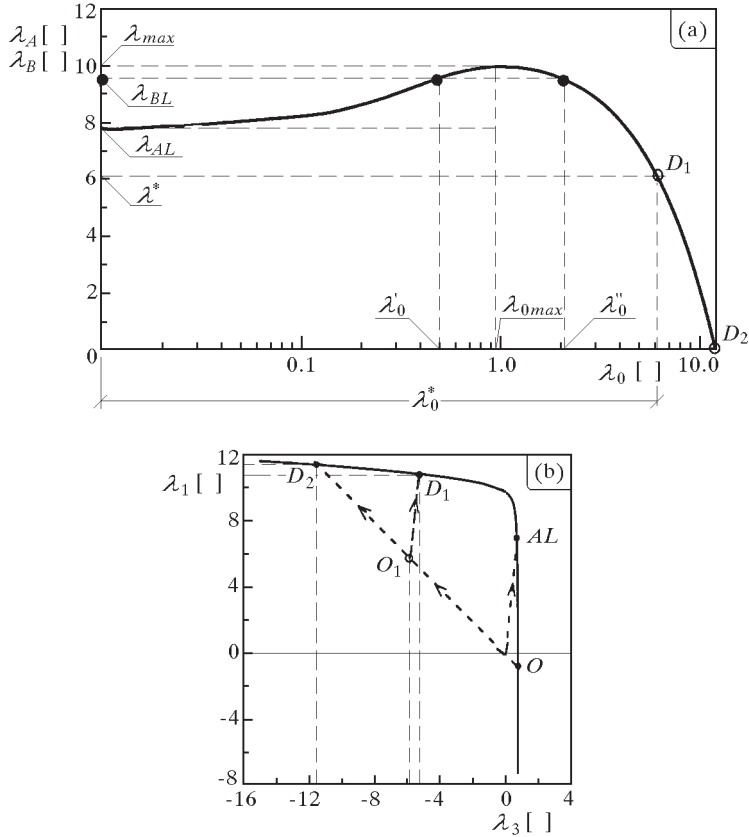

Fig. 3. The critical load parameter λ in relation to pre-stress λ_0 for APC column and asymmetry coefficient μ_L of the flexural rigidity

The numerical results obtained on the basis of equation (4.1), are presented in Fig. 3 and Fig. 4. The changes of the critical load parameter in relation to the λ_0 parameter describing the influence of prestressing (Fig. 3a and Fig. 4a), where

$$\lambda_0 = \frac{S_0 l^2}{\sum\limits_{i=1}^{3} E_i J_i} \tag{4.9}$$

and the range of changes of internal forces λ_1, λ_2 in the column rods (Fig. 3b and Fig. 4b) corresponding to the critical load at different prestressing rates, where

$$\lambda_1 = \frac{(S_1 + S_2)l^2}{\sum\limits_{i=1}^{3} E_i J_i} \qquad \lambda_3 = \frac{S_3 l^2}{\sum\limits_{i=1}^{3} E_i J_i} \qquad (4.10)$$
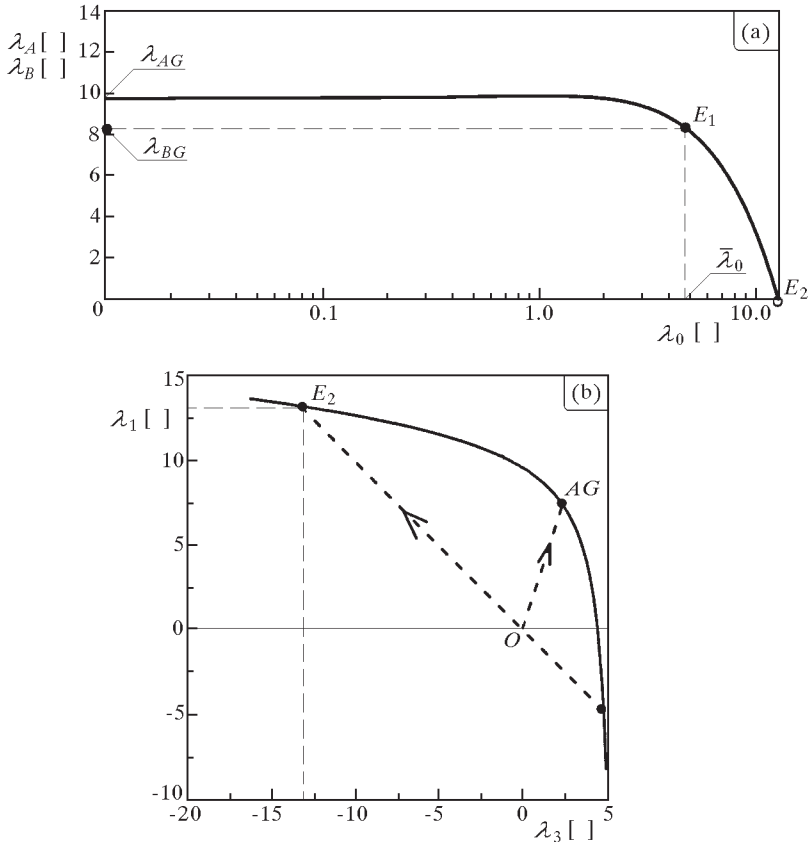
were also determined.

Fig. 4. The critical load parameter λ in relation to pre-stress λ_0 for APC column and asymmetry coefficient μ_G of the flexural rigidity

Additionally, the introduced nomenclature define:

$\lambda_{AL}, \lambda_{AG}$ – value of the critical load parameter for the geometrically non-linear column without pre-stress (AUC) adequately for local and global stability loss

$\lambda_{BL}, \lambda_{BG}$ – value of the critical load parameter for column B adequately for local and global stability loss.

The pre-stress of the geometrically non-linear column is realised along O-D_2 (Fig. 3b) and O-E_2 (Fig. 4b) lines and produces the internal force λ_0^* in individual rods of the system. The column rods with index $1, 2$ (Fig. 1) are compressed while rod 3 is stretched. Then, an external force is loaded to the system causing a change in the internal forces in the rods of the column along the O_1-D_1 path until the loss of the system stability at the load parameter λ_* is reached (see D_1 point in Fig. 3a).

The considered pre-stress in the presented model of the column causes an increase in the critical load of the system. This increase is more intensive if the asymmetry of the flexural stiffness of individual members of the system increases. It was stated (Fig. 3a) that such a range of λ_0 values $(\lambda_0 \in (\lambda_0', \lambda_0'')$ exists when the critical load parameter λ for the geometrically non-linear system (APC) is larger than the value of this parameter for the linear column B.

The thus defined phenomenon is the so-called "exit" of the system from local loss of stability. An increase in the critical local parameter for the initial values of the pre-stress is the result of unloading of the rod characterised by a lower flexural rigidity. The unloading of a part of the geometrically non-linear system enables a higher load to be transferred by the remaining member of the column. The value of λ_0, for which the biggest critical load is obtained (λ_{max} for $\mu = \mu_L$), was denoted as $\lambda_{0\,max}$. The column losses its stability for point D_2 only because of the pre-stress.

A noticeable increase in the critical load was not stated for larger coefficients of the asymmetry of the flexural stiffness distribution ($\mu = \mu_G$), e.g. for the column with the global loss. The opposite fact was stated for the column with the local stability loss. Additionally, it was revealed that the pre-stress of the geometrically non-linear column in the $\lambda_0 > \overline{\lambda}_0$ range is not advisable because it is characterised by the critical value of the loss lower than the corresponding value of the critical load for the linear system λ_{BG}.

5.   Test stand, constructional solution of columns

The test stand for experimental research on vibrations of the considered columns is presented in Fig. 5. The load head (1) can be moved horizontally on guides (2) ensuring an axial character of the load applied to the system. A dislocation of the belt system of the head causes the load of the considered column to appear. The load force is measured by dynamometer (3). Hinged supports of column (5) are mounted on slabs (4a,b). They provide the imposed boundary conditions to be fulfilled.
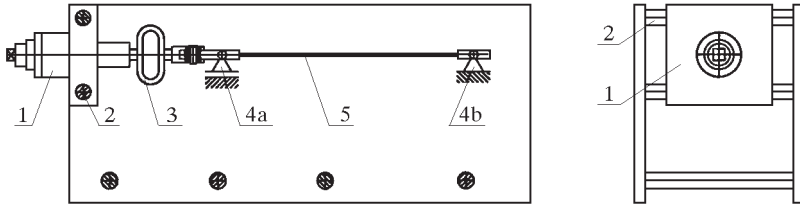
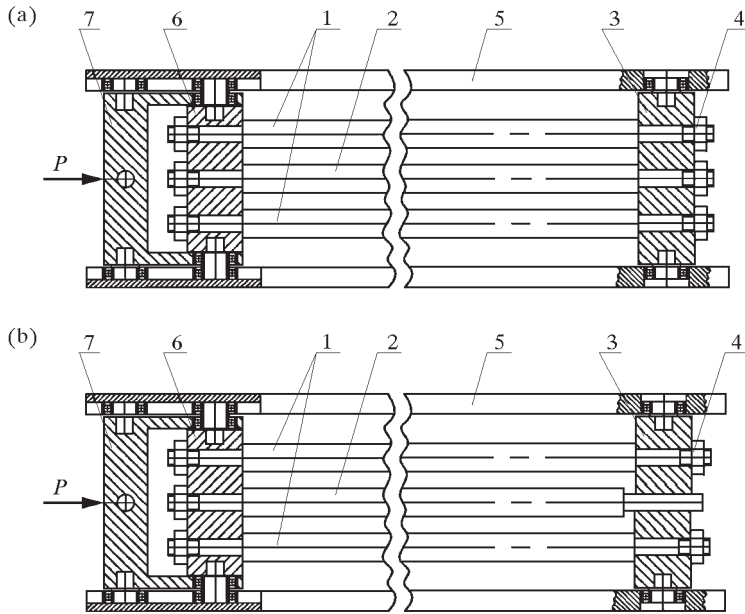Fig. 5. The test stand for experimental research on the column system

Fig. 6. Constructional model of column (a) geometrically non-linear (AUC),
(b) linear (C)

The construction (Fig. 6a) of column A consists of three circular rods. Two identical external rods (1) are made of steel with the flexural stiffness $E_1 J_1$, $E_2 J_2$, while central one (2) is made of duralumin with the flexural stiffness $E_3 J_3$. The rods are connected together with the use of rigid elements $(3, 6)$, the threaded ends of the rods are immobilised using nuts $(3, 6)$. The hinged clamping of the column is realised by rolling bearings, connecting rigid elements $(3, 6)$ and frame (5). Additionally, element (6) can be moved in the longitudinal direction under the external force P applied axially due to hinged-clamped buckle (Nayfeh, 1973). The internal rod (central) does not carry the longitudi-

nal force in the case of the geometrically linear system (Fig. 6b – column C). The end of rod (2) is not clamped with respect to rigid element (3) and can be axially moved. This model has the same total flexural stiffness as the system in Fig. 6a, but the external force is transmitted only by rods (1). Internal rod (2) is removed from the system in the constructional model column B (Fig. 1).

6. Frequency curves of free vibrations and vibration forms – numerical and experimental results (AUC, B, C columns)

The solution to the boundary value problem for the considered systems (Section 3 of this paper) leads to underestimated results with respect to frequency and free vibration forms. Two characteristic forms of the free vibrations (Fig. 7) can be distinguished for the geometrically non-linear Euler column (AUC column):

$N_1, N_2, ...$ – normal correct forms of the free vibrations. They are identical as for the geometrically non-linear column, which consists of two rods with $E_1 J_1$ and $E_3 J_3$ flexural stiffness and $\rho_1 A_1$, $\rho_3 A_3$ mass per unit length, and

$$E_I J_I = E_1 J_1 + E_2 J_2 \qquad \rho_I A_I = \rho_2 A_2 + \rho_2 A_2$$

Nm^s – additional symmetrical forms of the free vibrations. The presence of the above indicated vibration frequencies result from symmetrical division of the flexural stiffness of the external rods; and such a model is considered in this paper. The number of the rods for the m-th form is equal to $m - 1$. The frequncies of free vibrations are numerically equal to the frequencies of the one-rod column vibrations with the $E_1 J_1$, $E_2 J_2$ flexural stiffness. Two ends of the rod are rigidly fixed and the boundary conditions can be written as

$$y(0) = y'(0) = y(l) = y'(l) = 0$$

The forms of the frequencies are shown in Fig. 7 where continuous lines – correspond to the external rods, the dashed ones – to the internal rod.

The course of the free vibration frequency in relation to the external load for AUC, B, C columns was numerically calculated and experimentally verified on the test stand shown in Fig. 5. The geometrical and physical parameters of the above mentioned columns are presented in Tables 1 and 2.
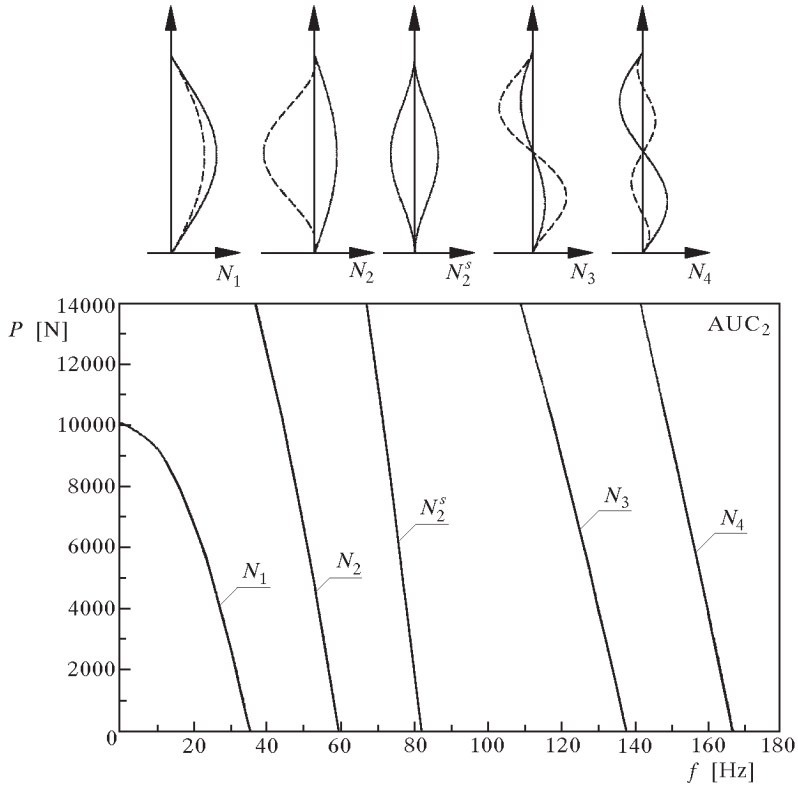
Fig. 7. Vibration forms of the AUC column and the corresponding free vibration
frequencies

Table 1

Column	d_1 [m]	d_2 [m]	E_1 [MPa]	E_2 [MPa]	ρ_1 [kg/m^3]	ρ_2 [kg/m^3]
B	0.014	0.014	$1.9 \cdot 10^5$	$1.9 \cdot 10^5$	7600	7600

Table 2

AUC, C column	d_1, d_2 [m]	d_3 [m]	E_1, E_2 [MPa]	E_3 [MPa]	ρ_1, ρ_2 [kg/m^3]	ρ_3 [kg/m^3]	μ [ ]
1	0.014	0.006	$1.9 \cdot 10^5$	$6.63 \cdot 10^4$	7600	2530	0.0059
2	0.014	0.01	$1.9 \cdot 10^5$	$7.19 \cdot 10^4$	7600	2790	0.049
3	0.014	0.0218	$1.9 \cdot 10^5$	$7.19 \cdot 10^4$	7600	2790	1

The column denotations, considered in the following part of the paper, are as follows:

AUC_1 – geometrically non-linear Euler's column without pre-stress for the geometrical and physical data from Table 2 (row 1)

B – linear column which consists of two rods with the $E_1J_1 + E_2J_2$ flexural stiffness for the geometrical and physical data from Table 1

C_2 – linear column which consists of three rods like the geometrically non-linear column, but the rod with the E_3J_3 stiffness does not transmit the longitudinal force for the geometrical and physical data from Table 2 (row 2).

The results of experimental investigations and the numerical simulations are presented in Fig. 8 and Fig 9. The first four free vibration frequencies of columns A and C and the first two free vibration frequencies of column B were taken into account in the considered systems. The additional denotations of the free vibration frequency curves in relation to the external load which determine vibration forms of the non-linear system (see Fig. 7) are as follows:

M^* – forms of vibrations of the non-linear column with the unloaded internal rod (column C)

M – forms of vibrations of the linear column which consists of two rods (column B).

The obtained numerical and analytical results are of good accuracy. The maximal relative difference between the numerical and analytical results equals 10%.

The choice of the physical and geometrical parameters was aimed to demonstrate the local loss of stability (Fig. 8a). The points on the first curve of the free vibration frequency for $f = 0$ represent the value of the critical load. The value of the critical load for the geometrically non-linear system is lower than the critical load of the linear column, which consists of two rods (comparison of eigenvalue curves M_1 and N_1). The changes in the free vibration frequencies in relation to the external load, presented in Fig. 8 and Fig. 9, are related to the global loss of stability of the geometrically non-linear Euler column.

7. Conclusions

The analysis of the obtained results for the geometrically non-linear Euler column allows one to state that the considered system is characterised by
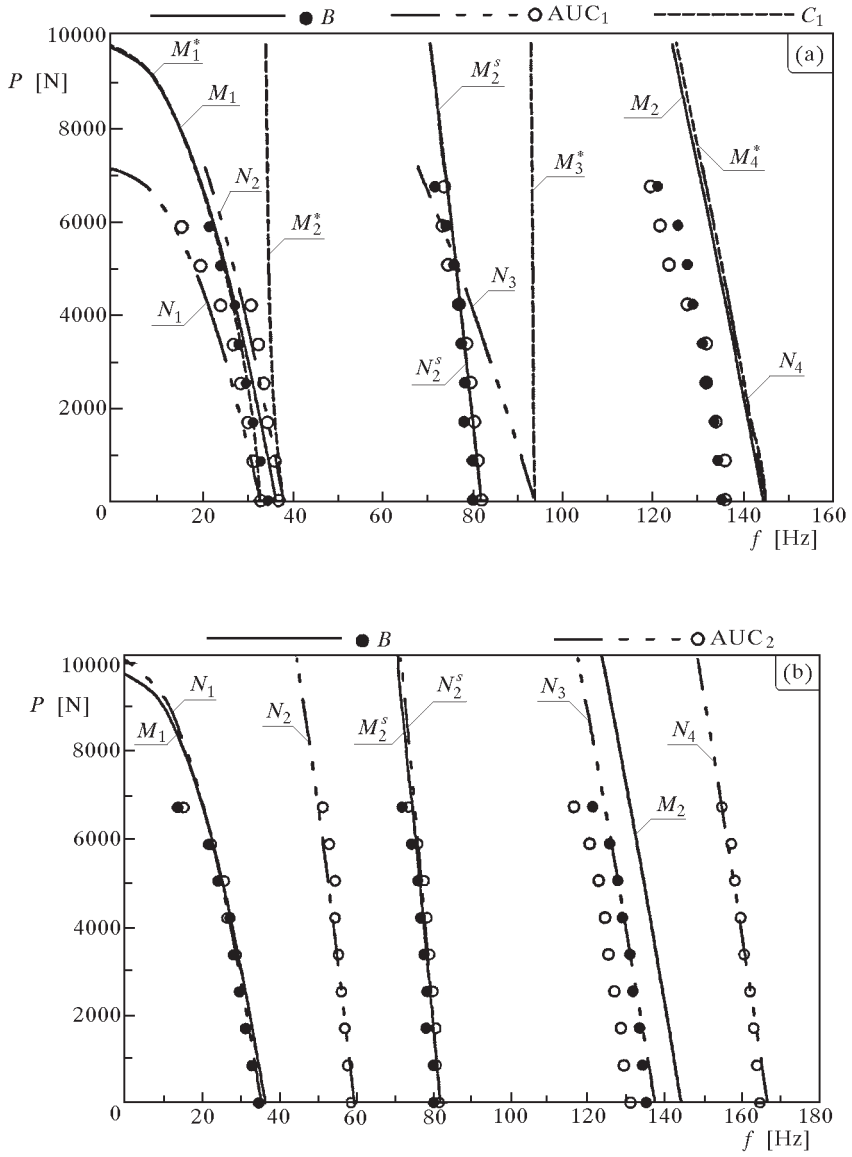
Fig. 8. The free vibration frequencies of the AUC_1, B, C_1 columns (a) and AUC_2, B columns (b)

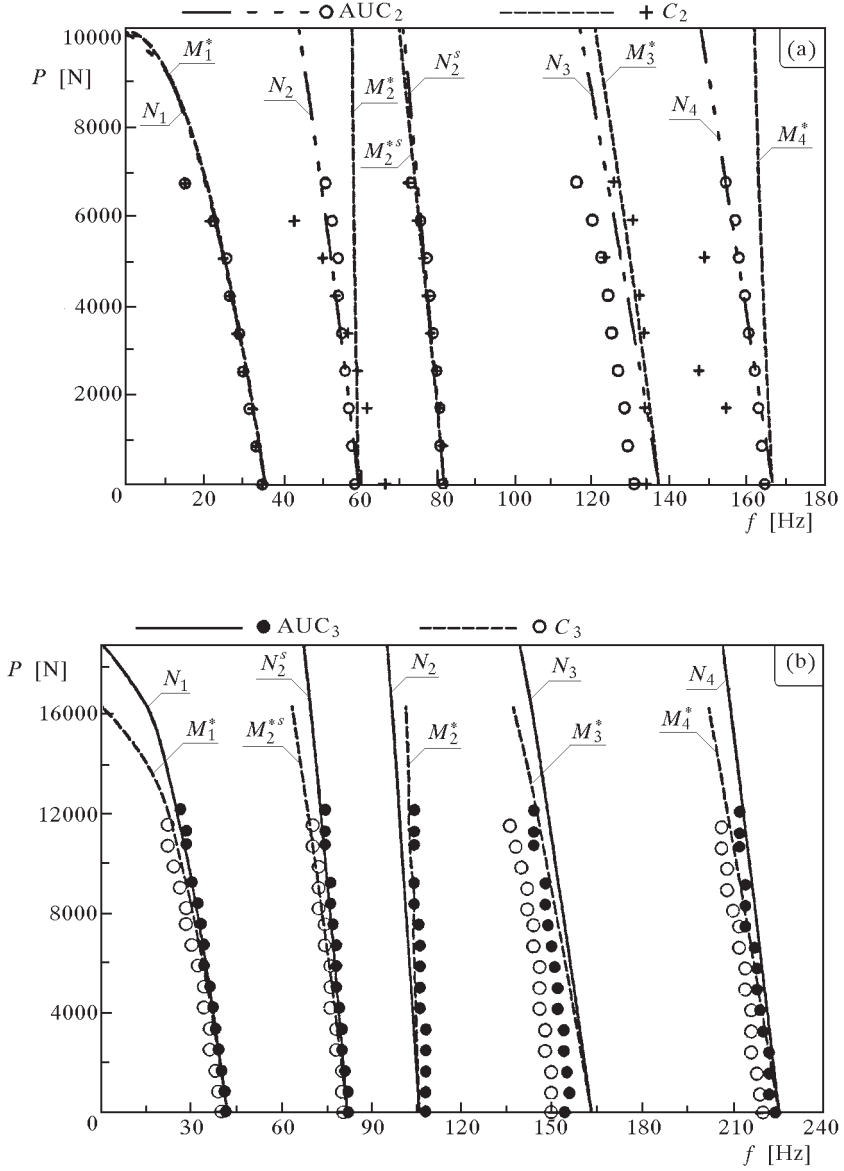Fig. 9. The free vibration frequencies of the AUC_2, C_2 columns (a) and AUC_3, C_3 columns (b)

a local or global loss of stability depending on the value of the asymmetry coefficient of the flexural stiffness. It has been proved that the part of the compound column with the lower flexural stiffness is responsible for the local loss of the system stability. Theoretical considerations of changes in the free vibration frequencies in relation to the external load have been supported by experimental investigations.

The range of pre-stress values, in which the increase in the critical load for the considered geometrically non-linear Euler column over the border of the local loss of stability takes place, has been determined. The pre-stress of the system in the whole possible range is undesirable from the point of view of the obtained value of the critical load. It especially concerns higher values of the pre-stress for which opposite to the expected results are found (a noticeable decrease in the critical load). The pre-stress should be applied to the columns with the local loss of stability. In the other case (global loss of stability) the influence of the pre-stress on the increase in the critical force is insignificant.

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Niestateczność lokalna i globalna oraz drgania przesztywnionej kolumny Eulera

Streszczenie

W pracy prezentuje się badania teoretyczne i eksperymentalne dotyczące stateczności i drgań własnych nieliniowej geometrycznie kolumny Eulera. Określa się wpływ asymetrii rozkładu sztywności na zginanie poszczególnych prętów kolumny na wartość siły krytycznej oraz istnienia obszarów lokalnej i globalnej utraty stateczności. Kolumna zbudowana jest z trzech prętów umieszczonych w płaszczyźnie. Dwa zewnętrzne pręty mają jednakową sztywność na zginanie, a trzeci środkowy inną. Pręty są połączone w taki sposób, że ich ugięcie w miejscu zamocowania jest równe zeru, a kąty ugięcia są jednakowe.