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FLOW MODELLING IN GAS TRANSMISSION NETWORKS. PART I – MATHEMATICAL MODEL[1]

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The mathematical problem, corresponding to the physical problem of the gas flow in transmission networks, is formulated in the first part. The nonstationary flow of viscous, compressible gas is modelled. All elements of the transmission networks which influence the flow, functions describing delivery and consumption of gas and control of the network are taken into account. The method of solution and algorithms are presented in the second part of the paper.

Key words: flow in nets, gas transmission modelling, gas flow

1. Introduction

Software packets modelling flow in gas transmission networks are parts of SCADA (Supervisory Control and Data Acquisition) systems. These packets are also used for planning and development. In the first case these packets work both in on-line or off-line modes. In planning and development departments, only off-line mode is applied. The packets enable calculation of gas supply reliability. The packets are used for teaching gas dispatchers how to act in normal conditions and in emergency. The coordination between computer systems in acquisition departments and the control system with the modelling packets are developed for handling gas supply applications. The most important proof that the modelling packets are useful is the fact, that they exist (Jenocek et al., 1997, Knieschewski and Reith, 1996, Blacharski et al., 2000).

[1]Part II – *Method of solution and algorithms* will be appear in JTAM, **41**, 1, 2003

In the paper the mathematical problem, corresponding to the physical problem of the gas flow in transmission networks, is formulated and the variants of physical simplifications are described.

In the second part "Solution method and algorithms" you will find:

- the reduction of the boundary-initial problem for the system of partial differential equations to the system of nonlinear algebraic equations,
- the algorithm which enables generation of this algebraic system for different networks,
- the method of solution of the system of nonlinear algebraic equations and main algorithms,
- our good and bad experiences concerning the methods of solution,
- plans of the packet development.

The algorithms are successfully applied in the PSD software packet Blacharski et al. (2000), Fedorowicz et al. (1998).

The problem which we are dealing with in the paper, was solved earlier but, with the exception of Kralik et al. (1988), we have found neither the description of the methods of solution nor the algorithms. These methods and algorithms have commercial value and are not published.

Transmission networks are some hundreds km wide and flowing gas is under high pressure (6 MPa). In such conditions, only transient model is applicable (Blacharski et al., 2000) because static models give quantitative and qualitative errors. Quantitative errors are caused by density (and pressure) strongly varying in time. More important is the fact that in such networks the amount of gas flowing in and flowing off is different at each moment of time and globally also for 24 hours. This fact contradicts the mass conservation principle applied to static models.

The phenomenon depends on: network properties, gas composition, network control, gas consumption by customers, and initial state of the flow. All the above mentioned factors are consecutively analyzed.

2. The factors

2.1. Network properties

From 66 types of the network elements enumerated in the software packet for network management (Fedorowicz, 2001), only 12 influence the flow in the network.

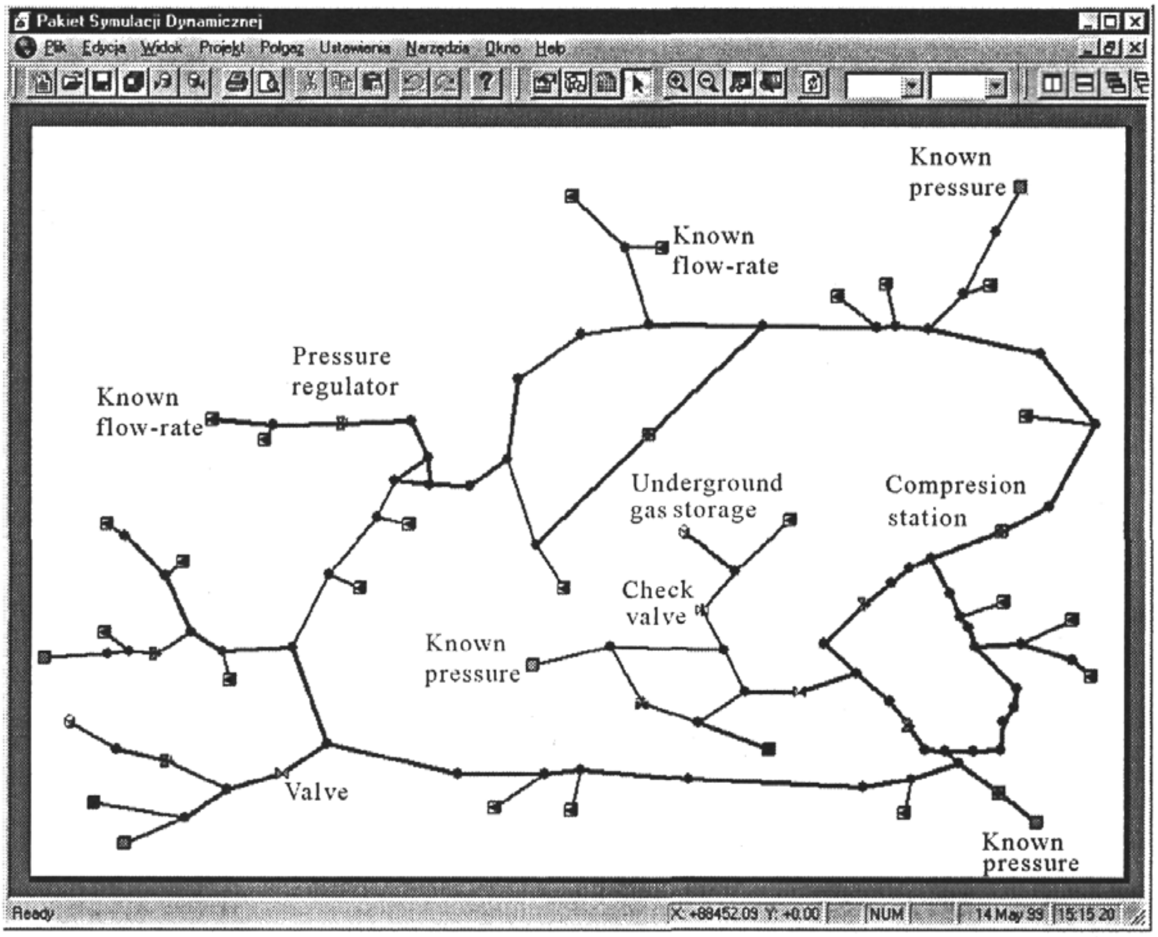

Fig. 1. View of a transmission network

We divide these elements into two groups: pipes and nodes. Each pipe is characterized by: internal diameter D, length L, roughness k, registration number, registration numbers of the first and second incident node.

The nodes are classified, for the modelling purpose, as: two-way, three-way, source, delivery (city gate stations), gasholder (reservoir), compressor station, valve, gate valve, check valve, regulator. Each node is characterized by: registration number and level. Two-way nodes and valves are also characterized by the coefficient of the local hydraulic resistance (explained in 3.1). The gasholder is treated as a source of gas or gas run-off node, and it has the attribute - control mode. The compressor stations inside the network has the "control mode" attribute and the flow direction attribute. The flow direction attribute is determined by the registration number of the directional pipe. The gas flows only into this pipe. The opposite direction is known as the forbidden direction. The regulators and check valves also have the attribute "flow direction". The flow in the network is limited by a system of one-side constraints, e.g. maximum or minimum pressure, maximum speed of flow, maximum flow rate. These limitations are also attributes of nodes and pipes.

2.2. Gas properties

In the network, the cleaned natural gas is transported. Polish standard PGNiG ZN-G-4002 defines the transported gas as a mixture of 36 components. Knowing the mass or mole fraction of the components we calculate the coefficient of compressibility Z_N, mass density ρ_N, coefficient of dynamic viscosity μ_N, molar mass, isentropic exponent, and heat of combustion, in normal conditions. The formulae from the earlier mentioned standard, enable calculation of the coefficient of dynamic viscosity $\mu(p,T)$ at temperature T and pressure p. Coefficient of compressibility $Z(\rho,T)$ is the function of mass density ρ and temperature T, and it is calculated from the formulae SGERG88 [17].

2.3. Network control

The compressor stations enable active control of gas transportation. The compressor station (shortly CS) may be inside the network or at the boundary of the network. The CS at the contour (boundary) of network is modelled as a source of gas or as a run-off node. The CS can be controlled in many different manners, e.g. through the pressure of the gas flowing-out the CS, through the flow rate, through the ratio of the pressures at outlet and inlet of the CS ("compression"). The above mentioned rules of control are implemented in the PSD packet. The other rules of control can also be implemented, e.g. through the pressure at the inlet of CS, through the pressure at the inlet of the remote reducer. The second rule of control require the tuned modelling packet working in-line with control system of the CS. Total efficiency of the CS has a small maximum as a function of the flow rate, the compression, and the pressure at the input of the CS. Optimal control of the whole network should assure the work of the CSs in the conditions related to the maximum efficiency. The tuned simulation packets facilitate such optimal control. The work of CS is limited by a system of one-side constraints, e.g. level of vibrations, flow rate maximum or minimum. The gasholders with CS also enable active control of the network.

Opening and closing valves, and gate valves, enable passive control of the network. Control of the pressure at the outlets of reducers also enable the passive control.

2.4. Network load

The gas flowing off from the network is measured at reduction stations of the first step (reduction from high to middle or small pressure, city gate stations) or at measuring stations. Flow rate in these nodes (delivery nodes from the gas consumer point of view) depends on the gas consumption (load) by communal and industrial customers. On the basis of previous data concerning the gas consumption, and on the basis of knowledge of the future weather conditions, it is possible to predict gas consumption (load). The simulation packet needs the 24 or 48 hour forecast.

It is expected that forecast output should be the curve $Q_p(t)$ – flow-off rate as a function of time at the node p. In Poland, most often, the daily reports on gas consumption give the one hour mean of flow-off ratio. The measuring-reduction stations with telemetry give data about the flow rate with e.g. 5 or 10 minutes sampling. We change these data into the data in such forms as in the daily reports, with small loss of precision. The 24 numbers defining mean flow-off ratio for each hour of a calendar day are remembered as a daily load Q_d and an index of profile. The daily load Q_d is a sum of the above mentioned 24 hour mean values of flow-off ratio. The profile defines 24 numbers, each number is equal to the hour mean of the flow-off ratio divided by Q_d. All profiles (thousands of them) are remembered in the small base of data about loads. We define the measure and the distance between the profiles.

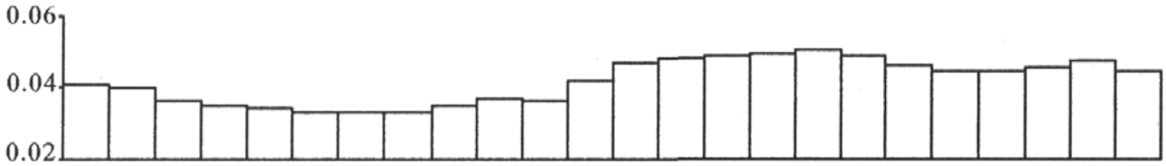

Fig. 2. Scheme of the profile

The profile is defined by its consecutive number α. It is a set of ordinates $y_i^{(\alpha)}$, $i = 1, 2, ..., 24$ such that $\sum_{i=1}^{24} y_i^{(\alpha)} = 1$. The measures of the profile $\sigma^{(\alpha)}$ and the distance between profiles $\sigma^{(\alpha\beta)}$ are modified Cartesian measures

$$\sigma^{(\alpha)} = \sqrt{\sum_{i=1}^{24} \frac{1}{24} (y_i^{(\alpha)})^2} \qquad \sigma^{(\alpha\beta)} = \sqrt{\sum_{i=1}^{24} \frac{1}{24} (y_i^{(\alpha)} - y_i^{(\beta)})^2} \tag{2.1}$$

We chose the critical distance. All profiles with mutual distance $\sigma^{(\alpha\beta)}$ less than the critical distance are treated as the same profile. The consecutive number in the base of different profiles is the profile index. So, the problem of

$Q_p(t)$ forecasting is changed into the problem of forecasting two numbers: Q_d and the profile index for the node. The details are discussed in Fedorowicz et al. (2001a).

3. The mathematical model of the physical problem of flow in transmission networks

3.1. Assumptions and simplifications of the mathematical model

3.1.1. One-dimensional model

The one-dimensional model is considered. The unknown functions depend on the Eulerian space variable x measured along the pipe, from the first to the second incident node, and the time variable t.

3.1.2. Turbulent-laminar flow

Most often the flow is turbulent, but at different pipes, at various moments of time, the flow can be laminar or transient from turbulent to laminar, or *vice versa.* This is the reason why different formulae for the coefficient of the equivalent hydraulic resistance λ must be used. The gradient of the pressure is proportional to the dynamic pressure $\rho V^2/2$, is proportional to the coefficient λ and is inversely proportional to the pipe diameter D. The symbol $\rho(x,t)$ for mass density of the gas, is averaged over the cross-section of the pipe. The symbol $V(x,t)$ is called velocity and is defined by the flow rate $Q(x,t)$ [kg/s]

$$V(x,t) = \frac{Q(x,t)}{\rho(x,t)A} \tag{3.1}$$

where $A = \pi D^2/4$.

The coefficient of the equivalent hydraulic resistance λ has been investigated experimentally and there are many formulae listed in Fedorowicz (1997). The coefficient λ depends on Reynold's number Re and relative roughness ε

$$\mathrm{Re} = \frac{VD}{\nu} \tag{3.2}$$

where ν is for the coefficient of kinematic viscosity, $\nu = \mu/\rho$

$$\varepsilon = \frac{k}{D} \tag{3.3}$$

Approximation of the formula $\lambda(\mathrm{Re}, \varepsilon)$ which avoids discontinuity for small Reynold's numbers is used Fedorowicz (1997)

$$\begin{aligned} \lambda &= \frac{64}{\mathrm{Re}} \qquad \text{for } \mathrm{Re} < R_{change} \\ \lambda &= 4.07\Big[-4\log\Big(\frac{\varepsilon}{3.7065} - \frac{5.0452}{\mathrm{Re}}\log A_4\Big)\Big]^{-2} \end{aligned} \tag{3.4}$$

where

$$\begin{aligned} A_4 &= \frac{\varepsilon^{1.1098}}{2.8257} + \Big(\frac{7.149}{\mathrm{Re}}\Big)^{0.8961} \qquad \mathrm{Re} \geqslant R_{change} \\ R_{change} &= [(-1.27234 \cdot 10^6\varepsilon + 214208)\varepsilon - 15112.9]\varepsilon + 1026.15 \end{aligned} \tag{3.5}$$

The formula for fully turbulent flow is the same as the Chen formula [3].

3.1.3. Local pressure drop

To the pressure drop in pipes, the local pressure drop (LPD) in internal nodes of network should be added. These LPDs are most important in open valves, gate valves. The LPDs in gate valves grow as the open cross-section of the gate valves diminishes. The LPD is proportional to the dynamic pressure $\rho V^2/2$. It is very difficult to get knowledge about the coefficients of the LPD. Tables of the coefficients of LPD for stationary flow are presented in [15]. The coefficients of LPD for three-way nodes depend on the flow direction, so it is really difficult to model the LPD in them. We propose to include into the network virtual two-way nodes, with known coefficients of LPD, at the pipes incident with the three-way node to model the LPD for the three-way nodes.

3.1.4. Load of the network

Gas consumption through the reduction stations of the first step (the city gate station) is forecasted. The forecast for each delivery node is determined by two numbers: daily load Q_d and the index of profile, as it was explained above. The pressure in a source node may also be forecasted. In such a case, the process is determined by middle pressure of the day and night and the index of the pressure profile in the base of profiles. For the calculations, the profiles are interpolated by the spline method of the third rank e.g. Flannery et al. (1998). The functions determining the controlled parameters, such as the outlet pressure of the compressor station, the flow ratio from the gasholder with compressor station, the state of the valve (open-closed), the output pressure of the reducer, are the right-side continuous step functions of time.

3.2. Equations

The principle of mass conservation $(3.6)_1$, momentum conservation $(3.6)_2$, energy conservation $(3.6)_3$ with constitutive equation $(3.6)_4$ generate the closed system of equation

$$
\begin{aligned}
&\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x}(\rho V) = 0\\
&\rho\Big(\frac{\partial V}{\partial t} + V\frac{\partial V}{\partial x}\Big) + \frac{\partial p}{\partial x} + \lambda\frac{\rho V^2}{2D} + \rho g\frac{\partial h(x)}{\partial x} = 0\\
&\rho c_v(T)\Big(\frac{\partial T}{\partial t} + V\frac{\partial T}{\partial x}\Big) = \frac{\partial}{\partial x}\Big(K(T)\frac{\partial T}{\partial x}\Big) - \frac{4}{D}k_T(T - T_{env}) + \frac{\Theta(x,t)}{A} - p\frac{\partial V}{\partial x}\\
&p = \rho Z(\rho, T)RT
\end{aligned}
\tag{3.6}
$$

where the function $Z(\rho, T)$ is calculated according to the formulae SGERG88 [17]. The formula takes into account all 36 components of the flowing gas. There is the function in the program, which calculates $Z(\rho, T)$ for the flowing gas.

The function $p(x, t)$ is for pressure averaged over the cross-section of the pipe, the function $h(x)$ is for the elevation over the see level, g is for gravitation constant. The function $T(x, t)$ denotes the absolute temperature, $c_v(T)$ is the specific heat of isochoric process, $K(T)$ is the Fourier constant, k_T is the thermal conductance coefficient, T_{env} is the temperature of the soil, $\Theta(x, t)$ is the external source of heat intensity, R is the gas constant.

3.3. Boundary conditions

3.3.1. Internal and external nodes

The nodes are classified as internal and external, when the boundary conditions are considered.

External nodes:

- Pressure source. The pressure in this node is a known function of time. Examples: compressor stations (CS) at the contour of the network controlled after the pressure of the gas flowing-out the CS, reservoirs with CSs controlled in the same way, reducers at the contour of the network. Through these nodes gas flows into the network.

- Source with known flow ratio. The flow ratio in this node is a known function of time. Examples: CSs at the contour of the network controlled after the flow ratio, reservoirs with CSs controlled in the same way. Through these nodes the gas flows into the network.

- Delivery node (gas run-off node, city gate). The flow ratio in this node is a known function of time. Examples: reduction stations of the first step, reservoirs with CSs controlled after the flow ratio. Through these nodes gas flows off from the network.

Internal nodes:

- Two-way node
- Three-way node
- Compressor station inside the network
- Pressure reducer inside the network
- Valve
- Gate valve
- Check valve.

3.3.2. Special cases

The list does not contain four-way nodes, five-way nodes, and so on. These rather rare nodes are virtually replaced by two or more three-way nodes linked by a very short pipe.

Boundary conditions for gasholders without compressor stations are very interesting. In this case, the boundary conditions are based at mass conservation principle, the equation of state $(3.6)_4$, and energy conservation principle. The flow direction depends on the pressure inside the gasholder and in the node. These boundary conditions link the flow parameters at different nodes belonging to this gasholder.

3.3.3. Boundary conditions at external nodes

At the pressure source

$$p(x_b, t) = F_p(t) \tag{3.7}$$

where x_b is the coordinate at the bounds of the pipe; $x_b = 0$ or $x_b = L$. The function $F_p(t)$ may be known from the forecast, so it is the spline-type function, or it may result from the control mode, then it is the step function.

At the source with a known flow ratio

$$Q(x_b = 0, t) = F_Q(t) \quad \text{or} \quad Q(x_b = L, t) = -F_Q(t) \tag{3.8}$$

The function $F_Q(t)$ may be both spline-type or step function. This function may be positive definite, then the gas flows into the network or negative definite, then the gas flows off.

At the delivery node

$$Q(x_b = 0, t) = -F_Q(t) \quad \text{or} \quad Q(x_b = L, t) = F_Q(t) \tag{3.9}$$

In this case, the function $F_Q(t)$ can be the spline-type, positive definite function only.

At external nodes we have only one boundary condition at each node.

3.3.4. Boundary conditions at two-way nodes

The boundary conditions at two-way nodes are based at mass and momentum conservation principles.

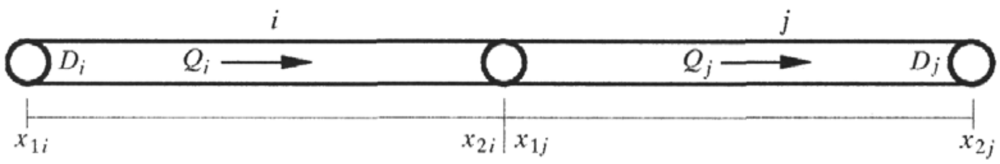

Fig. 3. Scheme of a two-way node

The symbol x_{1i} is for the space variable x at the point of the pipe where the first node is incident. $x_{1i} = 0$. The symbol $x_{2i} = L_i$ is for the placement of the second node. If the gas flows from the first node to the second, then $Q_i > 0$. The junction shown in Fig. 1 is specified as $(\Longrightarrow\Longrightarrow)$. The arrows $\Longleftarrow$ and $\Longrightarrow$ are for the direction of the pipe, not for the direction of the flow. If the flow is in the same direction as the direction of the pipe then $Q > 0$. In the case shown in the Fig. 3, from the mass conservation principle we get

$$Q_i(x_{2i}, t) - Q_j(x_{1j}, t) = 0 \tag{3.10}$$

and from the momentum conservation principle we get

$$p_i(x_{2i}, t) - p_j(x_{1j}, t) = \frac{\Lambda_{node}}{4}\Big[\frac{Q_i(x_{2i}, t)|Q_i(x_{2i}, t)|}{\rho_i(x_{2i}, t)A_i^2} + \frac{Q_j(x_{1j}, t)|Q_j(x_{1j}, t)|}{\rho_j(x_{1j}, t)A_j^2}\Big] \tag{3.11}$$

The formula (3.11) discloses, that the local pressure drop is proportional to the mean value of the dynamic pressure $\rho V^2/2$ from both sides of the node. The coefficient Λ_{node} is specific for each node. For the nodes incident to the pipes with different diameters, the coefficient Λ_{node} should depend on the direction of the flow.

The analogues formulae for the junction $(\Longrightarrow \Longleftarrow)$

$$
\begin{aligned}
&Q_i(x_{2i},t) + Q_j(x_{2j},t) = 0 \\
&p_i(x_{2i},t) - p_j(x_{2j},t) = \frac{\Lambda_{node}}{4}\Big[\frac{Q_i(x_{2i},t)|Q_i(x_{2i},t)|}{\rho_i(x_{2i},t)A_i^2} - \frac{Q_j(x_{2j},t)|Q_j(x_{2j},t)|}{\rho_j(x_{2j},t)A_j^2}\Big]
\end{aligned}
\tag{3.12}
$$

For $(\Longleftarrow \Longrightarrow)$ we get

$$
\begin{aligned}
&Q_i(x_{1i},t) + Q_j(x_{1j},t) = 0 \\
&p_i(x_{1i},t) - p_j(x_{1j},t) = -\frac{\Lambda_{node}}{4}\Big[\frac{Q_i(x_{1i},t)|Q_i(x_{1i},t)|}{\rho_i(x_{1i},t)A_i^2} - \frac{Q_j(x_{1j},t)|Q_j(x_{1j},t)|}{\rho_j(x_{1j},t)A_j^2}\Big]
\end{aligned}
\tag{3.13}
$$

For $(\Longleftarrow \Longleftarrow)$ we get

$$
\begin{aligned}
&Q_i(x_{1i},t) - Q_j(x_{2j},t) = 0 \\
&p_i(x_{1i},t) - p_j(x_{2j},t) = -\frac{\Lambda_{node}}{4}\Big[\frac{Q_i(x_{1i},t)|Q_i(x_{1i},t)|}{\rho_i(x_{1i},t)A_i^2} + \frac{Q_j(x_{2j},t)|Q_j(x_{2j},t)|}{\rho_j(x_{2j},t)A_j^2}\Big]
\end{aligned}
\tag{3.14}
$$

In the following parts of the paper, the method of generation of formulae for different junctions will be the same as for two-way nodes. For each two-way node, two boundary conditions will be generated.

3.3.5. Boundary conditions at three-way nodes

The boundary conditions at three-way nodes are based on mass and momentum conservation principles. The local pressure drop can be modelled by virtual two-way nodes as it was mentioned above.

For the junction shown in Fig. 4, from the mass conservation principle we get

$$
Q_i(x_{2i},t) - Q_j(x_{1j},t) - Q_k(x_{1k},t) = 0 \tag{3.15}
$$

From the momentum conservation principle we get

$$
\begin{aligned}
&p_i(x_{2i},t) - p_j(x_{1j},t) = 0 \\
&p_i(x_{2i},t) - p_k(x_{1k},t) = 0
\end{aligned}
\tag{3.16}
$$

For the 7 remaining cases of junction, the formulae are generated as it is explained for the two-way node. For each three-way node, three boundary conditions are generated.

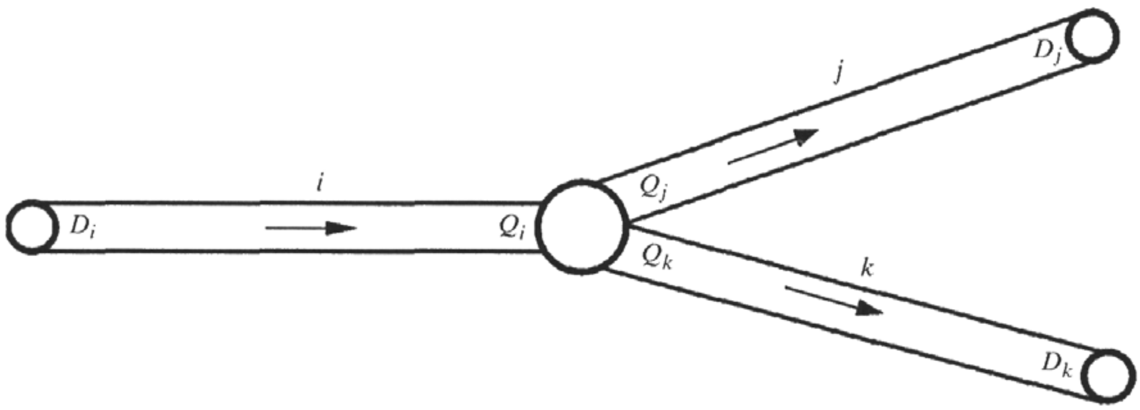

Fig. 4. Scheme of three-way node

3.3.6. Boundary conditions for compressor station inside the network

A compressor station is a system with complex and changeable internal structure, with even 50 operational modes. There are two main types of compressors; reciprocating (piston) and centrifugal, turbine driven. The filters, cyclones, oil separators, internal valves, and heat exchangers also influence the flow inside the compressor station. The software packet modeling, compressor station (Bryant, 1998) is an example of the packet which includes most of functionality in the compressor station. The boundary conditions for the station with turbine-driven centrifugal and reciprocating compressors are consecutively discussed.

Boundary conditions for the station with turbine-driven compressors

In order to get realistic results, we should use the performance curves of each units. We may get them from the producer or they should be verified during exploitation. In Fig. 5 the considered variants of control are illustrated.

The turbine-driven centrifugal compressor characteristic, seen in Fig. 5, depends slightly on the pressure of the gas at the inlet to the compressor. At the horizontal axis we have flow rate of the gas at the outlet of the compressor. At the vertical axis we have the pressure at the outlet of the compressor. The lines of constant rotational speed and level lines of the surface of the efficiency coefficient spread over the working area are shown in Fig. 5. In the PSD packet three modes of control are taken into account (see Section 2.3). The temperature and the pressure at the inlet of compressor and the work point P, which is known from the solution of the problem, determine the efficiency coefficient. Knowing the work point, we are able to calculate the

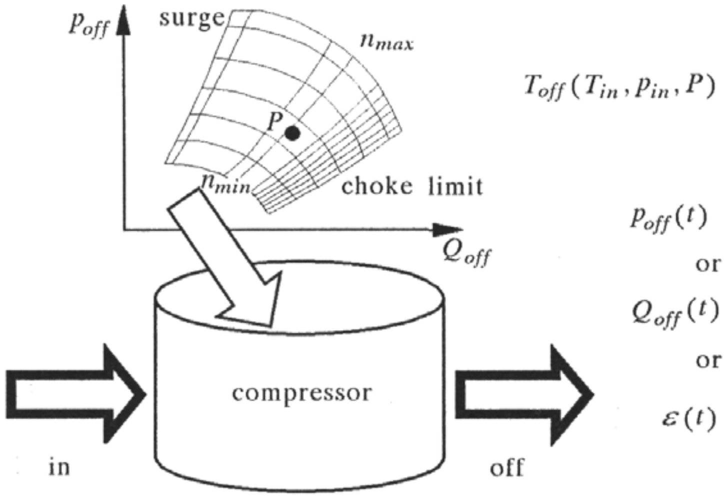

Fig. 5. Turbine-driven compressor characteristic

temperature at the outlet of the compressor T_{off} and the flow rate of the gas consumed by the turbine $Q_{off}(t) \cdot S(P, ...)$.

From the mass conservation principle we get the first boundary condition. For the junction with the compressor station seen in the Fig. 6, marked as ($\Longrightarrow \overrightarrow{T} \Longrightarrow$) we have

$$Q_i(x_{2i}, t) - Q_j(x_{1j}, t)[1 + S(P, ...)] = 0 \tag{3.17}$$

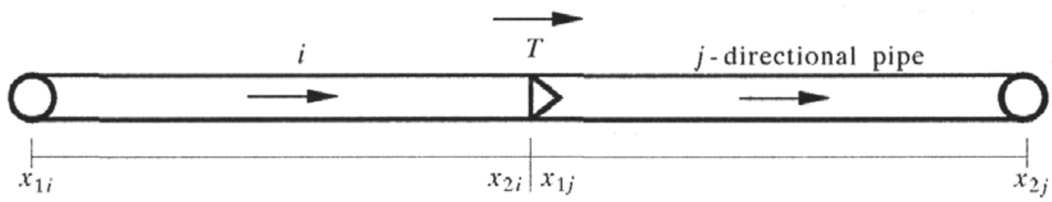

Fig. 6. Scheme of the junction with the compressor station

The arrows $\Longleftarrow$ and $\Longrightarrow$ are for the direction of the pipe. The arrow over the letter T is for the permissible direction of flow (opposite to the forbidden direction). For the 7 remaining cases of junction the formulae (3.17) are written as for two-way nodes. The compressor station has its own forbidden direction of flow, opposite to the directional pipe. The gas can flow off from the compressor station only into the directional pipe, so $Q_j(x_{1j}, t) > 0$ in this case. The second boundary condition depends at the mode of control.

- When the $p_{off}(t)$ is controlled, and for the junction seen in Fig. 6, we have

$$p(x_{1j}, t) = F_{p\,ctrl}(t) \tag{3.18}$$

where $F_{p\,ctrl}(t)$ is the step function of control (see Section 3.1.4). For the remaining 3 cases of junction $(\circ\overrightarrow{T} \Longleftarrow)$, $(\Longrightarrow \overleftarrow{T}\circ)$, $(\Longleftarrow \overleftarrow{T}\circ)$ the formula (3.18) also determines the pressure at the output of the compressor station.

- When $Q_{off}(t)$ is controlled, and for the junctions seen in Fig. 6 $(\Longrightarrow \overrightarrow{T} \Longrightarrow)$ and also $(\Longleftarrow \overrightarrow{T} \Longrightarrow)$, we have

$$Q_j(x_{1j}, t) = F_{Q\,ctrl}(t) \tag{3.19}$$

where $F_{Q\,ctrl}(t)$ is the step function of control. For $(\Longrightarrow \overrightarrow{T} \Longleftarrow)$ and $(\Longleftarrow \overrightarrow{T} \Longleftarrow)$ we have

$$Q_j(x_{2j}, t) = -F_{Q\,ctrl}(t) \tag{3.20}$$

For $(\Longrightarrow \overleftarrow{T} \Longleftarrow)$ and $(\Longrightarrow \overleftarrow{T} \Longrightarrow)$ we have

$$Q_j(x_{2i}, t) = -F_{Q\,ctrl}(t) \tag{3.21}$$

For $(\Longleftarrow \overleftarrow{T} \Longleftarrow)$ and $(\Longleftarrow \overleftarrow{T} \Longrightarrow)$ we have

$$Q_j(x_{1i}, t) = F_{Q\,ctrl}(t) \tag{3.22}$$

- When compression $\varepsilon(t)$ is controlled (see Section 2.3), and for the junctions shown in Fig. 6 $(\Longrightarrow \overrightarrow{T} \Longrightarrow)$, we have

$$p_i(x_{2i}, t)F_{\varepsilon\,ctrl}(t) - p_j(x_{1j}, t) = 0 \tag{3.23}$$

where $F_{\varepsilon\,ctrl}(t)$ is the step function of control. For $(\Longrightarrow \overleftarrow{T} \Longrightarrow)$ we have

$$p_i(x_{2i}, t) - F_{\varepsilon\,ctrl}(t)p_j(x_{1j}, t) = 0 \tag{3.24}$$

For the 6 remaining cases of junction in the formulae (3.23), the coordinates of the first or the second node are used and the function $F_{\varepsilon\,ctrl}(t)$ is properly situated. For the node with compression station, two boundary conditions are generated, one of the (3.17) variants and one on the basis of (3.18) or (3.19) or (3.23) variants.

The temperature of gas grows up considerably during compression. The temperature at the outlet of compressor depends on: the composition of the gas, the temperature of the gas at the inlet of compressor, the work point P. So this temperature depends on the solution of the problem. The gas is cooled at heat exchangers and this process is controlled, the temperature at the outlet of the compressor station is measured, so it is reasonable to treat the temperature as a known function of time

$$T(x_b, t) = F_{T\,ctrl}(t) \tag{3.25}$$

The symbol x_b is the same as in (3.7).

Boundary conditions for the station with reciprocating compressors

The reciprocating compressors are driven by internal combustion engines or by electric motors. If the engines are gas-fueled, we add the virtual node with controlled flow-off. The compression process is characterized by:

- the coefficient of pressure loss at filters and oil separators

$$p_{coeff} = \frac{p_{entrance} - p_{exit}}{p_{exit}} = f(Q) \tag{3.26}$$

  which is a known function on the basis of experiments or on the basis of producer's measurements. The loss of pressure is rather small ($\cong 0.1\,\mathrm{MPa}$) so it is reasonable to approximate this loss by an averaged value.

- the number of identical compressors m and their parameters.

The first boundary condition is derived from the mass conservation principle

$$Q_i(x_{2i},t) - Q_j(x_{1j},t) = 0 \tag{3.27}$$

The formula (3.27) is written for the junction seen in Fig. 6. For the 7 remaining cases of junction, the formulae (3.27) is written as for two-way nodes.

The second boundary condition is derived from the relation between the parameters at the inlet to the compressor unit, marked by index in, and the flow ratio. The pressure at the input to the compressor unit is equal to the pressure p_{exit} at the output of filter section. On the basis of Tuliszka (1976), the pump efficiency η_T is equal to

$$\eta_T = \eta_V \frac{T_{in}}{T_{off}} - \varsigma \tag{3.28}$$

where the volumetric efficiency η_V is equal to

$$\eta_V = 1 + \epsilon_C \Big[1 - \Big(\frac{p_{off}}{p_{in}}\Big)^{1/\kappa}\Big] \tag{3.29}$$

The symbol ς denotes the leak coefficient. Its value is (0.04 - 0.06). The coefficient $\epsilon_C = V_0/V_d$ where V_0 is the clearance volume, V_d is the piston-swept capacity. The symbol κ denotes the isentropic exponent or polytropic exponents specific for the process. The more precise formula has different exponent for the compression and decompression process (Osiadacz, 1976). After these preliminary formulae, the second boundary condition is

$$Q_{in} = mV_d \frac{n}{60} \eta_T \rho_{in} \tag{3.30}$$

From the polytropic curve we have

$$\frac{p_{off}}{p_{in}} = \Big(\frac{\rho_{off}}{\rho_{in}}\Big)^{\kappa} \tag{3.31}$$

From the constitutive equation (3.6) we have

$$T_{off} = T_{in}\Big(\frac{\rho_{off}}{\rho_{in}}\Big)^{\kappa-1}\frac{Z(\rho_{in}, T_{in})}{Z(\rho_{off}, T_{off})} \tag{3.32}$$

For the junction seen in Fig. 6 we have

$$\begin{aligned} p_{in} &= p_i(x_{2i}, t)\frac{1}{1 + f(Q_i(x_{2i}, t))} \\ T_{in} &= T_i(x_{2i}, t) \end{aligned} \tag{3.33}$$

and $p_{off} = p_j(x_{1j}, t)$, $T_{off} = T_j(x_{1j}, t)$. The second boundary condition for the reciprocating compressor is strongly non-linear.

3.3.7. Boundary conditions for pressure reducers

The control elements of the pressure reducer keep the pressure at the outlet of reducer not greater than $p_r(t)$. The pressure reducer has the forbidden direction of flow, opposite to the directional pipe. The gas can flow through the reducer to the directional pipe only, so $Q_j(x_{1j}, t) > 0$ in the junction ($\Longrightarrow \overrightarrow{R} \Longrightarrow$) shown in Fig. 7.

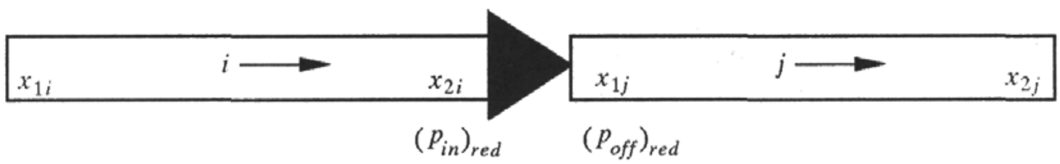

Fig. 7. Scheme of the junction with reducer

The first boundary condition is deduced from the mass conservation principle and is the same as for two-way nodes (3.10). The second boundary condition for $(p_{in})_{red} \leqslant p_r(t)$ is the same as for two-way nodes (3.11), but when $(p_{in})_{red} > p_r(t)$ we have (for the junction shown in Fig. 7)

$$p_j(x_{1j}, t) = p_r(t) \tag{3.34}$$

For the pressure reducer we have 8 variants of junction. The second boundary condition for different junctions is written as the compressor station at $p_{off}(t)$ control mode The pressure drop at the reducers inside the high-pressure network is rather small, so the drop of the gas temperature is also small and is not taken into account.

3.3.8. Boundary conditions for valves

The valve may be open or closed. The boundary conditions for the open valve are the same as for two-way nodes, (3.10), (3.11). The closed valve cuts the network. It may happen that after this cut, the subnetwork without the sources of gas will emerge, but we are able to calculate the flow in it. The above mentioned fact differs the transient model and the static models. When we cut the network, we change one node into two nodes, because each pipe must have the nodes at both ends. Each of the two nodes is an external node, and for each of them we have (see (3.8))

$$Q(x_b, t) = 0 \tag{3.35}$$

3.3.9. Boundary conditions for gate valves

The gate valve may be closed, partially open or fully open. If the gate valve is closed, the boundary coditions are the same as those for a closed valve. If the gate valve is partially open or fully open, the boundary conditions are the same as for two-way nodes or open valves but the coefficient of local pressure loss Λ_{node} is multiplied by the function $F_{op}(A_{open}/A_{full})$. The ratio of the area of cross-section of gate valve to the area of fully open gate valve is the argument of the function F_{op}.

3.3.10. Boundary conditions for check valves

The check valve may be open or closed. The boundary conditions for open or closed check valve are the same as for valve (see Section 3.3.8). We investigate during calculations:

- when the check valve is open, if it does not begin to close
- when the check valve is closed, if it does not begin to open.

This investigation makes the difference between the boundary conditions for valve and check valve. The process "open-close" has a small hysteresis loop which is included in the software packet PSD, but it is not described in the paper.

3.4. Initial and consistency conditions

3.4.1. Initial conditions

The state of flow in the network may be known from:

- solution of the stationary (static) flow problem for the same network

- results of the previous calculations of the transient flow problem
- results of the extrapolation of telemetric measurements.

The third case can be applied in the packet working in the network with many remote measurement stations. In the remaining cases, the packet must include the module for static flow calculations. The static calculation of the initial state of flow causes quantitative and qualitative errors. Influence of quantitative errors diminishes with time, because the initial state of flow is "swilied out" (Fedorowicz, 2001). The static model disables the calculations even in rather simple networks with compressor stations controlled in $Q_{off}(t)$ mode or in the network with two reducers in the same closed loop. Knowing these qualitative errors of the static model we are able to avoid them.

3.4.2. Consistency conditions

The information about the parameters of flow contained in the boundary conditions for $t \to 0^+$ should be consistent with the information from the initial conditions. The consistency conditions should be fulfilled to avoid the discontinuity waves in the flow. When we use the method of finite differences, the consistency conditions change into the conditions of small differences between the same quantities from the boundary and initial conditions. The 10% difference between the value of pressure from initial and boundary conditions may destabilize the calculations but the calculations in the network with the open valve are stable after some iterations, even if the initial conditions follow from the calculations with this valve being closed.

Equations (3.6), the boundary and initial conditions form the mathematical problem describing the physical phenomenon of flow in the transmission network.

4. Physical simplifications

4.1. Simplification of the energy equation

In the equation of energy conservation $(3.6)_3$, both: heat exchange with the environment $4k_T(T - T_{env})/D$, and energy dissipation along a pipe $\frac{\partial}{\partial x}[K(T)\frac{\partial T}{\partial x}]$, are taken into account. The Fourier term $\frac{\partial}{\partial x}[K(T)\frac{\partial T}{\partial x}]$ should be withdrawn from Eq. $(3.6)_3$.

1. If Fourier term is taken into account, two boundary conditions concerning temperature should be formulated. The temperature of the gas is

measured at the compression station outlet and at some measuring stations. The temperature is also measured at nearly all reduction stations of the first step (city gates), but the temperature is measured at the outlet of the gas-heaters, so we do not know the temperature of the gas at the inlets to these nodes. We have not information about the temperature at the external nodes. We may try to formulate the boundary condition of the third kind, the relation between the gradient of the temperature and the temperature, but the constants in this relation are not known.

2. The Fourier term is 10^6 - 10^{10} times less than the term corresponding to the heat exchange with environment. An attempt to solve the problem by the finite difference method has failed (Fedorowicz et al., 2001b). The asymptotic method, specific for the equation with small parameter at the highest order term, should be used in this case. Generally, it is useless to take into account the term million times smaller when you have problems with the knowledge about the first digit of the k_T coefficient.

3. The other authors have already observed that the Fourier term may be withdrawn (Osiadacz and Chaczykowski, 1999; Puzyrewski and Sawicki, 1998).

The energy equation $(3.6)_3$ changes into

$$\rho c_v(T)\Big(\frac{\partial T}{\partial t} + V\frac{\partial T}{\partial x}\Big) + \frac{4k_T}{D}(T - T_{env}) + p\frac{\partial V}{\partial x} = 0 \tag{4.1}$$

so the term $\Theta(x,t)/A$, corresponding to the heat exchange by radiation, is also neglected.

4.2. Postulate of the thermodynamic process versus energy equation

The key problem for modelling with the energy equation is the knowledge of the thermal conductance coefficient k_T. The value of this coefficient depends on the pipe material, thickness of the pipe wall, thickness, quality and state of the insulation layer, the surrounding soil. The value of this coefficient may be time-dependent. This dependence may be caused by rain or by the seasons of the year. The temperature of the surrounding soil T_{env} is known with certain accuracy. We analyze the real processes, not the results of the experiments in the laboratory. The temperature of the flowing gas is measured at few points only.

The problems, emerging when the transmission of the gas in networks is considered with the energy equation, exist because we are unable to get the value of the parameters. The growing complexity of the calculations is unimportant.

The modelling of the problem with the energy equation is useful for estimation of the k_T parameter on the basis of pressure, flow ratio, and temperature measurements (Fedorowicz et al., 2001b).

Two postulates of the thermodynamic process were analyzed. The postulate that the process is adiabatic, or polytropic with variable exponent κ

$$p = C\rho^{\kappa} \tag{4.2}$$

where C is a constant, causes that the calculated losses of pressure are greater than those measured. The idea that we should reinterpret the formula $\lambda(\mathrm{Re}, \varepsilon)$ is rather unrealistic. We should withdraw this postulate from application in the considered problem. The temperature of the flowing gas is rather constant or depends on the seasons of the year in pipes located far away from the compressor stations, so we may postulate the isothermal process

$$T = \text{const} \tag{4.3}$$

Slightly different is the postulate that the temperature should be known function of space variable x (Jenocek et al., 1997). The packet PSD is based on the postulate (4.3).

4.3. Other simplifications

The parameters of the mixture of different gases are calculated by the packet, but the composition of the gas during flow calculations is assumed to be constant.

The compressor stations with reciprocating compressors are not implemented in the packet.

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Modelowanie przepływu gazu w sieci przesyłowej.
Część I – Model matematyczny

Streszczenie

W tej częśći pracy przedstawiono model matematyczny adekwatny do opisu przepływu gazu w realnej sieci przesyłowej. Model uwzględnia ściśliwość i lepkość gazu i ich zależność od składu, temperatury i ciśnienia. Rozważono wpływ wszystkich elementów sieci oddziałujących na zjawisko. Pobory i dostawy gazu są traktowane jako efekt procesu prognozowania lub jako funkcje wynikające z procesu sterowania siecią. Szczegółowo opisane są warunki brzegowe w węzłach sieci, a także uproszczenia fizyczne dotyczące bilansu energii.

W części II, publikowanej w następnym numerze, znajduje się opis metody rozwiązania i istotnych algorytmów.

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