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STRUCTURAL FRICTION AND VISCOUS DAMPING IN A FRICTIONAL TORSION DAMPER

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The considerations concern the steady state of a non-linear discrete three-degree-of-freedom system containing a torsional damper. The system vibrates under harmonic excitation. The analysis takes into account structural friction and linear viscous friction of a ring floating in a plunger filled with a high-density silicon oil. A uniform pressure distribution between the friction discs is assumed. The influence of the main parameters such as: external load amplitude, unit pressures, linear viscous damping, geometric parameters and amplitude-frequency characteristics are analysed. The equations of motion of the examined power transmission system are solved by a slowly-varying-parameter method and digital simulation.

Key words: frictional torsion damper, non-linear vibration, viscous damping, structural friction

1. Introduction

Phenomena described in the paper are very important for engineers who aim at improving the construction of frictional dampers. Traditional professional literature treats frictional dampers, frictional clutches and brakes as joints of rigid bodies. Therefore, the efect of natural damping has been neglected. The autor of this paper takes into consideration the elasticity of the material of co-operating elements in a frictional damper. Frictional torsion dampers are widely applied to to many devices and machines, e.q. to fuel engines. In the paper a three-degree-of-freedom system, reduced to a two-degree-of freedom one is examined. The problem is investigated on the assumption of a uniform distribution pressures and uniform friction coefficient. Structural friction occurs between the cooperating surfaces of discs 2 and plunger 1 (Fig. 1). The discs 2 are pressed down to the plunger 1 by means of the springs 3. Viscous friction arises due to the ring 4, which is suspended and immersed in a high-density silicon oil that fills the plunger. An electrorheological fluid FL or ML as well as magnetostricitive suspension can be substituted for the silicon oil. In the case of electrorheological fluids and suspensions, the medium density can be changed, hence the damping by means of current control can be modified as well. Therefore, the damping of vibrations in the system can assume an active form. The influence of linear viscous damping and structural friction on the damping of vibrations in the system with the uniform distribution pressure is considered, too. The problem of deriving precise mathematical description of the structural friction is very complicated because of the complexity of the friction phenomena as well as difficulties in describing stress or strain states present in the slip zone. Therefore, the mathematical description is based on many simplifications. The friction forces are assumed to be in accordance with the Coulomb law. The discs are made of Hookean materials. The friction is fully developed in the slip zones and does not appear outside them. Theoretical results show good agreement with experimental data.

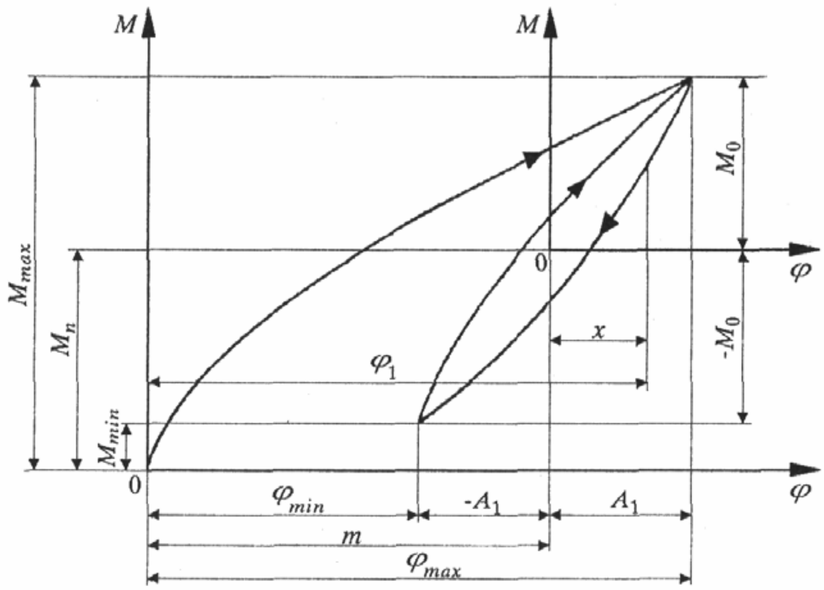

Fig. 1. Physical model with bilinear hysteretic and viscotic damping of the frictional torsion damper; $M = M(\varphi_1, A_1, \dot{\varphi}_1)$

More advanced techniques utilize viscoelastic properties of a given damper as the source of energy dissipation. A proper selection of geometric and dynamic parameters can considerably decrease vibration amplitudes near resonant frequencies or shift these dangerous regions away from operational frequencies.

To accomplish that one must create a mathematical model which can reflect, possibly precisely, most aspects of a real system.

The influence of geometric parameters and external loads on the system response are analysed. In this area, most of the work has been restricted to the analysis of a one-degree-of-freedom system (Gałkowski, 1981; Giergiel, 1990; Grudziński et al., 1992; Iwan, 1965; Osiński, 1993, 1998; Skup, 1991; Szadkowski and Morford, 1992; Zagrodzki, 1994).

In a series of three papers Caughey (1960) successfully investigated the response of a one-degree-of-freedom system to both harmonic and random excitations, and then went on to treat the problem of forced oscillation of a semi-infinite rod exhibiting weak bilinear hysteresis. In the paper by Kosior and Wróbel (1986) a 2-DOF system with an iron strip (dynamical vibration eliminator) with structural friction and viscotic damping was examined where the authors used a method of digital simulation.

2. Equations of motion of the system

We assume a three-mass model of the mechanical system which contains a frictional torsion damper as shown in Figure 1.

The equations of motion of the considered system can be written as follows

$$
\begin{aligned}
&I_1\ddot{\varphi}_{11} + M(\varphi_1, A_1, \dot{\varphi}_1) + C(\dot{\varphi}_{22} - \dot{\varphi}_{33}) = \overline{M}(t) + \overline{M}_m \\
&I_2\ddot{\varphi}_{22} - M(\varphi_1, A_1, \dot{\varphi}_1) = 0 \qquad\qquad (2.1)\\
&I_3\ddot{\varphi}_{33} - C(\dot{\varphi}_{22} - \dot{\varphi}_{33}) = 0
\end{aligned}
$$

where

I_1, I_2, I_3	–	mass moments of inertia of the driving and driven part, respectively
$\varphi_{11}, \varphi_{22}, \varphi_{33}$	–	angular displacements
$M(\varphi_1, A_1, \dot{\varphi}_1)$	–	damper torque in a cycle represented by a structural hysteresis loop (Fig. 2) dependent on the relative displacement and sign of the velocity
C	–	viscous damping coefficient (Fig. 1)
$\overline{M}(t) + \overline{M}_m$	–	variable engine torque described by the constant average value $\overline{M}_m$ and discrete torque $\overline{M}(t)$ in the form of harmonic excitation, i.e.

$$
\overline{M}(t) = M_0 \cos\omega t \qquad (2.2)
$$

and

M_0 – excitation amplitude
ω – angular velocity of the excitation torque
t – time.

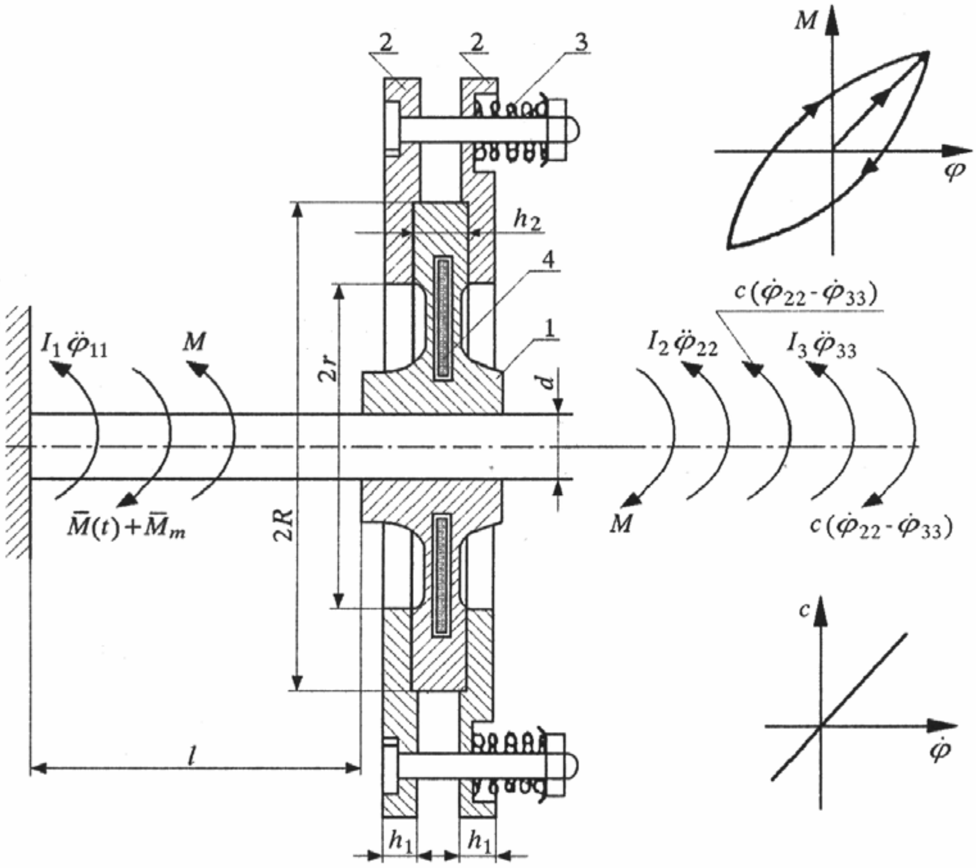

Fig. 2. Hysteresis loop in the frictional torsion damper

In order to determine the hysteresis clutch torque $M(\varphi_1, A_1, \dot{\varphi}_1)$, we shall apply the results obtained by Skup (2000), thus

$$M(\varphi_1, A_1, \dot{\varphi}_1) = \tag{2.3}$$
$$= \frac{1}{\sqrt{\eta_3}}\Big(\frac{1}{\sqrt{2}}\sqrt{A_1} + \operatorname{sgn}\dot{\varphi}_1\sqrt{A_1 + \phi_1 \operatorname{sgn}\dot{\varphi}_1} - \frac{1}{2}\sqrt{2A_1} - \frac{1}{2}\operatorname{sgn}\dot{\varphi}_1\sqrt{2A_1}\Big)$$

$$\eta_3 = \frac{\kappa_1\nu^2}{6} \qquad \kappa_1 = \frac{2\beta(k_1 + k_2)}{k_1 k_2} \qquad \nu = \frac{3}{2\pi\mu p R^3}$$
$$\beta = \frac{\mu p R}{6} \qquad k_1 = Gh_1 \qquad k_2 = Gh_2 \tag{2.4}$$

where

η_3 – nondimensional parameter
k_1 – stiffness of the discs
k_2 – stiffness of the plunger
h_1 – discs thickness
h_2 – plunger thickness
μ – friction coefficient
p – unit pressure
R – external radius of the discs
G – shear modulus.

Introducing new variables $\varphi_1 = \varphi_{11} - \varphi_{12}$ and $\varphi_2 = \varphi_{12} - \varphi_{13}$ in the form of relative angles of torsion, we can reduce equations (2.1) to two second-order nonlinear equations describing the relative torsional vibration

$$\begin{aligned} &\ddot{\varphi}_1 + p_1\dot{\varphi}_2 + f_1(\varphi_1, A_1, \dot{\varphi}_1) - B = z\cos\omega t \\ &\ddot{\varphi}_2 + n\dot{\varphi}_2 - f_1(\varphi_1, A_1, \dot{\varphi}_1) = 0 \end{aligned} \tag{2.5}$$

where

$$\begin{aligned} &\ddot{\varphi}_1 = \ddot{\varphi}_{11} - \ddot{\varphi}_{22} && \ddot{\varphi}_2 = \ddot{\varphi}_{22} - \ddot{\varphi}_{33} && \dot{\varphi}_2 = \dot{\varphi}_{22} - \dot{\varphi}_{33} \\ &p_1 = \frac{C}{I_1} && I_z = \frac{I_1 I_2}{I_1 + I_2} && B = \frac{\overline{M}_m}{I_1} \\ &z = \frac{M_0}{I_1} && f_1(\varphi_1, A_1, \dot{\varphi}_1) = \frac{M(\varphi_1, A_1, \dot{\varphi}_1)}{I_z} \\ &n = \frac{C}{I_3} && f_2(\varphi_1, A_1, \dot{\varphi}_1) = \frac{M(\varphi_1, A_1, \dot{\varphi}_1)}{I_2} \end{aligned} \tag{2.6}$$

3. Solution to equations of motion

Let the solution to system of equations (2.5) be approximated by

$$\varphi_i = A_i \cos\theta_i \qquad i = 1,2 \tag{3.1}$$

where

$$\theta_1 = \omega t - \phi_1 \qquad \theta_2 = \theta_1 - \phi_2 \tag{3.2}$$

and A_i, ϕ_i, $i = 1,2$ are slowly varying functions of time. Thus

$$\dot{\varphi}_i = \dot{A}_i \cos\theta_i + A_i\dot{\phi}_i \sin\theta_i - A_i\omega\sin\theta_i \qquad i = 1,2 \tag{3.3}$$

By analogy to Lagrange's method of the variation of a parameter, it is permissible to set

$$\dot{A}_i \cos\theta_i + A_i \dot{\phi}_i \sin\theta_i = 0 \qquad i = 1,2 \tag{3.4}$$

Thus

$$\ddot{\varphi}_i = \omega A_i \dot{\phi}_i \cos\theta_i - A_i \omega^2 \cos\theta_i - \omega \dot{A}_i \sin\theta_i \qquad i = 1,2 \tag{3.5}$$

Substituting equations (3.5) into the equations of motion (2.5), using formulas (3.2) and (3.4) as well as the slowly varying parameter method, we obtain

$$\begin{aligned}
&-\frac{p_1 A_2 \omega \cos\phi_2}{2} + \frac{1}{2\pi}\int_0^{2\pi} f_1(A_1,\theta_1)\sin\theta_1 \, d\theta_1 - \omega \dot{A}_1 = -\frac{1}{2} z \sin\phi_1 \\
&-\frac{1}{2} A_1 \omega^2 + \omega A_1 \dot{\phi}_1 + \frac{1}{2} p_1 A_2 \omega \sin\phi_2 + \\
&+\frac{1}{2\pi}\int_0^{2\pi} f_1(A_1,\theta_1)\cos\theta_1 \, d\theta_1 = -\frac{1}{2} z \cos\phi_1 \\
&-\omega \dot{A}_2 - \frac{1}{2} n A_2 \omega - \frac{\cos\phi_2}{2\pi}\int_0^{2\pi} f_2(A_1,\theta_1)\sin\theta_1 \, d\theta_1 + \\
&+\frac{\sin\phi_2}{2\pi}\int_0^{2\pi} f_2(A_1,\theta_1)\cos\theta_1 \, d\theta_1 = 0 \\
&\omega A_2 \dot{\phi}_2 - \frac{1}{2} A_2 \omega^2 - \frac{\cos\phi_2}{2\pi}\int_0^{2\pi} f_2(A_1,\theta_1)\cos\theta_1 \, d\theta_1 - \\
&-\frac{\sin\phi_2}{2\pi}\int_0^{2\pi} f_2(A_1,\theta_1)\sin\theta_1 \, d\theta_1 = 0
\end{aligned} \tag{3.6}$$

As the variables A_i and ϕ_i, $i = 1,2$ have been assumed to be slowly varying, they remain essentially constant over one cycle of θ_1. Thus, equations (3.6) can be averaged over the cycle of θ_1.

Steady-state equations (3.6) can be obtained when $\dot{A}_i = \dot{\phi}_i = 0$, $i = 1,2$. If we introduce the notation

$$C(A_1) = \frac{1}{\pi}\int_0^{2\pi} f_1(A_1,\theta_1)\cos\theta_1 \, d\theta_1 \qquad S(A_1) = \frac{1}{\pi}\int_0^{2\pi} f_1(A_1,\theta_1)\sin\theta_1 \, d\theta_1 \tag{3.7}$$

equations (3.6) take the following form

$$\begin{aligned} &\omega p_1 A_2 \cos\phi_2 - \frac{S(A_1)}{I_z} = z\sin\phi_1 \\ &-A_1\omega^2 + p_1 A_2 \omega \sin\phi_2 + \frac{C(A_1)}{I_z} = z\cos\phi_1 \end{aligned} \tag{3.8}$$

$$\begin{aligned} &nA_2\omega + \frac{S(A_1)\cos\phi_2}{I_2} - \frac{C(A_1)\sin\phi_2}{I_2} = 0 \\ &\omega^2 A_2 + \frac{C(A_1)\cos\phi_2}{I_2} + \frac{S(A_1)\sin\phi_2}{I_2} = 0 \end{aligned} \tag{3.9}$$

Because of the discontinuity of the function $M(\varphi_1, A_1, \dot{\varphi}_1)$ at $\dot{\varphi}$ we confine the analysis to one half-period of the vibrating motion. The integration $(0, 2\pi)$ will be divided into the following two intervals: $(0, \pi)$, when $\operatorname{sgn}\dot{\varphi}_1 < 0$ and $(\pi, 2\pi)$ when $\operatorname{sgn}\dot{\varphi}_1 > 0$. This is a procedure adopted by Giergiel (1990) and Osiński (1998).

Thus, by substituting formulas (2.3), (2.6) and (3.2) into equations (3.7), and by integrating, we obtain

$$\begin{aligned} C(A_1) &= \frac{1}{\pi}\int\limits_0^{2\pi} f_1(A_1,\theta_1)\cos\theta_1\, d\theta_1 = \\ &= \frac{1}{\pi I_z}\Big(\int\limits_0^{\pi} M\cos z\; dz_{\downarrow \operatorname{sgn}\dot{\varphi}_1<0} + \int\limits_{\pi}^{2\pi} M\cos z\; dz_{\downarrow \operatorname{sgn}\dot{\varphi}_1>0}\Big) = 2\kappa\sqrt{A_1} \\ S(A_1) &= \frac{1}{\pi}\int\limits_0^{2\pi} f_1(A_1,\theta_1)\sin\theta_1\, d\theta_1 = \\ &= \frac{1}{\pi I_z}\Big(\int\limits_0^{\pi} M\sin z\; dz_{\downarrow \operatorname{sgn}\dot{\varphi}_1<0} + \int\limits_{\pi}^{2\pi} M\sin z\; dz_{\downarrow \operatorname{sgn}\dot{\varphi}_1>0}\Big) = -\kappa\sqrt{A_1} \end{aligned} \tag{3.10}$$

where $M = M(\varphi_1, A_1, \dot{\varphi}_1)$ and

$$\kappa = \frac{2\sqrt{2}}{3\pi I_z\sqrt{\eta_3}} \tag{3.11}$$

The variable ϕ_1 can be eliminated from the foregoing equations by squaring and adding equations (3.8).

This gives

$$\alpha^2 A_2^2 + \omega^2 A_1^2(\omega^2 A_1 - 2\alpha A_2 \sin\phi_2) +$$
$$+2\beta_1\sqrt{A_1}[\alpha A_2(\cos\phi_2 + 2\sin\phi_2) - 2\omega^2 A_1] + 5\beta_1^2 A_1 = z^2 \tag{3.12}$$

where

$$\alpha = p_1\omega \qquad \beta_1 = \frac{\kappa}{I_z} \tag{3.13}$$

Equations (3.9) can be rewritten in the form

$$\sin\phi_2 = \frac{A_2 h}{\sqrt{A_1}} \qquad \cos\phi_2 = \frac{A_2 g}{\sqrt{A_1}} \tag{3.14}$$

where

$$h = \frac{v - \delta g}{2\delta} \qquad v = n\omega \qquad \delta = \frac{\kappa}{I_2} \qquad g = \frac{v - 2\omega^2}{5\delta} \tag{3.15}$$

By squaring and adding equations (3.14) we obtain

$$A_1 = wA_2^2 \tag{3.16}$$

where $w = h^2 + g^2$.

Equations (3.14) can be used to eliminate the variable ϕ_2 in equation (3.12). Thus, substitution of equations (3.14) and (3.16) into equation (3.12) and rearranging gives

$$A_2^4 + bA_2^3 + eA_2^2 + f = 0 \tag{3.17}$$

where

$$b = \frac{-2\sqrt{w}}{w^2\omega^2}(\alpha h + 2\beta_1 w) \qquad f = \frac{-z^2}{w^2\omega^4}$$
$$e = \frac{1}{w^2\omega^4}[\alpha^2 + 5w\beta_1^2 + 2\alpha\beta_1(g + 2h)] \tag{3.18}$$

Finally, we introduce the following notations

a_2	–	nondimensional vibration amplitude, $a_2 = A_2/\varphi_{st}$
φ_{st}	–	static displacement in the form of a relative angular displacement of the damper discs, $\varphi_{st} = M_0/k$
γ	–	nondimensional frequency, $\gamma = \omega/\omega_0$
ω_0	–	natural frequency of the system, $\omega_0 = \sqrt{k/I_z}$
k, G	–	shear modulus, $k = GI_0/l = \pi G d^4/(32l)$
l, d	–	diameter and length of the damper shaft

gives

$$a_2^4 + t_1 a_2^3 + u a_2^2 + m = 0 \qquad a_1 = \frac{wM_0^2}{k^2} a_2^2 \tag{3.19}$$

where

$$t_1 = \frac{bk}{M_0} \qquad u = \frac{ek^2}{M_0^2} \qquad m = \frac{fk^4}{M_0^4} \tag{3.20}$$

By substitution

$$a_2 = y - \frac{t_1}{4} \tag{3.21}$$

we reduce equation $(3.19)_1$ to the following form

$$y^4 + p_0 y^2 + q_0 y + r = 0 \tag{3.22}$$

where

$$\begin{aligned} p_0 &= u - \frac{3}{8} t_1^2 \qquad q_0 = \frac{1}{2} t_1 \Big(\frac{1}{4} t_1^2 - u \Big) \\ r &= m + \frac{1}{16} u t_1^2 - \frac{3}{256} t_1^4 \end{aligned} \tag{3.23}$$

Because $q_0 \neq 0$, the roots of equation (3.22) are determined from the following equations

$$\begin{aligned} y^2 + y\sqrt{\lambda - p_0} + \Big(\frac{\lambda}{2} - \frac{q_0}{2\sqrt{\lambda - p_0}} \Big) &= 0 \\ y^2 - y\sqrt{\lambda - p_0} + \Big(\frac{\lambda}{2} + \frac{q_0}{2\sqrt{\lambda - p_0}} \Big) &= 0 \end{aligned} \tag{3.24}$$

It results from the analysis of equations (3.24) that only one root of equation $(3.24)_1$ is a real number.

Therefore

$$y = \frac{\lambda - p_0 + \sqrt{-(\lambda^2 - p_0^2) - 2q_0\sqrt{\lambda - p_0}}}{2\sqrt{\lambda - p_0}} \tag{3.25}$$

where λ can be determined from the following equation

$$\lambda^3 - p_0\lambda^2 - 4r\lambda + (4p_0 r - q_0^2) = 0 \tag{3.26}$$

By substitution

$$\lambda = z_1 + w_1 \tag{3.27}$$

where $w_1 = p_0/3$, equation (3.26) can be reduced to the form

$$z_1^3 + \xi z_1 + \eta = 0 \tag{3.28}$$

where

$$\xi = -\frac{1}{3}p_0^2 - 4r \qquad \eta = \frac{2p_0}{3}\left(4r - \frac{p_0^2}{9}\right) - q_0^2 \tag{3.29}$$

Because

$$\frac{\eta^2}{4} + \frac{\xi^3}{27} > 0 \tag{3.30}$$

thus

$$z_1 = \sqrt[3]{-\frac{\eta}{2} + \sqrt{\frac{\eta^2}{4} + \frac{\xi^3}{27}}} + \sqrt[3]{-\frac{\eta}{2} - \sqrt{\frac{\eta^2}{4} + \frac{\xi^3}{27}}} \tag{3.31}$$

In this way the formulation of the steady-state problem has been reduced to a set of two equations (3.19) expressed in terms of two unknown nondimensional amplitudes a_1 and a_2.

Equation $(3.19)_1$ is solved by the Newton-Raphson Iterative method. One from the ten roots of equation $(3.19)_1$ satisfying the physical condition is chosen in the analysis. This root takes a specific value of the deformation amplitude in the examined system. For such a value a_2 the value a_1 is calculated with formulae (3.19) in the function of forced vibration frequency.

Numerical simulations carried out for the above formulas incorporated the following data:

$h_1 = 0.003$ m	$h_2 = 0.006$ m	$R = 0.08$ m
$l = 0.015$ m	$d = 0.055$ m	$I_1 = 0.35\ \mathrm{kg\,m^2}$
$I_2 = 0.09\ \mathrm{kg\,m^2}$	$M_0 = 100$ Nm	$\mu = 0.20$
$p = 0.6 \cdot 10^5\ \mathrm{N/m^2}$	$I_3 = 0.002\ \mathrm{kg\,m^2}$	

4. Conclusions

In the paper the effect of most important parameters of the vibrating frictional damper system on resonant amplitudes is discussed in detail.

On the basis of the obtained results it has been found that all resonance curves start from the nondimensional resonance amplitude and tend asymptotically to zero in the postresonance range. They also tend to a more smooth form in that range.

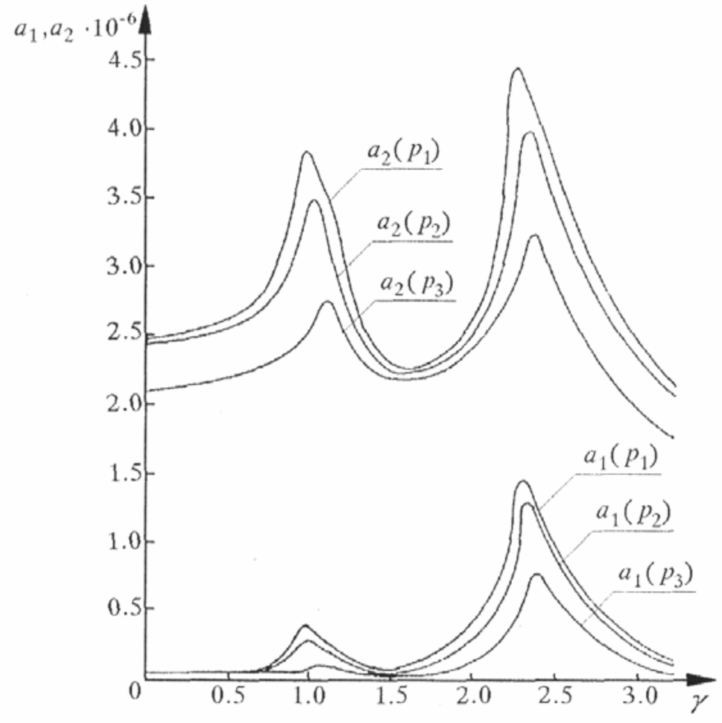

Fig. 3. Resonant curves for various values of the unit pressure p; $p_1 = 0.8 \cdot 10^5\,\mathrm{Nm}^{-2}$, $p_2 = 1.2 \cdot 10^5\,\mathrm{Nm}^{-2}$, $p_3 = 1.5 \cdot 10^5\,\mathrm{Nm}^{-2}$

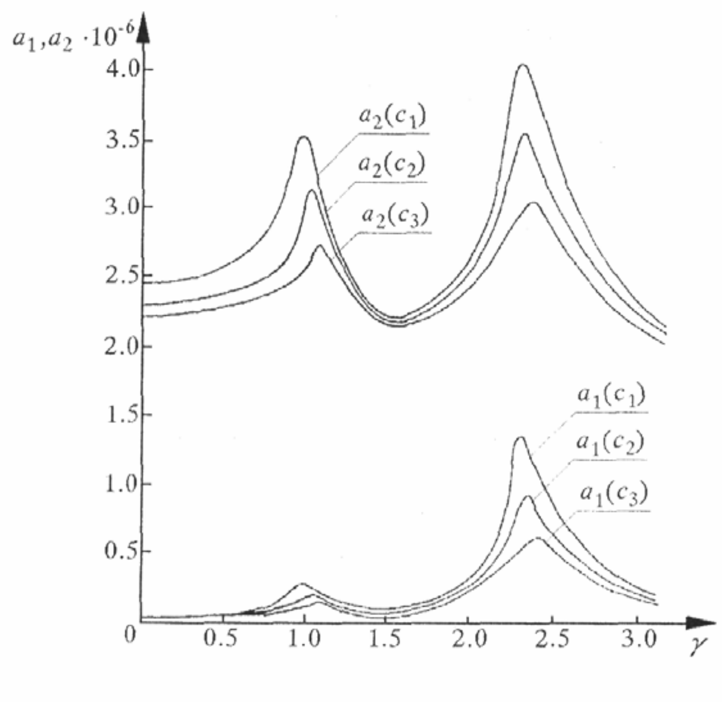

Fig. 4. Resonant curves for various values of the viscous damping coefficient c; $c_1 = 1.0\,\mathrm{Nms}$, $c_2 = 1.3\,\mathrm{Nms}$, $c_3 = 2.0\,\mathrm{Nms}$

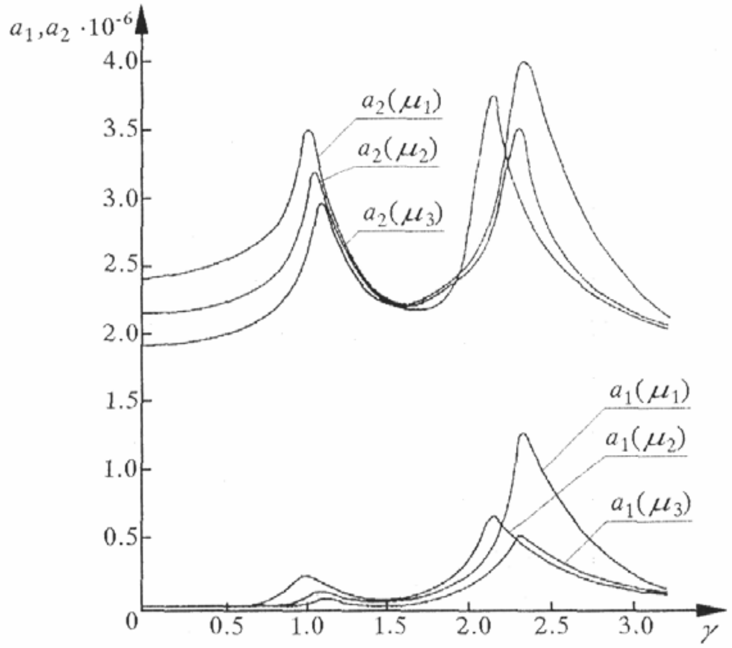

Fig. 5. Resonant curves for various values of the friction coefficient μ; $\mu_1 = 0.24$, $\mu_2 = 0.34$, $\mu_3 = 0.44$

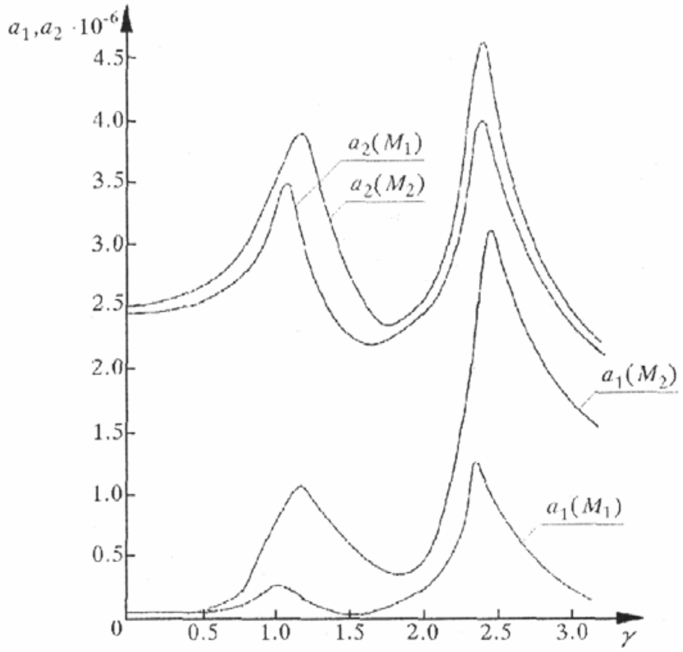

Fig. 6. Resonant curves for various values of the friction coefficient M; $M_1 = 120\,\mathrm{Nm}$, $M_2 = 200\,\mathrm{Nm}$

The graphs shown in Fig. 3 to Fig. 6 illustrate these phenomena. Moreover, one can observe a decrease in the resonance amplitudes and their rightwards shift for growing pressure (Fig. 3), viscous damping (Fig. 4), friction coefficient (Fig. 5) and excitation amplitude (Fig. 6) with the other parameters kept constant. The amplitudes of the first resonance are smaller than the vibration amplitudes of the second one. In the extra resonance range the vibration decreases rapidly to zero (but asymptotically). A distinctive feature of this system is that the vibration amplitudes of the frictional part of the clutch plates are smaller by an order than the amplitudes of the ring immersed in the liquid with a proper damping coefficient.

The effect of reduction of the vibration amplitude is caused by the existence of the optimal pressure and friction coefficient of the material, which enhances greater energy dissipation. The increase in the energy dissipation results in a more intensive vibration damping in the examined power transmission system. The effect of damping is especially big for an appropriate value of the friction force because the solid zone between the discs in the damper is the greatest. It is clearly seen in Fig 3 to Fig. 6 as they show it for the same geometric parameters and loading but varying pressure p, viscous damping c, friction coefficient μ and loading M. Non-linear effects occur at any frequency and for any vibration amplitude. For the forced frequency ω which is close to the natural frequency the non-dimensional amplitudes a assume big values.

The damping effect is the greatest for an appropriate value of the friction force because the zone of relative slip between the damper discs is the largest. The effects of structural friction and viscous damping can be used in order to improve the design methods of dynamic systems.

Finally, it can be said that the efficiency of vibration damping by means of a frictional damper is largely influenced by the following factors: excitation amplitude, stiffness of the shaft and discs, unit pressure, friction and viscous damping coefficient. It seems that the proposed model is interesting from the engineering point of view.

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Tłumienie drgań skrętnych poprzez tłumik cierny przy uwzględnieniu tarcia konstrukcyjnego i wiskotycznego

Streszczenie

Rozważania przeprowadzono dla stanu ustalonego nieliniowego dyskretnego układu mechanicznego zawierającego tłumik drgań skrętnych o trzech stopniach swobody. Drgania występują pod wpływem wymuszenia harmonicznego. Badany układ zawiera specjalny tłumik cierny, uwzględnia tarcie konstrukcyjne oraz liniowe tłumienie wiskotyczne pływającego pierścienia wewnątrz bezwładnika wypełnionego olejem silikonowym o dużej gęstości. Zagadnienie rozpatrywane jest przy założeniu równomiernego rozkładu nacisków występujących pomiędzy współpracującymi powierzchniami tarcz ciernych. Zbadano wpływ amplitudy obciążenia zewnętrznego, nacisków jednostkowych, liniowego tłumienia wiskotycznego oraz współczynnika tarcia na charakterystykę amplitudowo-częstotliwościową układu. Równania ruchu badanego układu mechanicznego rozwiązano metodą wolno zmieniających się parametrów oraz symulacji cyfrowej.

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