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DETERMINATION OF PERMEABILITY AND TORTUOSITY OF PERMEABLE MEDIA BY ULTRASONIC METHOD. STUDIES FOR SINTERED BRONZE

Jan Kochański
Mariusz Kaczmarek
Józef Kubik

Department of Environmental Mechanics, Bydgoszcz University

e-mail: mkk@rose.man.poznan.pl

The paper presents a combined graphical and analytical method of determination of two structural parameters of a permeable medium: permeability and tortuosity. The method is based on a two-phase model of permeable material and uses experimental data from ultrasonic studies of wave parameters in alcohol-saturated sintered bronze.

Key words: saturated porous materials, ultrasonic waves, attenuation

1. Introduction

The fundamental parameters characterizing geometrical structure of porous permeable materials are: volume porosity f_v, permeability k, and tortuosity α. Such parameters play an important role in the motion equations of the two-phase model of permeable media and particularly, they determine interaction forces $\boldsymbol{R}^s$ and $\boldsymbol{R}^f$ between the pore fluid and the porous skeleton (Biot, 1956; Kaczmarek and Kubik, 1993)

$$\boldsymbol{R}^s = -\boldsymbol{R}^f = b(\boldsymbol{v}^f - \boldsymbol{v}^s) + \overline{\rho}^f(\alpha - 1)\frac{\partial}{\partial t}(\boldsymbol{v}^f - \boldsymbol{v}^s) \tag{1.1}$$

where $b = \eta f_v^2/k$, $\boldsymbol{v}^f$ and $\boldsymbol{v}^s$ denote the velocities of fluid and solid skeleton, $\overline{\rho}^f$ is the partial density of fluid, and η is the dynamic viscosity of fluid.

The values of the above pore structure parameters of permeable materials can be evaluated by the analysis of solutions of wave equations for the case of harmonic excitation, obtained within the two-phase model of a saturated porous material (Kaczmarek and Kubik, 1993; Kochański, 1998). Then, the following general relationships must be satisfied

$$\{V_j, A_j\} = \{V_j, A_j\}\{A, Q, R, N, \eta, \rho^s, \rho^f, f_v, \alpha, k, \omega\} \tag{1.2}$$

where V_j and A_j denote wave velocities and coefficients of attenuation for the three modes propagating in porous media (fast, slow, and shear wave, respectively). A, Q, R, N are the coefficients which determine elastic properties of a saturated medium, ρ^f, ρ^s are partial densities of fluid and solid, and ω is the angular frequency. Using the relation (1.2) and the appropriate experimental data for wave parameters (phase velocity and attenuation coefficient), one can identify the structural parameters of a porous material if an appropriate searching procedure is applied.

This paper presents a procedure of determination of the two structural parameters: permeability and tortuosity for porous sintered bronze.

2. Experimental results

Experimental results for the model of porous materials made from sintered bronze particles of the average grain diameter equal to $70\,\mu$m and saturated with ethylene alcohol are performed. The experimental technique is based on the broad-band ultrasonic transmission method (Kaczmarek et al., 1998; Kochański, 1998). A cylindrical sample of sintered bronze is placed between transmitting and receiving transducers immersed in liquid (alcohol). The sample can be rotated to change the angle of wave incidence which serves as a way to separate different wave modes (fast, slow and shear wave) (Plona, 1980; Kochański, 1998). From the three registrated wave modes the data obtained for the slow wave have been used to identify the structural parameters. The phase velocity and coefficient of attenuation of the wave are shown in Fig. 1.

3. Determination of structural parameters

In order to determine the permeability k and tortuosity α, a combined graphical and analytical method based on searching for the common point (point of intersection) of zero error functions of the wave velocity ΔV and attenuation ΔA is used. In Fig. 2 the error functions for fixed frequency

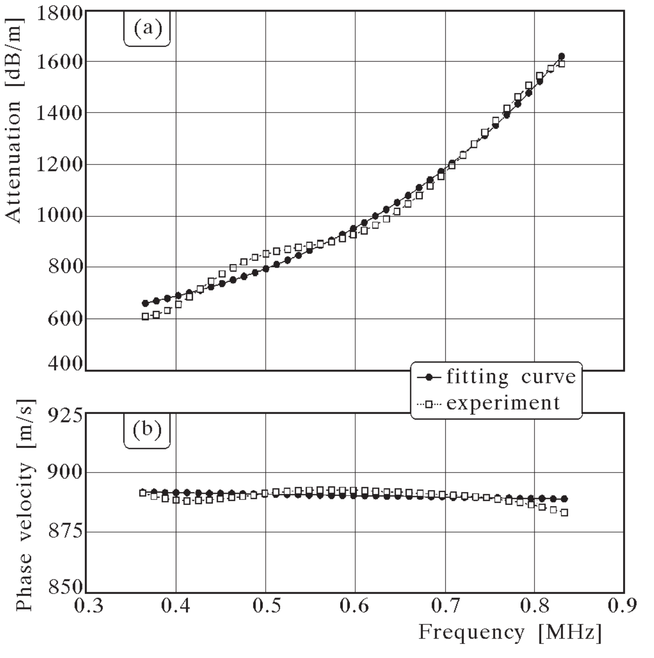

Fig. 1. Attenuation coefficient (a) and phase velocity (b) of a slow wave for alcohol-saturated bronze with the average grain size 70 μm

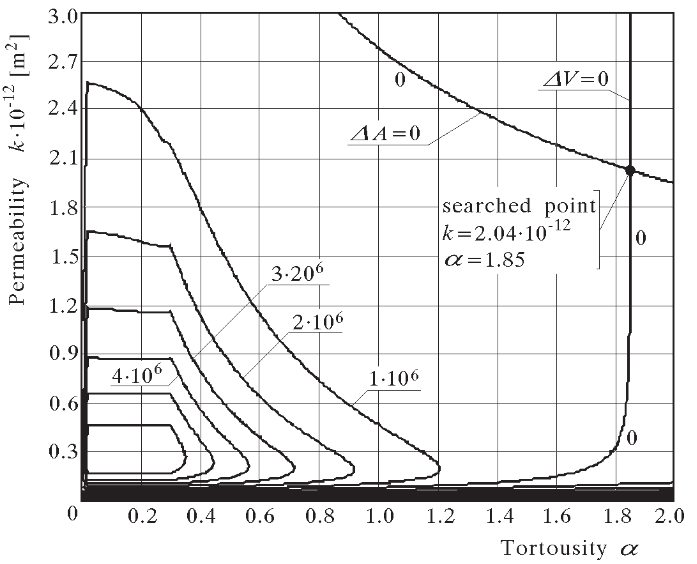

Fig. 2. Illustration of the steps of the proposed method of determination of structural parameters. Determination of the common point for $\Delta V = 0$ and $\Delta A = 0$

(0.4 Mhz) are plotted versus permeability k and tortuosity α. The error functions are defined as follow

$$\Delta V = [V_{pom} - V_{mod}(\alpha, k)]_{(\omega=\text{const})}$$

$$\Delta A = [A_{pom} - A_{mod}(\alpha, k)]_{(\omega=\text{const})}$$

where V_{pom}, $V_{mod}(\alpha, k)$, A_{pom}, $A_{mod}(\alpha, k)$ are the velocities and attenuation coefficients determined from experiments and calculated from the two-phase model as the functions of α and k (see Kaczmarek and Kubik (1993) for the review of dynamical methods of determination of material parameters of the two-phase model). The proposed method applied with experimental data obtained for the sintered bronze and frequency equal to 0.4 MHz allowed us to find the following values of permeability and tortuosity: $k = 2.04 \cdot 10^{-12}\,\text{m}^2$, $\alpha = 1.85$.

4. Analysis of the obtained results and discussion

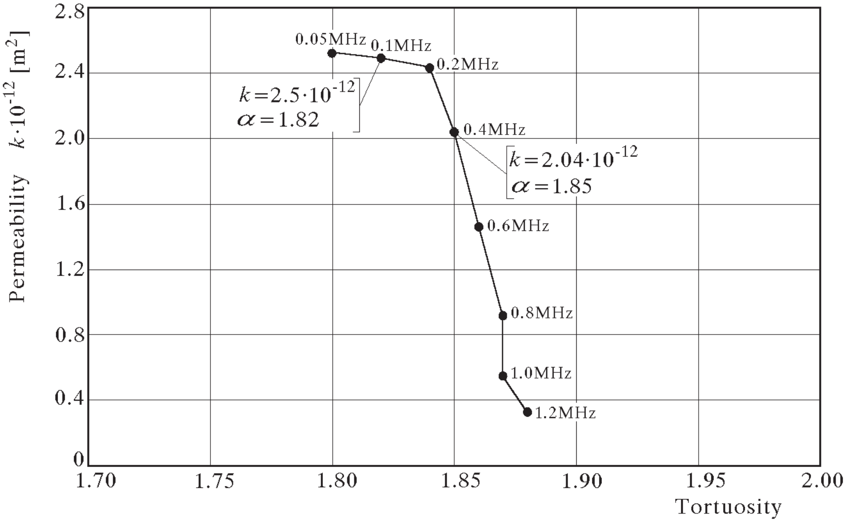

Fig. 3. Influence of the frequency on the values of structural parameters (the structural parameters should be identified with coordinates of the points)

In order to test the sensitivity of the proposed method to the frequency, for which the structural parameters are identified, the same procedure was applied for various values of frequency from 0.05 to 1.2 MHz. The results of calculations are presented in Fig. 3.

It can be seen that for relatively low frequencies (below 0.2 MHz), the results for permeability and tortuosity are very close to each other and remain within the range from $2.45 \cdot 10^{-12}$ to $2.52 \cdot 10^{-12}\,\mathrm{m}^2$ for permeability, and from 1.8 to 1.84 for tortuosity. For the frequencies above 0.2 MHz, the tortuosity changes are still insignificant (from 1.85 to 1.88) but permeability reaches the values from $0.33{\cdot}10^{-12}$ to $2.45{\cdot}10^{-12}\,\mathrm{m}^2$. Thus, it can be assumed that the values of permeability and tortuosity derived for frequencies below 0.2 MHz are close to the real values of the material studied. One can expect that the observed dispersion of the parameters for higher frequencies are the result of scattering of the waves on grains of the material (Kaczmarek et al., 1998). The latter effect is not included into the classical model of the two-phase material and thus it may essentially influence the errors of the identified structural parameters. In order to incorporate the influence of scattering on the wave parameters, we propose (at the present stage of the modelling it is a postulate) a modification of the model by the following frequency correction of the parameter b

$$b = \eta\frac{f_v^2}{k}[1+\chi(f)] \qquad\qquad \chi(f) = lf^2 + nf^4 \tag{4.1}$$

The best fit to the experimental data is obtained when $l = 10\,\mathrm{MHz}^{-2}$ and $n = 2.5\,\mathrm{MHz}^{-4}$. Application of the function $(4.1)_1$ to the two-phase model of porous medium allows one to minimize the dispersion of the structural parameters derived for various frequencies. The values of permeability are between $2.46 \cdot 10^{-12}$ and $2.54 \cdot 10^{-12}\,\mathrm{m}^2$ while the values of tortuosity range from 1.81 to 1.87, see Fig. 4.

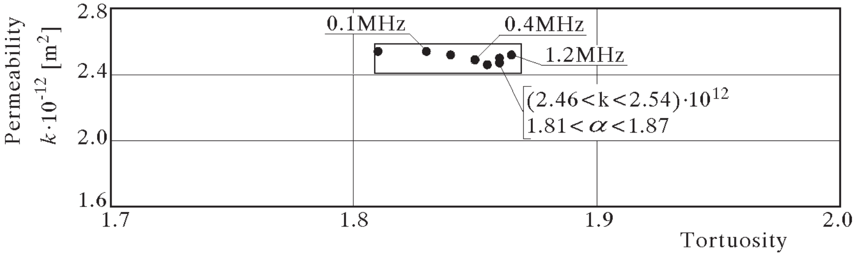

Fig. 4. Distribution of structural parameters for corrected form of coefficient b

The phase velocity and attenuation coefficient of the slow wave predicted by the model with uncorrected and corrected coefficient b, for $k = 2.5 \cdot 10^{-12}\,\mathrm{m}^2$, and $\alpha = 1.85$, were compared with the experimental data giving an excellent agreement of the model predictions with the experimental results.

References

1. BIOT M.A., 1956, Theory of Propagation of Elastic Waves in a Fluid Saturated Porous Solid. I. Low Frequency Range, *J. Acoust. Soc. Am.*, **28**, 2, 168-178
2. KACZMAREK M., KUBIK J., 1993, Dynamiczne metody wyznaczania charakterystyk materiałowych nasyconych ośrodków Porowatych, *Prace IPPT*, **16**
3. KACZMAREK M., KOCHAŃSKI J., KUBIK J., 1998, Ultrasonic Waves in Saturated Porous Materials. Discussion of Modeling and Experimental Results, *J. Theor. Appl. Mechanics*, **36**, 597-618
4. KOCHAŃSKI J., 1998, Wyznaczanie parametrów struktury ośrodka przepuszczalnego na podstawie pomiarów fal ultradźxwiękowych, Rozprawa Doktorska, Politechnika Poznańska
5. PLONA T.J., 1980, Observation of a Second Compressional Waves in a Porous Medium at Ultrasonic Frequencies, *Appl. Phys. Lett.*, **36**, 259-261

Wyznaczanie przepuszczalności i krętości ośrodka przepuszczalnego przy wykorzystaniu wolnej fali ultradźwiękowej. Badania dla porowatego spiekanego brązu

Streszczenie

W pracy przedstawiono analityczno-graficzną metodę wyznaczania dwóch parametrów struktury ośrodka przepuszczalnego: przepuszczalności i krętości. Proponowana metoda bazuje na dwufazowym modelu ośrodka przepuszczalnego i na wynikach pomiaru parametrów propagacji fali ultradźwiękowej – wolnej w nasyconym ośrodku porowatym. Pomiary wykonano dla porowatego brązu nasyconego alkoholem.

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