JOURNAL OF THEORETICAL
AND APPLIED MECHANICS
4, **38**, 2000

SENSITIVITY AND DESIGN OF FIBER LAYOUT IN COMPOSITES

Krzysztof Dems

Department of Technical Mechanics, Łódź Technical University
e-mail: dems@ck-sg.p.lodz.pl

For a linear structure reinforced with elastic fibers, the first variation of an arbitrary behavioral functional corresponding to variation of shape or orientation of fibers is derived by using the direct approach to sensitivity analysis. The relevant optimality conditions for optimal design and identification problems are then derived. Both the static and dynamic loading cases are considered. Some simple examples illustrate the applicability of the presented approach.

Key words: composites, fiber layout, sensitivity analysis, optimization

1. Introduction

The full advantage of fiber-reinforced materials are obtained when fibers are distributed and oriented optimally with respect to the assumed objective behavioral measure in the optimization process. To fulfill the assumed optimization goals for a certain structure, we can modify some parameters of the structural material, such as fiber plies thickness, fiber density, shape and orientation, stacking sequence etc. A possible way to satisfy the optimal design requirements is to derive the optimality conditions and sensitivity formulae, which can be used either directly or in the iterative optimization procedure.

The optimization of composite and fiber-reinforced materials has found a growing interest in the literature. Such materials are treated on the macroscale level as anisotropic or orthotropic materials, and their stiffness parameters are subjected to the optimization process. Optimal design of fiber-reinforced materials and structures were discussed, for instance, in Dems (1996, 2000), Matheus et al. (1991), Olhoff and Thomsen (1990), Thomsen (1991). The maximum stiffness was used as objective function in optimal design of structures made of fiber-reinforced composite materials by Bojczuk (1992) and Thomsen (1991). The problem of optimal design of a disk reinforced by a number of fibers was considered by Mróz and Dems (1992).

The fiber-reinforced laminate will be treated on the macroscale level as an orthotropic linear material, and sensitivity analysis and optimal design will be performed with respect to the parameters defining the fibers shape or orientation in each play.

2. Problem formulation

Consider a thin plane disk made of symmetric laminate with the middle plane, Fig.1. The transverse boundary surfaces of the disk are cylindrical and parallel to the x_3-axis. The disk is subjected to generalized body forces $\boldsymbol{f}$ acting within its plane, and generalized in-plane tractions $\boldsymbol{T}^0$ acting along the boundary portion S_T of external boundary S. On the remaining portion S_u, the in-plane displacement $\boldsymbol{u}^0$ is specified. All the specified boundary conditions can be in general time-dependent.

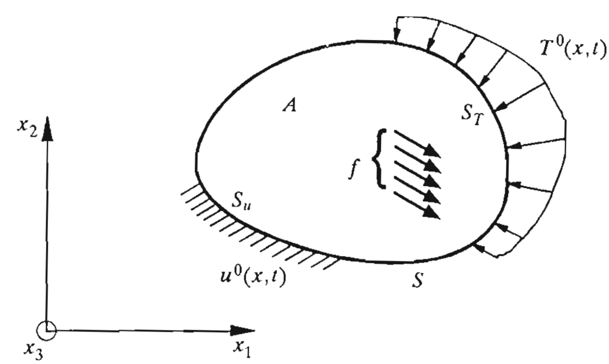

Fig. 1. Thin disk made of composite material

The material of a disk is a symmetric laminate made of a stack of layers, which are symmetric in both the geometric and material properties about the middle plane of the disk. Each layer, placed symmetrically with respect to the middle plane, constitutes a lamina made of matrix reinforced by a ply of unidirectional fibers of arbitrary shape. The orientation of fibers in each layer can vary with respect to the global co-ordinate system x_1, x_2. Let us denote the mid-plane strain field within a disk by $\boldsymbol{e}^\top = \{e_1, e_2, e_{12}\}$, and the generalized stress field by $\boldsymbol{N}^\top = \{N_1, N_2, N_{12}\}$. Using the classical lamination theory, the generalized stress and mid-plane strain are interrelated by Hooke's law of the form

$$\boldsymbol{N} = \mathbf{D}\boldsymbol{e} \tag{2.1}$$

where $\mathbf{D}$ is a symmetric and positive definite extensional stiffness matrix of the laminate. The components of this matrix are expressed by

$$D_{ij} = 2\sum_{k=0}^{n} \overline{Q}_{ij}^k h_k \tag{2.2}$$

where

- h_k – thickness of the kth layer (thickness of the middle layer is assumed to be equal to $h_0/2$)
- $\overline{Q}_{ij}^k$ – components of the stiffness matrix for orthotropic layer with respect to the global reference system x_1, x_2.

The components of matrix $\mathbf{D}$ can be expressed in terms of engineering constants E_1^k, E_2^k, G_{12}^k, and ν_{12}^k for each particular layer and fiber shape or orientation parameters within this layer. The E_1^k and E_2^k are the generalized Young's moduli in fiber direction y_1 and direction y_2 transverse to the fibers, cf Fig.2, while G_{ij}^k denotes the in-plane generalized shear modulus of a lamina and ν_{12}^k is the so-called major Poisson's ratio. The stiffness matrix appearing in (2.2) can be written in the form

$$\overline{\mathbf{Q}}^k = \mathbf{T}_k^{-1}\mathbf{Q}^k\mathbf{T}_k^{-\top} \tag{2.3}$$

where $\mathbf{Q}^k$ denotes the lamina stiffness matrix with respect to material axes y_1, y_2 and $\mathbf{T}_k$ is the transformation matrix from the global coordinate system x_1, x_2 to the material axes y_1, y_2 of the kth layer. Let us note that $\mathbf{Q}^k$ depends on engineering constants of the kth layer, while $\mathbf{T}_k$ is a matrix function of fiber orientation in a particular layer.

Using the simplest model of lamina (cf Jones, 1975), the engineering constants in each layer follow from the rule of mixtures. Thus, the extensional stiffness matrix $\mathbf{D}$ defined by Eqs (2.2) and (2.3) depends on mechanical properties of the fiber and matrix as well as the fiber density, shape and orientation in each layer and on layer thickness. Let us note that, besides the above presented simplest model of fiber-reinforced composite material, there exist more complicated and more accurate models of a laminate composed of uni- and bidirectionally reinforced layer. But even for these more complicated models, the generalized stiffness matrix $\mathbf{D}$ depends on the same parameters as for the model presented here. The approach to sensitivity analysis and optimal

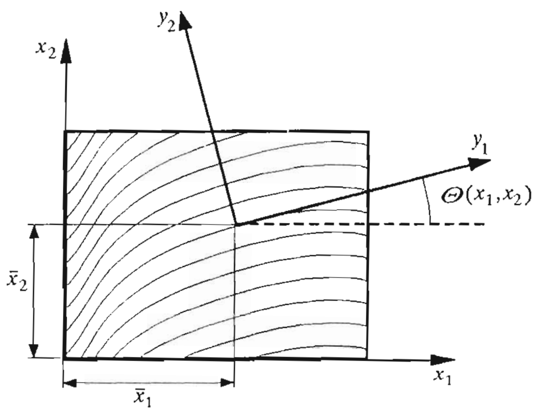

Fig. 2. Global $\{x_i\}$ and material $\{y_i\}$ axes in the laminate layer

design, discussed in the following sections, can be easily applied also for more complicated models of the composite material, not discussed here.

Let us now assume that the number of layers and mechanical properties of the fibers and matrix as well as the fiber density are given in advance and are constant in each layer. On the other hand, the fiber orientation Θ^k, $(k = 0,...,n)$, can vary in each layer. The orientation of fibers in each particular layer can be either constant and then the fibers are rectilinear through the layer, or their shape can vary through the layer. In this latter case, the fibers are placed curvilinearly in matrix, see Fig.2. As a result, the fiber orientation at any point of the layer domain can depend on a set of fiber shape parameters $\boldsymbol{b}_k$, i.e. $\Theta^k = \Theta^k(\boldsymbol{x}, \boldsymbol{b}_k)$. Then the extensional stiffness matrix D of the disk material appearing in (2.2) can be written in the form

$$\mathsf{D}(\boldsymbol{x}, \boldsymbol{b}) = 2\sum_{k=0}^{n} h_k \overline{\mathsf{Q}}^k(\boldsymbol{x}, \boldsymbol{b}_k) = 2\sum_{k=0}^{n} h_k \mathsf{T}_k^{-1}(\boldsymbol{x}, \boldsymbol{b}_k)\mathsf{Q}^k\mathsf{T}_k^{-\top}(\boldsymbol{x}, \boldsymbol{b}_k) \tag{2.4}$$

The change of components of vector $\boldsymbol{b}$ will then influence the material properties of the resultant material of disk, and the sensitivity analysis and optimal design will be carried out with respect to this design vector.

3. Sensitivity analysis for an arbitrary functional

In this section we extend the analysis presented by Dems and Mróz (1983), Dems (1996) to the case of dynamically loaded disk made from fiber-reinforced material for which the generalized stresses and strains are interrelated by Eq (2.1), with extensional stiffness matrix defined by Eq (2.2). This stiffness matrix depends on vector $\boldsymbol{b}$ of the design parameters defining the shape or orientation of the fiber in each layer.

The equation of motion of a disk under the applied load in A and on S_T and support conditions on S_u, can be expressed by the following set of equations

$$\begin{aligned} &\operatorname{div}\boldsymbol{N}(\boldsymbol{x},t)+\boldsymbol{f}(\boldsymbol{x},t)=\rho(\boldsymbol{x})\ddot{\boldsymbol{u}}(\boldsymbol{x},t) && \text{in} \quad A\times[0,t_f]\\ &\boldsymbol{N}(\boldsymbol{x},t)\boldsymbol{n}(\boldsymbol{x})=\boldsymbol{T}^0(\boldsymbol{x},t) && \text{on} \quad S_T\times[0,t_f]\\ &\boldsymbol{u}(\boldsymbol{x},t)=\boldsymbol{u}^0(\boldsymbol{x},t) && \text{on} \quad S_u\times[0,t_f]\\ &\boldsymbol{u}(\boldsymbol{x},0)=\boldsymbol{u}_0(\boldsymbol{x}) && \text{in} \quad A\times[0]\\ &\dot{\boldsymbol{u}}(\boldsymbol{x},0)=\boldsymbol{v}(\boldsymbol{x}) && \text{in} \quad A\times[0] \end{aligned} \tag{3.1}$$

where $\rho(\boldsymbol{x})$ denotes generalized density of the disk material and t_f is the terminal time of dynamic process. Moreover, the dot over a symbol denotes the derivative with respect to time t. The strains in the disk are related to the displacements through the strain-displacement relation

$$\boldsymbol{e}=\mathsf{B}\boldsymbol{u} \tag{3.2}$$

where B is a linear operator. Solution of (3.1), (3.2), along with (2.1), is performed (with prescribed values of design parameters $\boldsymbol{b}$) and results in determining the generalized stress and strain distribution over the domain of the disk.

Let us now consider the problem of evaluating the first-order sensitivities of an arbitrary behavioral functional G defined over the fixed time period $[0,t_f]$, given of the form

$$G=\int_0^{t_f}\Big\{\int \varPhi[\boldsymbol{N}(\boldsymbol{x},t),\boldsymbol{e}(\boldsymbol{x},t),\boldsymbol{u}(\boldsymbol{x},t),\dot{\boldsymbol{u}}(\boldsymbol{x},t)]\;dA+\int h[\boldsymbol{T}(\boldsymbol{x},t),\boldsymbol{u}(\boldsymbol{x},t)]\;dS\Big\}\;dt \tag{3.3}$$

depending on generalized stress, strain and displacement fields within the disk domain A as well as on the surface tractions and displacements along the disk boundary S. We should note that the functional G can express any behavioral measure of the disk. It can be a measure of the disk mean stiffness or compliance. But G can also express the global measure of stress, strain or displacement intensity within the disk domain or along its boundary calculated over a given period of time, or even the local quantities such as stress, strain or displacement components at a given point of disk.

When Φ and h in (3.3) are continuous functions of their arguments, the variation of G equals

$$\delta G = \boldsymbol{G_b}\delta\boldsymbol{b} = \sum_{k=0}^{n} \boldsymbol{G_{b_k}}\delta\boldsymbol{b}_k \tag{3.4}$$

where $\boldsymbol{G_b}$ is the first-order sensitivity vector of G with respect to the design parameters $\boldsymbol{b}$ and $\delta\boldsymbol{b}$ denotes variation of the design vector $\boldsymbol{b}$.

Assuming that the surface traction $\boldsymbol{T}^0$ on S_T and displacement $\boldsymbol{u}^0$ on S_u as well as the initial conditions in A are design-independent, the first-order sensitivity vector of G can be written in the form

$$\begin{aligned} \boldsymbol{G}_b = \int \Phi,_{\dot{\boldsymbol{u}}}\dot{\boldsymbol{u}},_{\boldsymbol{b}}\, dA\Big|_{t=t_f} + \int_0^{t_f} \Big\{ \int \{\Phi,_{\boldsymbol{N}}\boldsymbol{N},_{\boldsymbol{b}} + \Phi,_{\boldsymbol{e}}\boldsymbol{e},_{\boldsymbol{b}} + \\ +[\Phi,_{\boldsymbol{u}} - (\Phi,_{\dot{\boldsymbol{u}}})^{\cdot}]\boldsymbol{u},_{\boldsymbol{b}}\}\, dA + \int h,_{\boldsymbol{u}}\boldsymbol{u},_{\boldsymbol{b}}\, dS_T + \int h,_{\boldsymbol{T}}\boldsymbol{T},_{\boldsymbol{b}}\, dS_u \Big\}\, dt \end{aligned} \tag{3.5}$$

As it can be seen from (3.5) the knowledge of sensitivities of state fields is necessary to calculate G. These sensitivities can be either obtained by using the direct approach to sensitivity analysis, or eliminated from (3.5) by means of adjoint approach.

Let us consider first the generalized stress-strain relation (2.1) and derive the sensitivities of extensional stiffness matrix D with respect to components of the design vector $\boldsymbol{b}$. Selecting an arbitrary component b of the design vector $\boldsymbol{b}$ and differentiating (2.1) with respect to this component, we obtain

$$\boldsymbol{N},_b = \mathsf{D}\boldsymbol{e},_b + \mathsf{D},_b\,\boldsymbol{e} = \mathsf{D}\boldsymbol{e},_b + \boldsymbol{N}_b^i \qquad b = b_1,...,b_P \tag{3.6}$$

where P denotes the total number of all design parameters in $\boldsymbol{b}$, and the second term on the right-hand side of (3.6) can be regarded as a generalized initial stress applied within the additional disk due to the change of stiffness matrix D with respect to the design parameter b. Note furthermore that the relation between the sensitivity fields $\boldsymbol{N}_b$ and $\boldsymbol{e}_b$ is still linear with the same extensional stiffness matrix as that for primary disk.

For b coinciding with fiber shape parameter b_{kj} of the fiber shape set $\boldsymbol{b}_k$ in the kth layer, the sensitivity of D follows from (2.4) and equals

$$\mathsf{D},_{b_{kj}} = 2h\frac{\partial\overline{\mathbf{Q}}^k}{\partial\theta^k}\frac{d\theta^k}{db_{kj}} = 2h\overline{\mathbf{Q}}^k,_{\theta^k}(\theta^k),_{b_{jk}} \qquad \begin{aligned} k &= 0,...,n \\ j &= 1,...,J_k \end{aligned} \tag{3.7}$$

where now J_k denotes the number of fiber shape parameters in the kth layer. All sensitivities of matrix D can be easily calculated when the design function $\Theta^k = \Theta^k(\boldsymbol{x}, \boldsymbol{b}_k)$ is assumed in advance for each layer in the particular design.

Using now the direct approach to sensitivity analysis, the sensitivity fields $\boldsymbol{u}_{,b}$, $\boldsymbol{e}_{,b}$, $\boldsymbol{N}_{,b}$ and $\boldsymbol{T}_{,b}$ can be obtained as the solutions of an additional boundary-value problems associated with all components of the design vector $\boldsymbol{b}$. Differentiating the state equations (3.1) with respect to b and assuming moreover that the body forces are independent of fiber orientation, we obtain

$$\begin{aligned}
&\operatorname{div} \boldsymbol{N}_{,b} = \rho \ddot{\boldsymbol{u}}_{,b} && \text{in} \quad A \times [0, t_f] \\
&\boldsymbol{N}_{,b}\, \boldsymbol{n} = \mathbf{0} && \text{on} \quad S_T \times [0, t_f] \\
&\boldsymbol{u}_{,b} = \mathbf{0} && \text{on} \quad S_u \times [0, t_f] \\
&\boldsymbol{u}_{,b}(\boldsymbol{x}, 0) = \mathbf{0} && \text{in} \quad A \times [0] \\
&\dot{\boldsymbol{u}}_{,b}(\boldsymbol{x}, 0) = \mathbf{0} && \text{in} \quad A \times [0]
\end{aligned} \tag{3.8}$$

The equations (3.8) constitute the set of state equations for a linear-elastic disk with an imposed field of initial stress $\boldsymbol{N}^i_b = \mathsf{D}_{,e}\, \boldsymbol{e}$ and its material satisfying the constitutive equation (3.6). The state fields obtained as a result of solution of the additional problem (3.8) are the sought sensitivities $\boldsymbol{u}_{,b}(\boldsymbol{x}, t)$, $\boldsymbol{e}_{,b}(\boldsymbol{x}, t)$, $\boldsymbol{N}_{,b}(\boldsymbol{x}, t)$ and $\boldsymbol{T}_{,b}(\boldsymbol{x}, t)$.

Knowing the sensitivities of primary state fields with respect to the particular design parameter b, the sensitivity of functional G can be next evaluated from equation (3.5). The above process has to be repeated for evaluating the sensitivities with respect to all design parameters contained in design vector $\boldsymbol{b}$. Let us note, however, that calculating the state fields sensitivities by means of the direct approach, we are able to evaluate the sensitivity vectors for any number of functionals G without any additional effort.

The alternative method to evaluate the sensitivity vector of G is to use the solution for the adjoint structure in order to eliminate the state fields sensitivities from (3.5). However, this approach will not be discussed here, but the details can be found in Dems (2000).

The transition to the case of statically loaded structures can be easily performed by assuming that the boundary conditions specified in (3.1) and (3.8) are time-independent and functional G specified by (3.3) is reduced to the form

$$G = \int \Phi(\boldsymbol{N}, \boldsymbol{e}, \boldsymbol{u})\, dA + \int h(\boldsymbol{T}, \boldsymbol{u})\, dS \tag{3.9}$$

The sensitivities of (3.9) follow from (3.5) and now they are simplified as follows

$$
\boldsymbol{G_b} = \int \Phi,_{\boldsymbol{N}}\boldsymbol{N},_{\boldsymbol{b}} + \Phi,_{\boldsymbol{e}}\boldsymbol{e},_{\boldsymbol{b}} + \Phi,_{\boldsymbol{u}}\boldsymbol{u},_{\boldsymbol{b}}) \; dA + \int h,_{\boldsymbol{u}}\boldsymbol{u},_{\boldsymbol{b}} \; dS_T + \int h,_{\boldsymbol{T}}\boldsymbol{T},_{\boldsymbol{b}} \; dS_u \qquad (3.10)
$$

A detailed analysis of this static case is presented in Dems (1996).

4. Optimality conditions

One of typical formulations of the optimal design problem would require the minimization (or maximization) of functional G with the upper bound set on the structure cost, thus

$$
\text{Minimize} \quad G \quad \text{subject to} \quad C - C_0 \leqslant 0 \qquad (4.1)
$$

where G can be assumed in a general form (3.3) and C_0 denotes the upper bound of the structural cost.

Introducing the Lagrange functional $G' = G - \lambda(C - C_0 + \mu^2)$, where λ denotes the Lagrange multiplier and μ^2 is a slack variable, its stationarity yields the optimality conditions for the problem (4.1), namely

$$
\begin{aligned}
&G_b - \lambda C_b = 0 \qquad && b = b_1, ..., b_P \\
&\delta\lambda(C - C_0 + \mu^2) = 0 \qquad && 2\lambda\mu = 0
\end{aligned} \qquad (4.2)
$$

where G_b follows from (3.5) and C_b denotes the sensitivities of the cost function with respect to design parameter b.

The identification problems differ from the optimal design problem only by the absence of cost function, which is usually not associated with the identification procedure. Assume, for instance, that the displacement field $\boldsymbol{u}_M$ was measured over some control boundary part S_M within time interval $[0, t_t]$. Thus, the identification problem is reduced to determining a set of parameters influencing the stiffness matrix, $\mathsf{D} = \mathsf{D}(\boldsymbol{b})$, so that the distance between the measured and calculated displacement $\boldsymbol{u}$ is minimized. Such measure can be selected as

$$
G = \int_0^{t_f} \Big[\frac{1}{2} \int \alpha(\boldsymbol{u} - \boldsymbol{u}_M)^2 \; dS_M \Big] \; dt \to \min_b \qquad (4.3)
$$

where α is some weighting factor. The necessary optimality conditions following from the stationary requirements of (4.3), in view of (3.5), will now be expressed as

$$\int_0^{t_f} \left\{ \int \alpha [\boldsymbol{u}(\boldsymbol{x},t) - \boldsymbol{u}_M(\boldsymbol{x},t)] \boldsymbol{u}_{,b} \; dS_M \right\} dt = 0 \qquad b = b_1, ..., b_P \tag{4.4}$$

where all sensitivities $\boldsymbol{u}_{,b}$ should be derived as the solutions of P additional problems (3.8).

5. Illustrative example

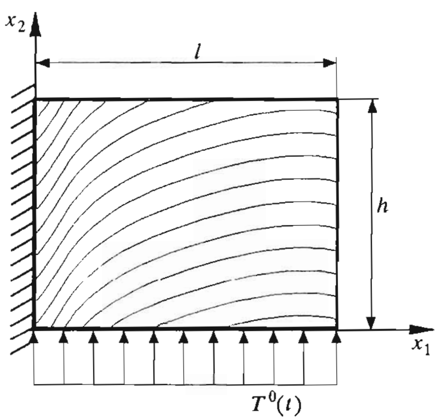

Fig. 3. Disk uniformly loaded along the x_1 axis

A simple example will be discussed in this section in order to show the applicability of the derived sensitivity expressions to sensitivity analysis and optimal design of fiber-reinforced disk. Let us consider then a rectangular disk with $l = 1.0\,\mathrm{m}$, $h = 1.0\,\mathrm{m}$, supported along the edge $x_1 = 0$ and loaded along the edge $x_2 = 0$, cf Fig.3, by harmonically varying traction $T^0(\boldsymbol{x},t) = \widehat{T}^0[\sin(\pi 5/t) + 1]$, where $\widehat{T}^0$ denotes the traction amplitude and $\pi/5\,\mathrm{s}^{-1}$ is the given frequency. The matrix material of the disk is reinforced by one fiber field of constant density $\rho_f = 0.4$. The material data for epoxy matrix and glass fibers are as follows: $E_m = 0.34 \cdot 10^4\,\mathrm{MPa}$, $\nu_m = 0.35$, $E_f = 7.3 \cdot 10^4\,\mathrm{MPa}$, $\nu_f = 0.22$. Let us assume that all fibers will constitute a one- or two-parameter family of the same shape that will be described by the shape of the so-called "directional fiber". The parameters defining the shape of directional fiber will be treated as design parameters, and their optimal values for the case of mean compliance design of the disk within the assumed period of time $t \in [0,10]$ will be derived. For initial time $t = 0$, both the displacement $\boldsymbol{u}(\boldsymbol{x},0)$ and velocities $\dot{\boldsymbol{u}}(\boldsymbol{x},0)$ are assumed to vanish. Thus, the optimization problem can be stated as follows

$$\min G = \int_0^{10} \Big(\int \boldsymbol{T}\boldsymbol{u}\, dS_T \Big)\, dt \qquad \text{with respect to } \boldsymbol{B} \tag{5.1}$$

The sensitivities of objective functional are expressed by (3.5), and then the optimality conditions for the problem at hand can be written in the very simple form

$$\int_0^{10} \Big(\int \boldsymbol{T}\boldsymbol{u}_{,b_j}\, dS_T \Big)\, dt = 0 \qquad j = 1,2,...,J \tag{5.2}$$

where J denotes the number of fiber shape parameters b_j. The additional displacement fields $\boldsymbol{u}_{,b}$ appearing in (5.2) were calculated using the finite element method, and the disk was divided into 15×15, 8-node rectangular plane stress elements.

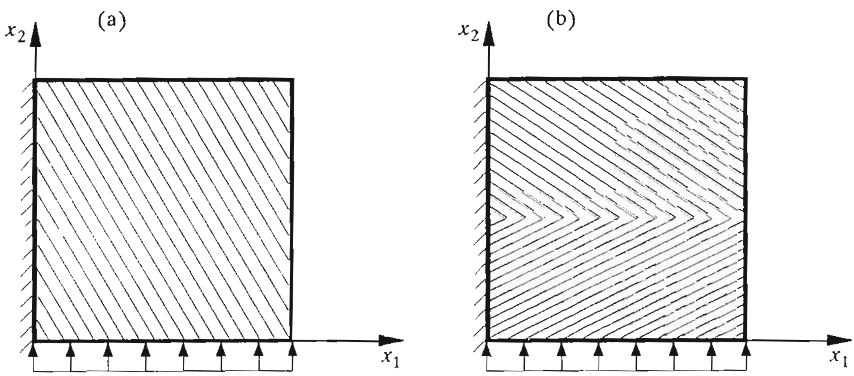

Fig. 4. Optimal disk with one family (a) and two families (b) of straight fibers

In the first case, the fiber field was composed of straight fibers for which the angle of orientation θ was the only design parameter. The optimal value of θ equals 121.1°, cf Fig.4a. This design was associated with reduction of the global disk compliance by 16.9% relative to the design with all fibers parallel to x_1 axis ($\theta = 0°$), and by 26.1% relative to the design with all fibers parallel to the x_2 axis. In the second case, two families of straight fibers were introduced as shown in Fig.4b and their angles of orientation were treated as two independent design parameters. The calculated optimal values of θ_1 and θ_2 are $26.1°$ and $146.8°$, respectively. This optimal design corresponds to reduction of the disk compliance by 38.7% in comparison to the design with both fiber families parallel to the x_1 axis, and by 45.5% for the case of fibers parallel to x_2 axis.

6. Concluding remarks

The present paper extends the results of previous works and provides a systematic approach to the sensitivity analysis and optimal design for dynamically and statically loaded disks of fixed shape made of fiber-reinforced composite materials. The concept of direct approach provides an effective tool in generating the sensitivities of an arbitrary functional with respect to parameters defining the extensional stiffness matrix of symmetric laminate material.

Acknowledgment

This research work was supported by the Committee of Scientific Research (KBN) Grant No. 7T08E09214.

References

1. BOJCZUK D., 1992, Optimization of Fiber Orientation in Composites – Optimality Conditions for Selected Problems, *IUTAM Symposium on Optimal Design with Advanced Materials*, Lyngby, Denmark

2. DEMS K., 1996, Sensitivity Analysis and Optimal Design for Fiber Reinforced Composite Disks, *Struct. Optimization*, **11**, 178-186

3. DEMS K., 2000, Sensitivity and Optimal Design of Fibers Shape in Composite Disk under Dynamic Load, *Fibers and Textiles in Eastern Europe*, January/March, 52-56

4. DEMS K., MRÓZ Z., 1983, Variational Approach by Means of Adjoint Systems to Structural Optimization and Sensitivity Analysis, Part I, *Int. J. Solids Struct.*, **19**, 677-692

5. JONES R.M., 1975, *Mechanics of Composite Materials*, New York, McGraw-Hill

6. MATHEUS H.C., MOTA SOARES C.M., MOTA SOARES C.A., 1991, Sensitivity Analysis and Optimal Design of Thin Laminated Composite Structures, *Procc. Course on CAOD of Structures*, Trento

7. MRÓZ Z., DEMS K., 1992, Discrete and Continuous Reinforcement of Materials, in *Optimal Design with Advanced Materials*, Edit. P. Pedersen, 383-406, Elsevier, Amsterdam

8. OLHOFF N., THOMSEN J., 1990, Optimization of Fiber Orientation and Concentration in Composites, *Procc. CEEC Course on Design and Analysis of Structures Made of Composite Materials*, Milano, Italy

9. THOMSEN J., 1991, Optimization of Composite Discs, *Struct. Optimization*, **3**, 89-98

Wrażliwość i projektowanie ułożenia włókien w kompozycie

Streszczenie

W pracy rozpatrzono pierwszą wariację dowolnego funkcjonału związaną z wariacją kształtu lub orientacji wzmacniających włókien w płaskiej liniowo-sprężystej tarczy. Wrażliwości rozpatrywanego funkcjonału wyznaczono wykorzystując metodę bezpośrednią analizy wrażliwości. Następnie sformułowano typowy problem optymalnego projektowania oraz problem identyfikacyjny i wyznaczono odpowiednie warunki optymalności. Rozpatrzono zarówno przypadek obciążeń statycznych, jak i dynamicznych.

Manuscript received March 28, 2000; accepted for print June 6, 2000