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ON APPLICATION OF PIEZOELECTRIC SHEAR EFFECT TO ACTIVE DAMPING OF TRANSVERSE VIBRATION IN BEAMS

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In the paper the problem of active damping of transverse vibration of a beam by making use of piezoelectric elements with shear piezoelectric effect employed is taken up. The analysis was motivated by the fact that popular piezoelectric materials made of lead zirconate titanate ceramics (PZT) exhibit very strong shear effect, being roughly three times greater than the longitudinal effect. The shear effect can be applied to beam-like systems by gluing piezoelectric sensors and actuators not to external surfaces of a given structure but placing them e.g. between two parallel strips as it is realised in three-layer sandwich structures, in which the middle layer is to dissipate vibration energy. In the presented model the middle viscoelastic core is replaced with a purely elastic piezoelectric element. Some preliminary results obtained for the proposed model based on a cantilever Timoshenko's beam made of PZT are presented. The method of control, generalised with respect to the piezoelectric shear effect boundary conditions, and finally, the resonance characteristics corresponding to excitation by external harmonic force are discussed. Particularly advantageous effect of the active damping is confirmed for lower resonances, where the natural damping by internal friction in the material is weak.

Key words: piezoelectricity, shear effect, active damping, Tomishenko's beam

1. Introduction

For several years piezoelectric elements have kept winning in the field of mechatronics as most popular materials, easy-to-use and not expensive for sensor and actuator applications. One can eventually come across them in beams, plates or shells, where they serve as the elements actively damping transverse vibration in such structures. Some fundamentals of active damping via piezoelectric elements in beam-like systems were set forth in the middle 1980s by Bailey and Hubbard (1985). Their work was then continuously developed by numerous researchers; e.g., by Crawley and de Luis (1987) who discussed different types of physical connection between beams and actuators (bonding and embedding), and by Crawley and Anderson (1990) who gave a thorough insight into the problems of piezoelectric actuation in beams. Other papers, such as by Tzou (1991) and van Niekerk et al. (1995) took up analogous problems in shells and plates. Recently, piezoelectric elements have been introduced to rotating systems to protect them from dangerous self-excited vibration that appears while exceeding critical rotation speed, see Przybyłowicz (1999). Some researchers have begun to explore efficiency of active stabilisation or vibration damping in view of its sensitivity to a quality of connection between piezoelements and a given structure. Tylikowski (1993) and Pietrzakowski (1997) discussed problems of imperfect attachment of piezoactuators and the effect of their local delamination on dynamic response of beams.

The papers mentioned above and carried out investigations have something in common. Namely, they examine the employment of piezoelectric elements as sensors and actuators making use of the direct and converse longitudinal piezoelectric effects. It consists in contracting or elongating of a cube-shaped piezoelectric element under an electric field in the directions of principal orthotropy axes of the considered piezoelectric material. The longitudinal effect, producing, e.g. in lead zirconate titanate (PZT) ceramics an elongation/contraction up to 170×10^{-12} m per one volt occurred to be strong enough for PZTs to become very popular piezoelectric materials. Furthermore, the PZT piezoceramics is known to exhibit another effect, the shear effect that converts an electric field into mechanical shear stress/strain and vice versa. This effect is worth mentioning the more so as it is about three times stronger than the longitudinal one. Both the effects are schematically presented in Fig.1.

Up to now, the shear effect have found applications to torsional systems. Meng-Kao Yeh et al. (1994) discussed applicability of piezoelectric materials to sensing torsional vibration in circular shafts. Chia-Chi Sung et al. (1994) studied dynamics of a clamped-free tube with active torsional vibration control by a PZT element. The effect of rheological properties of the layer bonding piezoelectric rings with the external surface of a shaft on a quality of such a control was examined by Przybyłowicz (1995).

A few years ago Spearritt and Asokanthan (1996) presented an interesting approach as they took advantage of the longitudinal piezoelectric effect for

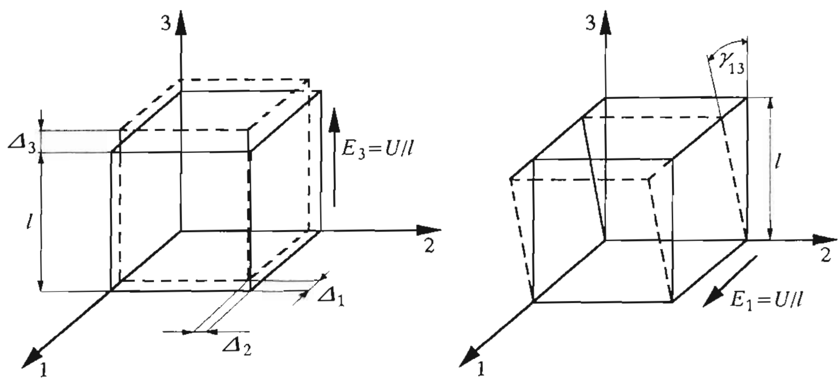

Fig. 1. Longitudinal and shear piezoelectric effect

controlling torsional vibration. They used piezoelectric PVDF polymer films, that produce only the longitudinal effect, and wrapped them around a tube at 45°. Some analogous concept will be discussed in the present paper, yet this time the shear effect is to control transverse vibration, not the longitudinal one to reduce torsional vibration. The idea originated from encouragingly great magnitude of the electromechanical coupling constant of PZTs (as it was mentioned nearly three times as much as that of the constant corresponding to the longitudinal effect). Another motivation was the fact that thick beams dissipate part of vibration energy by the shear effect – the fundamental on which the concept of passive damping of transverse vibration in composite plates and beams, especially in three-layer sandwich structures, is based. A sandwich beam consists of two thin strips, called skins, separated by a thick layer (core) revealing strong dissipative properties. A very interesting concept of attenuating vibration in slender beams by shear deformation enhanced with active piezoelectric elements was introduced by Kapadia and Kawiecki. They presented and experimentally proved efficiency of the so-called Active Constrained Layer Damping treatment consisting of a viscoelastic layer sandwiched between two piezoelectric layers and bonded to the base structure. The presence of piezoelectric sensor and actuator with reversed polarity increased magnitude of the shear deformation in the viscoelastic layer much more as in the passive dampers acting that way (cf Kapadia and Kawiecki, 1996, 1997).

This paper focuses on indicating the possibility of making use of the shear piezoelectric effect directly as a means of active control of transverse vibration to enhance natural damping capability of material of a beam that results from the presence of internal friction. The work has an introductory character and presents some first steps towards formulating the control strategy of active damping of transverse vibration in thick beams.

A literature survey of the problems associated with sandwich structures provides an abundance of different approaches and descriptions of their dynamic properties. Theories of moderate-thickness plates by Reissner and Mindlin, known since the 1940s, were developed by many researchers, and found various formal descriptions in terms of various number of equations of motion. Fundamentals of first modern theories of three-layer plates were due to Yu (1959) who took advantage of Mindlin's assumption of constant transverse displacement along thickness of a beam, and linear relationship between displacement in the beam plane and vertical coordinate. A generalisation of that theory to plates was proposed by Yan and Dowell (1974), and Durocher and Solecki (1976). A combination of Kirchhoff's and Mindlin's assumptions, presenting another concept, was due to Mead (1982).

In order to extract the results brought about by the sole piezoelectric shear effect a simplified model, based on Timoshenko's beam, is discussed in the paper. The beam is a thick cantilever, made entirely of PZT piezoceramics. Another simplification is negligence of rotary inertia effect and analysis of the shear alone. The equations of motion for the thus simplified Timoshenko's beam resemble, in terms of mathematical formalism, the equations by Yan and Dowell.

2. Description of the model and equations of motion

The system under consideration is a thick cantilever Timoshenko's beam made of PZT of length l, rectangular cross-section of area A, and transverse dimensions b, h. Over the upper and lower surfaces of the beam the electrodes separated into three bands are dusted. The middle electrode, see the grey band in Fig.2, is sensor measuring current and global (in integral sense) shear strain in the beam. The outer electrodes supply actuating voltage to the beam. The beam itself is not divided, which makes by design the control system collocated.

Consider an infinitely small section of the beam, as shown in Fig.2. The shear effect taken into account in Timoshenko's beam entails additional inclination of the cross-sectional surface of the angle denoted by γ, see Fig.3. Thus, the resultant slope ϕ equals $w_{,x} + \gamma$, where w is the transverse displacement of the beam, and a symbol following the comma (x in this case) means partial differentiation with respect to the quantity represented by that

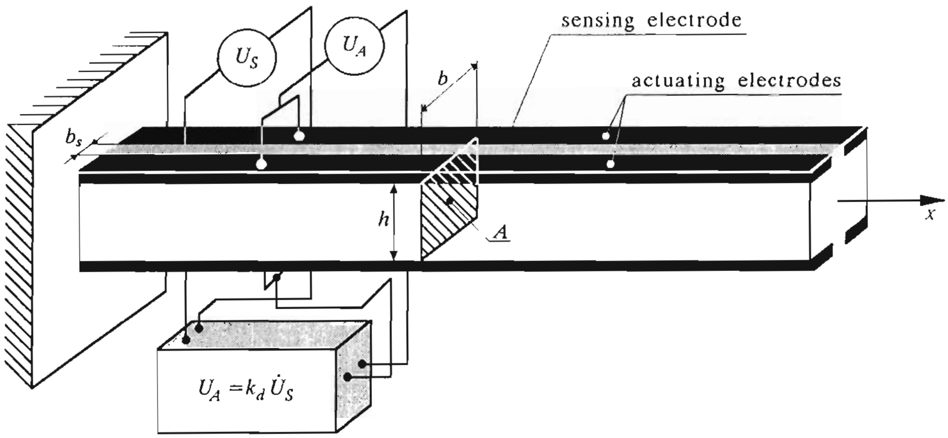

Fig. 2. Model of piezoelectric cantilever

symbol. By balancing the forces shown in Fig.2 in the transverse direction, and adding balance of their moments one obtains

$$\rho A \frac{\partial^2 w}{\partial t^2} = -\frac{\partial T}{\partial x} - q \qquad t = \frac{\partial M}{\partial x} \tag{2.1}$$

where $M = YJ\partial\phi/\partial x$ and the shear force, according to Timoshenko's correction is $T = \tau A = kGA$ and where k is the correction coefficient, being 5/6 for a rectangular cross-section, see Cowper (1966). Obviously ρ stands for the mass density, and YJ for the flexural stiffness of beam (Y is Young's modulus).

The above reasoning holds true provided no voltage is applied to the beam (classical Timoshenko's beam). In order to take into account the piezoelectric shear effect it is necessary to get back to the constitutive equations of piezoelectric materials and put them down in such a way so that the shear alone can be expressed.

The converse law is given by the following equation, see Nye (1985)

$$\varepsilon_i = s_{ij}^{(E)}\sigma_j + d_{ij}^{\top}E_j \tag{2.2}$$

where

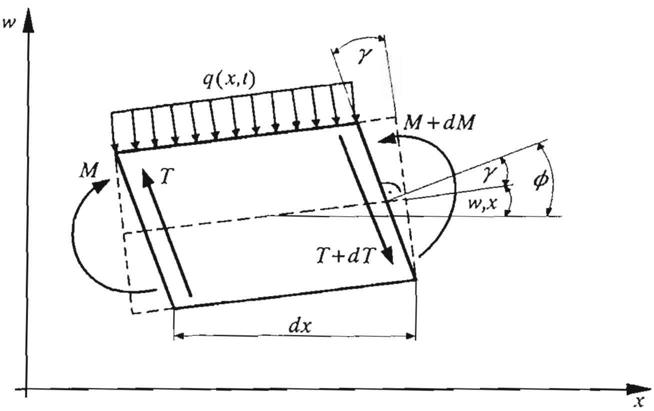

Fig. 3. Internal and external loading in infinitesimal section of the beam

ε_i	–	strain in the ith direction
σ_i	–	stress
$s_{ij}^{(E)}$	–	compliance matrix of the piezoelectric material measured for a constant electric field E
$d_{ij}^{\top}$	–	transposed matrix of the so-called electromechanical coupling coefficients.

This effect is illustrated in Fig.4.

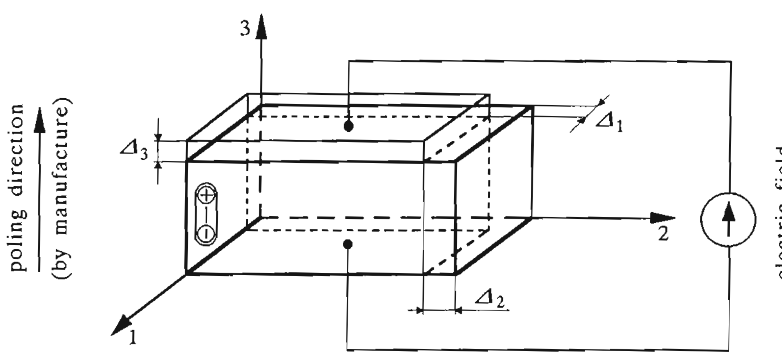

Fig. 4. Converse (longitudinal) piezoelectric effect

Suppose, at the moment, that the beam made of PZT is cut out in the way that the manufacturing polarisation coincides with the axis denoted by 3 – see Fig.5.

Keeping in mind that the index $i = 4, 5, 6$ means in fact a stress/strain in

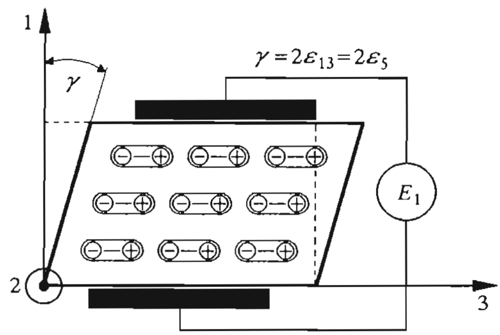

Fig. 5. Orientation of the coordinate system and the way of voltage application

the 2-3, 1-3, and 1-2 planes, respectively, one rewrites Eq (2.2) to the described situation shown in Fig.5 in the form

$$\varepsilon_5 = s_{55}^{(E)}\sigma_5 + d_{51}^{\top}E_1 \tag{2.3}$$

Let the shear stress σ_5 be denoted by $\sigma_{13} = \tau$, $s_{55}^{(E)} = (2G)^{-1}$, and $\varepsilon_5 = \varepsilon_{13} = \gamma/2$. This way the relationship between the shear stress and strain with Timoshenko's correction coefficient is

$$\tau = G(k\gamma - 2d_{15}E_1) \tag{2.4}$$

Substituting Eq (2.4) into Eqs (2.1) one obtains the unresolved equations of motion of the piezoelectric Timoshenko's beam

$$\begin{aligned}
\rho\frac{\partial^2 w}{\partial t^2} &= -G\Big\{k\frac{\partial}{\partial x}\Big(\phi - \frac{\partial w}{\partial x}\Big) - 2d_{15}\frac{\partial E_1}{\partial x}\Big\} - \frac{q}{A}\\
kGA\frac{\partial}{\partial x}\Big\{k\frac{\partial}{\partial x}\Big(\phi - \frac{\partial w}{\partial x}\Big) - 2d_{15}\frac{\partial E_1}{\partial x}\Big\} &= YJ\frac{\partial^2 \phi}{\partial x^2}
\end{aligned} \tag{2.5}$$

3. Concept of active control via piezoelectric shear effect

Before Eqs (2.5) become resolved with respect to e.g. transverse displacement w it is necessary to express explicitly the unknown function E_1 that describes the electric field applied to the actuating electrodes. Assume, at the moment, that the electrodes are dusted over the beam surfaces and form a rectangular shape. This entails the function E_1 independence the spatial coordinate x (denoted also by 3) $E_1(x,t) = E_1(t)$. This assumption immediately eliminates the presence of E_1 in Eqs (2.5). Now, they can be resolved, as in the classical Timoshenko beam. Neglecting the rotary inertia one obtains

$$\rho A\frac{\partial^2 w}{\partial t^2} + YJ\frac{\partial^4 w}{\partial x^4} - \frac{YJ\rho}{kG}\frac{\partial^4 w}{\partial x^2 \partial t^2} = q - \frac{J}{kAG}\Big(Y\frac{\partial^2 q}{\partial x^2} + \rho\frac{\partial^2 q}{\partial t^2}\Big) \tag{3.1}$$

As it is clearly seen, there is no terms in Eq (3.1) that would reflect the presence of any active control incorporated into the system. In fact, such terms appear while formulating boundary conditions. Before doing this, let us introduce a concept of the active control.

Assume one of the possibly simplest control strategies known in the control theory, i.e. based on velocity feedback. The electric field E_1 generated by actuating electrodes is equal to the voltage U_A applied to them and divided by the distance h between them. The velocity feedback implies that $U_A = -k_d\dot{U}_S$, where U_S is the voltage measured by the sensor, k_d is the gain factor, and "$-$" indicates that the actuating voltage is out of phase with respect to that swept by the sensor. In order to close a formal description of the control system operating one needs to determine the measured voltage U_S first.

From the constitutive equations of piezoelectric materials it is known as well that

$$D_1 = d_{15}\sigma_5 \tag{3.2}$$

where D_1 is the electric displacement in the direction 1, provided no other electric is applied to the sensing electrodes (that would introduce an additional term to Eq (3.2)). Since the electric charge Q is a surface integral of the dielectric displacement one obtains

$$Q = \int\limits_A D_1\;dS = \int\limits_A d_{15}\tau\;dS = \int\limits_A d_{15}kG\gamma\;dS = kGd_{15}b_S\int\limits_0^l \Big(\phi - \frac{\partial w}{\partial x}\Big)\;dx \tag{3.3}$$

where b_S is the sensing electrode width, see Fig.2. Now, as the electrodes separated by the beam body pose in fact a rectangular capacitor, the voltage between them is

$$V = \frac{Q}{C} \tag{3.4}$$

where the capacity

$$C = \frac{\epsilon_0\epsilon_{pe}S}{h}$$

and $\epsilon_0\epsilon_{pe}$ is the absolute dielectric permittivity of PZT piezoceramic material, and $S = b_S l$. Hence, the measured voltage is represented by the following integral

$$U_S = \frac{kGd_{15}h}{\epsilon_0\epsilon_{pe}l} \int\limits_0^l \Big(\phi - \frac{\partial w}{\partial x}\Big)\, dx \tag{3.5}$$

and finally the actuating electric field

$$E_1 = -k_d\frac{kGd_{15}}{\epsilon_0\epsilon_{pe}l} \int\limits_0^l \Big(\dot{\phi} - \frac{\partial \dot{w}}{\partial x}\Big)\, dx \tag{3.6}$$

It is worth mentioning that the actuating field is a global value (integral) – the function E_1 does not depend on the spatial coordinate x, and can only be a function of time.

Consider now the boundary conditions to be satisfied by the cantilever Timoshenko's beam. At the clamped end there is no transverse displacement and no slope. The free end records no bending moment and no shear force. Thus:

$$\begin{aligned} & w(0,t) = 0 \qquad\qquad \phi(0,t) = 0 \\ & T(l,t) = GA\Big\{k\Big[\phi(l,t) - \frac{\partial w(l,t)}{\partial x}\Big] - 2d_{15}E_1\Big\} = 0 \\ & M(l,t) = YJ\frac{\partial \phi(l,y)}{\partial x} = 0 \end{aligned} \tag{3.7}$$

In general, for Timoshenko's beams one cannot resolve the boundary conditions so that they could be expressed in terms of either the transverse displacement w or slope ϕ. It can be done, however, in some cases assuming certain simplifying assumptions about e.g. external excitation. Let dynamic characteristics presenting amplitude-frequency response of the piezoelectric Timoshenko's beam be of the interest in the following analysis. Assume, therefore, that the external excitation is a uniformly distributed and harmonically variable in the time-force type excitation of intensity q_0. Having it substituted to Eq (3.1) one obtains

$$\frac{\partial^2 w}{\partial t^2} + a^2\frac{\partial^4 w}{\partial x^4} - a^2b^2\frac{\partial^4 w}{\partial x^2 \partial t^2} = f_0 \sin \nu t \tag{3.8}$$

where

$$a^2 = \frac{YJ}{\rho A} \qquad\qquad b^2 = \frac{\rho}{kG} \qquad\qquad f_0 = \frac{q_0}{\rho A}\Big(1 - \frac{J\nu^2\rho}{kGA}\Big) \tag{3.9}$$

and where ν is the excitation frequency. Such an assumption allows one to predict dynamic response of the beam in form of the harmonic function: $w(x,t) = W(x)\mathrm{e}^{\mathrm{i}\nu t}$, where the complex description has been chosen for convenience. By replacing $\sin \nu t$ with $\mathrm{e}^{\mathrm{i}\nu t}$ in the excitation function, then substituting it into Eq (3.8), and finally dividing by $\mathrm{e}^{\mathrm{i}\nu t}$ one finds the ordinary differential equation with respect to the spatial function of the transverse vibration of the beam

$$-\nu^2 W(x) + a^2 W''''(x) + a^2 b^2 \nu^2 W'' = f_0 \tag{3.10}$$

the solution to which can be put down as

$$W(x) = C_1 \mathrm{e}^{k_1 x} + C_2 \mathrm{e}^{-k_1 x} + C_3 \mathrm{e}^{\mathrm{i}k_2 x} + C_4 \mathrm{e}^{-\mathrm{i}k_2 x} - \frac{f_0}{\nu^2} \tag{3.11}$$

The primes in Eq (3.10) denote differentiation with respect to x. Occurring in Eq (3.11) the constants k_1 and k_2 have the following explicit form

$$k_1 = \frac{b\nu}{\sqrt{2}}\sqrt{-1 + \sqrt{1 + \frac{4}{a^2 b^4 \nu^2}}} \qquad k_2 = \frac{b\nu}{\sqrt{2}}\sqrt{1 + \sqrt{1 + \frac{4}{a^2 b^4 \nu^2}}} \tag{3.12}$$

It is to be remembered that the function $W(x)$ together with the constants $C_1, ..., C_4$ are complex. Resolve, now, the boundary conditions given by Eqs (3.7) with respect to $W(x)$. To accomplish this one has to find the slope $\Phi(x)$ expressed in terms of $W(x)$ and its derivatives (obviously $\Phi:\ \phi(x,t) = \Phi(x)\mathrm{e}^{\mathrm{i}\nu t}$) what can be done by making use of Eqs (2.1). Remembering that $q(x,t)$ is in fact $q(t)$ one obtains after a few of simple transformations

$$\Phi = W'(1 + a^2 b^4 \nu^2) + a^2 b^2 W''' \tag{3.13}$$

Substituting Eqs (3.12) into the boundary conditions, see Eqs (3.7), one gets

$$\begin{aligned} &W(0) = 0 \\ &\Phi(0) = 0 \Longrightarrow W'(0)(1 + a^2 b^4 \nu^2) + a^2 b^2 W'''(0) = 0 \\ &\Phi'(l) = 0 \Longrightarrow W''(l)(1 + a^2 b^4 \nu^2) + a^2 b^2 W''''(l) = 0 \end{aligned} \tag{3.14}$$

and the "piezoelectric" boundary condition

$$T(l,t) = 0 \Longrightarrow \phi(l,t) - \frac{\partial w(l,t)}{\partial x} - k_d \frac{2d_{15}^2 G}{\epsilon_0 \epsilon_{pe} l} \int\limits_0^l \Big(\dot{\phi}(l,t) - \frac{\partial \dot{w}(l,t)}{\partial x}\Big)\, dx = 0 \tag{3.15}$$

The last condition must also be resolved. For this purpose the integral $\int_0^l(\dot{\phi} - \dot{w}_{,x})\,dx$ should be found first

$$\int\limits_0^l \Big(\dot{\phi} - \frac{\partial \dot{w}}{\partial x}\Big)\,dx = \mathrm{i}\nu \int\limits_0^\nu \Big[\varPhi(x) - W'(x)\Big]\,dx = \mathrm{i}\nu \int\limits_0^l \varPhi(x)\,dx - \varDelta W \tag{3.16}$$

where $\varDelta W = W(l) - W(0)$. Then resolving the function $\varPhi$, see Eq (3.13), and substituting to the third one of the boundary conditions (3.7) one finally obtains

$$W'''(l) + b^2 n u^2 W'(l) = -\overline{k}_d \mathrm{i}\nu[W''(l) - W''(0) + b^2\nu^2 W(l)] \tag{3.17}$$

where

$$\overline{k}_d = k_d \frac{2d_{15}^2 G}{\epsilon_0 \epsilon_{pe} l}$$

since $W(0) = 0$. Having found the form of the boundary conditions applicable to Eq (3.10) and substituted to the predicted solution (3.11) the following matrix equation is derived, from which the complex constants $C_1, ..., C_4$ can be calculated. This equation is

$$\begin{bmatrix} 1 & 1 & 1 & 1 \\ a_{21} & -a_{21} & \mathrm{i}a_{23} & -\mathrm{i}a_{23} \\ a_{31}\mathrm{e}^{k_1 l} & a_{32}\mathrm{e}^{-k_1 l} & \mathrm{i}a_{33}\mathrm{e}^{\mathrm{i}k_2 l} & \mathrm{i}a_{34}\mathrm{e}^{-\mathrm{i}k_2 l} \\ a_{41}\mathrm{e}^{k_1 l} & a_{41}\mathrm{e}^{-k_1 l} & a_{43}\mathrm{e}^{\mathrm{i}k_2 l} & a_{43}\mathrm{e}^{-\mathrm{i}k_2 l} \end{bmatrix} \begin{bmatrix} C_1 \\ C_2 \\ C_3 \\ C_4 \end{bmatrix} = \begin{bmatrix} \dfrac{f_0}{\nu_2} \\ 0 \\ 0 \\ 0 \end{bmatrix} \tag{3.18}$$

where

$$\begin{aligned}
a_{21} &= k_1[1 + a^2b^2(b^2\nu^2 + k_1^2)] \\
a_{23} &= k_2[1 + a^2b^2(b^2\nu^2 + k_2^2)] \\
a_{31} &= (k_1^2 + b^2\nu^2)(k_1 + \mathrm{i}\nu\overline{k}_d)\mathrm{e}^{k_1 l} - \mathrm{i}\nu\overline{k}_d k_1^2 \\
a_{32} &= (k_1^2 + b^2\nu^2)(-k_1 + \mathrm{i}\nu\overline{k}_d)\mathrm{e}^{-k_1 l} - \mathrm{i}\nu\overline{k}_d k_1^2 \\
a_{33} &= (-k_2^2 + b^2\nu^2)(k_2 + \nu\overline{k}_d)\mathrm{e}^{\mathrm{i}k_2 l} + \nu\overline{k}_d k_2^2 \\
a_{34} &= (-k_2^2 + b^2\nu^2)(-k_2 + \nu\overline{k}_d)\mathrm{e}^{-\mathrm{i}k_2 l} + \nu\overline{k}_d k_2^2 \\
a_{41} &= a_{21} \\
a_{43} &= k_2[-1 + a^2b^2(k_2^2 - b^2\nu^2)]
\end{aligned} \tag{3.19}$$

4. Results of simulations

As it was mentioned in the previous section the analysis aims at determination of resonance characteristics of the considered system. Let the amplitude of the transverse vibration A_V be recorded at the free end of the beam, hence

$$A_V = \Big|W(l)\Big| = \Big|C_1 e^{k_1 l} + C_2 e^{-k_1 l} + C_3 e^{ik_2 l} + C_4 e^{-ik_2 l} - \frac{f_0}{\nu^2}\Big| \tag{4.1}$$

Before the resonance curves will be presented, let us consider first natural damping properties resulting from the presence of internal friction in the beam material. The dissipative capability of the beam material can be formally expressed by substituting an operator form of the Kirchhoff and Young moduli into the equation of motion. Assuming the simplest rheological model that regards viscoelastic properties of the material, i.e. the Kelvin-Voigt model one writes the moduli down as

$$Y^* = Y\Big(1 + \beta_{33}\frac{\partial}{\partial t}\Big) \qquad\qquad G^* = G\Big(1 + \beta_{13}\frac{\partial}{\partial t}\Big) \tag{4.2}$$

where β_{33} and β_{13} denote time constants of the Kelvin-Voigt model for tension/compression and shear, respectively. In the literature one can find various assumptions about the magnitudes of β_{33} and β_{13}. Quite often the energy amounts dissipated via both effects assumed to be equal, see Alam and Asnani (1986), but most commonly in sandwich structures β_{33} is negligible when compared to β_{13}. In this paper β_{33} is one order of magnitude less than β_{13}.

The corresponding to Eq (3.10) equation for the spatial function $W(x)$ in a beam with intrinsic damping properties assumes the following form

$$W''''a^2(1 + i\beta_{33}\nu) + W''a^2b^2\nu^2\frac{1 + i\beta_{33}\nu}{1 + i\beta_{13}\nu} - \nu^2 W = f_0 \tag{4.3}$$

The solution to Eqs (4.2) under the external excitation $q(t)$ can be found by making use of the same boundary conditions provided that the gain factor $\overline{k}_d$ is zero.

In the paper two cases are analysed. In the first case the piezoelectric Timoshenko's beam is passively damped by internal friction it is endowed with (disabled control system), and in the second case the beam is damped actively, but no internal friction is the taken into account. The resonance curves corresponding to the first case are presented by thin continuous lines, while those corresponding to the active damping – by thin dashed lines. As a reference the undamped response of the Timoshenko beam is drawn by thick continuous lines in the following figures.

It is well known that intensity of dissipating vibration energy due to internal damping grows with frequency of succeeding resonances (with square of the frequency). That means the first resonances are poorly damped. It is shown in Fig.6, where the two first resonances are presented.

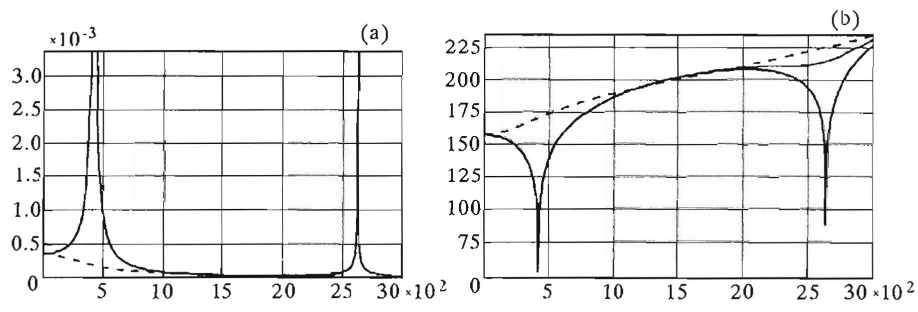

Fig. 6. Efficiency of the active and passive damping. The two first resonances: (a) absolute value, (b) in a logarithmic scale

Admittedly, the curves corresponding to the beam without and with passive damping, respectively, are practically in line for the first resonance (the differences are a matter of thickness of the curves) – the internal friction is too weak to damp effectively this resonance. At the same time it turns out that application of the active control distinctly reduces the first and the second resonances (practically eliminates them) even without enhancing the effect brought about by the internal friction. This is a very important conclusion that application of piezoelectric elements to the beam and making use of the shear effect proves to be efficient way of reducing transverse vibration, especially for low vibration modes of the cantilever. That advantageous effect is not so well pronounced for higher resonances, as some peaks can occur in the dynamic characteristics – see Fig.7b, but their absolute amplitude, are nearly zero – compare Fig.7a.

As mentioned before, particularly good damping capability for higher resonances is brought about by internal friction in the beam material. This is clearly seen in Fig.7b as the thin continuous line, corresponding to the passive damping, flattens.

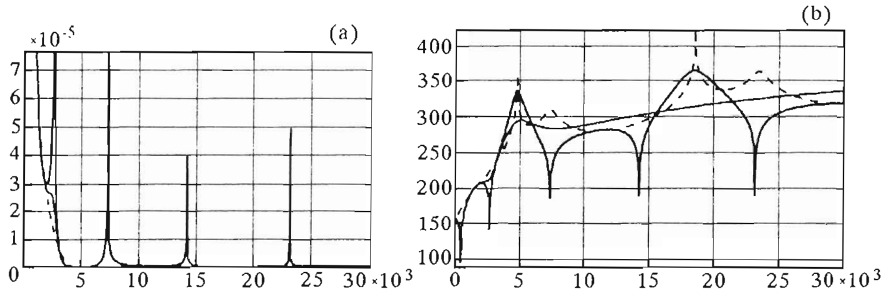

Fig. 7. Efficiency of active and passive damping in a wide range of excitation frequency: (a) absolute value, (b) in a logarithmic scale

5. Concluding remarks

In the paper the analysis of application of the piezoelectric shear effect to damping of transverse vibration of a thick beam has been presented. The beam has been assumed to be entirely made of piezoelectric lead zirconate titanate ceramics with electrodes stretched over the upper and lower surfaces. In order to underline the damping due to shear piezoelectric effect Timoshenko's beam with the shear alone (no rotary inertia) has been used for description of the model. The beam works as sensor and actuator element at the same time. Electric field developed by the actuating part is coupled with voltage measured by the sensor following a simple control strategy based on velocity feedback. It has turned out that the measured voltage has a global character as posed by an integral (see Eq (3.5)), and thus does not appear explicitly in equations of motion. The piezoelectric effect is introduced into dynamics of the system by boundary conditions.

A cantilever Timoshenko's beam has been taken into consideration. Consistently, the piezoelectric shear effect occurred in the boundary condition imposing no shear force at the free end of the beam. The obtained simulation results, corresponding to the harmonic force-type excitation, have clearly shown that the piezoelectric shear effect can be used in reducing transverse vibration of cantilevers. Particularly advantageous effect has been observed for lower resonances, poorly damped by internal friction in the beam material. Obviously, under real conditions there is no way to exclude the internal damping what only enhances efficiency of the active control for higher frequencies (not as good as the passive damping in that region). Hopefully, the presented analysis has an introductory character as it might be extended to cover multi-layer composite structures, especially sandwich beams and plates with embedded layer (core) made of piezoceramics. Another challenge to be met constitutes the problem of segmentation of PZT core (or simply segmentation of the electrodes only) to extract some dynamic phenomena that are lost due to a global character of the proposed strategy. Similar conclusions regarding segmentation were drawn by Kapadia and Kawiecki (1996, 1997) as they stressed that the segmentation and its optimisation might be the only way of improving effectiveness of the active damping, and pointed to finite element models that would allow one to predict an optimal length of piezoelectric segments. This seems to be a promising approach towards simply supported beams as well, where the presented method would fail for the even modes of the transverse vibration.

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O zastosowaniu postaciowego efektu piezoelektrycznego do aktywnego tłumienia drgań poprzecznych belek

Streszczenie

W pracy podjęto problem aktywnego tłumienia drgań poprzecznych belki za pomocą elementów piezoelektrycznych przy wykorzystaniu postaciowego efektu piezoelektrycznego. Motywację podjętych badań stanowi fakt, że popularne piezoelektryki ceramiczne wykonane na bazie spieków tlenków cyrkonu i tytanu (PZT) wykazują bardzo silny efekt postaciowy, około trzykrotnie większy od efektu wzdłużnego. Efekt postaciowy może zostać wykorzystany poprzez zastosowanie elementów piezoelektrycznych nie przez naklejanie aktuatorów na zewnętrznych powierzchniach belki, lecz umieszczenie w strukturze samej belki, np. między dwiema równoległymi listwami, podobnie jak w konstrukcjach typu sandwich, gdzie warstwą silnie rozpraszającą energię drgań jest warstwa środkowa – tu zastąpiona piezoelektrykiem. W pracy przedstawiono wstępne wyniki badań bazujące na modelu wspornikowej belki Timoszenki wykonanej z ceramiki PZT. Omówiono metodę sterowania, uogólnione na piezoelektryczny efekt postaciowy warunki brzegowe oraz, jako rezultat końcowy, charakterystyki rezonansowe przy wymuszeniu harmoniczną siłą zewnętrzną. Stwierdzono szczególnie korzystne działanie aktywnego tłumienia wykorzystującego efekt postaciowy dla niskich częstości rezonansowych, słabo tłumionych przez naturalne właściwości dyssypatywne materiału belki.

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