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NUMERICAL INVESTIGATION INTO FLIGHT DYNAMICS OF AN AGILE AIRCRAFT

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The aim of this study is to prove practical applicability of a numerical simulation technique in highlights some of the features of the flight under extreme conditions. Dynamics of spatial motion of an agile aircraft is considered. After a brief description of an aircraft mathematical model, the numerical analyses are presented. There have been considered various flight events (such as airdrop or aircraft accident) and manoeuvres (in particular, those performed at high angles of attack).

Key words: non-linear flight dynamics, trajectory analysis

Notation

V – airspeed
α – angle of attack
β – slip angle
Φ, Θ, Ψ – roll, pitch and yaw angles, respectively
$\delta_H, \delta_A, \delta_V, \delta_F$ – displacement of elevator, ailerons, rudder, and cockpit power lever, respectively
α_{z_H} – longitudinal decalage
ω – angular velocity of engine rotor
τ_0, τ_1, τ_2 – time-constants
K – amplification factor
Q_p – discharge of fuel for the current engine angular velocity and for the current aircraft altitude and airspeed
Q_{p0} – discharge of fuel, calculated for the sea level conditions and for $V = 0$.

1. Introduction

The need for improving the quality simulation of aircraft flight mechanics is constantly growing. The present investigations include real time piloted simulations as well as non-real time simulations. The real time piloted simulations are usually used for handling quality studies of a new aircraft, improvement of the existing aircraft and accident investigations, whereas the non-real time simulations are required needed for flight control design, aeroelastic calculations or vibration analysis.

The form of an aircraft physical model results directly from the specified range of its applications and physical properties of the considered object. In most general study, the physical model and the corresponding mathematical model representing the aircraft flight can be expressed by two sets of equations. The first one is the set of classical equations of three-dimensional motion of a rigid aircraft, and the second one consists of equations of elastic deformations of the airframe elements resulting from aerodynamic loads. A physical model of a modern, highly augmented aircraft can be presented with a satisfactory accuracy as a rigid body with moving elements of control surfaces. The gyroscopic moment of rotating masses of the engines should be included into this model. The mathematical form of physical model of the aircraft motion dynamics has been widely known for over eighty years (cf Painlave and Borel, 1911). This mathematical model is formed of a set of six non-linear differential equations representing spatial aircraft motion and six differential equations of kinematic relations. A series of applications of the classical quasi-Eulerian kinematic relations have been defined for the operational range of motion parameters. In some ranges of motion parameters, the quasi-Euler kinematic relations introduced very big errors, which makes this method for determination of the altitude in aerospace to be useless. Much better results can be obtained using a quaternion form of kinematic relations, with normalisation of correction employed (Gajda, 1990), Goldiez and Kuo-Chi Lin (1991). The equations of motion can be completed with the relations of stability and control augmentation system.

The aim of this study is to prove practical applicability of the numerical simulation technique in highlights some of the features of flight under extreme conditions. After a brief description of the aircraft mathematical model, the numerical analyse are presented for the PZL I-22 "Iryda" training aircraft, the Su-22 fighter bomber aircraft and the Mig-29 fighter aircraft. There have been considered various flight events (such as airdrop or aircraft accident) and manoeuvres (in particular, those performed at high angles of attack).

2. Non-linear equations of motion

Non-linear equations of motion of an aeroplane and the kinematics relations are expressed relative to moving co-ordinate systems, the common origin of which is located at the aeroplane centre of mass (cf Dżygadło and Sibilski, 1988, 1990; Maryniak, 1975; Sibilski, 1998b). The relative positions of the systems of co-ordinates; i.e., inertial and aircraft body-fixed is described by the Euler angles, while the relative position of the aircraft body-fixed system and that attached to the airflow – by the angle of attack α and slip angle β (Maryniak, 1975; Sibilski, 1997b).

The mathematical model of controlled motion of a rigid aircraft, can be formulated in the following form (Sibilski, 1998b)

$$\frac{d\boldsymbol{x}}{dt} = f(\boldsymbol{x}(t), \boldsymbol{u}(t)) \qquad \boldsymbol{x}(0) = \boldsymbol{x}_0 \tag{2.1}$$

where

- state vector

$$\boldsymbol{x} = [V, \alpha, \beta, p, q, r, \Phi, \Theta, \Psi, x_g, y_g, z_g]^\top \tag{2.2}$$

- control vector

$$\boldsymbol{u} = [\alpha_{zH}, \delta_H, \delta_A, \delta_V, \delta_F]^\top \tag{2.3}$$

The equations of aircraft motion should be completed with the equations of engine dynamics

$$\tau_1\tau_2\ddot{\omega} + (\tau_1 + \tau_2)\dot{\omega} = K[Q_p(t - \tau_0) - Q_{p0}]\omega \tag{2.4}$$

The time-constants are non-linear functions of engine angular velocity, aircraft altitude, air density, pressure, temperature and flight altitude. The discharge of fuel is the following function

$$Q_p(t) = f_f(\delta_F(t)) \tag{2.5}$$

where δ_F – displacement of the cockpit power lever.

The equation of thrust reads

$$F = F_0(\omega)\Big(\frac{\rho}{\rho_0}\Big)^{0.7}(K_0 + K_1\mathrm{Ma} + K_2\mathrm{Ma}^2) \tag{2.6}$$

where K_0, K_1, K_2 – constants depending on an engine model. Usually, the engine model is adapted basing on the code, data, and flow charts provided by the manufacturer.

The equations of motion are completed by the kinematics relations determining the position of the airship relative to the inertial system of co-ordinates (cf Maryniak, 1975). Since the altitude equations represented by the Euler angles reveal singularities at $\Theta = \pm 90°$, it was necessary to use quaternions to represent the orientation of the aircraft body-fixed axes with relative to the earth-fixed frame of reference frame (cf Gajda, 1990; Goldiez and Kuo-Chi-Lin, 1991).

The aerodynamic characteristics and derivatives, both for the steady and unsteady aerodynamic models (including deep stall conditions) are calculated using a modified wing section theory (Martina, 1947a,b; Sibilski, 1998a). The non-linear airfoil characteristics, which were used, were calculated using the ONERA deep stall model, using the algorithms circumscribed by Narkiewicz (1994). More exact and detailed description of the methods and algorithms employed in computer codes used for a calculation of aircraft aerodynamic loads can be found in Narkiewicz (1994), Sibilski (1998a,b).

3. Numerical examples

The results of numerical simulations of selected flight events and manoeuvres are presented below, for several aircraft models (i.e., PZL I-22 "Iryda" training, Su-22 fighter-bomber, and Mig-29 fighter aircraft). Their geometrical and weight parameters are assumed after [23,24].

3.1. Asymmetric drop of suspended loads

Dynamics of the I-22 training-combat aircraft motion, after drop of loads has been studied, on the assumption that the displacements of elevator, rudder and ailerons and thrust remain unchanged (uncontrolled flight), and in the case of controlled flight (the control surfaces are displaced and the thrust varies). There have been the following five events considered, in the case of uncontrolled flight:

1. Load remained in the left wing external beam grip
2. Load remained in the left wing internal beam grip
3. Load remained in the right wing external beam grip
4. Load remained in the right wing internal beam grip
5. Load remained in the left wing internal and external beam grips.

The results of the analysis are shown in Fig.1 ÷ Fig.5 the form five variations of: angles of attack (Fig.1); normal load factor (Fig.2); roll, pitch and yaw angles (Fig.3 ÷ Fig.5). It can be concluded that in the case of asymmetric airdrop in an uncontrolled horizontal flight, substantial variations in all the flight parameters occur. If the flight after airdrop is controlled, the angle of attack α becomes steady and variations in the angles Θ, Φ, Ψ and other flying parameters are limited to a considerable extent.

3.2. Reconstruction of an accident

Fig.6 ÷ Fig.8 show the results of numerical simulation of the Su-22M4 fighter aircraft accident. The Military Aviation Safety Board in their accident rapport gave the following description of the flight event. Two Su-22M4 bomber aircraft performed the flight over the firing ground. During improvement of the position in the aircraft formation, the attention of the second aircraft pilot was distracted and the rapidly pulled the stick and deflected the rudder, due to the danger of mid-air collision with the leading aircraft. Such an action caused the stall of the aircraft. Due to a low flight altitude (1200 m) the pilot of the second aircraft had to catapult and the aircraft crashed into the ground. Fig.6 shows time-variations of the angle of attack α and normal load factor n_z (the moments of stalling and catapulting are marked down). Fig.7 shows time-variations of the airspeed and altitude. Fig.8 presents deflections of the control surfaces (reconstruction of pilot actions).

3.3. Simulation of flight under conditions of a deep stall

The spin can be classified as a flight under conditions of a deep stall. Some results of numerical simulation of the PZL I-22 trainer aircraft spin are shown in Fig.9 ÷ Fig.12. Time-variation of the normal load factor n_z is shown in Fig.9. Variations of the angular velocities p, q and r are shown in Fig.10. Fig.11 presents the airspeed variation. Deflections of the control surfaces are shown in Fig.12.

3.4. Super- and hyper-manoeuvrability (simulation of selected manoeuvres high angles of attack manoeuvres)

At subsonic speeds, the manoeuvres at supercritical angles of attack can be realised under two different flight conditions. These states of flight are referred to as:

- Super-manoeuvrability – the aircraft can fly at angles of attack of 60° ÷ 70° maintaining the controllability in all channels

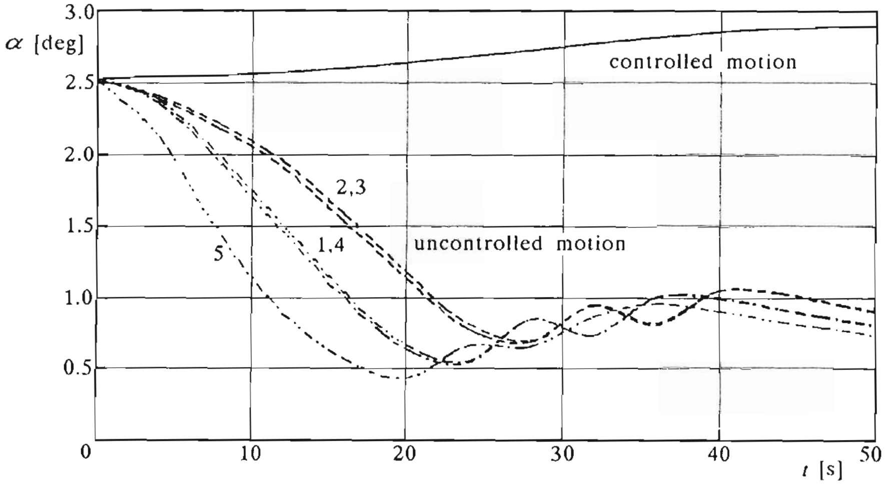

Fig. 1. Airdrop, controlled and uncontrolled flights – variations of the angle of attack α

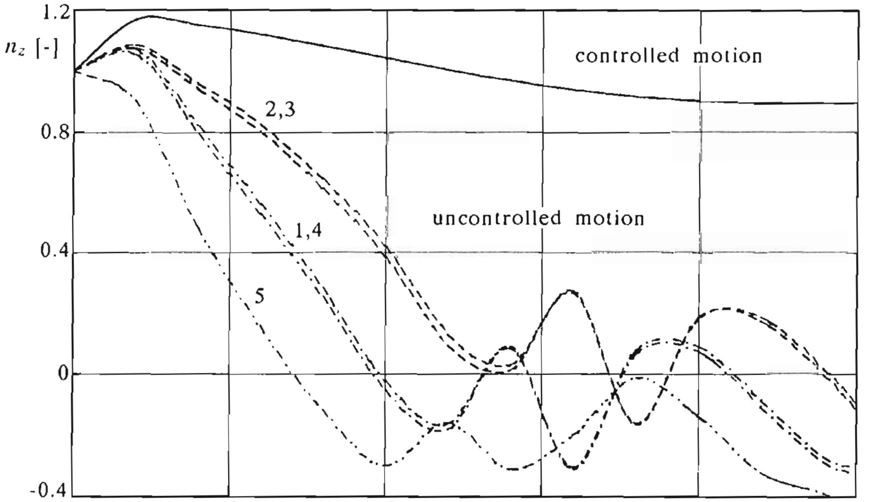

Fig. 2. Airdrop, controlled and uncontrolled flights – variations of the normal load factor n_z

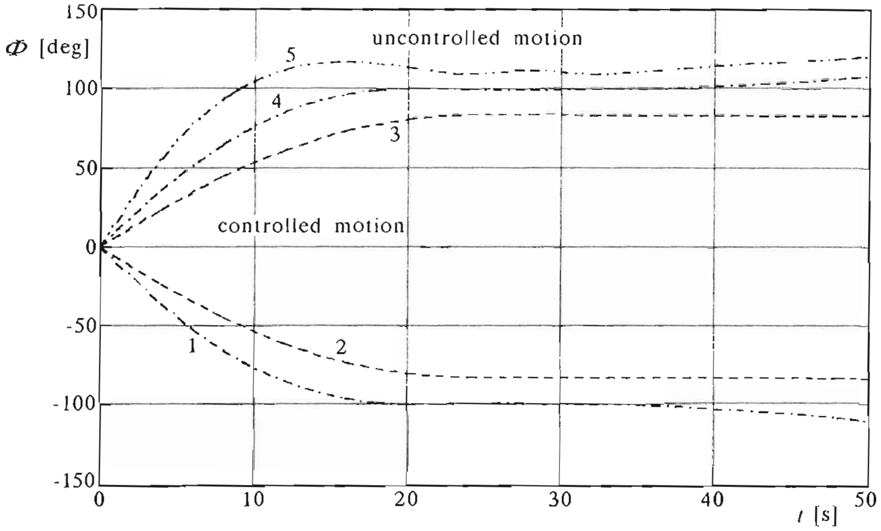

Fig. 3. Airdrop, controlled and uncontrolled flights - variations of the roll angle Φ

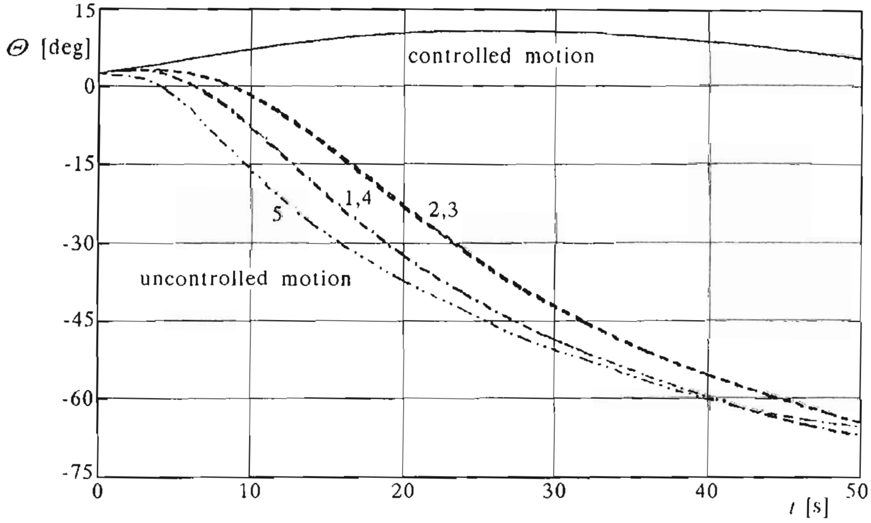

Fig. 4. Airdrop, controlled and uncontrolled flights – variations of the pitch angle Θ

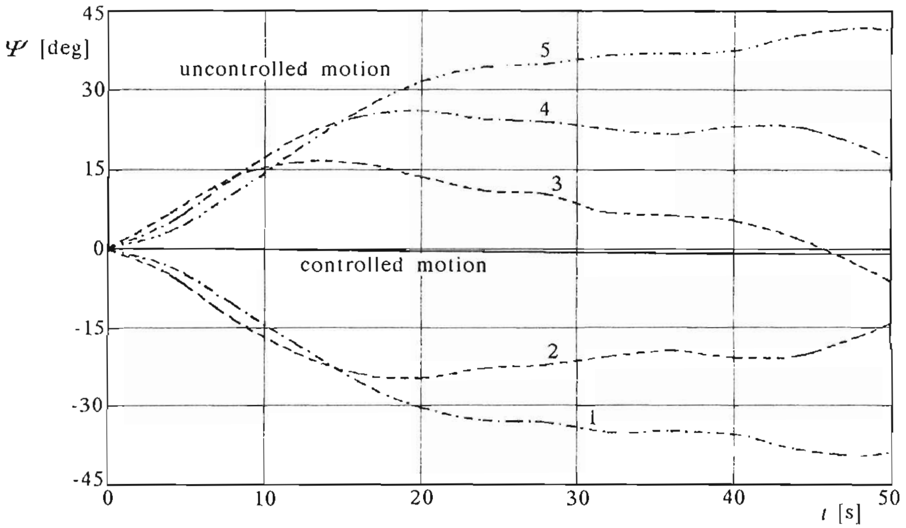

Fig. 5. Airdrop, controlled and uncontrolled flights – variations of the yaw angle Ψ

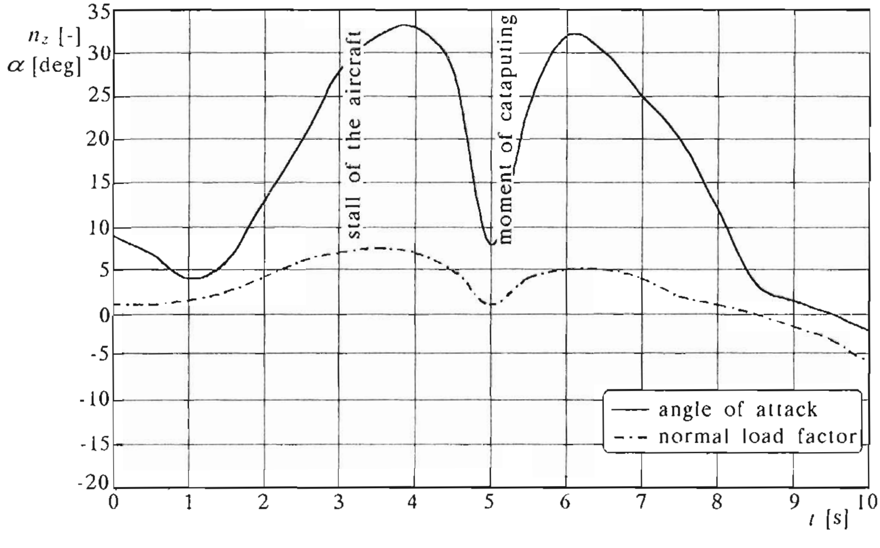

Fig. 6. Stall of the SU-22M4 aircraft – variations of the angle of attack α and normal load factor n_z

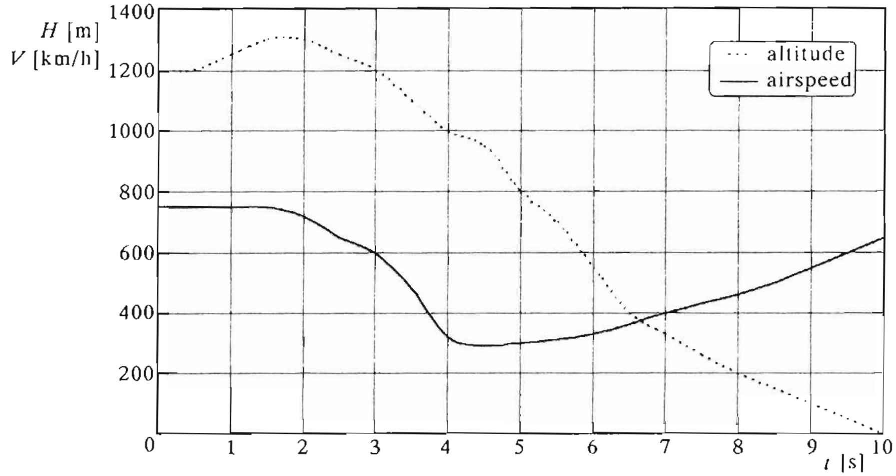

Fig. 7. Stall of the SU-22M4 aircraft – variations of the airspeed V and altitude H

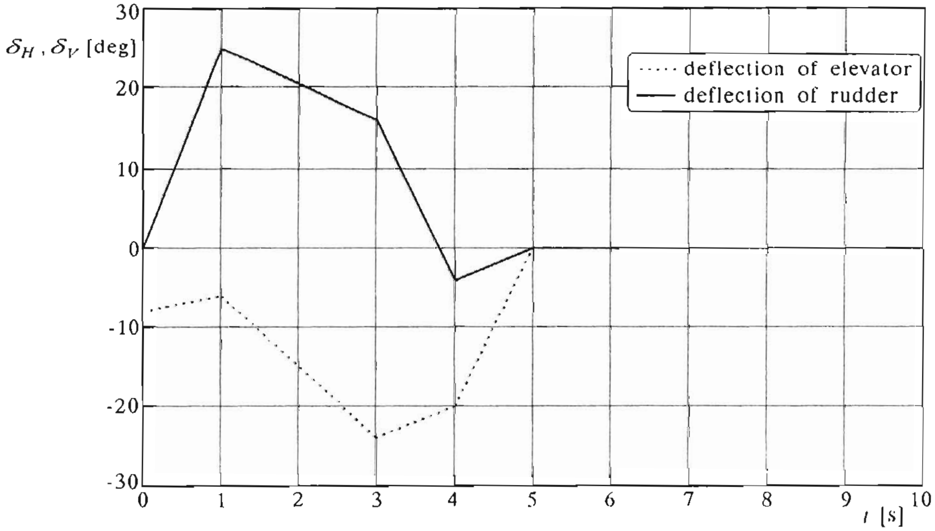

Fig. 8. Stall of the SU-22M4 aircraft – variations of angles of elevator δ_H and rudder δ_V

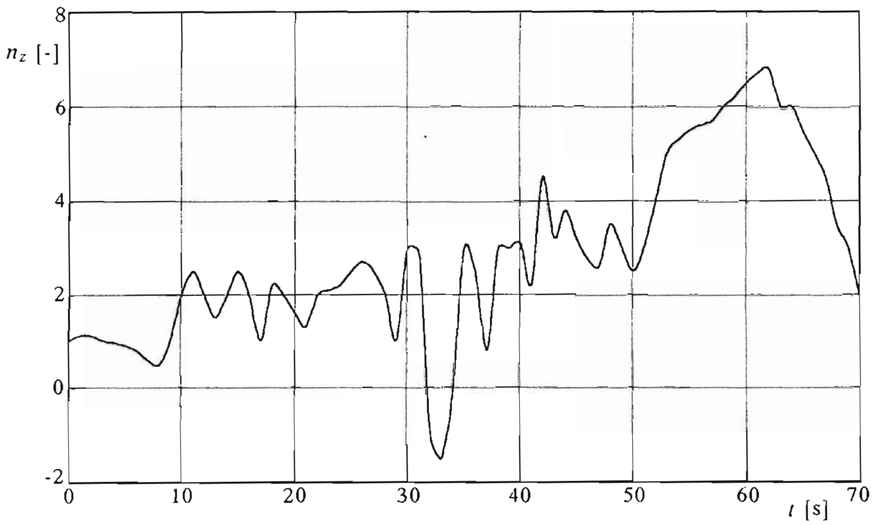

Fig. 9. Spin – variation of the normal load factor n_z

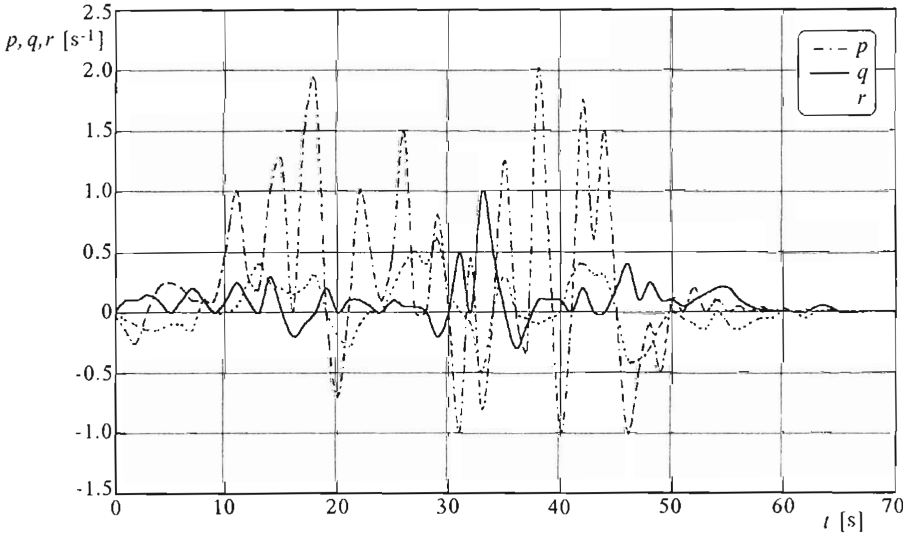

Fig. 10. Spin – variation of the angular velocities p, q, r

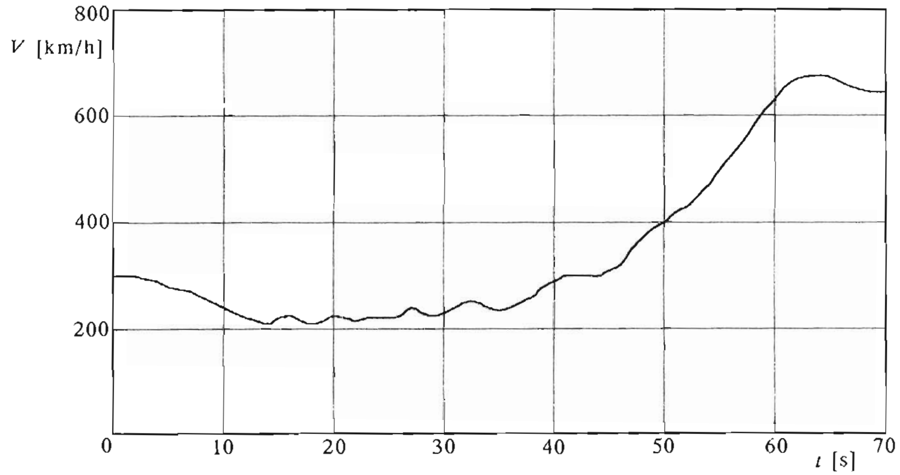

Fig. 11. Spin – variation of the air speed V

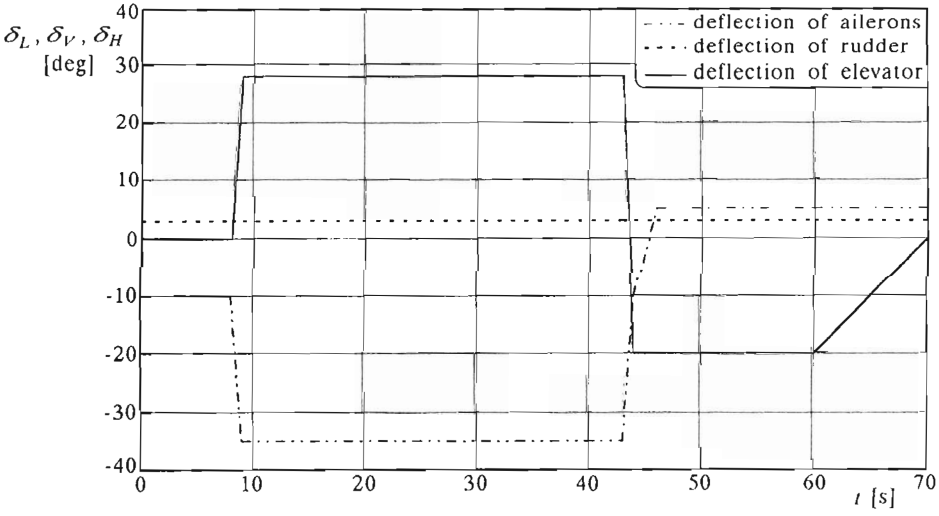

Fig. 12. Spin – deflections of the control surfaces

- Hyper-manoeuvrability – the aircraft can fly at angles of attack $80^\circ \div 120^\circ$ maintaining to stability in all channels. In this flight regime, the controllability is usually lost.

It can be noted, that the super-manoeuvrability does permit a decrease in the radius for a steady turn up to $2 \div 2.5$ times.

The famous "Cobra" manoeuvre, first performed by Vladimir Pugatshov in 1991 at the Paris Air Show, is a good example of the advanced flying manoeuvre and has entered the history of aviation as the first manoeuvre from the hyper-manoeuvrability range whenever shown. Herbst (1980) predicted the possibility of flight within the hyper-manoeuvrability range.

Theoretical investigation of the Cobra manoeuvre has been undertaken in Poland in the 1990s (cf Dżygadło et al., 1996). Important parameters, characterising the Cobra manoeuvre versus time are shown in Fig.13 $\div$ Fig.16. These figures present the results of numerical simulation of the Cobra manoeuvre. One of peculiarities of a dynamic entrance into the high angles of attack is the air breaking and a rapid decrease in the aircraft speed (Fig.16). The speed loss during the manoeuvre affects not only the flight safety but also just after the Cobra manoeuvre.

After Samoylovich (1997) it can be stated, that the necessary conditions to perform this manoeuvre are as follows:

- Aircraft should be statically unstable within the range of moderate angles of attack
- Balance of the aircraft should be attainable for angles of attack of the value greater then the critical one
- Pitching moment for the diving has a margin value within the range of angles of attack $\alpha \in (30^\circ, 60^\circ)$
- Natural pitching moment for the diving should be great enough within the range of angles of attack greater than 60°
- Limiter of the extreme flying parameters should be turned off
- Aircraft should be highly insensitive to the spin tendency, especially due to the lack of lateral stability at high angles of attack.

The "fire-pole" known also as the "helicopter fire" manoeuvre first performed by the X-31A experimental fighter aircraft, is a good example of the advanced flying within the super-manoeuvrability range. The aircraft performing this manoeuvre turns around a vertical axis with a substantial loss in altitude. This manoeuvre has a great impact on tactical utility. During dogfights, closing in onto the adversary both aircraft try to establish a gun tracking

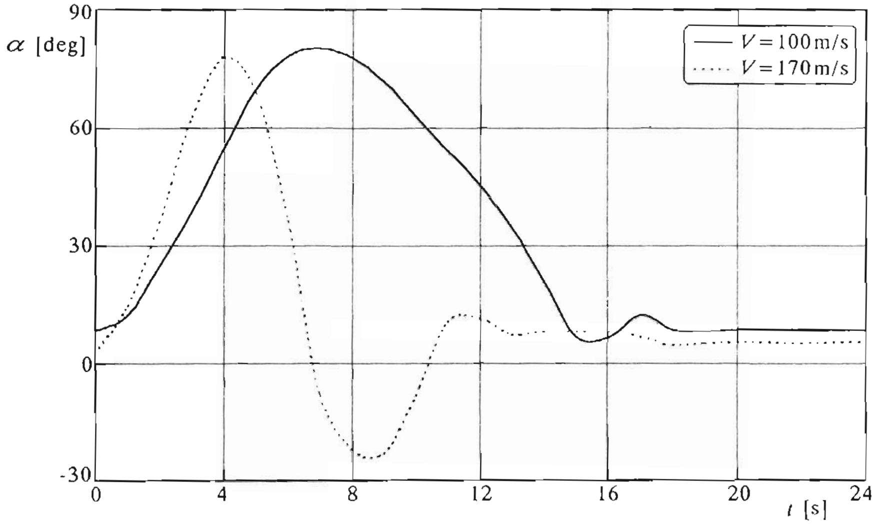

Fig. 13. Cobra manoeuvre (cf Dżygadło et al., 1996) – variations of the angle of attack α

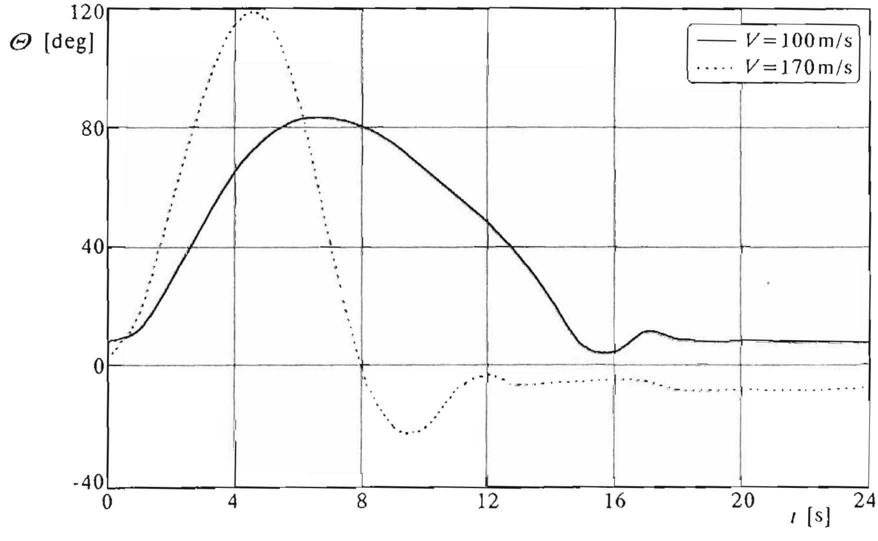

Fig. 14. Cobra manoeuvre (cf Dżygadło et al., 1996) – variation of the pitch angle Θ

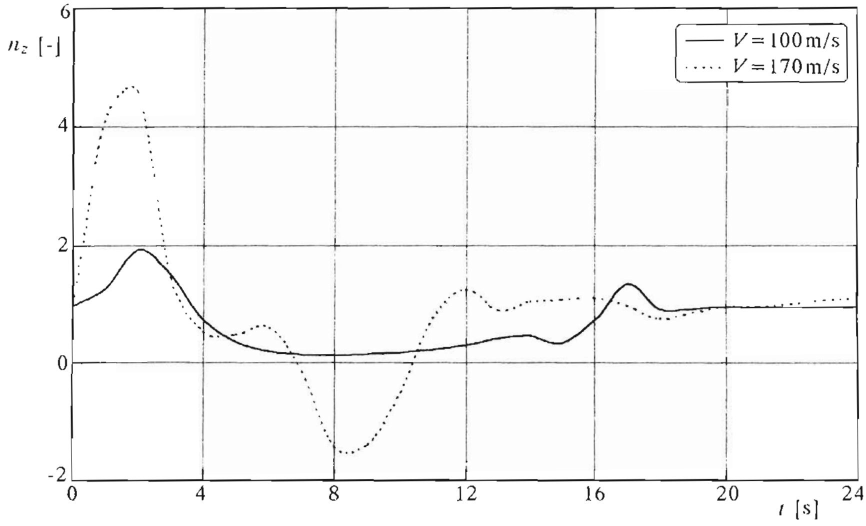

Fig. 15. Cobra manoeuvre (cf Dżygadło et al., 1996) – variation of the normal load factor n_z

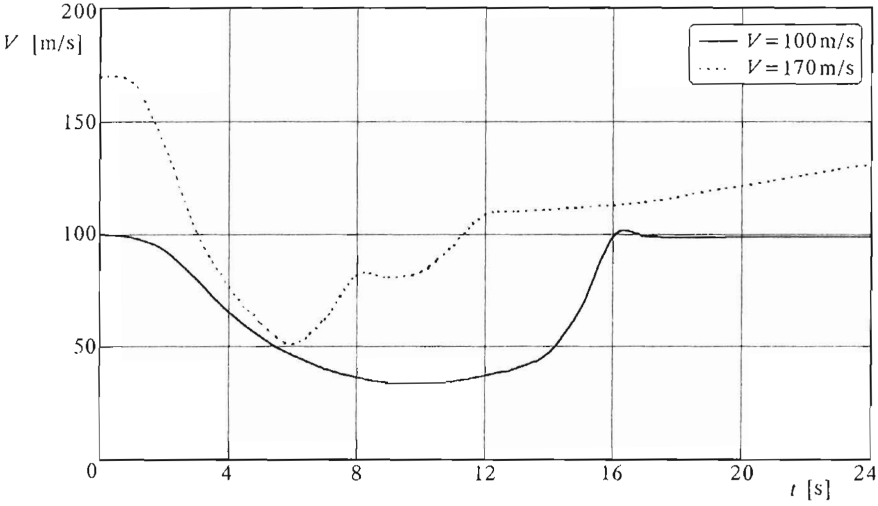

Fig. 16. Cobra manoeuvre (cf Dżygadło et al., 1996) – variation of the airspeed V

solution. While the adversary's turn radius is limited by its maximum possible load factor i.e., the maximum lift, the aircraft in the hyper-manoeuvrability can exploit its capabilities. The adversary aircraft bleeds of speed aiming to achieve smaller turn radii and to establish a gun tracking solution. Having reached its minimum turn radius, the adversary aircraft is restrained to circling at this turn radius. Thus, its flight path describes the surface of a "funnel" with a cylindrical lower part. The aircraft in the hyper-manoeuvrability, however, slides down a "fire pole". With its extremely tight turn radii and its reference line decoupled from its flight path, the aircraft can permanently threaten the adversary. The results of numerical simulation of the "fire pole" manoeuvre are shown in Fig.17÷ Fig.22. Time plots of the parameters characterising this manoeuvre are shown in the figures; i.e., angle of attack – Fig.17, normal load factor n_z – Fig.18, yaw, pitch, and bank angles – Fig.19, angular velocities p, q, r – Fig.20, airspeed – Fig.21, and flight altitude – Fig.22, respectively.

4. Final remarks

The aim of the presented work was to study the problems of aircraft dynamics under extreme flight conditions. After a brief presentation of the mathematical aircraft model, the dynamics of spatial motion of an aeroplane is considered. The investigations are carried out using computer simulation. The numerical analyses of PZL I-22 "Iryda", Su22M4 fighter-bomber and MiG-29 fighter aircraft are presented. The ONERA deep stall model together with proposed modification of the classical wing section theory makes it possible to predict the aircraft forces and moments at high angles of attack (cf Sibilski, 1998). Nevertheless, it showed emphasized that in the high angle of attack regime, fighter aircraft behaviour is so complex that it is very difficult to predict it exhaustively. An analysis of such phenomena is very dificult due to their complexity and apparently random character.

The flight tests as well as numerical simulations have shown that the dynamic entrance into high angles of attack can be done safely, including the cases when initial parameters are typical for a real dogfight. The numerical analysis shows that the dynamic entrance is possible in the offensive as well as defensive manoeuvres. Flight tests analyses have shown that the dynamic entrance exerts slight influence on the effectiveness of the dogfight, first of all because this manoeuvre takes extremely short time (cf Samoylovich, 1997). However, the dynamic entrance can be recommended as a new tactics, as a

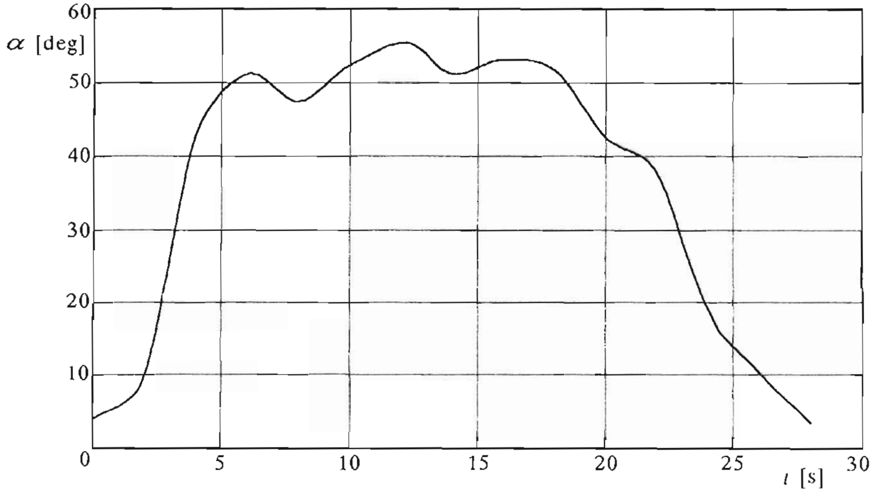

Fig. 17. "Fire pole" manoeuvre – variation of the angle of attack α

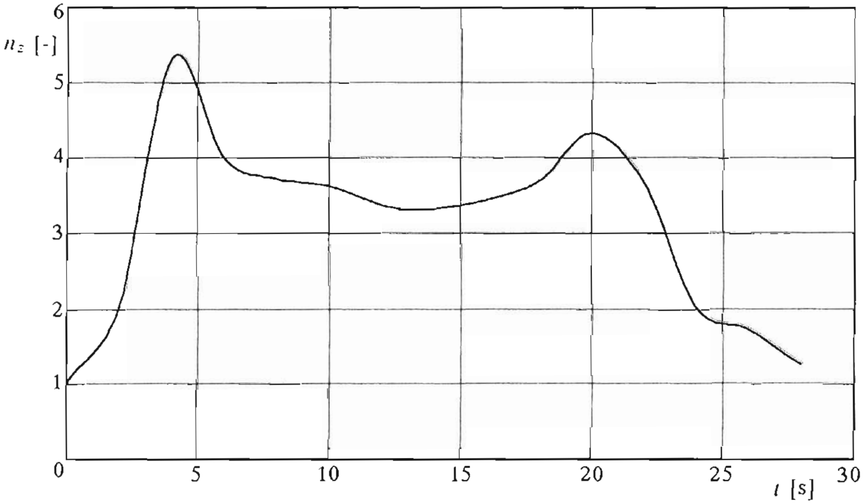

Fig. 18. "Fire pole" manoeuvre – variation of the normal load factor n_z

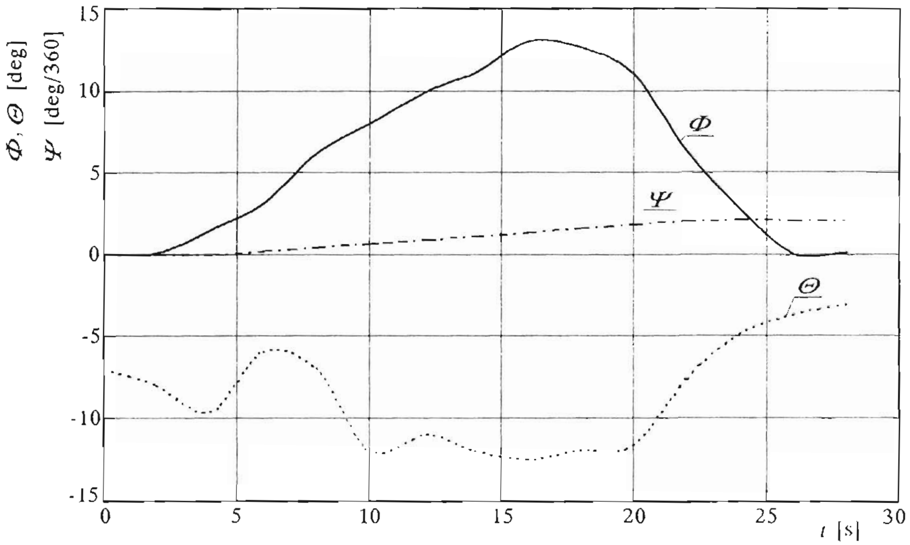

Fig. 19. "Fire pole" manoeuvre – variation of the yaw, pitch, and bank angles Ψ, Θ, Φ

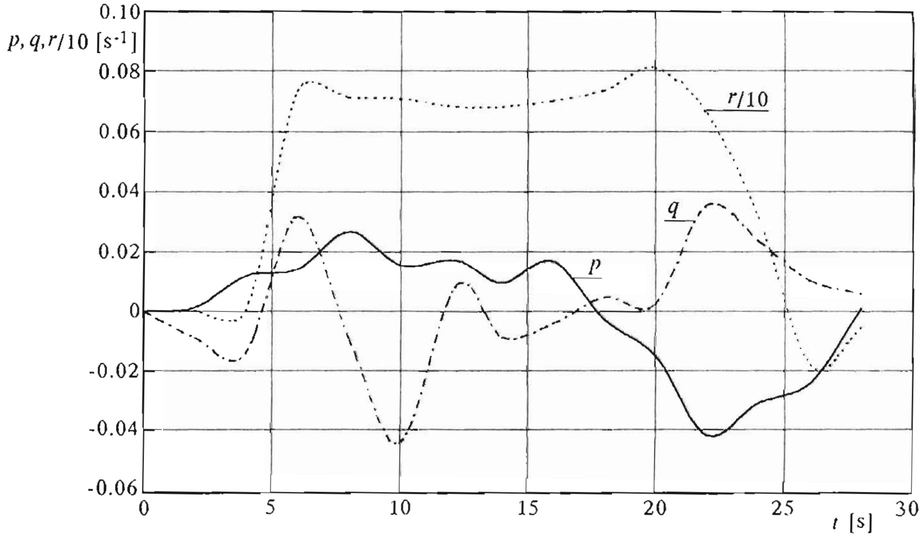

Fig. 20. "Fire pole" manoeuvre – variation of the angular velocities p, q, r

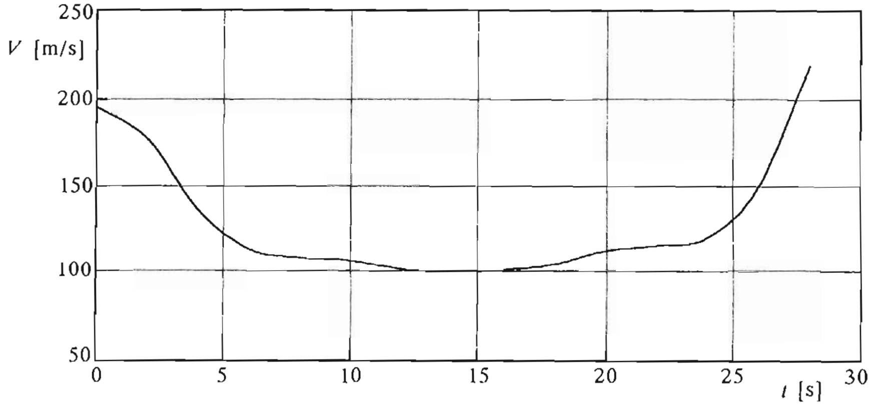

Fig. 21. "Fire pole" manoeuvre – variation of the air speed V

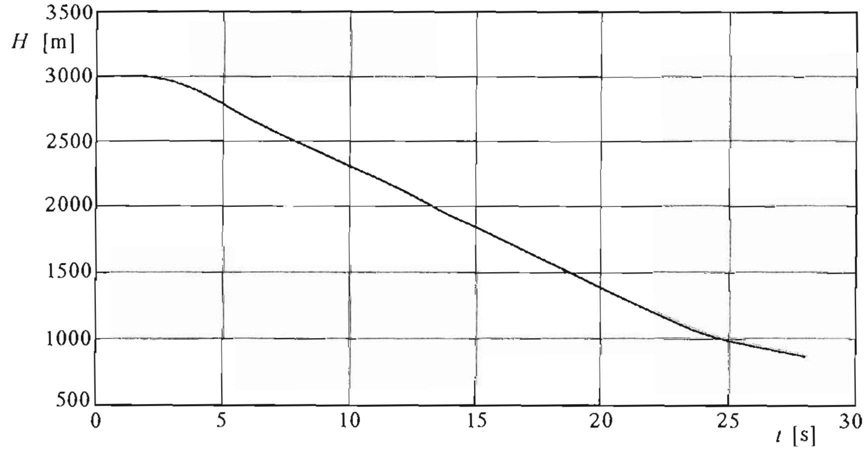

Fig. 22. "Fire pole" manoeuvre – variation of the altitude H

phase predicting the real attack. It can be stated that the radical extension of aircraft agility can be reached using the Thrust Vectoring Control System.

The results presented above prove the applicability numerical simulation techniques to prediction of the aircraft behaviour in critical flight events and manoeuvres. However, one should remember that the quality of calculations is directly connected with the accuracy of the aerodynamic data base put in the aircraft model.

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Numeryczne badanie dynamiki lotu samolotów o podwyższonej sterowności

Streszczenie

Rozpatrzono dynamikę lotu samolotu o podwyższonej sterowności. Badania przeprowadzono stosując symulację komputerową. Zaprezentowano wyniki analizy dynamiki ruchu samolotu w różnych stanach lotu i manewrach.

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