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MODERN TECHNIQUES FOR ACTIVE MODIFICATION OF THE AIRCRAFT DYNAMIC BEHAVIOUR

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In the paper an attempt has been made to show crucial ideas of modification theory. This theory, though just in statu nascendi, provides the flight dynamics engineer with a very valuable design tool, because it covers the diverse topics that have attracted utmost attention in current researches. This paper considers the problem of improving the aircraft dynamic characteristics using feedback control. The starting point is a linear quadratic problem. Next, several applications of control theory to aircraft control law developments are described. The applications include solutions to the artificial stabilization and wing rock control problems. Finally, some issues to be explored are shown.

Key words: stability and control characteristics, artificial stabilization, wing rock control

1. Introduction

Whenever a set of specifications has been laid down for the dynamic behaviour of an aircraft, and when those specifications cannot be met, a modification problem arises. If the required dynamic behaviour has to be achieved then additional equipment must be used with the basic aircraft. Stability and control are two of the major technical challenges in the aircraft design. The failure of many aircraft projects in the past resulted from inadequate solutions to the stability and control problem. New aircraft designs, which typically are driven at requirements for reduced operating cost, in the case of civil transport aircraft, and reduced radar signature, in the case of military aircraft, present strong challenge to the flight dynamics engineer.

The research aims at a more systematic study of the aircraft modification problem. The paper highlights the important role the feedback control plays when solving this problem. The importance of feedback control in furnishing the required stability and control characteristics for aircraft has been firmly established. All high-performance aircraft produced today employ some form of the feedback control, e.g., in the form of an autopilot, a limited authority command and stability augmentation system, or a full authority fly-by-wire system.

For most aircraft flying today the control laws were accomplished using classical techniques, e.g., the root locus technique. However, over the last 25 years new multivariable control law analysis and synthesis techniques have been proposed. These techniques have their roots in the theories of optimal control developed by Pontryagin et al. (1962) and Bellman (1957). There has been a proliferation of extensions and variations, which have kept the academic community well occupied. The proponents of multivariable control theory claim that the modern techniques can handle multiloop control problems in a formal and systematic manner.

The paper partially summaries some of the analytical control design experience during the last 10 years gaived by the Movable Objects Dynamics Team at Warsaw University of Technology.

2. Formulation of the problem

2.1. Classical versus modern design techniques

With the rapid development of high-speed computers during the recent decades, a new approach to the control system design has evolved; this approach is commonly called *modern control theory*. This theory permits a more systematic approach to the problem of control system design. In modern control theory, the control system is specified as a system of first-order differential equations. By formulating the problem in this manner, the control system designer can fully exploit the digital computer when solving complex control problems. Another advantage of modern control theory is that optimisation techniques can be applied to design of optimal control systems. The purpose of the paper is to expose to some of the concepts of modern control theory and then apply the procedures to design of aircraft flight control systems.

2.2. Basic concepts of modern control system design

In attempt to improve aircraft designs and to explore the effects and particularly the benefits of the act, the obvious step is to present philosophy of modern control system design. Two concepts are crucial for this design. The first one: the design is based directly on the state-variable model

$$\dot{\boldsymbol{x}} = \boldsymbol{f}\Big(\boldsymbol{x}(t), \boldsymbol{u}(t)\Big) \tag{2.1}$$

where $\boldsymbol{f}$ is, in general, a nonlinear function of the state vector $\boldsymbol{x}(t)$ of dimension n, and the control vector $\boldsymbol{u}(t)$ of dimension m.

The second basic concept: the formula for performance specifications in terms of a mathematically precise scalar functional of the general form

$$I = \int\limits_{t_0}^{t_f} \varphi(\boldsymbol{x}, \boldsymbol{u})\; dt \tag{2.2}$$

termed *performance index*, where $(t_f - t_0)$ is the operation time.

In many practical situations instead of the model (2.1) it is sufficient to use a little simpler model

$$\dot{\boldsymbol{x}} = \mathbf{A}\boldsymbol{x} + \boldsymbol{\vartheta}(\boldsymbol{x}) + \mathbf{B}\boldsymbol{u}(t) \tag{2.3}$$

where $\mathbf{A}$ and $\mathbf{B}$ are constant $n \times n$ and $n \times m$ matrices, respectively.

Therefore, it makes sense to apply the index (2.2) in the form

$$\varphi = \frac{1}{2}(\boldsymbol{x}^\top \mathbf{Q}\boldsymbol{x} + \boldsymbol{u}^\top \mathbf{R}\boldsymbol{u}) \tag{2.4}$$

where the weighting matrices $\mathbf{Q}$ and $\mathbf{R}$ are $n \times n$ non-negative and $m \times m$ positive definite symmetric matrices, respectively. In the case, when $\dim \boldsymbol{x} = 2$, and $\dim \boldsymbol{u} = 1$, we have the relations

$$\mathbf{Q} = [q_1, q_2]^\top \qquad\qquad \mathbf{R} \equiv r > 0 \tag{2.5}$$

often presented in the papers.

2.3. Aircraft modification problem

An important task in aircraft dynamics is modification of the aircraft parameters for specified dynamic behaviour to be attained. According to the best of our knowledge, the term *aircraft modification problem* has never been used, although the term *modification* can be found in the various contexts of aircraft dynamics. Such terms as *alleviation, control, enhancement, improving, reduction*, will be treated as equivalents of one universal word *modification*. The objective of the modification is to improve the operational capabilities of the aircraft with respect to range, take-off and landing distances, time-to-climb, etc. Thus, we would term *modification* (precisely *active modification*) the mathematical process by which one realises the act functions within the framework of modern control theory, not necessary the optimal one (in the sense of index (2.2)).

The general modification problem can be stated as follows: given a dynamic system represented by Eq (2.1), the following output equation

$$\boldsymbol{y} = \boldsymbol{h}(\boldsymbol{x}) \tag{2.6}$$

and a required output trajectory $\boldsymbol{y}_d(t)$, find a control law $\boldsymbol{u}(t)$ such that the tracking error

$$\boldsymbol{e} = \boldsymbol{y}(t) - \boldsymbol{y}_d(t) \tag{2.7}$$

tends to zero, as the whole state $\boldsymbol{x}(t)$ remains bounded. When we take into consideration the index (2.2), then we have the optimal modification problem.

2.4. Standard modification technique

As a basis for discussion, consider in the new light, the well-known Kalman-Letov problem: the linear time-invariant controllable system is described by

$$\dot{\boldsymbol{x}} = \mathsf{A}\boldsymbol{x} + \mathsf{B}\boldsymbol{u} \tag{2.8}$$

The system performance is expressed by the functional

$$I = \frac{1}{2}\int\limits_0^\infty (\boldsymbol{x}^\top \mathsf{Q}\boldsymbol{x} + \boldsymbol{u}^\top \mathsf{R}\boldsymbol{u})\; dt \tag{2.9}$$

If we apply the principles of calculus of variations to minimisation of the performance index (2.9), we obtain the feedback control law in the form

$$\boldsymbol{u}(t) = \mathsf{F}\boldsymbol{x}(t) \qquad\qquad \mathsf{F} = -\mathsf{R}^{-1}\mathsf{B}^\top\mathsf{K} \tag{2.10}$$

where the feedback gain F can be obtained after solving the following matrix algebraic Riccati equation

$$\mathsf{PBR}^{-1}\mathsf{B}^\top\mathsf{P} - \mathsf{PA} - \mathsf{A}^\top\mathsf{P} - \mathsf{Q} = \mathsf{0} \tag{2.11}$$

where P is a positive-definite symmetric matrix. Except for simplest examples, solution of Eq (2.11) requires sophisticated computer codes. Finally, the resulting closed-loop system becomes

$$\dot{\boldsymbol{x}} = \mathsf{L}\boldsymbol{x} \qquad\qquad \mathsf{L} = \mathsf{A} + \mathsf{BF} \tag{2.12}$$

Kalman showed that for a controllable system this solution yields a stable closed-loop system, i.e., the eigenvalues of L, denoted as $\lambda_j(\mathsf{L})$, $j = 1, ..., n$, $n = \dim \boldsymbol{x}$, lie in the open left-half plane of the complex plane. In short

$$\mathrm{Re}\lambda_j(\mathsf{L}) < 0 \tag{2.13}$$

The Kalman-Letov problem is often referred to as the Linear Regulator Problem (LRP). The reason for choosing this problem was as follows: the systems to be controlled may be unstable without control, and one of the tasks of the control is to maintain stability in the entire working region of the process. But there is still a more important reason for this presentation: the LRP technique provides a prototype of systematic approach to the aircraft modification problem.

3. Techniques for linear control systems

3.1. General remarks

The design technique presented above is however, suitable only for technically simple problems. The principal difficulty lies not in the solution, but in the choice of a suitable performance index. The control law (2.12) is optimal in the sense that the chosen performance index (2.9) is minimised, but different optimal controls can be obtained by altering the matrices Q and R. A pilot using a subjective criterion ultimately judges the performance of aircraft control system. The designer must rely upon his experience and judgement to indicate the response curves that the pilot will find satisfactory. Since only a tenuous relationship exists between the performance index and the desired performance, a certain amount of trials and errors can not be avoided. Therefore, in the following section we focus on advanced design methods, which are suitable for use when solving the aircraft modification problem, including eigenstructure assignment and model-following ones. As we will see, each of these techniques has its advantages and disadvantages, and therefore each has its proponents and antagonists.

3.2. Eigenstructure assignment

The eigenstructure assignment (pole placement) technique allows one to assign the poles in the MIMO (multi-input/multi-output) systems to desired locations in one step by solving equations for the feedback gains. The required pole locations for aircraft design may be found in the military flying quality specifications (see Stevens and Lewis, 1992). However, while discussing flying quality requirements, we have noted that the time response depends not only on the pole locations; it also depends on the zero value of the individual SISO (single-input/single-output) transfer function, or equivalently on the eigenvectors. Thus, the capability of modern control system to select both the closed-loop poles and eigenvectors is relevant in aircraft design.

In this design approach, the system to be controlled is represented by Eq (2.8). For simplicity of the presentation we take into consideration only full state feedback. Thus, the control law is a linear function of the state vector (see Eq (2.10))

$$\boldsymbol{u} = -\mathsf{G}\boldsymbol{x} \tag{3.1}$$

The feedback gain matrix G selected ensuring that this control law results in the intended location of closed-loop eigenvalues λ_i and shaping of closed-loop eigenvectors $\boldsymbol{v}_i$; in short, we seek the closed-loop eigenstructure $\{\lambda_i, \boldsymbol{v}_i\}$.

Substituting the control (3.1) into Eq (2.8) yields the closed-loop system

$$\dot{\boldsymbol{x}} = (\mathsf{A} - \mathsf{BG})\boldsymbol{x} \tag{3.2}$$

To select G so that a desired eigenstructure is assigned to the closed-loop system, suppose that we can find a vector $\boldsymbol{w}_i$ that satisfies the equation

$$[\lambda_i \mathsf{I} - \mathsf{A}, \mathsf{B}] \begin{bmatrix} \boldsymbol{v}_i \\ \boldsymbol{w}_i \end{bmatrix} = \mathbf{0} \tag{3.3}$$

Now, choose the feedback gain to satisfy

$$\mathsf{G}\boldsymbol{v}_i = \boldsymbol{w}_i \tag{3.4}$$

Using Eqs (3.3) and (3.4), we may obtain the equation

$$[\lambda_i \mathsf{I} - (\mathsf{A} - \mathsf{BG}]\boldsymbol{v}_i = \mathbf{0} \tag{3.5}$$

which states that $\boldsymbol{v}_i$ is assigned as a closed-loop eigenvector for a desired eigenvalue λ_i.

To complete the presentation, we define the modal matrices V and W (see Porter and Crossley, 1972), and finally we calculate G from the relation

$$\mathsf{GV} = \mathsf{W} \tag{3.6}$$

3.3. Model-following design

This technique is an important approach to the control design, where it is required that the aircraft behave like an ideal (or model) system with required flying qualities (cf Tomczyk, 1997), a quadratic performance index in which the difference between the controlled aircraft and model responses are minimised.

In many problems, particularly in the field of flight control, one would like the closed-loop system (2.12) to be as close as possible to a system given by the differential equation

$$\dot{\boldsymbol{z}} = \mathbf{A}_d \boldsymbol{z} \tag{3.7}$$

that represents the model of desirable dynamics; such as, transient behaviour, handling qualities, etc. Since, in general, $\dim \boldsymbol{x} \neq \dim \boldsymbol{z}$, one compares $\boldsymbol{z}$ with the output vector $\boldsymbol{y}$ ($\dim \boldsymbol{z} = \dim \boldsymbol{y}$) that is related to $\boldsymbol{x}$ by a constant matrix $\mathbf{C}$

$$\boldsymbol{y} = \mathbf{C}\boldsymbol{x} \tag{3.8}$$

There are two fundamentally different sorts of model-following control, explicit and implicit, which result in the controllers of different structure (Stevens and Lewis, 1992). In the explicit model-following control, index (2.9) is modified as

$$I_{ex} = \frac{1}{2} \int\limits_0^\infty [(\boldsymbol{y} - \boldsymbol{z})^\top \mathbf{Q}_{ex} (\boldsymbol{y} - \boldsymbol{z}) + \boldsymbol{u}^\top \mathbf{R} \boldsymbol{u}] \; dt \tag{3.9}$$

Index (3.9) can be transformed to the standard index (2.9). In the implicit model following control, index (2.9) is modified as

$$I_{im} = \frac{1}{2} \int\limits_0^\infty [(\dot{\boldsymbol{y}} - \mathbf{A}_d \boldsymbol{y})^\top \mathbf{Q}_{im} (\dot{\boldsymbol{y}} - \mathbf{A}_d \boldsymbol{y}) + \boldsymbol{u}^\top \mathbf{R} \boldsymbol{u}] \; dt \tag{3.10}$$

By substituting Eqs (2.8) and (3.8) into Eq (3.10), this index becomes equivalent to one of the standard type but with some cross-product (see Michalski and Pietrucha, 1996).

3.4. Illustrative example: artificial stabilization

3.4.1. Stabilization of unstable aeroplane

The design of any aircraft depends on aerodynamic stability requirements. Artificial stability is obtained essentially by feedback of sensed aircraft motion to provide suitably phased stabilising control moments as is sometimes referred to as relaxed static stability. If the aircraft is "natural" unstable, then the following relation arises

$$\mathrm{Re}\lambda_j(\mathbf{A}) > 0 \tag{3.11}$$

The aim of active control is to establish such a control law (2.10), that the closed-loop system (2.12) is stable, i.e., Eq (2.13) holds for all $j = 1, ..., m$, $m = \dim \mathbf{L}$. More precise definition can be achieved by imposing the requirement that the time-to-half-amplitude

$$T_{1/2} = \frac{-\ln 2}{\mathrm{Re}\lambda_j(\mathbf{A})} \tag{3.12}$$

take the specified value.

3.4.2. Mathematical model of a short-period motion

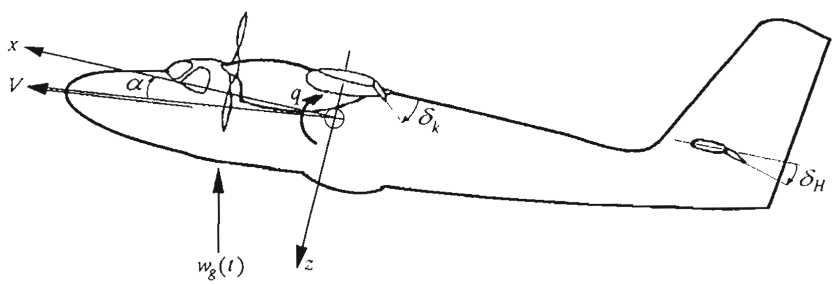

Fig. 1. Body system of axes and the sign convention

The linear equations of longitudinal motion that approximate the short-period mode are used in this example. This mode of motion occurs at almost constant forward speed; therefore "the X-equation" can be neglected, as it does not contribute much to the short-period motion. The frame of reference for the aircraft motion is the system of body axes (see Fig.1). With these assumptions, the equations of motion are similar to those given by Stevens and Lewis (1992)

$$\begin{aligned}\dot{\alpha}(t) &= Z_\alpha \alpha(t) + Z_q q(t) + Z_H \delta_H(t) + Z_k \delta_k(t)\\ \dot{q}(t) &= M_\alpha \alpha(t) + M_q q(t) + M_H \delta_H(t) + M_k \delta_k(t)\end{aligned} \tag{3.13}$$

where

$\alpha(t)$	–	angle of attack
$q(t)$	–	pitch rate
$\delta_H(t)$	–	elevator deflection angle
$\delta_k(t)$	–	flap deflection angle
$Z_\alpha, M_\alpha, \ldots$	–	dimensional aerodynamic coefficients.

The initial condition for Eq (3.7) is

$$\boldsymbol{x}_0 = [\alpha_0, q_0]^\top = \left[\arctan\frac{w}{V}, 0\right]^\top \tag{3.14}$$

where

$$\boldsymbol{x} = [\phi, p]^\top \qquad \boldsymbol{u} = [\delta_H, \delta_k]^\top$$
$$\mathsf{A} = \begin{bmatrix} Z_\alpha & 1 \\ M_\alpha & M_q \end{bmatrix} \qquad \mathsf{B} = \begin{bmatrix} Z_H & Z_k \\ M_H & M_k \end{bmatrix} \tag{3.15}$$

3.4.3. Numerical example

Calculations for a model of controlled aeroplane were made for diagonal weighted matrices with identical constant elements, i.e.

$$\mathsf{Q} = w_q\mathsf{I} \qquad \mathsf{R} = w_r\mathsf{I} \qquad w_q, w_r \neq 0 \tag{3.16}$$

were chosen, where I is the unit matrix. In this case only one parameter can be introduced

$$\beta = \frac{w_r}{w_q} \tag{3.17}$$

which is some kind of measure of economising control as far as constraints limits imposed on the state of system are concerned. In such a case, the performance index can be presented in the form

$$I = \frac{1}{2}\int\limits_0^\infty (\boldsymbol{x}^\top\boldsymbol{x} + \beta\boldsymbol{u}^\top\boldsymbol{u})\,dt \tag{3.18}$$

A decrease in the parameter β value; i.e., allowance for more expensive control elements is accompanied by an increase in the absolute values of feedback matrix F and eigenvalues λ. This means obtaining systems with faster and faster changing transient processes – which is confirmed by the monotonic decreasing time-to-half-amplitude. It is worth noticing that in all cases of actively controlled plane the eigenvalues are real, so the motion is aperiodic.

The results of calculations are shown in Table 1 for the models of uncontrolled plane ($N = 0$) and actively controlled plane (remaining N), respectively. One of the eigenvalues in the case with controls locked is positive. According to the condition (3.11) this shows instability. The time-to-half-amplitude corresponding to this value is negative. The minus has here the physical sense: it indicates that deviation from equilibrium state is rising, instead of declining (Fig.2).

Table 1

No.	β	Elements of **P**		Elements of **F**		λ	$T_{1/2}$
0	–	–		–		−41770	0.166
						0.1130	−6.135
1	5.0	0.3108	0.0589	0.2456	0.3578	−10.4251	0.066
		0.0589	0.0888	−0.0557	−0.1452	−2.0664	0.335
2	1.0	0.2519	0.0198	0.4354	0.8573	−22.1304	0.031
		0.0198	0.0426	0.0165	−0.3549	−2.2738	0.305
3	0.5	0.4759	0.0255	0.5853	1.2365	−31.0902	0.022
		0.0255	0.0615	0.1338	−0.5143	−2.4060	0.288
4	0.1	1.9605	0.0488	1.2837	2.8437	−69.1507	0.010
		0.0488	0.1415	1.0394	−1.1908	−3.2091	0.216
5	0.05	0.3395	0.0065	1.8290	4.0494	−97.7287	0.007
		0.0065	0.0202	1.9703	−1.6994	−3.9843	0.174

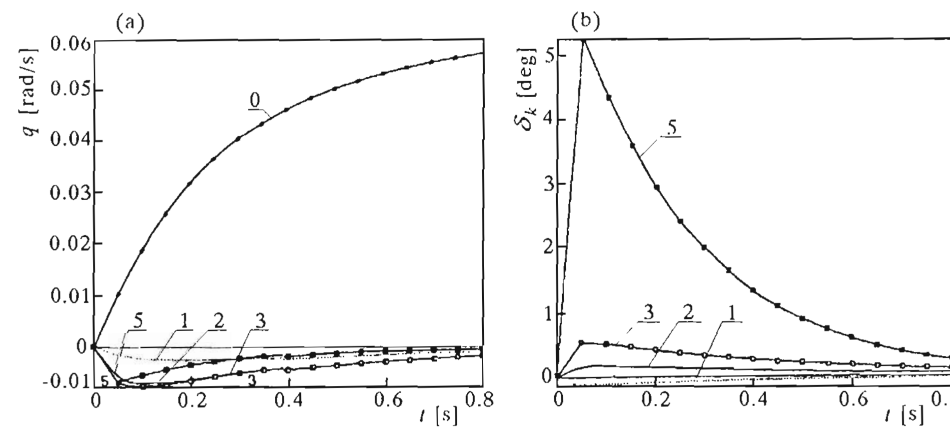

Fig. 2. (a) – Histories of the pitch rate during the stabilization; (b) – histories of the flap deflection during the stabilization

In the table there is also presented the solution **P** of Eq (2.11) used to determine the optimal control (2.10). It can be easily seen that, as it was required and expected – they are positive determine and symmetric. It is worthwhile to note that almost all corresponding to each other elements of matrix **F** have the same sign. This is accompanied by an identical (in quality) behaviour of the control surfaces (Fig.2b). An exception to this rule is the second row of this matrix, in the case of economising control ($N = 1$). As a result the flaps should deflect in oposit direction than in other cases.

3.4.4. Conclusions

The figures presented show that the determined control, which we will call stabilising, achieves the goal (Fig.2a). But we cannot stop on this. The histories of accelerations (Fig.3) show that it can be harmful to the staff and airplane. A special attention is paid to a rapid increase in the normal acceleration immediately after the plane enters a gust. So a new issue appears: minimisation of the normal acceleration (see Michalski and Pietrucha, 1996).

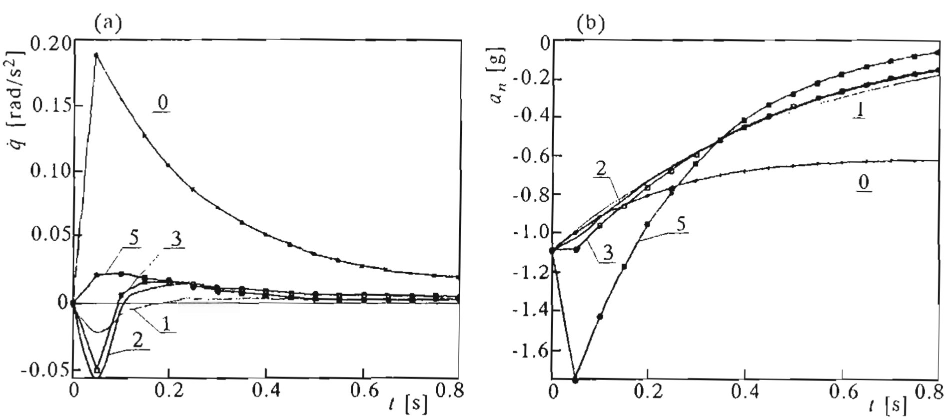

Fig. 3. (a) – Histories of the pitch acceleration during the stabilization; (b) – histories of the normal acceleration during the stabilization

4. Design techniques for nonlinear control systems

4.1. General remarks

Many dynamic systems, particularly the aerospace systems, are nonlinear and/or time-depended, and the techniques for design of linear time-invariant control systems are, in general, not applicable to such more complicated systems.

The linear systems have been investigated in full details because its characteristics allows a detailed analysis. This is not the case when the nonlinear systems are considered, for there are no general approaches to the solution of nonlinear differential equations. Generally, there are two approaches that may be taken. The first, and most traditional one consists in postulating a system and analysing its behaviour. Changes are made if the behaviour is not proper, and the process is repeated. In other words, this is the trial-and-error method. For example, the phase plane method, and describing method can be used for this purpose. Experience and intuition are crucial for this process. For complex systems, however, this approach often fails.

The second possible approach is that of direct synthesis. An objective is established and an attempt is made to find such a controller that best attains the objective in the best way when subject to appropriate restrictions. In this section only the methods for synthesis of control systems to modify certain characteristics will be considered. The following methods are evaluated: Pontryagin maximum principle; Lyapunov function method; nonlinear inverse dynamics.

The features of these methods are analysed on the example of Wing Rock (WR) motion. Since the wing rock is a nonlinear phenomenon, a method of control based on overall or local linearization is not aplicable. Indeed, roll-coupling instabilities first appeared in flight, often with fatal results, because the linearized equations of motion used for analysis at that time did not contain the instability. The main objective of the section is modification of WR characteristics by active control.

4.2. Pontryagin Maximum Principle (PMP)

The PMP can be viewed as an extension of the classical calculus of variations to cover the optimal control problem (Elbert, 1984). Basically, the PMP gives a set of local necessary conditions for optimality that in turn provide a test, which determines whether or not any given control is a candidate for optimality.

The solution to the optimal problem (see Eqs (2.1) and (2.2)) is given in terms of the Hamiltonian function defined as

$$H = H(\boldsymbol{x}, \boldsymbol{u}, \boldsymbol{\psi}) = -\varphi + \boldsymbol{f}^\top \boldsymbol{\psi} \tag{4.1}$$

where

φ – loss function in Eq (2.2)
$\boldsymbol{\psi}(t)$ – vector of adjoin variables that are defined by the equations

$$\dot{\psi} = -\frac{\partial H}{\partial \boldsymbol{x}} = \frac{\partial \varphi}{\partial \boldsymbol{x}} - \frac{\partial \boldsymbol{f}^\top}{\boldsymbol{x}}\boldsymbol{\psi} \tag{4.2}$$

According to PMP, the problem described by Eqs (2.1) and (2.2) has a solution only if the Hamiltonian function has an absolute extremum with respect to $\boldsymbol{u}(t)$ for every fixed value of $(\boldsymbol{x}, \boldsymbol{\psi})$. If the control vector $\boldsymbol{u}(t)$ is not constrained in any way, the necessary condition is

$$\frac{\partial H}{\partial \boldsymbol{u}} = \mathbf{0} \tag{4.3}$$

In practice, this situation is seldom encountered because saturation effects are almost present to some extent, since all practical controllers can deliver only a finite amount of energy. In order to consider this important aspect of optimal control, the problem presented in Section 2 should be reformulated as follows: it is required to select the control vector $\boldsymbol{u}(t)$ with the additional constraint (Elbert, 1984)

$$u_p(t) \leqslant C_p \qquad p = 1, ..., P \tag{4.4}$$

The details of the solution to this important problem are beyond the scope of the present contribution, and can be found e.g. in Dubiel and Homziuk (1990). Here, it is sufficient to say: application of the PMP yields the following result

$$\begin{aligned} \frac{\partial H}{\partial u_p} &= 0 \qquad &\text{if} \quad u_p \leqslant C_p \\ \frac{\partial H}{\partial u_p} &\leqslant 0 \qquad &\text{if} \quad u_p = C_p \end{aligned} \tag{4.5}$$

4.3. Lyapunov Function Method (LFM)

The basic Lyapunov theory comprises the two methods introduced by Lyapunov, i.e., indirect and direct methods. The direct method is a powerful tool for nonlinear system analysis, and therefore the so-called Lyapunov analysis often actually refers to the direct method. The direct method is a generalisation of the energetic concepts associated with the mechanical system: a motion of such a system is stable if its total mechanical energy decreases all the time. When using the direct method to analyse the stability of a nonlinear system, the idea is to construct a scalar energy-like function (a Lyapunov function) for the system, and to see whether it decreases.

Although Lyapunov's direct method is originally a method of stability analysis, it can be used for solving other problems in nonlinear control. Design of nonlinear controllers is one of the important applications. The idea is to formulate somehow a scalar positive function of the system states, and then choose a control law to function decrease. The nonlinear control system designed that way will ensure the stability.

The methodology presented by Pietrucha and Złocka (1995) leads to the set of two algebraic equations

$$\frac{\partial V}{\partial t} + (\operatorname{grad} V)^\top \boldsymbol{f} + \varphi = 0 \qquad\qquad (\operatorname{grad} V)^\top \frac{\partial \boldsymbol{f}}{\partial \boldsymbol{u}} + \frac{\partial \varphi}{\partial \boldsymbol{u}} = 0 \tag{4.6}$$

If $\dot{V}$ can only be rendered negative semi-definite, it must be verified that $\dot{V}$ remains zero at the required final state. The control function we can obtain from Eq $(4.6)_2$. Introducing the right-hand-side of Eqs (2.3) and (2.4) into Eq $(4.6)_1$, we obtain the differential equation

$$(\operatorname{grad} V)^\top \mathbf{A}\boldsymbol{x}\frac{1}{2}\mathbf{R}^{-1}(\operatorname{grad} V)^\top \mathbf{B}\mathbf{B}^\top \operatorname{grad} V + (\operatorname{grad} V)^\top \boldsymbol{\vartheta}(\boldsymbol{x}) + \boldsymbol{x}^\top \mathbf{Q}\boldsymbol{x} = 0 \tag{4.7}$$

for determination of needed a Lyapunov function.

Of course, the nature of control law depends on the type of V chosen. A very popular choice is as follows

$$V(\boldsymbol{x}) = \sum_{n=2}^{\infty} V_n(\boldsymbol{x}) \tag{4.8}$$

where V_n is the homogenous form of nth order. Unknown coefficients of V_n must be found. To this end, the form (4.8) should be substituted into Eq (4.7). Then, the resulting algebraic equations are obtained setting the sum of coefficients of similar terms equal to zero.

4.4. Nonlinear Inverse Dynamics (NID)

The purpose of NID is to develop the feedback control law linearizing the aircraft response to commands. This technique is based on the construction of inverse dynamics as presented by Goszczyński et al. (1997). The basic idea of the approach is to transform a nonlinear system into a linear system via decoupling, and then use the well-known linear control techniques to construct controllers. The class of systems under consideration can be represented as

$$\dot{\boldsymbol{x}} = \mathsf{F}(\boldsymbol{x}) + \sum_{j=1}^{m} \mathsf{G}(\boldsymbol{x})u_j \qquad\qquad y_j(t) = h_j(\boldsymbol{x}) \tag{4.9}$$

where $\dim \boldsymbol{x} = n$, $\dim \boldsymbol{u} = \dim \boldsymbol{y} = m$.

Assume that our objective is to satisfy the following condition

$$\boldsymbol{y}(t) := \boldsymbol{y}_d(t) \tag{4.10}$$

where $\boldsymbol{y}_d(t)$ is the required trajectory. So, it is a tracking control problem. Its solution seems to be simple: each controlled output, y_i $(j = 1, ..., m)$, is differentiated until an input term with $\boldsymbol{u}$ appears. The system will be decoupled using nonlinear state variable feedback of the form

$$\boldsymbol{y}^{(r_j)} = \mathsf{N}(\boldsymbol{x}) + \mathsf{D}(\boldsymbol{x}) \tag{4.11}$$

where

$$\mathsf{N} = \mathsf{L}_F^{r_j} h_j(\boldsymbol{x})$$

$$\mathsf{D} = \begin{bmatrix} L_{G1}L_F^{r_1-1}h_1 & \dots & L_{Gm}L_F^{r_1-1}h_1 \\ \vdots & \vdots & \vdots \\ L_{G1}L_F^{r_m-1}h_m & \dots & L_{Gm}L_F^{r_m-1}h_m \end{bmatrix} \tag{4.12}$$

The symbols L with superscripts and subscripts denote the Lie derivative.

The matrix $\mathsf{D}(\boldsymbol{x})$ is called the decoupling matrix (Slotine and Li, 1991) for the system (4.9). If the decoupling matrix is non-singular, then the input transformation

$$\boldsymbol{u} = \mathsf{D}^{-1}(\boldsymbol{x})[\boldsymbol{v} - \mathsf{N}(\boldsymbol{x})] \tag{4.13}$$

yields a linear differential relation between the output $\boldsymbol{y}$ and a new input $\boldsymbol{v}$

$$\boldsymbol{y}^{(r_j)} = \boldsymbol{v} \tag{4.14}$$

Note that v_j affects only the corresponding output y_j, but not the others. Therefore, a control law of the form (4.13) is called *decoupling control law.* As a result of the decoupling, one can use single the input-output design to construct tracking controllers.

4.5. Illustrative example: wing rock control

4.5.1. Wing rock phenomenon

There is continuing interest in improving the performance of present and future fighters and missiles by increasing their high Angle of Attack (AoA) capability. However, serious lateral directional stability problems have been encountered at high AoA. The extension of an aeroplane performance envelope to enter the high AoA region to improve manoeuvrability often carries the penalty of an undesirable motion. A frequently encountered instability is the limit-cycle oscillation, wing rock, which is driven by strong, concentrated vortices originating from the leading edges of highly swept lifting surfaces. The wing rock is a concern for combat aircraft because it may have opposite effects on manoeuvrability, tracking accuracy, and operational safety. Typically, it occurs at a moderate to high AoA and involves mainly the roll degree of freedom (Fig.4).

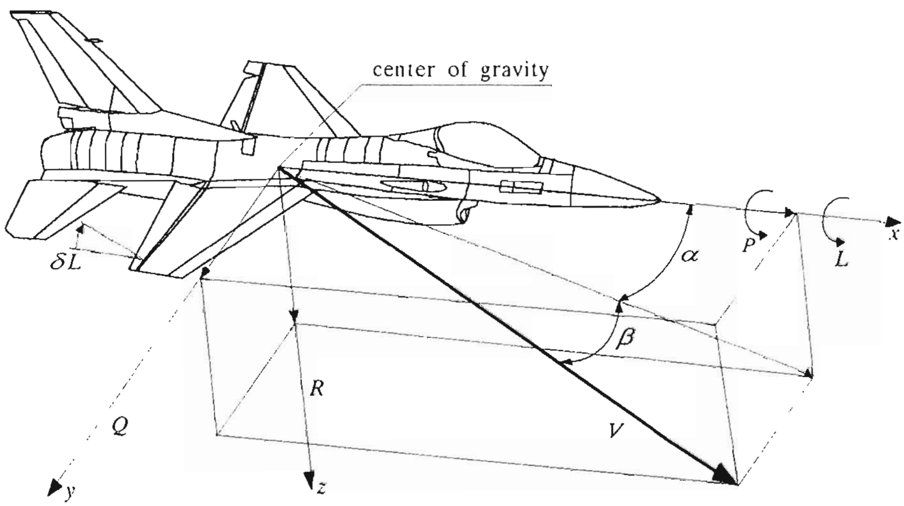

Fig. 4. Body fixed axes and the related angles

4.5.2. *Mathematical model of the wing rock phenomena*

In this study, we consider a nonlinear mathematical model of the wing rock based on the results (cf Pietrucha and Złocka, 1995). On the following assumptions (see also Fig.4): $\alpha = \text{const}$, $V \approx \text{const}$, $Q = 0$, $R = 0$, $h = \text{const}$

$$\dot{\beta} \approx \sin\alpha \tag{4.15}$$

we may obtain the following one degree of freedom differential equation for the WR motion

$$I_x \frac{d^2\phi}{dt^2} = L = \frac{1}{2}\rho V^2 SbC_l(t) \tag{4.16}$$

where

I_x – mass moment of inertia of the wing about the x-axis
ϕ – roll angle
L – aerodynamic roll-moment
ρ – density of air
S – wing area
b – characteristic length.

Basing on the test data (Arena and Nelson, 1994) the total rolling moment coefficient can be written as

$$C_l = l_1\phi + l_2p + l_3\phi^2p + l_4\phi p^2 + l_5p^3 \tag{4.17}$$

where

$$\dot{\phi} = p \qquad\qquad p = \frac{Pb}{V} \tag{4.18}$$

The values of the l_i $(i = 1, ..., 5)$ appearing in Eq (4.17) depend on the AoA. The quantity p in Eq (4.18) is a dimensionless reduced roll rate. The dot over ϕ implies the derivative with respect to the non-dimentional time

$$\tau = \frac{Vb}{t} \tag{4.19}$$

After substitution of Eq (4.17) into Eq (4.16), we have finally the WR equation

$$\ddot{\phi} = c_1\phi + c_2p + c_3\phi^2p + c_4\phi p^2 + c_5p^3 \tag{4.20}$$

where

$$c_i = \frac{1}{2}\rho\frac{Sb^3}{I_x}l_i \qquad\qquad i = 1, ..., 5$$

The roll angle and roll rates as a function of time are depicted in Fig.5.

4.5.3. Model of wing rock control

As can be seen from Fig.5, the problem of wing rock suppression becomes of a present interest. Therefore, the main objective of the example is modification of WR motion by active control: we apply the active control by means of ailerons that induce the control moment

$$L_{control} = q_\infty SbC_{l\delta_L}\delta_L \tag{4.21}$$

where $C_{l\delta_L}$ is the rolling moment derivative, and δ_L is the aileron deflection (see Fig.4).

Defining

$$x_1 = \phi \qquad\qquad x_2 = p \qquad\qquad u = \delta_L \tag{4.22}$$

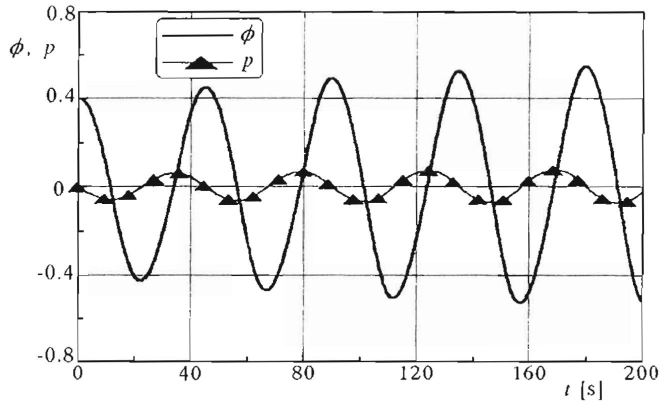

Fig. 5. Wing rock motion (histories of roll angle and roll rate)

we have the standard model of the control theory (see Eq (2.3))

$$\dot{\boldsymbol{x}} = \mathsf{A}\boldsymbol{x} + \boldsymbol{\vartheta}(\boldsymbol{x}) + \boldsymbol{b}u \tag{4.23}$$

where

$$\mathsf{A} = \begin{bmatrix} 0 & 1 \\ c_1 & c_2 \end{bmatrix} \qquad \boldsymbol{\vartheta} = \begin{bmatrix} 0 \\ c(\boldsymbol{x}) \end{bmatrix} \qquad \boldsymbol{b} = \begin{bmatrix} 0 \\ g \end{bmatrix} \tag{4.24}$$

and

$$\boldsymbol{x} = [\phi, p]^\top \qquad g = \frac{1}{2}\rho\frac{Sb^3}{I_x}C_{l\delta_L}$$

$$c(\boldsymbol{x}) = c(\phi, p) = c_3\phi^2 p + c_4\phi p^2 + c_5 p^3$$

4.5.4. Wing rock control using the Pontryagin Maximum Principle (PMP)

We take the explicit form of Eq (4.23)

$$\ddot{\phi} = c_1\phi + c_2\dot{\phi} + c_3\phi^2\dot{\phi} + c_4\phi\dot{\phi}^2 + c_5\dot{\phi}^3 + g\delta_L \tag{4.25}$$

The Hamiltonian (4.1) for Eq (4.25) with notations (4.22), and the weighting factors (2.5) reads

$$H = -(q_1\phi^2 + q_2\dot{\phi}^2 + r\delta_L^2) + \dot{\phi}\psi_1 + [c_1\phi + c_2\dot{\phi} + c(\phi, \dot{\phi}) + g\delta_L]\psi_2 \tag{4.26}$$

Thus, the condition (4.3) by virtue of Eq (4.23) with (2.5) gives the formula

$$u = \frac{1}{2r}\boldsymbol{b}^\top\boldsymbol{\psi} \tag{4.27}$$

An unknown function $\boldsymbol{\psi}$ can be determined from Eq (4.2), which for Eq (4.23) takes the form

$$\dot{\boldsymbol{\psi}} = \mathbf{Q}\boldsymbol{x} - \Big(\mathsf{A}^\top + \frac{\partial \boldsymbol{\vartheta}^\top}{\partial \boldsymbol{x}}\Big)\boldsymbol{\psi} \tag{4.28}$$

and from Eq (4.27) we have

$$\delta_L = \frac{1}{2r}\psi_2 \tag{4.29}$$

By virtue of Eq (4.28) with Eqs (4.24) and (4.29) we have

$$\begin{aligned} \dot{\psi}_1 &= -(c_1 + 2c_3\phi\dot{\phi} + c_4\dot{\phi}^2)\psi_2 + q_1\phi \\ \dot{\psi}_2 &= -\psi_1 - (c_2 + c_3\phi^2 + 2c_4\phi\dot{\phi} + 3c_5\dot{\phi}^2)\psi_2 + q_2\dot{\phi} \end{aligned} \tag{4.30}$$

Now, the point is to solve the set of nonlinear differential equations (4.25) and (4.30). To this end we use the Krylov-Bogoliubov (K-B) technique – this a method of harmonic averaging is well known in nonlinear mechanics (Bogoliubov and Mitropolski, 1958). The K-B approach splits the given equation into an in-phase part for the frequency, and an out-of-phase one for the amplitude.

To solve the nonlinear differential equations (4.25) and (4.30) using the K-B approach, it is assumed that

$$\phi = A\cos\theta \qquad \psi_i = A(\xi_i \sin\theta + \eta_i\cos\theta) \qquad i = 1, 2 \tag{4.31}$$

where A, θ, ξ_1, ξ_2, η_1 and η_2 are functions of time. Differentiating Eqs (4.31), introducing the following notations

$$\mu = \frac{\dot{A}}{A} \qquad \dot{\theta} = \omega \qquad (\cdot)' = \frac{d(\cdot)}{dA} \tag{4.32}$$

and assuming for the first approximation that

$$\mu\lambda' = (\omega^2)' = \xi_1' = \eta_1' = \xi_2' = \eta_2' = 0 \tag{4.33}$$

we obtain

$$\begin{aligned} &\dot{\phi} = A(\mu\cos\theta - \omega\sin\theta) \qquad \ddot{\phi} = A[(\lambda^2 - \omega^2)\cos\theta - 2\lambda\omega\sin\theta)] \\ &\dot{\psi}_i = A[(\lambda\xi_i - \omega\eta_i)\sin\theta + (\lambda\eta_i + \omega\xi_i)\cos\theta] \qquad i = 1, 2 \end{aligned} \tag{4.34}$$

Substituting Eqs (4.31) and (4.34) into Eqs (4.25) and (4.30), and assuming that the amplitude A and frequency ω do not vary greatly over one oscillation cycle, the harmonic averaging K-B technique allows one to obtain an implicit differential equation of the form

$$\boldsymbol{f}(\dot{\boldsymbol{y}}, \boldsymbol{y}) = \boldsymbol{0} \tag{4.35}$$

where $\boldsymbol{y} = [A, \omega]^\top$.

Therefore, the results can be summarised as follows: after specifying the coefficients in Eqs (4.17) and (2.5), Eq (4.35) should be integrated for $A(t)$ and $\theta(t)$. Then, the function $\psi_2(t)$ and in turn the control δ_L can be determined from Eq (4.29).

4.5.5. Wing rock control using the Lyapunov Function Method (LFM)

Assume that the Lyapunov function has the homogenous form

$$V = p_{11}\phi^2 + 2p_{12}\phi p + p_{22}p^2 + D_1\phi^4 + D_2\phi^3 p + D_3\phi^2 p^2 + D_4\phi^3 p + D_5 p^4 \tag{4.36}$$

where the coefficients $p_{11}, ...,$ $D_1, ...$ must be found. To this end, the form (4.36) should be substituted into Eq (4.7), which for Eq (4.23) with (4.24) yields

$$-\frac{g^2}{4r}\Big(\frac{\partial V}{\partial p}\Big)^2 + [c_1\phi + c_2 p + c(\phi, p)]\frac{\partial V}{\partial p} + p\frac{\partial V}{\partial \phi} + q_1\phi^2 + q_2 p^2 = 0 \tag{4.37}$$

Then, the resulting algebraic equations are obtained setting the sum of coefficients of similar terms equal to zero. After the Lyapunov function is determined, the control law (for Eq (4.23) with (2.5)) is

$$\delta_L = -\frac{g}{4r}\frac{\partial V}{\partial p} \tag{4.38}$$

or in the explicit form

$$\delta_L = -\frac{g}{4r}\Big(p_{12}\phi + p_{22}p + D_3\phi^2 p + \frac{3}{2}D_4\phi p^2 + 2D_5 p^3\Big) \tag{4.39}$$

The WR model is numerically solved for the delta wings data. The following values of weight coefficients (2.5) are chosen

$$q_1 = 1 \qquad\qquad q_2 = 1 \qquad\qquad r = 1 \tag{4.40}$$

The results of the presented WR model are shown in Fig.6 and Fig.7.

In order to compare our results with those obtained by Shue et al. (1996) in Fig.6 is depicted the control

$$w = g\delta_L \tag{4.41}$$

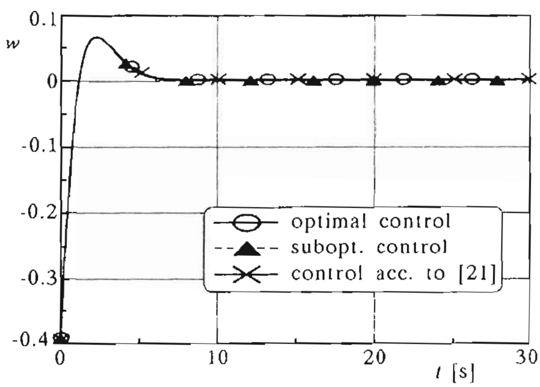

Fig. 6. Comparison between the three control laws

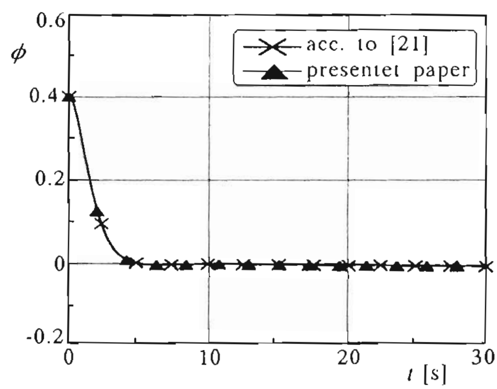

Fig. 7. Histories of roll angles after two different papers

4.5.6. Wing rock control using the Nonlinear Inverse Dynamics (NID)

The NID requires determining the output equation. We assume that the measured value will be the roll angle. So, the output equation takes the form

$$y = \phi \tag{4.42}$$

According to the methodology of NID we differentiale them twice $(d = 2)$. Thus, the equation necessary for determination of the vector $\boldsymbol{v}$ (see Slotine and Li, 1991) takes the form

$$\ddot{y} = P_1\dot{y} + P_0 y = 0 \tag{4.43}$$

as we assume that $y_z = 0$ (in order to compare with the LFM). The calculations were done (cf Goszczyński et al., 1997) for the two sets of data:

"stronger" control – $P_1 = 3 \quad P_0 = 2$
"weaker" control – $P_1 = 4 \quad P_0 = 1$

For Eq (4.43) the following initial conditions were assumed

$$\phi(0) = \phi_0 = 0.4 \qquad\qquad p(0) = 0$$

The results are depicted in Fig.8.

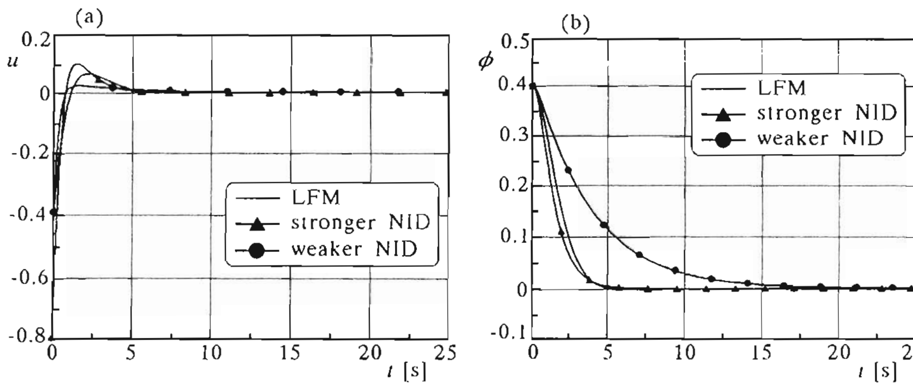

Fig. 8. (a) – Histories of the synthesised control; (b) – histories of the roll angle control (acc. to Goszczyński et al., 1997)

4.5.7. Concluding remarks

- The wing-rock model is numerically solved for the delta wings data. The results indicate that it is sufficient to use a linear feedback of the state variables to suppress the wing-rock motion. In this context, the Lyapunov technique is accurate in analysing the dynamic motion and determining the optimal control input.
- The Krylov-Bogoliubov harmonic averaging technique is rather complicated in implementation and leads to very complicated expressions, which are troublesome both to analytically investigate and calculate numerical. Nevertheless, it would be interesting to comparise between the results obtained by means of the Lyapunov Function Method and Pontryagin Maximum Principle.
- Although the Nonlinear Inverse Dynamics has been used successfully in solving some practical problems, our experiences in dealing with this approach are rather unsatisfactory because it does not give a rational base

to the construction of good procedure, particularly for more complex models of the wing rock (e.g., for three degrees of freedom).

5. Conclusions

- Modern design techniques have affected significantly the aircraft industry in recent years and the role of feedback control is becoming more and more important as it is a key to meet the performance objectives of new aircraft. The era of high-performance aircraft poses many new challenges to flight dynamics engineers, who must now think in terms of guidance and control. Many relatively new techniques are required, including robustness to parameter variations and adaptive techniques. Furthermore, the engineer can no longer work in isolation; many other specialists should be closely incorporated into a design.
- Aircraft stability, particularly for high-performance aircraft, may be extremely sensitive to the aircraft dynamics and flight conditions. *Robust stability* considers the characteristics of an aircraft model when subject to perturbations. Those perturbations might represent unmeasured forces acting on the aircraft or errors associated with the model. Some of the recent efforts on the robust control approach were presented by Ackermann (1993). Probably, the most rational method is to incorporate the flight data into the model development process.
- Modern aircraft have flight envelopes, which are so extensive that the changes in aircraft dynamics are too big to be handled by the control laws served in Sections 3 and 4. In such situations, the use of adaptive control is advocated as effectiveness of the adaptive control systems lies in its ability to rapidly assess the performance and to make required modifications in the control gains. Since the dynamic equations of these systems are nonlinear, the Nonlinear Inverse Dynamics approach might be applicable here.
- The ability to manipulate a flow field to effect a desired change using feedback we will name the Active Flow Control Technology (AFCT). The desired goals of external flow modifications are; e.g., drag reduction, separation/reattachment control, lift enhancement, transition delay/advancement, and noise control. These objectives are not necessarily mutually exclusive, either. If the boundary layer becomes turbulent, its resistance to separation is enhanced, and more lift could be obtained at

increased incidence. Because there are a number of experiments and numerical simultations that validate this approach, the AFCT may become a new modification methodology (see e.g., Pietrucha and Wojciechowski, 1997).

- The usual approach to the structure-control system synthesis consists in designing of the structure, with constraints imposed on weight, static and dynamic displacements and stresses, and natural frequencies, and then to design a control system for this structure. However, because of the strong interaction (synergetic effects) between the structure and control system, simmultaneous optimal design of both systems may be necessary in order to obtain optimum performance at minimum cost. Some of the past efforts on the integrated design approach given by Khot (1988). The modern investigations have shown that the relationship between the stability robustness and the optimum structural weight is nonlinear. Therefore, one can use the control techniques presented in Section 4.
- The performance index (2.9) or even (3.10), currently in use in control system design are far from being complete (Pietrucha, 1999). They consider only the dynamic performance of the close-loop system and do not account for numerous other factors that affect the control system design. Since there are a number of aspects to be considered, multiple (or vector) performance index might be suggested.
- The problem of modelling of aircraft manoeuvres is usually solved using the control approach. Blajer (1990) applied another approach to the problem: the requirements imposed on aircraft motion are treated as the program constraints on the aircraft. Consequently, the resultant motion of the aircraft is considered as a program motion of the controlled system. The point now is: what is the relationship between that approach and the general modification problem, posed in Section 2.3?
- Al Azab and Maryniak (1994) proposed a very interesting approach to the flight dynamics of a self-guided air-to-air missile: the set of control laws is presented in a form of the nonholonomic constraints. In this case the question is: is it possible to solve this problem using the Blajer method, or vice versa?

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Współczesne metody modyfikacji dynamicznego zachowania się statków powietrznych

Streszczenie

W pracy podjęto próbę przedstawienia kluczowych idei teorii modyfikacji. Chociaż teoria ta znajduje się jeszcze *in statu nascendi*, to już teraz dostarcza inżynierom lotnictwa bardzo wartościowego narzędzia projektowania, dzięki temu, że "obsługuje" rozmaite zagadnienia, które pojawiają się w bieżących badaniach. Praca niniejsza dotyczy zagadnienia poprawiania charakterystyk dynamicznych statków powietrznych za pomocą układów sterowania. Punktem wyjścia do prezentacji jest zagadnienie liniowo-kwadratowe. Następnie opisane są mało znane metody dla układów liniowych. Jądrem pracy są jednak metody dla układów nieliniowych, a mianowicie: Zasada Maksimum Pontriagina, Metoda Funkcji Lapunowa i Nieliniowa Dynamika Odwrotna. Niektóre metody zostały zilustrowane zagadnieniem ustateczniania samolotu niestatecznego oraz zagadnieniem sterownia ruchem "wing-rock".

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