JOURNAL OF THEORETICAL
AND APPLIED MECHANICS
1, 38, 2000

ESTIMATION BEFORE MODELLING AS THE METHOD FOR IDENTIFICATION OF THE AIRCRAFT AERODYNAMIC CHARACTERISTICS IN NONLINEAR FLIGHT REGIME

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The paper presents a successful application of the Estimation Before Modelling (EBM) methodology, known also as the two-step method, to a broad spectrum of the identification problems of aircraft aerodynamic characteristics at high angles of attack and sideslip. The paper presents some of the basic concepts and briefly surveys principal features of the method. From the results presented in the paper it can be concluded that the EBM approach have reached such a level of maturity that it is an integral part of any aircraft development and assessment program.

Key words: flight dynamics, parameters identification, aerodynamic characteristics

1. Introduction

Modern combat aircraft in order to survive on the contemporary field of battle are expected to have the ability to fly in an efficient manner, taking advantage of nonlinear aerodynamic effects. An engineer is supposed to design an object that allows performing the manoeuvres like cobra, kulbit or Herbst manoeuvre (Fig.1), which are realised at high angles of attack and sideslip (Batterson and Klein, 1989; Hamel and Jategaonkar, 1996; Sri-Jayantha and Stengel, 1988; Stalford, 1979; Stalford et al., 1977). The flight control systems of such an aircraft have to make use of the on-line information on its aerodynamic characteristics. Only in such a case the co-operation between those systems and, e.g., the vectored-thrust control is possible. Using some empirical and theoretical applications of measurement results processing technologies enables one to employ on-line recorded (real-time) flight data in identification of real, highly nonlinear aerodynamic characteristics of an aircraft (Goszczyński, 1998; Klein, 1989; Manerowski, 1990).

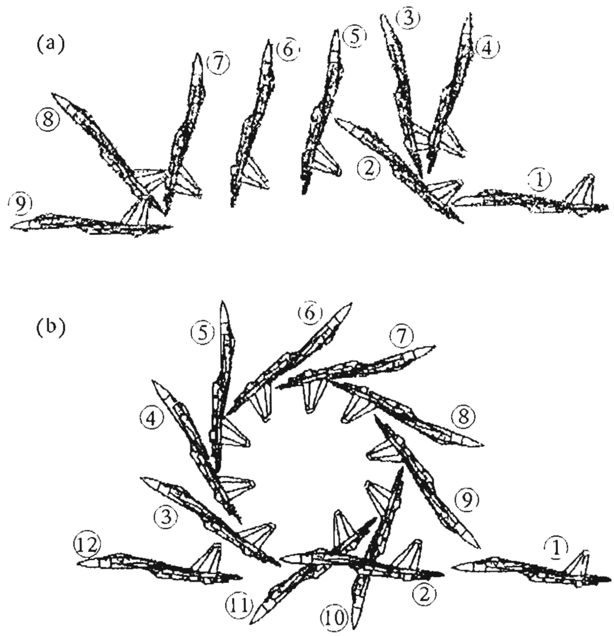

Fig. 1. Cobra (a) and kulbit (b) manoeuvres of the SU-37 fighter

To avoid long and expensive seeking for optimal solutions, one should have an appropriate mathematical model at one's disposal. The procedure for construction of the mathematical model, which represents a technical object operation (like flying aircraft), making use of the results from experimental investigations of a real object (or its scale model), is termed *identification*. The purpose of identification is to obtain the mathematical descriptions that properly represent the real aircraft properties (Manerowski, 1990). In effect, if we know a priori a certain mathematical model of the flying aircraft, the problem consists in improvement of its accuracy by appropriate using of research experimental or results.

In spite of enormous literature about dynamic systems identification, still gaps in the field of aerodynamic characteristics identification exist. One of the most interesting techniques of carrying out the aerodynamic characteristics identification is a technique founded on the Estimation-Before-Modelling (EBM) (Hoff and Cook, 1996; Sri-Jayantha and Stengel, 1988; Stalford, 1979; Stalford et al., 1977). This term means the estimation of the aircraft states before modelling of the aerodynamic coefficients.

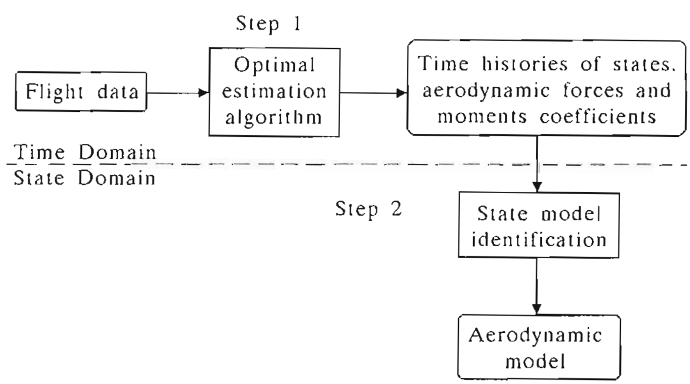

Fig. 2. Overview of the EBM method (Stalford et al., 1977)

Thus, the EBM technique is a two-step approach to the identification (Fig.2). First, the estimation is made using state and measurement (output) equations, measurement and their stochastic characteristics, and noise stochastic characteristics. In this step, time courses of the forces and moments are estimated using stochastic filtering and smoothing techniques. In the second step, the aerodynamic coefficients of the aircraft model are identified. One should pay attention to the mentioned above expansion for not to be surprised that the aircraft model appears before estimation step. Against other identification methods particularly the following properties stand out:

- Estimation in the time domain provides better data for the modelling phase

- No a priori knowledge about a state-dependent model (see Fig.2) need to be selected to perform the state estimation and identification of aerodynamic derivatives

- Multiple manoeuvres can be processed simultaneously to identify aerodynamic derivatives more accurately

- Global state-dependent modelling region can be subdivided into small regions (subspaces). The identification of aerodynamic derivatives can be performed within each subspace separately (Batterson and Klein, 1989; Stalford, 1979; Stalford et al., 1977). It has a considerable impact on carrying out identification within a wide range of variables affecting the values of parameters.

The EBM research conducted in 1978-79 in the USA by Sri-Jayantha and Stengel (1988), Stalford (1979), Stalford et al. (1977) yielded very good results. Especially in view of hardly measurable from the wind tunnel (or when using other identification techniques) the so-called dynamic derivatives, i.e. the dynamic components of aerodynamic forces and moments, e.g., C_{np} and C_{nr} (Fig.3).

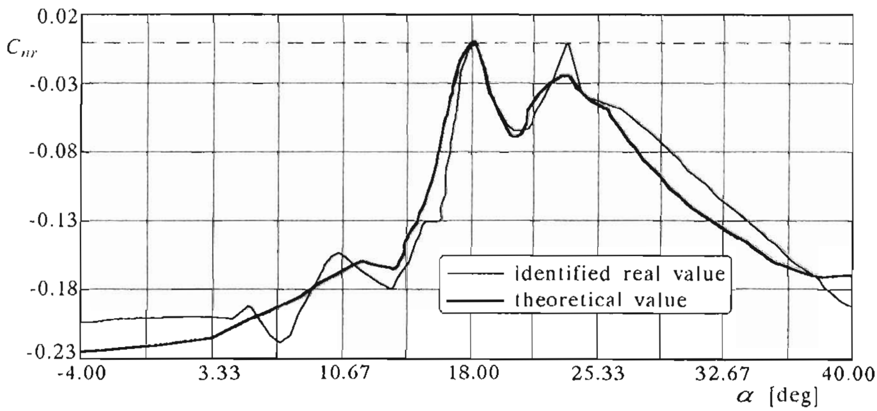

Fig. 3. Comparison of the identified value of derivative C_{nr} with the predicted values determined theoretically (after Stalford, 1979)

2. Mathematical model of aircraft motion

An aircraft is treated as a 6DoFs rigid body with movable, weightless control surfaces. Consider the aircraft in the standard configuration. Mathematical model is presented in body-fixed system. Within the framework of

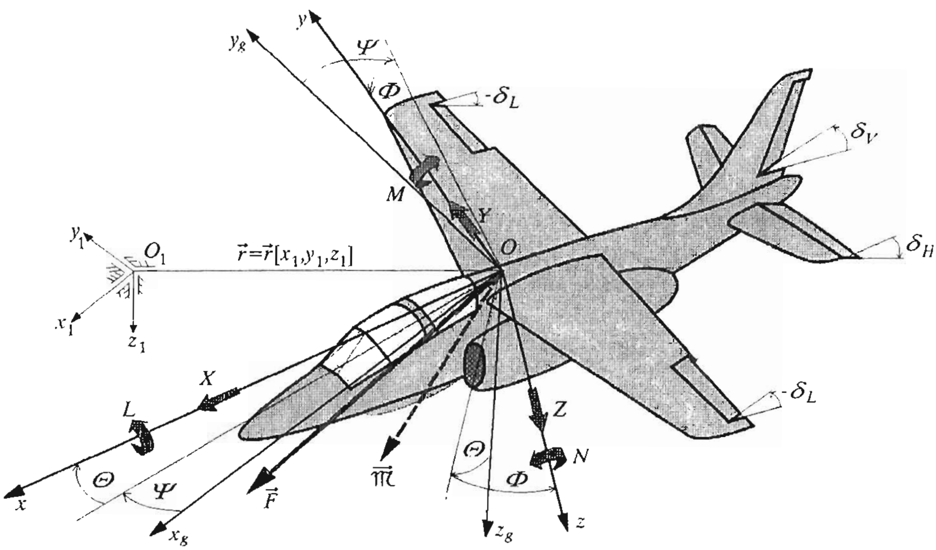

Fig. 4. Aircraft forces and its moments with control surfaces deflections during a flight

theoretical mechanics one can obtain the following equations of motion (Goszczyński, 1998; Manerowski, 1990; Maryniak, 1985; Sibilski, 1998)

$$\dot{\boldsymbol{x}}_d = \mathbf{B}^{-1}(\mathbf{V}_\omega \mathbf{B} \boldsymbol{x}_d + \boldsymbol{F}_M) \tag{2.1}$$

where

- $\boldsymbol{x}_d$ – dynamic part of the state vector, $\boldsymbol{x}_d = [U, V, W, P, Q, R]^\top$
- $\mathbf{B}$ – matrix of inertia
- $\mathbf{V}_\omega$ – matrix of linear and angular velocities
- $\boldsymbol{F}_M$ – vector of external forces and moments (Fig.4)

$$\boldsymbol{F}_M = \begin{bmatrix} \boldsymbol{F} \\ \mathfrak{m} \end{bmatrix} = [X, Y, Z, L, M, N]^\top$$

and

$$\dot{\boldsymbol{x}}_k = \mathbf{T}(\boldsymbol{x}_k)\boldsymbol{x}_d \tag{2.2}$$

where

- $\mathbf{T}$ – transformation matrix from the body-fixed system to inertial system, respectively
- $\boldsymbol{x}_k$ – kinematics constrains, $\boldsymbol{x}_k = [\Phi, \Theta, \Psi, x_1, y_1, z_1]^\top$.

The vector $\boldsymbol{F}_M$ is represented as a sum of gravity, thrust and aerodynamic forces and moments

$$\boldsymbol{F}_M = \boldsymbol{F}_M^G + \boldsymbol{F}_M^T + \boldsymbol{F}_M^A \tag{2.3}$$

Assume that the gravity and thrust forces and moments are known, and the aerodynamic forces and moments

$$\boldsymbol{F}_M^A = [P_x^a, P_y^a, P_z^a, M_x^a, M_y^a, M_z^a]^\top \tag{2.4}$$

are estimated from the recorded digital flight data by means of filtering and smoothing techniques.

The aerodynamic forces can be represented as follows

$$\begin{aligned} P_{(*)}^a &= \frac{1}{2}\rho V_0^* S C_{(*)}(\alpha, \beta, \mathrm{Ma}, \mathrm{Re}) \\ M_{(*)}^a &= \frac{1}{2}\rho V_0^* S l C_{(*)}(\alpha, \beta, \mathrm{Ma}, \mathrm{Re}) \end{aligned} \tag{2.5}$$

where

$P_{(*)}^a$ – aerodynamic force
$M_{(*)}^a$ – aerodynamic moment
$C_{(*)}$ – dimensionless aerodynamic coefficient
$(*)$ – co-ordinates of forces and their moments
ρ – air density
V_0 – aircraft velocity
S – reference area
l – characteristic dimension
α – angle of attack
β – sideslip angle
Ma – Mach number
Re – Reynolds number.

Occurring in Eqs (2.5) the dimensionless aerodynamic coefficients of drag, side and lift forces, roll, pitch and yaw moments are determined for an aircraft at the design stage, from the wind tunnel data and engineering calculations (Goszczyński et al., 1998; Goszczyński, 1998).

The dimensionless aerodynamic coefficients, taking into account the terms coming from control inputs and changes of linear and angular velocities, can be expanded in the Taylor series, extracting the so-called static and dynamic components

$$C_{(*)} = C_{(*)STAT} + C_{(*)DYN} \tag{2.6}$$

E.g. for the dimensionless aerodynamic coefficient of side force one obtains (Goszczyński, 1998)

$$
\begin{aligned}
C_{y_{STAT}} &= C_{y_\beta}(\alpha,\beta,\mathrm{Ma})(\beta-\beta_0) + C_{y_{\delta_a}}(\mathrm{Ma})\delta_a + C_{y_{\delta_r}}(\mathrm{Ma})\delta_r \\
C_{y_{DYN}} &= C_{y_{P\alpha}}(\mathrm{Ma})P\alpha + C_{y_R}(\mathrm{Ma})R + C_{y_{\dot\beta}}(\mathrm{Ma})\dot\beta
\end{aligned}
\tag{2.7}
$$

where the aerodynamic derivatives read

$$
\begin{array}{lll}
C_{y_\beta} = \dfrac{\partial C_y}{\partial \beta} & C_{y_{\delta_a}} = \dfrac{\partial C_y}{\partial \delta_a} & C_{y_{\delta_r}} = \dfrac{\partial C_y}{\partial \delta_r} \\
C_{y_{P\alpha}} = \dfrac{\partial\left(\frac{\partial C_y}{\partial P}\right)}{\partial \alpha} & C_{y_R} = \dfrac{\partial C_y}{\partial R} & C_{y_{\dot\beta}} = \dfrac{\partial C_y}{\partial \dot\beta}
\end{array}
$$

Assuming a polynomial interpolation, one can represent particular derivatives, e.g. those of the coefficient of side force in the following manner

$$
C_{y_{index}} \equiv Y_k = \sum_{i=0}^{3}\sum_{j=0}^{3} p_{ij}\alpha^i \mathrm{Ma}^j \qquad index := \beta, \delta_a, ..., \dot\beta \qquad k = 1,...,6
\tag{2.8}
$$

the unknown coefficients p_{ij} are to be identified.

Finally, the total aerodynamic side force can be written down accordingly to (2.5) $\div$ (2.8) (Goszczyński, 1998)

$$
P_y^a = \frac{1}{2}\rho V_0^2 S(Y_1\beta + Y_2\delta_a + Y_3\delta_r + Y_4\alpha P + Y_5 R + Y_6\dot\beta)
\tag{2.9}
$$

3. EBM characteristics

3.1. Estimation

The first step of the method is state estimation. The estimation is conducted by means of spline or filtering techniques. The latter one is recognised to be more efficient. Usually, an extended Kalman filter is applied (Manerowski, 1990) that requires a stochastic model of aircraft motion.

When compared to the model described in Section 2 this one have to include stochastic noise (Fig.5)

$$
\dot{\boldsymbol{x}} = f\Big(\boldsymbol{x}(t), \boldsymbol{u}(t), \boldsymbol{p}(t)\Big) + \boldsymbol{w}(t)
\tag{3.1}
$$

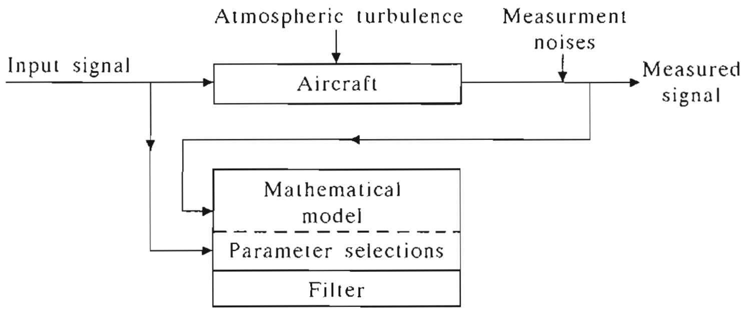

Fig. 5. Identification by means of the EBM (Goszczyński et al., 1998)

where

$\boldsymbol{x}$ – state vector

$\boldsymbol{u}$ – control vector

$\boldsymbol{w}$ – state noise vector.

The estimation is conducted on the basis of measurement

$$\boldsymbol{y} = h\Big(\boldsymbol{x}(t), \boldsymbol{u}(t), \boldsymbol{p}(t)\Big) + \boldsymbol{v}(t) \tag{3.2}$$

where

$\boldsymbol{y}$ – output vector

$\boldsymbol{p}$ – vector of unknown parameters

$\boldsymbol{v}$ – measurement noise vector.

The vectors $\boldsymbol{w}$ and $\boldsymbol{v}$ are Gaussian white noises with the following covariance matrices

$$\begin{aligned} E\Big\{\boldsymbol{w}(t)\boldsymbol{v}^\top(\tau)\Big\} &= \mathbf{0} \\ E\Big\{\boldsymbol{w}(t)\boldsymbol{w}^\top(\tau)\Big\} &= \mathbf{Q}\delta(t-\tau) \\ E\Big\{\boldsymbol{v}(t)\boldsymbol{v}^\top(\tau)\Big\} &= \mathbf{R}\delta(t-\tau) \end{aligned} \tag{3.3}$$

where $\mathbf{Q}$ and $\mathbf{R}$ are constant matrices and δ is the Dirac function.

Application of extended Kalman filter to the state (3.1) and output (3.2) equations, respectively, allows obtaining optimal state estimates. Before starting computations estimates of the initial values of the state variables are required. Initial values for the measured variables are obtained from the trim flight preceding each manoeuvre, while those for notmeasured variables have to be specified.

Because the extended Kalman filter state estimator introduces bias, usually a linear data-smoothing filter is used. A commonly used smoother is the modified Bryson-Frazier filter. Alternatively, a fixed lag filter can be used, which can be simpler and less computationally demanding. Its main advantage is that it allows for simultaneous data smoothing and state derivative estimation, and thus avoids the need for a separate differentiation algorithm. An important feature is that the noise characteristics are determined before the smoothing process, while a disadvantage consists in its inability to reduce the effects of bias.

An important question is the way of considering of measurement process noises and measurement errors. The covariance matrices are assumed to be diagonal. Evaluation of their elements is an iterative process, referred to as tuning, in which the object is to the residual sequences minimise and to ensure convergence of the filter gain and covariance matrices. An attempt is made to choose the initial values of noise, which are consistent with the observed disturbances in the recorded time courses. As a hint, the initial estimates of process noise are 10 to 100 times larger than the corresponding measurement noise estimates. Too large initial values of the noise cause the extended Kalman filter to output state estimates, which are the same as the measured variables.

Stochastic aerodynamic modelling plays a crucial role in the EBM method. The aerodynamic equations, in order to be used in the Kalman filtering have to be transformed into the state equation

$$\dot{\boldsymbol{x}}_i(t) = \mathsf{K}_i(t)\boldsymbol{x}_i(t) + \mathsf{G}_i\boldsymbol{\zeta}(t)\Big) \qquad \boldsymbol{x}_i(0) = \boldsymbol{x}_{i0} \qquad i = 1, ..., 6 \tag{3.4}$$

where

- ζ_i – white noise (Gaussian)
- G_i – input matrix
- $\boldsymbol{x}_i$ – state vector
- K_i – state matrix

$$\mathsf{K}_i = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix} \tag{3.5}$$

The state estimated in the first step becomes the input in the next step of the method.

3.2. Modelling

In order to use efficiently the linear regression with constant coefficients when analysing the data measured at aircraft manoeuvres performed in a broad range of parameter changes, it is necessary to divide the domain of variation of aerodynamic coefficients to little variation sub-spaces, to reduce of

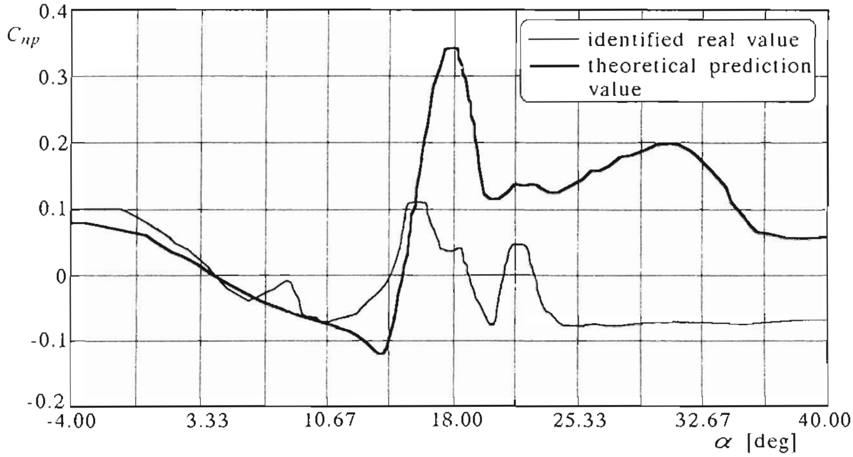

Fig. 6. Identified value of the derivative C_{np} versus the angle of attack (after Stalford, 1979)

the problem dimension. This allows treating coefficients as constants within each sub-space. The value of an argument within a sub-space is constant and equal to its average value a given sub-space. In this way, the area in which identification should be done is split to a number of sub-spaces. The identification is performed in each of these sub-spaces independently. Stalford (cf Stalford, 1979; Stalford et al., 1977) used this approach – he investigated variations of aerodynamic coefficients with respect to the angle of attack (Fig.6). This requires division of the identified characteristics into small pieces. If the division is not dense enough with respect to the independent variable, we cannot assume that the coefficient is constant within a sub-space. Stalford, investigating variation of aerodynamic coefficients with respect to the angle of sideslip, used the spline functions, e.g., a non-dimensional aerodynamic coefficient of the sideslip force it can be presented as follows (Stalford, 1979)

$$C_{y_\beta}(\alpha,\beta) = C_{y_{\beta_0}}(\alpha) + \sum_{i=1}^{N} C_{y_{\beta_i^3}}(\alpha)\frac{\partial f}{\partial \beta}(\beta,\beta_i,\beta_{i+1}) \tag{3.6}$$

where

$$\frac{\partial f}{\partial \beta}(\beta,\beta_i,\beta_{i+1}) = \begin{cases} 0 & 0 \leqslant \beta \leqslant \beta_i \\ (\beta-\beta_i)^2 & \beta_i \leqslant \beta \leqslant \beta_{i+1} \\ (\beta_{i+1}-\beta_i)^2 & \beta_{i+1} \leqslant \beta \end{cases}$$

$$\frac{\partial f}{\partial \beta}(-\beta, \beta_i, \beta_{i+1}) = \frac{\partial f}{\partial \beta}(\beta, \beta_i, \beta_{i+1}) \qquad \beta > 0$$

From the analysis of available data there may result a necessity for division into the aforementional sub-spaces and the way of doing this division is very important. Unless one is certain about this necessity, we propose to use a general, more complete solution. In terms of polynomials is presented in Section 2. It is however more complicated from the calculation point of view. Since the proposed way requires identification of a number of polynomial coefficients for every aerodynamic coefficient. However, it gives a chance to receive one model of aerodynamic forces and their moments (so-called aerodynamic model) for a broad range of the angles of attack and sideslip (Fig.7).

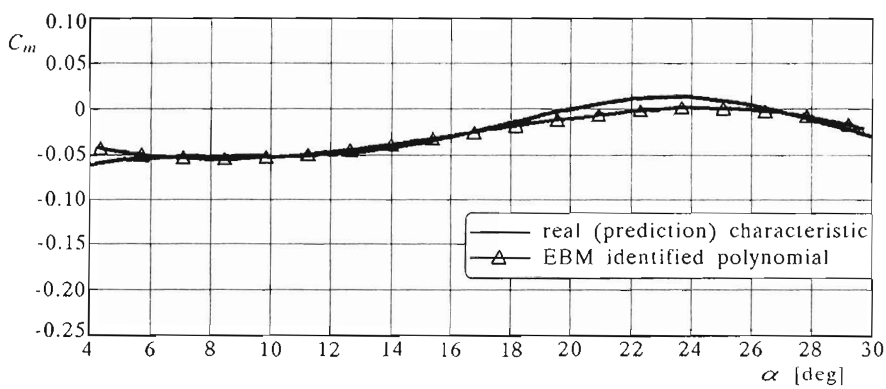

Fig. 7. Identified non-dimensional aerodynamic coefficient of the pitch moment C_m versus the angle of attack (Goszczyński, 1998)

Whatever way from the two listed above we choose, obtained the aerodynamic model – containing unknown coefficients – can be solved by the linear regression technique. This model – using N measurements for the determination of n parameters $(N \geqslant n)$ – is the simplest type of parametric model in the form

$$\boldsymbol{y}_i = \mathsf{X}_i \boldsymbol{p}_i + \boldsymbol{e}_i \qquad i = 1, ..., 6 \tag{3.7}$$

where

- $\boldsymbol{y}_i$ – vector of aerodynamic force or moment of N order
- $\boldsymbol{X}_i$ – matrix of independent variables of $N \times n$ order
- $\boldsymbol{p}_i$ – vector of unknown parameters of n order
- $\boldsymbol{e}_i$ – error vector of N order.

On the basis of the least squares method we get the formula

$$\widehat{\boldsymbol{p}}_i = (\mathsf{X}_i^\top \mathsf{X}_i)^{-1} \mathsf{X}_i^\top \boldsymbol{y}_i \tag{3.8}$$

presenting the result of identification in the explicit form (Fig.8) where $\widehat{\boldsymbol{p}}_i$ are the final estimation of unknown parameters.

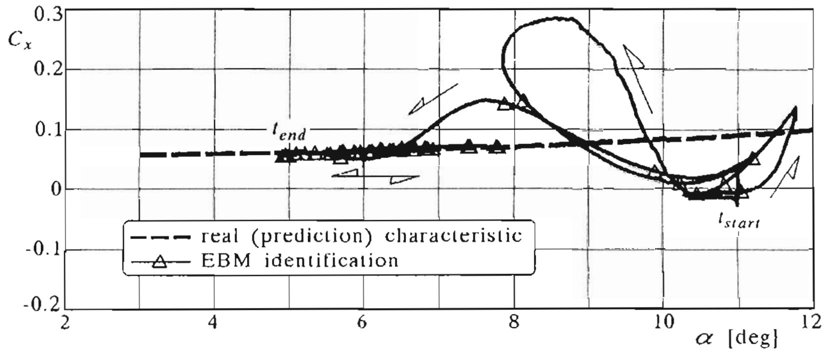

Fig. 8. Identification of non-dimensional aerodynamic coefficient of drag C_x versus angle of attack (Goszczyński, 1998)

4. Final remarks

Information from the literature and experience of the authors show that:

- Unlike other methods, the EBM is two-step, i.e., estimation in time domain is done before modelling in state domain that broadens the range of using the method.
- The EBM technique provides the model of aircraft aerodynamics on the basis of available data.
- The use of EBM enables one to establish requirements for measurement accuracy of essential flight parameters.

Practical implementation of identification technique with the use of EBM requires the determination of particular stages of algorithms testing. The first stage consists in numerical simulations of the aircraft flight (using available aerodynamic characteristics obtained in wind tunnel, etc.), then the results are disturbed by white noise and yields the stock to EBM. As a result of identification we can define the accuracy of the implemented algorithm comparing the input data with result of identification.

The algorithms have already been prepared to apply to the PZL I-22 aircraft identification. After numerical tests are completed, which should enable to correct software, these algorithms will be used in identification of characteristics of I-22 basing on the real measurement data registered in experimental flights (Goszczyński, 1998).

Practical implementation of the EBM technique can become very usefull tool in computer-aid of flight-path reconstruction, e.a., in investigating plane crashes or the so-called Objective Flight Control in pilot education and training programs (Sibilski, 1998).

Acknowledgement

The research was partially supported by the State Committee for Scientific Research under grant No. 0T00A 036 17.

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Estymacja przed modelowaniem jako metoda identyfikacji charakterystyk aerodynamicznych samolotu w nieliniowych stanach lotu

Streszczenie

W niniejszej pracy przedstawiono metodę estymacji przed modelowaniem (EBM), znaną również pod nazwą metody dwu etapowej identyfikacji charakterystyk aerodynamicznych (i ich pochodnych). Przedstawiona technika jest szczególnie przydatna do identyfikacji charakterystyk samolotu poruszającego się na dużych kątach natarcia i ślizgu. W pracy przedstawiono podstawowe cechy i zależności metody.

Uzyskane wyniki, wraz z posiadaną wiedzą o zakończonych badaniach innych zespołów, pozwalają określić przedstawioną technikę jako potencjalnie integralną część badań rozwojowych i oceny każdego samolotu.

Manuscript received August 27, 1999; accepted for print September 28, 1999