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MODELLING OF CRACK GROWTH INITIATION IN A LINEAR VISCOELASTIC MATERIAL

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Widespread application of viscoelastic materials the structures, which have to operate for a long time, requires better understanding of their mechanical behaviour and fracture properties. It has been observed that time dependence is of great importance in determining the rate of crack growth. The effects of viscoelastic characteristics on creep crack growth initiation are studied in the paper by using the finite element method. In order to define displacements and stresses around the crack tip in linear isotropic viscoelastic media new formulation in the time domain has been produced. We formulate a new constitutive equation in terms of the stress and crack opening intensity factors, using the correspondence principle by means of Volterra integrals. Afterwards, the fracture parameters are computed with an incremental viscoelastic formulation.

Key words: viscoelasticity, fracture mechanic, crack growth initiation

1. Introduction

When the linear viscoelasticity theory is applied to the analysis of fracture characteristics of a cracked body, it is important to predict the critical time which expresses the crack growth initiation. Though for a creep loading from the correspondence principle (Schapery, 1984) it follows that stresses remain constant in time, the experimental results indicate that cracks in viscoelastic materials do grow, even under constant loads well beneath the elastic stress intensity factors. The effect is due to the existence of a failure zone (Schapery, 1975) where the crack tip can propagate. The failure zone size is proportional to the stress intensity factor level which reveals a local highly non-linear behaviour. If we consider the amount of energy dissipated in the crack growth process, the crack tip speed depends on the viscoelastic characteristics and the stress intensity factor value. In order to study the crack initiation phenomena, several authors have used a far field solution by means of the J-integral type approach. Though this technique is effective in critical time evaluation, mechanical fields in the crack tip vicinity cannot be determined precisely in the time domain.Therefore, we should introduce the energy release rate into our considerations for the amount of energy dissipated in the crack tip propagation to be known. We cannot, however, predict the failure zone size and the crack growth speed. In order to develop the existing numerical techniques, this paper deals with a new incremental fracture law which enables us to evaluate the energy release rate and mechanical fields around the crack tip in the time domain. The concepts of Schapery and Brincker are used to apply the finite element method for viscoelastic fracture to the determination of the conditions for crack growth initiation and local mechanical fields which are defined by the stress and crack opening intensity factors.

First, a review of the Brincker formulation is presented. The stress and strain intensity factors enable us to define the singularities of stress and strain fields, respectively. In order to realise the coupling between global energy release rate and local mechanical fields, we introduce new crack opening intensity factors which represent the crack lips state and allow for evaluation of the amount of energy dissipated in the crack growth process.

In Section 2, we recall the linear viscoelastic incremental formulation by means the finite element method (Ghazlan et al., 1995a,b). The coupling with the local fracture characteristics is proposed by using a spectral decomposition of the reduced viscoelastic function. Coupled with the J-integral computation, this formulation allows for determination, in the time domain, of the energy release rate, stress and crack opening intensity factors courses.

Finally, the last part presents a numerical application which demonstrates the validity of the formulation. Basing on the creep crack growth initiation, numerical results are compared with the analytic ones.

2. Crack tip parameters

2.1. Local mechanical fields

According to the linear viscoelastic theory, Brincker (1992) showed that, for isotropic materials, mechanical fields in the crack tip vicinity can be defined by two stress intensity factors $K_\gamma^{(\sigma)}$ and four strain intensity factors C_β and D_β, $((\beta,\gamma) \in [1,2]^2)$. Having the polar co-ordinates (r,θ) with the origin at the crack tip the stress and displacement fields are defined as

$$\begin{aligned} \sigma_{\alpha\beta}(r,\theta,t) &= K_\gamma^{(\sigma)}(\mu,k,t)\frac{f_{\alpha\beta\gamma}(\theta)}{\sqrt{2\pi r}} \\ u_\alpha(r,\theta,t) &= \Big[g_{\alpha\beta}(\theta)C_\beta(\mu,k,t) + h_{\alpha\beta}(\theta)D_\beta(\mu,k,t)\Big]\sqrt{\frac{r}{2\pi}} \end{aligned} \tag{2.1}$$

where k denotes compression and μ stands for the shear modulus. $f_{\alpha\beta\gamma}$, $g_{\alpha\beta}$ and $h_{\alpha\beta}$ are well known angular functions. According to the correspondence principle, the relationships, between stress and strain intensity factors, can be defined by the Boltzmann form

$$\begin{aligned} C_\beta(t) &= \int\limits_{0^-}^{t} \frac{1}{2\mu(t-\tau)}\frac{\partial K_\beta^{(\sigma)}}{\partial\tau}\,d\tau \\ D_\beta(t) &= \int\limits_{0^-}^{t} \frac{\lambda(t-\tau)}{2\mu(t-\tau)}\frac{\partial K_\beta^{(\sigma)}}{\partial\tau}\,d\tau \end{aligned} \tag{2.2}$$

If ν designates the Poisson ratio, λ is defined as

$$\lambda = \begin{cases} 3-4\nu & \text{for plane strains} \\ (3-\nu)/(1+\nu) & \text{for plane stresses} \end{cases}$$

2.2. Crack growth initiation criteria

The theory of Schapery takes into account the existence of a failure zone in the crack tip vicinity (Schapery, 1975). Consider the following two integrals (see Fig.1)

$$\begin{aligned} J_v &= \int\limits_{C_1} (\Phi dx_2 - \sigma_{ij}n_j u_{i,1})\,dS \\ J_f &= \int\limits_{C_2} (\Phi dx_2 - \sigma_{ij}n_j u_{i,1})\,dS \end{aligned} \tag{2.3}$$

where Φ denotes the pseudo-strain energy density and will be detailed in the next section. By virtue of the properties of line integral invariance and taking

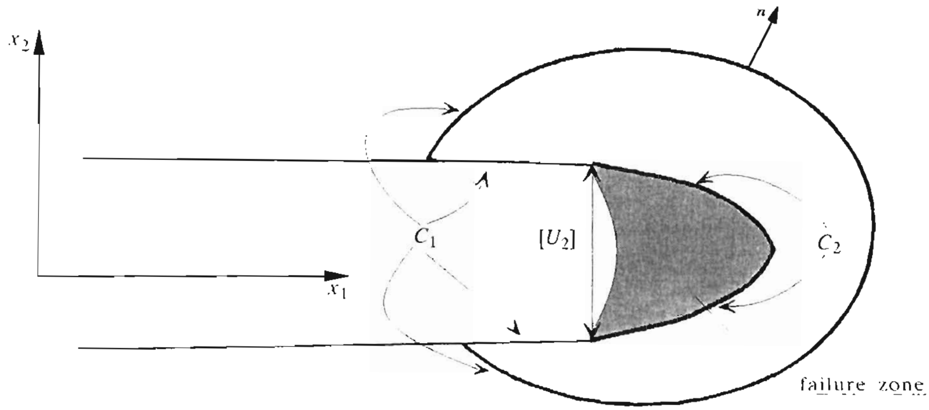

Fig. 1. Integration domain and failure zone

into account that fact that we integrate along the paths C_1 and C_2, we have (Schapery, 1984)

$$J_v = J_f \tag{2.4}$$

If σ_m represents the stress distribution in the failure zone for a crack opening mode, J_f is defined as the work W_f required to create the crack extension in this zone. If $[U_2]$ designates the crack opening displacement (see Fig.1) W_f can be evaluated as follows

$$J_f = \sigma_m[U_2] = W_f \tag{2.5}$$

Eq (2.4) is very important because it allows for direct determination of W_f with no necessity for forming any hypothesis on the size and behaviour of the failure zone. Therefore, J_v can be calculated by means of a line integral where the material is considered as linear and viscoelastic. So, at the time t_i, considering Eqs (2.4) and (2.5), the crack growth initiation is determined by

$$J_v(t_i) = W_f \tag{2.6}$$

2.3. Crack opening intensity

We can also evaluate J_v by using the well-known formula (Kanninen and Popelar, 1985)

$$J_v(t) = -\frac{\partial \Phi(t)}{\partial a} = \lim_{\Delta a \to 0} -\frac{\Delta \Phi}{\Delta a} \tag{2.7}$$

J_v is the amount of energy dissipated at an unit step of crack propagation and J_v denotes, in other terms, the energy release rate. Then, Φ stands for the free energy which is not dissipated in the material. Then, according to e.g. Blackburn (1972), Φ can be defined by

$$\Phi = \frac{1}{2}\sigma_{\alpha\beta}\varepsilon_{\alpha\beta} \tag{2.8}$$

In order to obtain a relationship between the energy release rate and singular fields around the crack tip, we rewrite Eq (2.7) as follows

$$\Delta\Phi = \frac{1}{2}\int\limits_0^{\Delta a} \sigma_{\alpha 2}(\xi,t)[U_\alpha](\xi',t)\, d\xi' \tag{2.9}$$

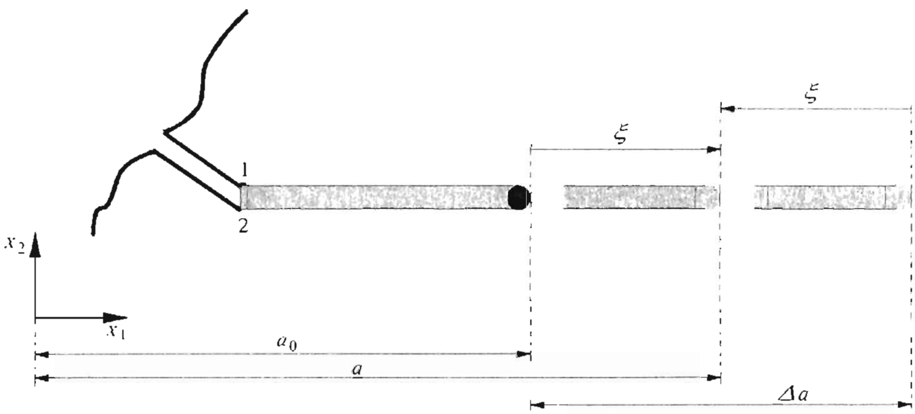

Fig. 2. Virtual crack extension in opening mode

Different notation is used in Fig.2. According to Dubois et al. (1996), the most common way to evaluate relative displacements of crack lips is to define crack opening intensity factors $K_\beta^{(\varepsilon)}$

$$[U_1](\xi',t) = K_2^{(\varepsilon)}\sqrt{\frac{\xi'}{2\pi}} \qquad [U_2](\xi',t) = K_1^{(\varepsilon)}\sqrt{\frac{\xi'}{2\pi}} \tag{2.10}$$

where ξ' is the crack lips length between the crack tip and the considered point. By virtue of Eqs (2.1) in (2.10) the relationship between the crack opening and strain intensity factors is given by

$$K_\beta^{(\varepsilon)}(t) = 2[C_\beta(t) + D_\beta(t)] \tag{2.11}$$

Substituting Eq (2.11) into Eqs (2.2), we obtain the following integral

$$K_\beta^{(\varepsilon)}(t) = \int_{0^-}^{t} C(t-\tau)\frac{\partial K_\beta^{(\sigma)}}{\partial \tau}\,d\tau \tag{2.12}$$

where $C(t)$ is the reduced viscoelastic compliance introduced by Schapery (1975)

$$C(t) = \frac{1+\lambda(t)}{\mu(t)} \tag{2.13}$$

To simplify integration of Eq (2.9), we introduce the opening crack growth Δa. Upon the definition of crack opening intensity factors (Eqs (2.11)), the dissipated energy $\Delta\Phi$ is

$$\Delta\Phi = -\frac{K_1^{(\sigma)}(t)K_1^{(\varepsilon)}(t)}{8}\Delta a \tag{2.14}$$

However, by employing the superposition principle we can generalize this approach to cover a mixed mode problem. Then, by virtue of Eqs (2.14) and (2.7) we can write

$$J_v(t) = \frac{K_1^{(\sigma)}(t)K_1^{(\varepsilon)}(t)}{8} + \frac{K_2^{(\sigma)}(t)K_2^{(\varepsilon)}(t)}{8} \tag{2.15}$$

3. Numerical procedure

The energy release rate, stress and crack opening intensity factors courses in the time domain have been implemented into the finite element software.

3.1. Linear viscoelasticity formulation

For a linear viscoelastic material the constitutive relation may be represented in terms of the Volterra integral. When considering a non-ageing material we have (Salençon, 1983)

$$\varepsilon_{\alpha\beta} = \int_{0^-}^{t} J_{\alpha\beta\gamma\delta}\frac{\partial \sigma_{\gamma\delta}}{\partial \tau}\,d\tau \tag{3.1}$$

where $J_{\alpha\beta\gamma\delta}$ is the generalized creep compliance function and t, τ represent time. To obtain the stress and strain state at any time we apply the spectral decomposition technique proposed by Mandel (1966) to each creep tensor component

$$J_{\alpha\beta\gamma\delta} = \frac{1}{k^{(0)}_{\alpha\beta\gamma\delta}} + \frac{t}{\eta^{(\infty)}_{\alpha\beta\gamma\delta}} + \sum_{m=1}^{M} \Big[\frac{1}{k^{(m)}_{\alpha\beta\gamma\delta}}(1 - \mathrm{e}^{-\lambda^{(m)}_{\alpha\beta\gamma\delta}t})\Big] \tag{3.2}$$

where $\lambda^{(m)}_{\alpha\beta\gamma\delta} = k^{(m)}_{\alpha\beta\gamma\delta}/\eta^{(m)}_{\alpha\beta\gamma\delta}$. $k^{(i)}_{\alpha\beta\gamma\delta}$, $(i \in [0,1,...,M])$ and $\eta^{(j)}_{\alpha\beta\gamma\delta}$, $(j \in [\infty,1,...,M])$ denote the spring modulus and dash-pot viscosities,respectively. The decomposition enables us to represent $J_{\alpha\beta\gamma\delta}$ in terms of a generalized Kelvin-Voigt model comprising M elementary cells (see Fig.3).

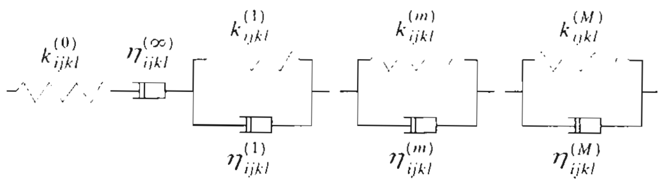

Fig. 3. Generalized Kelvin-Voigt model

Using a finite difference integration and linear approximation of stresses at each time step $(\Delta t_n = t_n - t_{n-1})$, the incremental constitutive equation reads (cf Ghazlan et al., 1995a,b)

$$(\Delta\varepsilon_{\alpha\beta})_n = M_{\alpha\beta\gamma\delta}(\Delta\sigma_{\gamma\delta})_n + \widetilde{\varepsilon}_{\alpha\beta}(t_{n-1}) \tag{3.3}$$

where $(\Delta\varepsilon_{\alpha\beta})_n$ and $(\Delta\sigma_{\gamma\delta})_n$ are the strain and stress increments in the time step Δt_n, respectively. The memory effect of all mechanical fields is stored in a pseudo-strain $\widetilde{\varepsilon}_{\alpha\beta}(t_{n-1})$ which is released in the last increment

$$\begin{aligned}\widetilde{\varepsilon}_{\alpha\beta}(t_{n-1}) = \Big\{\frac{\Delta t_n}{\eta^{(\infty)}_{\alpha\beta\gamma\delta}} + \sum_{m=1}^{M}\Big[\frac{1}{k^{(m)}_{\alpha\beta\gamma\delta}}\Big(1 - \mathrm{e}^{-\lambda^{(m)}_{\alpha\beta\gamma\delta}\Delta t_n}\Big)\Big]\Big\}\sigma_{\gamma\delta}(t_{n-1}) + \\ + \sum_{m=1}^{M} \varepsilon^{(m)}_{\alpha\beta\gamma\delta}(t_{n-1})\Big(\mathrm{e}^{-\lambda^{(m)}_{\alpha\beta\gamma\delta}\Delta t_n} - 1\Big)\end{aligned} \tag{3.4}$$

where $\varepsilon^{(m)}_{\alpha\beta\gamma\delta}(t_{n-1})$ is the strain part of the mth Kelvin-Voigt cell of the creep component $J_{\alpha\beta\gamma\delta}$ taking the following value

$$
\begin{aligned}
\varepsilon^{(m)}_{\alpha\beta\gamma\delta}(t_n) = & \frac{1}{k^{(m)}_{\alpha\beta\gamma\delta}}\Big(1 - \mathrm{e}^{-\lambda^{(m)}_{\alpha\beta\gamma\delta}\Delta t_n}\Big)\sigma_{\gamma\delta}(t_{n-1}) + \\
& + \frac{1}{k^{(m)}_{\alpha\beta\gamma\delta}}\Big[1 - \frac{1}{\lambda^{(m)}_{\alpha\beta\gamma\delta}\Delta t_n}\Big(1 - \mathrm{e}^{-\lambda^{(m)}_{\alpha\beta\gamma\delta}\Delta t_n}\Big)\Big](\Delta\sigma_{\gamma\delta})_n + \\
& + \mathrm{e}^{-\lambda^{(m)}_{\alpha\beta\gamma\delta}\Delta t_n}\varepsilon^{(m)}_{\alpha\beta\gamma\delta}(t_{n-1}) \qquad \text{(without summation)}
\end{aligned}
\tag{3.5}
$$

where $M_{\alpha\beta\gamma\delta}$ stands for the component of a pseudo-compliance tensor

$$
\begin{aligned}
M_{\alpha\beta\gamma\delta} = & \frac{1}{k^{(0)}_{\alpha\beta\gamma\delta}} + \frac{\Delta t_n}{2\eta^{(\infty)}_{\alpha\beta\gamma\delta}} + \\
& + \sum_{m=1}^{M} \frac{1}{k^{(m)}_{\alpha\beta\gamma\delta}}\Big[1 - \frac{1}{\lambda^{(m)}_{\alpha\beta\gamma\delta}\Delta t_n}\Big(1 - \mathrm{e}^{-\lambda^{(m)}_{\alpha\beta\gamma\delta}\Delta t_n}\Big)\Big]
\end{aligned}
\tag{3.6}
$$

In order to take into consideration the hereditary phenomena caused by a suplementary loading, we propose to invert the incremental constitutive law, represented by Eq (3.3)

$$
(\Delta\sigma_{\alpha\beta})_n = D_{\alpha\beta\gamma\delta}(\Delta\varepsilon_{\gamma\delta})_n - \widetilde{\sigma}_{\alpha\beta}(t_{n-1}) \tag{3.7}
$$

This inversion allows us to define $D_{\alpha\beta\gamma\delta}$ as the inverse of the compliance tensor and $\widetilde{\sigma}_{\alpha\beta}(t_{n-1})$ as the pseudo-stress given by

$$
\widetilde{\sigma}_{\alpha\beta}(t_{n-1}) = D_{\alpha\beta\gamma\delta}\widetilde{\varepsilon}_{\gamma\delta}(t_{n-1}) \qquad \text{with} \qquad \mathbf{D} = \mathbf{M}^{-1} \tag{3.8}
$$

The study of a complex problem demands numerical treatment. Then, the incremental equation (3.8) can be introduced into the finite element algorithm derived from the principle of virtual displacements, (see Ghazlan et al., 1995a,b). If the increment of nodal displacement vector is denoted as $\{\Delta u\}_n$, the equilibrium equation can be written as

$$
\mathbf{K}_T\{\Delta u\}_n = \{\Delta F^{ext}\}_n + \{F^{vis}\}(t_{n-1}) \tag{3.9}
$$

$\{\Delta F^{ext}\}_n$ denotes the increment of nodal force vector. $\{F^{vis}\}(t_{n-1})$ denotes the suplementary viscous load vector, which represents the complete mechanical history. If $\mathbf{B}$ is the strain displacement matrix and Ω is the discretized domain, the vector is defined by

$$
\{F^{vis}\}(t_{n-1}) = \int_{\Omega} \mathbf{B}^{\top}\mathbf{M}\{\widetilde{\varepsilon}\}(t_{n-1})\, d\Omega \tag{3.10}
$$

$\mathbf{K}_T$ designates the equivallent stiffness matrix

$$\mathbf{K}_T = \int_{\Omega} \mathbf{B}^{\top}\mathbf{M}^{-1}\mathbf{B}\, d\Omega \tag{3.11}$$

Now, we propose to adapt the behaviour formulation for the crack tip vicinity.

3.2. Incremental fracture formulation

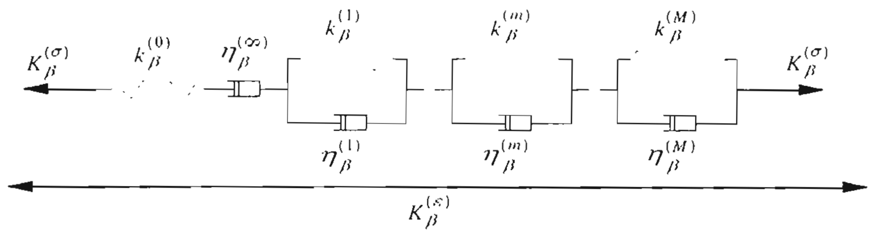

Fig. 4. Spectral decomposition of $C(t)$

Considering the similarity between the mechanical behaviour (3.1) and the relationship (2.12), between the stress and crack opening intensity factors development of the spectral decomposition technique is straightforward (cf Dubois et al., 1997). We propose a generalized Kelvin-Voigt model which allows us to determine the reduced viscoelastic compliance evolution in time. If the input data is the stress intensity factor $K_\beta^{(\sigma)}$ while the output is the strain intensity factor $K_\beta^{(\varepsilon)}$, according to the notation given in Fig.4, the reduced viscoelastic compliance $C(t)$ reads

$$C(t) = \frac{1}{k_c^{(0)}} + \frac{t}{\eta_c^{(\infty)}} + \sum_{m=1}^{M} \Big[\frac{1}{k_c^{(m)}}\Big(1 - e^{-\lambda_c^{(m)} t}\Big)\Big] \tag{3.12}$$

where $\lambda_c^{(m)} = k_c^{(m)}/\eta_c^{(m)}$.

Different constants appearing in Eq (3.12) must be compatible with those appearing in the time evolution (2.13). Using an extention of the behaviour law modelling (3.3), the incremental fracture formulation enables us to relate, for each time step Δt_n, the increment of crack opening intensity factors $(\Delta K_\beta^{(\varepsilon)})_n$ with the stress intensity factors increment $(\Delta K_\beta^{(\sigma)})_n$ as follows

$$(\Delta K_\beta^{(\varepsilon)})_n = C'_n(\Delta K_\beta^{(\sigma)})_n + \widetilde{K}_\beta^{(\varepsilon)}(t_{n-1}) \tag{3.13}$$

where C'_n is a pseudo-reduced function of viscoelastic compliance which depends on the spectral decomposition of $C(t)$ and the time increment Δt_n. By analogy with Eq (3.6) assuming that the stress intensity factor is linear in each increment, C'_n takes the following form

$$C'_n = \frac{1}{k_c^{(0)}} + \frac{\Delta t_n}{2\eta_c^{(\infty)}} + \sum_{m=1}^{M} \frac{1}{k_c^{(m)}}\Big[1 - \frac{1}{\lambda_c \Delta t_n}\Big(1 - e^{-\lambda_c^{(m)}\Delta t_n}\Big)\Big] \tag{3.14}$$

The hereditary behaviour is taken into account in the crack tip vicinity through $\widetilde{K}_\beta^{(\varepsilon)}(t_{n-1})$ which reflects the influence of the complete history of crack opening intensity factors.

Upon generalization of Eq (3.4) $\widetilde{K}_\beta^{(\varepsilon)}(t_{n-1})$ can be defined as follows

$$\begin{aligned}\widetilde{K}_\beta^{(\varepsilon)}(t_{n-1}) = \Big\{\frac{\Delta t_n}{\eta_c^{(\infty)}} + \sum_{m=1}^{M}\Big[\frac{1}{k_c^{(m)}}\Big(1 - e^{-\lambda_c^{(m)}\Delta t_n}\Big)\Big]\Big\} K_\beta^{(\sigma)}(t_{n-1}) + \\ + \sum_{m=1}^{M} K_\beta^{(m)}(t_{n-1})\Big(e^{-\lambda_c^{(m)}\Delta t_n} - 1\Big)\end{aligned} \tag{3.15}$$

where

$$\begin{aligned} K_\beta^{(m)}(t_n) &= \frac{1}{k_c^{(m)}}\Big(1 - e^{-\lambda_c^{(m)}\Delta t_n}\Big) K_\beta^{(\sigma)}(t_{n-1}) + \\ &+ \frac{1}{k_c^{(m)}}\Big[1 - \frac{1}{\lambda_c^{(m)}\Delta t_n}\Big(1 - e^{-\lambda_c^{(m)}\Delta t_n}\Big)\Big](\Delta K_\beta^{(\sigma)})_n + \\ &+ e^{-\lambda_c^{(m)}\Delta t_n} K_\beta^{(m)}(t_{n-1}) \end{aligned} \tag{3.16}$$

Finally, in order to compute the energy release rate and kinematical and statical fields around the crack tip, it is necessary to combine Eqs (2.15) and (3.13). If only the opening mode is considered for each value of Δt_n, the system of equations proposed by Chazal et al. (1998) can be used

$$J_v(t_n) = a[(\Delta K_1^{(\varepsilon)})_n]^2 + b(\Delta K_1^{(\varepsilon)})_n + c \tag{3.17}$$

where a, b and c are constants, released at each time increment

$$\begin{aligned} a &= \frac{1}{8C'_n} \\ b &= \frac{C'_n K_1^{(\sigma)}(t_{n-1}) + K_1^{(\varepsilon)}(t_{n-1}) - \widetilde{K}_1^{(\varepsilon)}(t_{n-1})}{8C'_n} \\ c &= K_1^{(\varepsilon)}(t_{n-1})\frac{C'_n K_1^{(\sigma)}(t_{n-1}) - \widetilde{K}_1^{(\varepsilon)}(t_{n-1})}{8C'_n} \end{aligned} \tag{3.18}$$

To solve Eqs (3.18), we can develop the kinematic opening displacement method. Using a local displacement interpolation, by virtue of Eqs (2.10), it is easy to compute the crack opening intensity factors for a complex mode configuration. However, this local method is accurate if the mesh around the crack tip is highly refined. To overcome this obstacle, we opt for a global technique which is based on the J_v-integral computation.

3.3. Energy release rate computation

The J_v integral, which is based on a path independent line integral, poses difficulties in the integration of the fields, defined at the Gaussian points. However, in contrast to the J-integral technique, the $G\theta$ method, used in elastic, viscoplastic or dynamic fracture, allowsus to determine the energy release rate by a surface integral, see Destuynder and Djaoua (1981), Destuynder (1983). By using the pseudo strain energy Φ, this integral can be easily extended and defined as

$$G\theta = \int\limits_V (-\Phi\theta_{k,k} + \sigma_{\alpha\beta}u_{\alpha,k}\theta_{k,\beta})\, dV \tag{3.19}$$

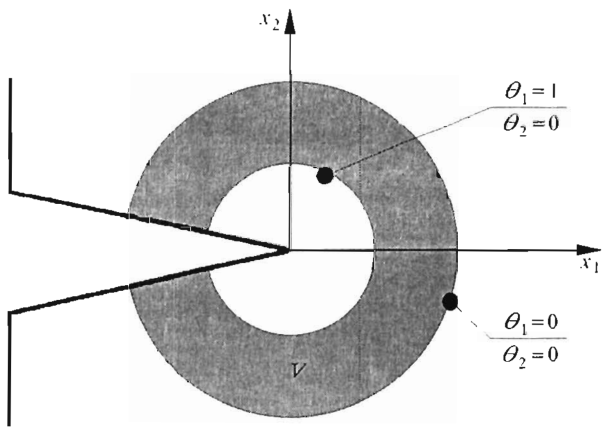

Fig. 5. Integration domain of $G\theta$

The integration domain V is a ring (see Fig.5) which is bounded by two contours defined by $\vec{\theta}$. This mapping function is continuously diferentiable as in the case of $\theta_1 = 1$ and $\theta_2 = 0$ inside the ring and $\vec{\theta} = \vec{0}$ outside it. This technique is very effective for simple fracture modes, for a mixed mode fracture it reveals, however, an energetic separation, which is dealt with in this paper.

4. Numerical applications

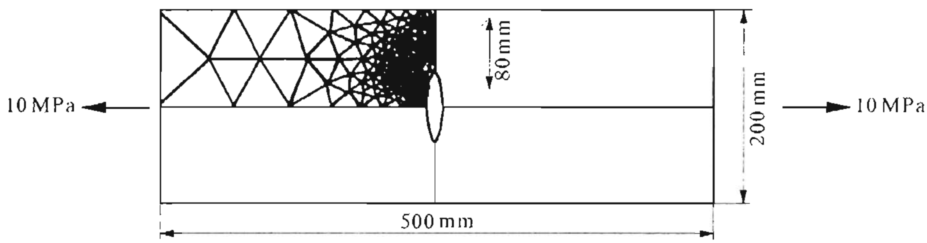

Fig. 6. Central cracked plate mesh

The general formulation, developed in this paper, is implemented into the finite element software Castem 2000 (produced by the French Energy Atomic Commission CEA) (Charvet-Quemin et al., 1986). In order to validate this approach, we consider a viscoelastic plate CTT of 500 mm in length, 200 mm in width, under the uniform tension of 10 MPa with a central crack of 80 mm in lenght perpendicular to the direction of loading, as shown in Fig.6. In this configuration, the crack is loaded in an open crack mode. To simplify the analytic approach, the isotropic material is supposed to behave as a linear viscoelastic one with a constant Poisson ratio ($\nu = 0.3$) (Ghazlan et al., 1995a,b). In this case, only the Young modulus $E(t)$ depends on time. In a plane stress configuration, we use the generalized Kelvin-Voigt model, presented in Fig.7, which represents the time evolution of $1/E(t)$.

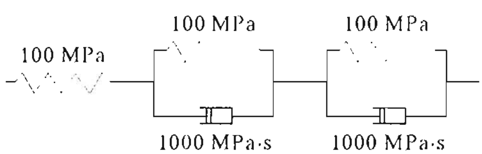

Fig. 7. Spectral decomposition of $1/E(t)$

Numerical solutions are compared with the results obtained by Masuero and Creus (1993) as well as with the analytical solution. The elastic value of J_v, in this case, is 154.85 N/mm. By using Eqs (2.12) and (2.15),the elastic fields in the crack tip vicinity are defined by ${}^eK_1^{(\sigma)}$ and ${}^eK_1^{(\sigma)}$ which take the following values

$$ {}^eK_1^{(\sigma)} = 118.7\ \mathrm{N\,mm}^{-3/2} \qquad {}^eK_1^{(\varepsilon)} = 10.44\ \mathrm{mm}^{1/2} \tag{4.1} $$

In view of the creep plane stress state the stress intensity must be constant. This consideration enables us to generalize (4.1)

$$K_{\rm I}^{(\sigma)}(t) = 118.7\ {\rm N\,mm}^{-3/2} \tag{4.2}$$

Substituting Eq (4.2) into the local mechanical behaviour (2.12), the crack opening intensity factor evolution can be predicted by

$$K_{\rm I}^{(\varepsilon)}(t) = {}^{e}K_{\rm I}^{(\varepsilon)}C(t) \tag{4.3}$$

By considering the spectral decomposition shown in Fig.7, the reduced viscoelastic function $C(t)$ can be written as follows

$$C(t) = 2 - {\rm e}^{\frac{-t}{10}} \tag{4.4}$$

Substituting Eqs (4.4), (4.3) and (4.2) into Eq (2.15) yields

$$J_v(t) = 154.84\Big(2 - {\rm e}^{\frac{-t}{10}}\Big)\ {\rm N/mm} \tag{4.5}$$

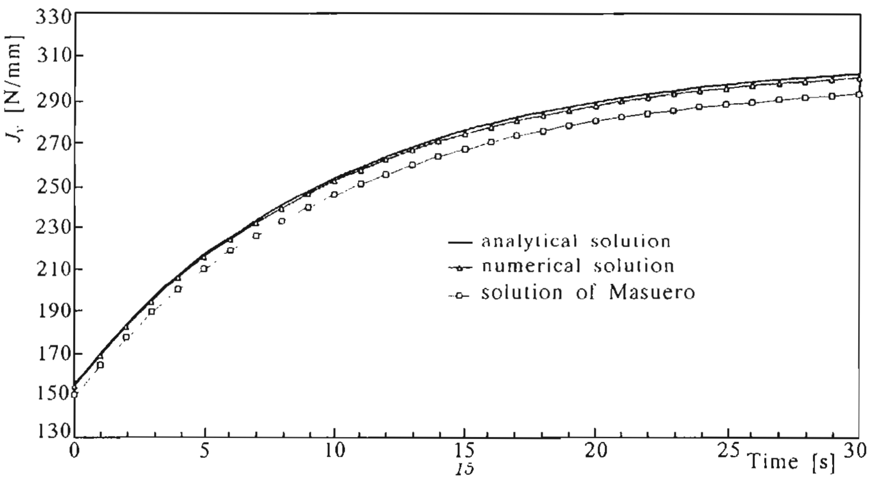

Fig. 8. Variation of J_v

The results in terms of J_v are shown in Fig.8, where the numerical and analytical solutions are plotted. By comparing our numerical technique and the results of Masuero a difference can be found due to different coarse meshes used. However, the small error observed is constant over the entire process. Now, in order to consider the stress and crack opening intensity factors the spectral decomposition technique results are presented, in Fig.9 and Fig.10, respectively. One can note a satisfactory agreement between the numerical results and analytical solutions.

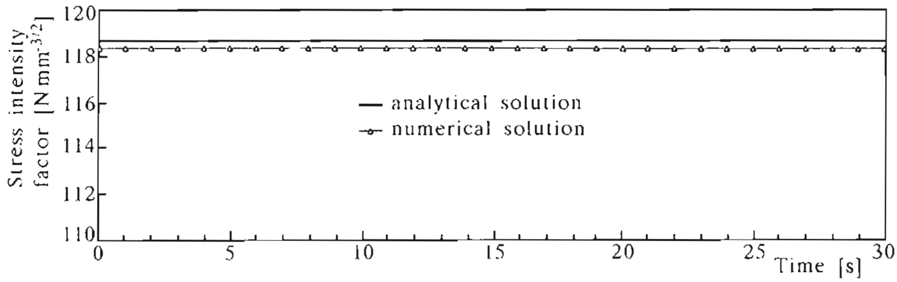

Fig. 9. Variation of the stress intensity factor $K_1^{(\sigma)}$

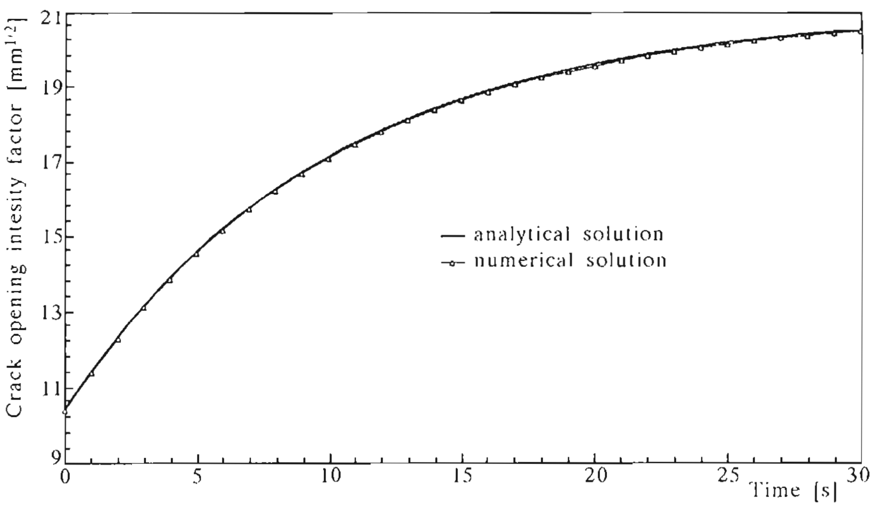

Fig. 10. Variation of the crack opening intensity factor $K_1^{(\varepsilon)}$

5. Conclusions and perspectives

In order to examine the influence of history-dependent phenomena on the crack growth initiation process in linear viscoelastic media the new formulation in the time domain has been developed. By introducing the crack opening intensity factor, the coupling between the energy release rate and the stress intensity factor is possible in the case of simple fracture mode. If strain intensity factors introduced by Brincker, allow for the strain state determination, the crack opening intensity factors define the kinematical state of crack lips that can be used directly in computation of theamount of energy dissipated in the crack growth process. The numerical approach, in terms of finite elements, allows for consideration of a variety of geometrical representations, boundary conditions and loads. This technique, based on the spectral decomposition of reduced viscoelastic functions, can easily be generalized to cover anisotropic materials and advanced to deal with the crack growth process in complex structures for a mixed mode fracture.

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Modelowanie inicjacji szerokiego pęknięcia dla materiałów liniowo sprężystolepkich

Streszczenie

Szeroko rozpowszechnione konstrukcje z materiałów lepkosprężystych, które muszą działać przez długi czas, wymagają lepszego zrozumienia ich mechanicznego zachowania i właściwości przy zniszczeniu. Zaobserwowano, że zależność od czasu ma ogromne znaczenie przy wyznaczaniu prędkości wzrostu pęknięcia. W pracy rozpatrzono efekt charakterystyki sprężysto-lepkiej na inicjację wzrostu pęknięcia przy pełzaniu wykorzystując metodę elementów skończonych. Zaproponowano nowe sformułowania w przestrzeni czasu dla liniowych, izotropowych ośrodków sprężystoplastycznych w celu wyznaczenia przemieszczeń i naprężeń wokół wierzchołka szczeliny. Zaproponowano nowe równania konstytutywne zależności współczynników intensywności naprężeń i otwarcia szczeliny wykorzystując odpowiadającą zasadę całek Volterra. W rezultacie obliczono parametry zniszczenia w procesie sprzężonym z przyrostowym sformułowaniem sprężystolepkim.

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