ACOUSTIC RADIATION OF VIBRATING PLATE STRUCTURES SUBMERGED IN WATER

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The paper presents results of the theoretical and numerical investigation on acoustic radiation of vibrating plate structures submerged in water. The current state of the art on the considered issues is briefly reviewed. Then, the method for determining eigenmode shape functions and eigenfrequencies of plate vibrating in water that has been used in presented study is introduced. The constitutive equations for solid domain and the pressure acoustic equation for liquid domain are coupled via boundary conditions and solved numerically using the finite element method. Structural mode shapes and eigenfrequencies computed for plate submerged in water are compared to analogous results obtained for air and for the in vacuo case. It is assumed, that the plate is rectangle shaped and that it is placed in an infinite rigid baffle. Three-dimensional near- and far-field acoustic radiation characteristics for the plate vibrating in water are introduced. Possibilities of implementation of the active control system for reduction of the hydroacoustic emission are briefly discussed.

INTRODUCTION

Acoustic radiation of vibrating structures is the topic of many scientific investigations. It is also very often an undesirable effect appearing in many technical applications. Most of the work dealing with this issue focuses on vibroacoustic emission in air, considering simple but representative structural elements, such as membranes, beams and plates with different boundary conditions. Due to the relatively low density of air, the effect of additional inertial load of the structure is most often omitted, simplifying the considered problem to the in-vacuo case. Such assumption usually has no significant effect on the results in mentioned issues but it is inadmissible when considering structures submerged in relatively heavy fluids, such as water. That makes the analysis of such problems much more complicated.

Influence of the fluid loading on the acoustic radiation of the structural elements has been investigated by many scientists. Some analytical solutions for specific cases were developed by Maidanik and Kerwin [2] and Stuart [1][12]. Many authors considered the impact of the submersion in water on the plate's eiegenfrequencies and eigenmode shapes. Different methods have been developed to solve this problem. Some of them assume that the structural mode shapes remain unchanged and focus only on the alteration of the natural frequencies of the plates [3][4]. Additional factors are often introduced to model the inertial loading by mass incrementation of the vibrating structure [3][4][5][7]. Other, more general approaches, use Rayleigh-Ritz method to compute both: mode shapes and eigenfrequencies [4][8]. The influence of fluid loading is different for plates having contact with water on one or both sides.

To analyze the hydroacoustic emission of the considered structures it is very convenient to introduce the decomposition of the plate vibrations into its structural modes. That is why the determination of natural frequencies and eigenmode shapes is important in point of view of the considered issues. The structural modes do not radiate sound independently, so the cross-modal contributions need to be taken into account.

One of the main areas in which hydroacoustic emission is crucial is military technology. In past decades lots of effort has been put into reducing noise generated by marine and navy vessels. This is especially true of the submarines, for which stealth operation is the most desirable. Some recently released papers [9][10] clearly point out, that this topic is important and still being developed.

The present paper aims to investigate the near- and far-field radiation characteristics of plate structures having contact with water on the one side. A fully-coupled multiphysics 3D numerical model is used to determine the vibration characteristics of the considered structure.

1. VIBRATIONS OF SUBMERGED PLATES

It is assumed, that considered plate is made of alumina and that it is rectangle shaped. Its dimensions are chosen to be as follows: length 30 cm, width 20 cm, height 1 mm. The plate is placed in an infinite, rigid baffle and it is in contact with water on the one side. Free boundary conditions are assigned to all the four edges. The surrounding medium is modeled as a semi-sphere with radius large enough to use the far-field approximation at the borders. The described problem geometry is presented on Fig.1.

The solid domain – represented by the plate – is described with the following constitutive equation:

$$-\rho_p \omega^2 \mathbf{u} - \nabla \cdot \sigma = \mathbf{F}, \tag{1}$$

where ρ_p is the materials density, ω is the angular frequency, $\boldsymbol{u}$ is the displacement vector, σ is the stress tensor and $\boldsymbol{F}$ is a vector of the external loads.

The liquid domain – represented by the surrounding water – is described with following pressure acoustics equation:

$$\nabla \cdot \frac{1}{\rho_c}(\nabla p) - \frac{\omega^2 p}{c_c \rho_c} = 0, \tag{2}$$

where p is the acoustic pressure, ρ_c is the density of water and c_c is the speed of sound in water.

Equations (1) and (2) are coupled via following boundary conditions assigned on the solid-fluid interface:

$$-\mathbf{n}\cdot\left(\nabla\cdot\frac{1}{\rho_c}(\nabla p)\right)=-\mathbf{n}\cdot\frac{\partial^2\mathbf{u}}{\partial t^2}, \tag{3}$$

$$\sigma\cdot\mathbf{n}=p\cdot\mathbf{n}, \tag{4}$$

where **n** is an unit vector normal to the interface boundary, and t denotes time.

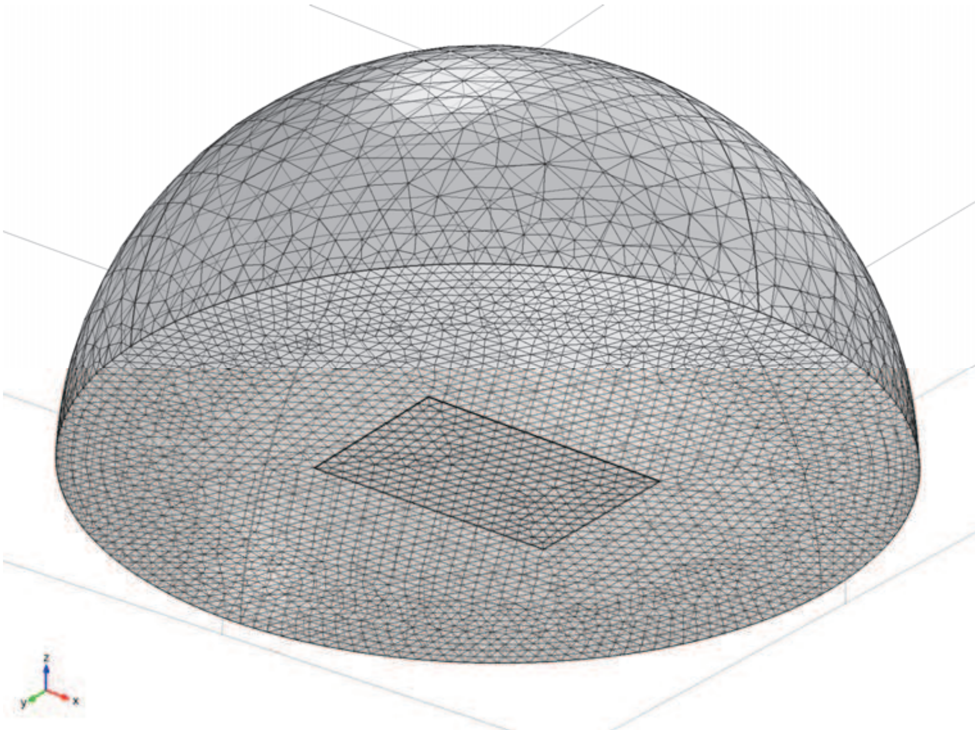

Fig.1. Geometry and discretization of the considered problem.

Equations were solved to find the natural frequencies of the submerged plate and the corresponding structural mode shapes. Numerical simulations have been performed for three different situations: plate with surrounding water, plate with surrounding air and for plate without any external load assumed. Low-frequency range only was considered, taking into account first eight eigenmodes (without rigid body motion solution at *f=0*). One of the reasons justifying such limitation is that if the analysis is aimed for developing an active control system for reduction of the hydroacoustic emission, then the frequency range of interest is below few hundreds Hz. That is because much cheaper and simpler passive vibration absorbers are ineffective in this range. Moreover, acoustic waves of such wavelengths encounter only relatively low damping in surrounding medium and many of the mechanical systems like vessel engines generate strong low-frequency vibrations.

The solution has been obtained using the finite element method. Considered domains have been discretized as it is shown on Fig.1. Computations were performed using Comsol Multiphysics.

It has been found, that the shapes of the corresponding structural modes in all three considered cases – for air, water and the in vacuo case - showed no significant differences. Obtained results are presented on Fig.2.

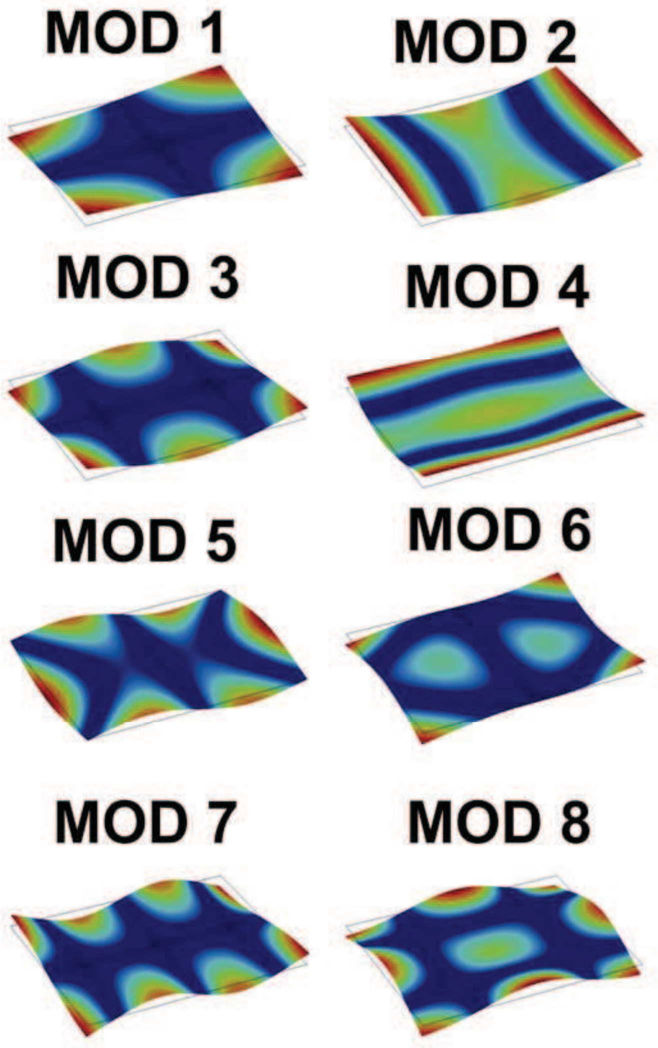

Fig.2. Shapes of the first eight eigenmodes of the considered plate.

Tab.1. Eigenfrequencies computed for different plate loads

Mode no.	No load	Air	Water
1	54,77 Hz	54,55 Hz	15,57 Hz
2	58,44 Hz	58,26 Hz	15,97 Hz
3	126,46 Hz	126,74 Hz	40,01 Hz
4	136,47 Hz	136,4 Hz	42,63 Hz
5	157,95 Hz	161,33 Hz	51,73 Hz
6	183,43 Hz	184,38 Hz	60,61 Hz
7	234,9 Hz	239,5 Hz	82,54 Hz
8	269,98 Hz	274,75 Hz	97,07 Hz

Eigenfrequencies of first eight structural modes, computed for all three considered cases, are presented in Tab.1. Frequencies computed for the air and for the in vacuo case are almost the same, differences are small enough to be considered as numerical errors. This conclusion confirms, that omitting the influence of air in case of relatively small structural elements and in low frequency range does not introduce any significant error in vibration analysis. Comparing those results to the values computed for the plate in contact with water it can be noticed that all of the natural frequencies in the latter case have been reduced approximately by the factor of 3. This significant change has to be taken into account in analysis of the hydroacoustic radiation. Omitting the influence of external load introduced by the surrounding water would lead to unacceptable errors in modeling the system dynamics.

2. HYDROACOUSTIC EMISSION OF VIBRATING PLATE STRUCTURES

Equation (2) was solved in the liquid domain using the finite element method to obtain the acoustic pressure distribution in the near-field. The results computed for selected structural modes at the corresponding natural frequencies are presented on Fig.3. The shapes of the hydroacoustic radiation characteristics close to the plate are complex and they vary with the distance.

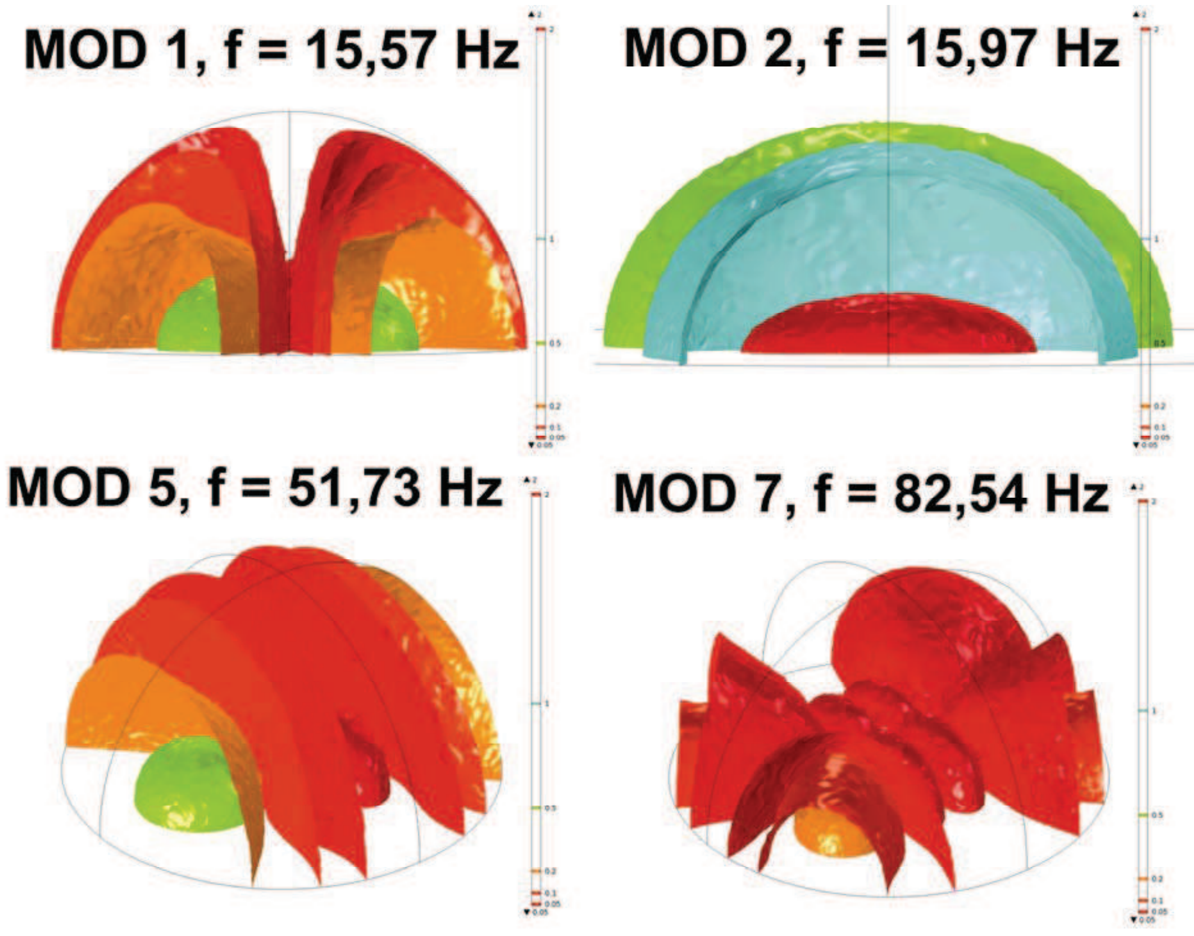

Fig.3. 3D near-field hydroacoustic radiation characteristics of selected structural modes of the considered plate at corresponding natural frequencies.

To compute the 3D far-field hydroacoustic beam patterns, the eigenmodes shapes and natural frequencies obtained using the finite element method were exported to the text file and then loaded using the developed Matlab script. The far-field acoustic pressure distributions for different structural modes, radiating at different frequencies were computed using Rayleigh`s integral [10]:

$$p(\mathbf{r}) = \int_S \frac{j\omega\rho_c v(\mathbf{r}_s) e^{-jkR}}{R} dS \,, \qquad (5)$$

where k is the acoustic wavenumber, S is the surface of the plate, $v(\mathbf{r}_s)$ is the component of the complex velocity normal to the S and $R = |\mathbf{r} - \mathbf{r}_s|$ is the distance between observation point and integration point on the surface S.

The velocity distributions at the plate surface were designated from the normal components of the exported displacement field vector taking into account the harmonic vibrations of the system. Lots of numerical simulations were performed to investigate the changes of hydroacoustic radiation beam pattern of every mode with the frequency. Examples of the obtained results are presented on Fig.4.

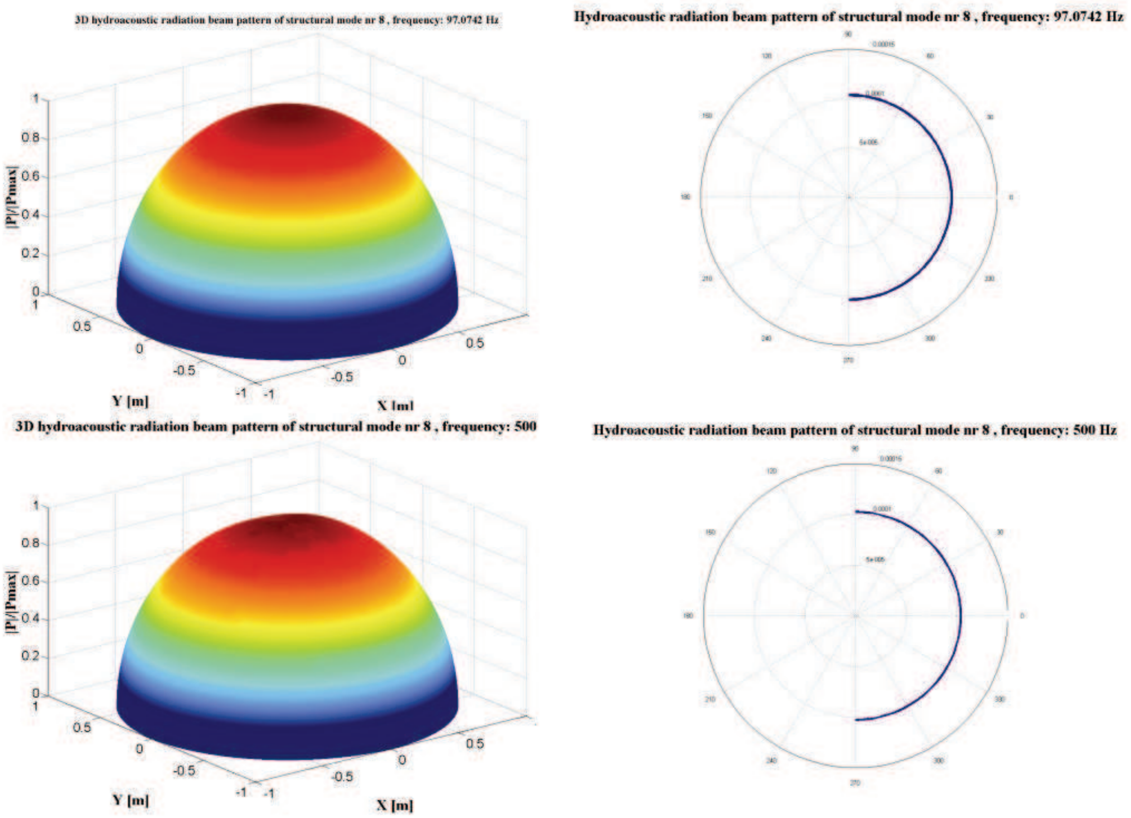

Fig.4. 2D and3D far-field hydroacoustic radiation beam patterns of structural mode nr 8, radiating at its natural frequency (upper graphs) and at frequency f=500 Hz (lower graphs).

The obtained results indicate that in the considered low-frequency range all of the structural modes show monopole-like acoustic beam patterns. The explanation of this phenomenon lies in fact that the dimensions of the considered plate are relatively small compared to the acoustic wavelength in water. Significant changes in the shapes of radiation characteristics are observed only in the frequency range of above tens of kHz. However, contribution of considered low-order structural modes in such high-frequency vibrations would be negligible.

It is also worth noticing that since for considered plate structure and frequency range the hydroacoustic beam patterns are very similar, substantial spill-over effect will be observed. None of the considered structural modes can be assumed to radiate sound independently from any other. This conclusion is important in terms of analyzing hydroacoustic pressure field generated by vibrating structures using modal decomposition.

3. COCLUSIONS

The method for analyzing acoustic radiation of vibrating plate structures submerged in water has been introduced. The hydroacoustic emission is clearly connected with the dynamic properties of the plate and with the parameters of the disturbance source that causes the vibrations. The response of the plate to the external excitation may be designated using modal decomposition. The eigenfrequencies values are strongly affected by the inertial loading introduced by the surrounding water, while the corresponding eigenmodes shapes seem to be insensitive to this factor.

The far-field hydroacoustic beam patterns of the vibrating plate structures in low frequency range are regular and monopole-like. Substantial modal spill-over effect occurs for all considered low-order structural modes.

The presented results of the conducted investigation form the basis for the development of an active control system that could be used, for example, to reduce the hydroacoustic emission generated by the vibrating elements of the submarine's or ship's hull. Analysis of the dynamic properties of the considered system would be required before the implementation. Then, taking into account the effects of fluid loading and modal hydroacoustic beam patterns optimal parameters of such system could be computed to achieve minimum radiation efficiency.

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