SIMULATION ALGORITHMS FOR DOA ESTIMATION WITH UCA

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Multi-sensors passive arrays are used to detect different objects emitting wideband acoustic signal to aquatic medium. The most important design problem is to proper match the size of the antenna and number of sensors to the wideband frequency range. Especially the multi-mode operation significantly increases the detection efficiency. In case of cylindrical antennas interesting is object detection at two frequencies. The first frequency results directly from the antenna radius, whereas the second one is doubled. The article presents detection capabilities of antennas composed of several hydrophones.

1. AMBIQUITY OF DIRECTION ESTIMATION

In the simples case the antenna is composed of two sensors as presented in the figure 1. Then the signals detected by hydrophones are depicted by formulas:

$$\begin{aligned} s1 &= F \cdot e^{j(\omega t+\varphi_1)} \\ s2 &= F \cdot e^{j(\omega t+\varphi_2)} \\ s1 \cdot s2^* &= |F|^2 e^{j(\varphi_1-\varphi_2)} \end{aligned} \tag{1}$$

The searched object is located on the hyperbole defined equivalently as the lotus of points where the difference of the distances to the two foci is a constant. This constant equals:

$$2 \cdot a = \frac{\varphi_1 - \varphi_2}{2\pi} \cdot \lambda \tag{2}$$

For far away distances from antenna the object direction is calculated using following formula:

$$\frac{a}{r} = \cos(\Theta - \alpha) \tag{3}$$

This equation has two solutions:

$$\Theta - \alpha = \pm \arccos\left(\frac{a}{r}\right)$$

$$\Theta' - \alpha = \mp \arccos\left(\frac{a}{r}\right) \qquad (4)$$

$$\Theta' = 2 \cdot \alpha - \Theta$$

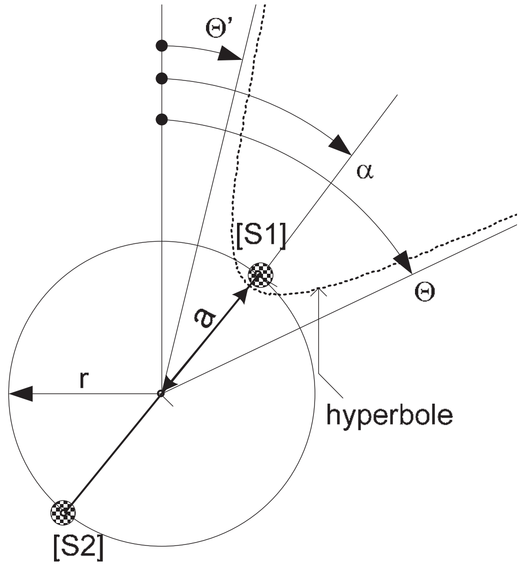

Fig.1. Ambiguity of target detection.

This ambiguity can be removed by application of the second set of sensors provided that they create the new baseline of different angle. The false direction Θ ' can be removed as shown in the figure 2.

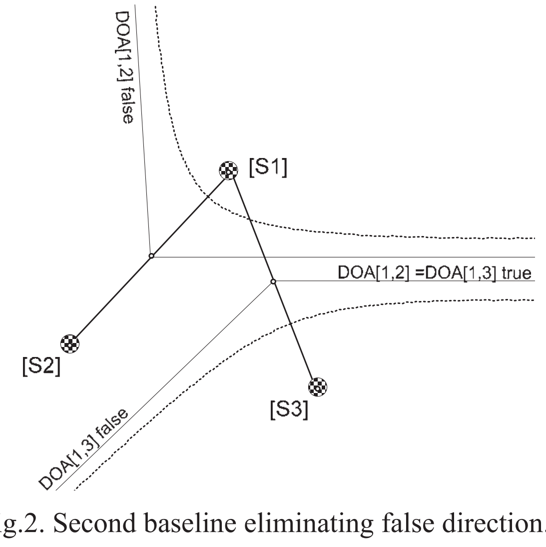

Fig.2. Second baseline eliminating false direction.

When the antenna size is bigger then the half of the wavelength the new hyperbolas occur. It introduces further ambiguities in DOA determination. Even the antenna composed of four sensors does not eliminate these ambiguities as shown in figure 3.

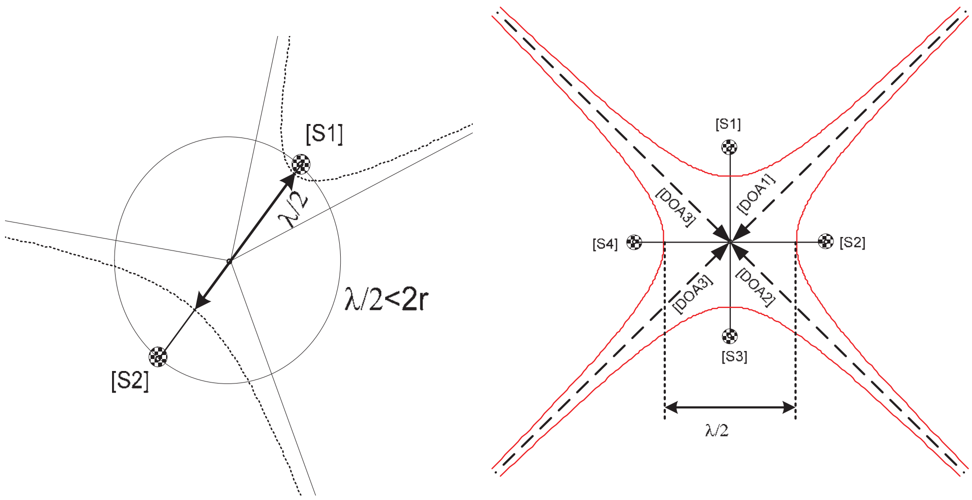

Fig.3. Ambiguity of target detection in case of 1 object detected by 4 sensors antenna.

The bigger number of objects more complicates the problem. The signal arriving to antenna expresses formula:

$$\vec{x} = \sum_{m=0}^{M-1} F_m \vec{a}_m + \vec{n} \quad , M < N \tag{5}$$

where:

- M – number of targets
- m – index of target in range [0..M-1]
- N – number of antena sensors
- F_m – random, complex amplitude corresponding to m object
- $\vec{a}_m$ – steering vector corresponding to object of polar coordinates (ρ_m, Θ_m)
- $\vec{n}$ – uncorrelated noise vector

Covariance matrix of this signal equals:

$$C - \sigma^2 I_N = \sum_{m=0}^{M-1} E(|F_m|^2)\vec{a}_m \vec{a}_m^H \tag{6}$$

It is the Hermite matrix. The spectral equation of this matrix is following:

$$C - \sigma^2 I_N = \sum_{m=0}^{M-1} \lambda_m \vec{u}_m \vec{u}_m^H + 0 \cdot \sum_{m=M}^{N-1} \vec{u}_m \vec{u}_m^H \tag{7}$$

$$I_N = \sum_{m=0}^{N-1} \vec{u}_m \vec{u}_m^H$$

The $\lambda_m, \vec{u}_m$ values correspond to eigenvalues and eigenvectors of C matrix. The eigenvectors are invariant in regards to the ambient noise dispersion σ.

The eigenvectors from range of [M..N-1] are orthogonal in regards to eigenvectors from range of [0..M-1].

$$\vec{a}_k^H \vec{u}_m = 0 \quad , k \in [0..M-1], m \in [M..N-1] \tag{8}$$

The trace of the C matrix equals:

$$trace(C - \sigma^2 I_N) = \sum_{m=0}^{M-1} E\left(|F_m|^2\right)|\vec{a}_m|^2 = \sum_{m=0}^{M-1} \lambda_m \tag{9}$$

Moreover:

$$\sum_{m=M}^{N-1} \left|\vec{a}_k^H \vec{u}_m\right|^2 = 0 \quad , k \in [0..M-1] \tag{10}$$

Practically for solution important is the second of above equations. It can be transformed to searching for function minimum:

$$\min_{\rho,\Theta} \sum_{m=M}^{N-1} \left|\vec{a}^H(\rho,\Theta)\vec{u}_m\right|^2 \tag{11}$$

The assumption $\rho = \infty$ is made for DOA estimation.

2. MATHEMATICAL MODEL

The mathematical model used by Smith's classical method relays on separation of the directional sound sources from uncorrelated noise background. The covariance matrix is the sum two matrixes. The first one corresponds to the directional target whereas the second one refers to the background noise. Many articles assumed that the known covariance matrix is diagonal. The proposed model assumes a large number (up to 100) of directional targets equally displaced in all directions. The amplitude of majority targets is significantly lower (several decibels) in comparison to one, two or three strong targets. This model more accurately presents the real underwater environment. The received noise signal describes following formula:

$$\vec{x} = F_1\vec{s}_1 + F_2\vec{s}_2 + \ldots + F_K\vec{s}_K + \vec{n}, \quad \vec{s}_i = \vec{a}\left(i \cdot \frac{2\pi}{K}\right) \quad K >> N \tag{12}$$

The values of $F_1, \ldots, F_K$ are the random variables of constant dispersion for all directions:

$$E\left(|F_i|^2\right) = F^2 \tag{13}$$

It is assumed that all sound sources are independent. In this case the covariance matrix estimated for antenna is the sum of covariance matrixes from all sources:

$$E(\vec{x} \cdot \vec{x}^H) = E(|F_1|^2)\vec{s}_1 \cdot \vec{s}_1^H + E(|F_2|^2)\vec{s}_2 \cdot \vec{s}_2^H + \ldots + E(|F_K|^2)\vec{s}_K \cdot \vec{s}_K^H + I \cdot \sigma^2 \tag{14}$$

The covariance matrix corresponding to weak noise signal characterizes dominant diagonal. The main diagonal of the Hermite matrix is the sum of positive real numbers, whereas the components situated outside this diagonal are the sum of complex numbers with random phase. Therefore the influence of majority weak directional noises is tantamount to vector noise of weak-cross-correlated components. The strong noise sources are decisive for DOA estimation. The parametric methods are based on narrowband signals. In reality the noises generated by maritime objects are broadband. So the separation of the narrowband signal from broadband noise signal requires application of the I/Q detection. The complete simulation path is composed of two parts:

- simulation of underwater environment,
- simulation of detector.

The amplitude of each target spectrum is random type, whereas the phase spectrum is modulated according to $\vec{a}(i \cdot 2\pi / K)$ steering vector. Summation of spectrums derived from all (weak and strong) targets provides to antenna spectrum. Implementation of Invert Discrete Fourier Transform reproduces the real snapshots for all sensors. This finishes the simulation of underwater environment.

Common generator of reference signal restores the parametric vector $\vec{x}$. The samples series of this vector are used to create the estimate of the covariance matrix. The MUSIC algorithm produces the criterion function (Power MUSIC) tuned to single, two or three objects. The analyse of this criterion function is the baseline for DOA estimation.

3. AMBIGUITY AND ANGULAR RESOLUTION OF UNIFORM CYLINDRICAL ARRAYS

Two ambiguity and angular resolution depend on two parameters:

- numer of hydrophones,
- r/ λ ration.

Below function corresponds to these parameters:

$$Diff(\Theta 1, \Theta 2) = \left\| \vec{a}(\Theta 1) - \vec{a}(\Theta 2) \right\| \tag{15}$$

The graph of this function is 3D type in $(\Theta 1, \Theta 2, \mathrm{Diff})$ coordinates presented in 4th figure. This function always equals 0 when $\Theta 1 = \Theta 2$. The ambiguity of solutions is visible in this graph as the additional zero point.

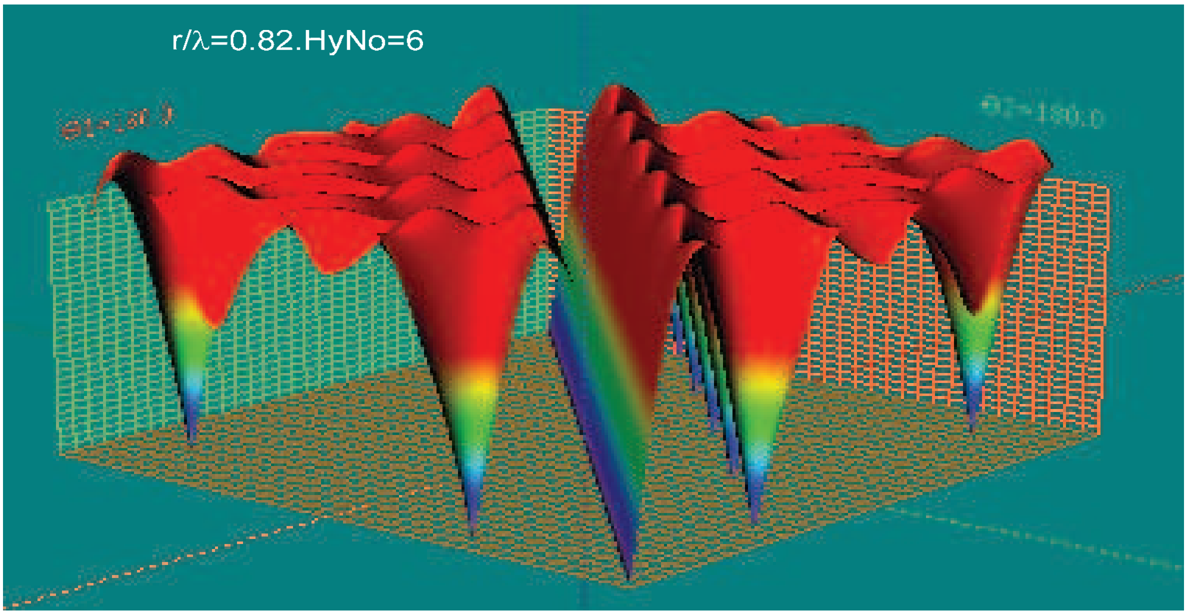

Fig.4. Ambiguity of direction determination in case of 6-sensor antenna.

The resolution of direction determination is visible as the function gradient in the vicinity of the $\Theta 1 = \Theta 2$ straight.

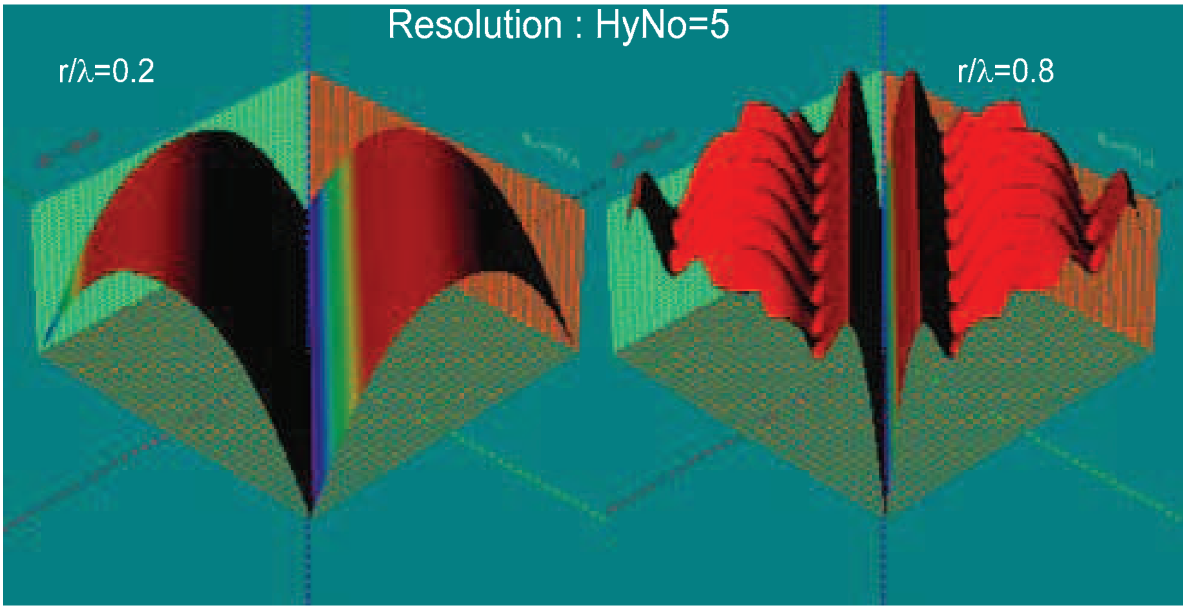

Fig.5. The angular resolution in case of 5-sensor antenna.

The antenna composed of more then 2 sensors has not this ambiguity for $r/\lambda<0.25$ ratio. The small value of this ratio means low angular resolution. Increase of r/λ improves this resolution but conducts to ambiguity. In this case ambiguity removal requires increase of sensors quantity. Thus the optimal antenna is the compromise between the numbers of hydrophones and achieved angular resolution with the lack of ambiguity.

4. TWO-SENSOR ANTENNA

The simulation results are shown in the figure 6. Only two directions (0° and 180°) have unique solution for $r/\lambda<0.25$ ratio. In case of the remaining angles many local maxims are observed, however the direction cannot be determined due to ambiguity.

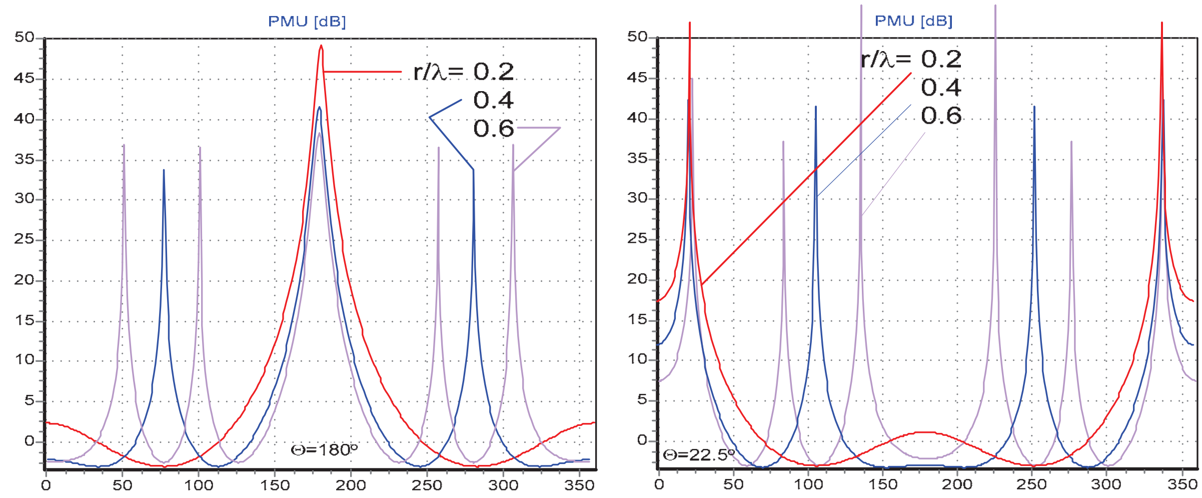

Fig.6. PMU criterion for 2-sensor antenna.

5. THREE-SENSOR ANTENNA

The simulation results are shown in the figure 7. Unique determination of direction is possible only for r/λ<0.25 ratio. Practically detection of two targets is impossible.

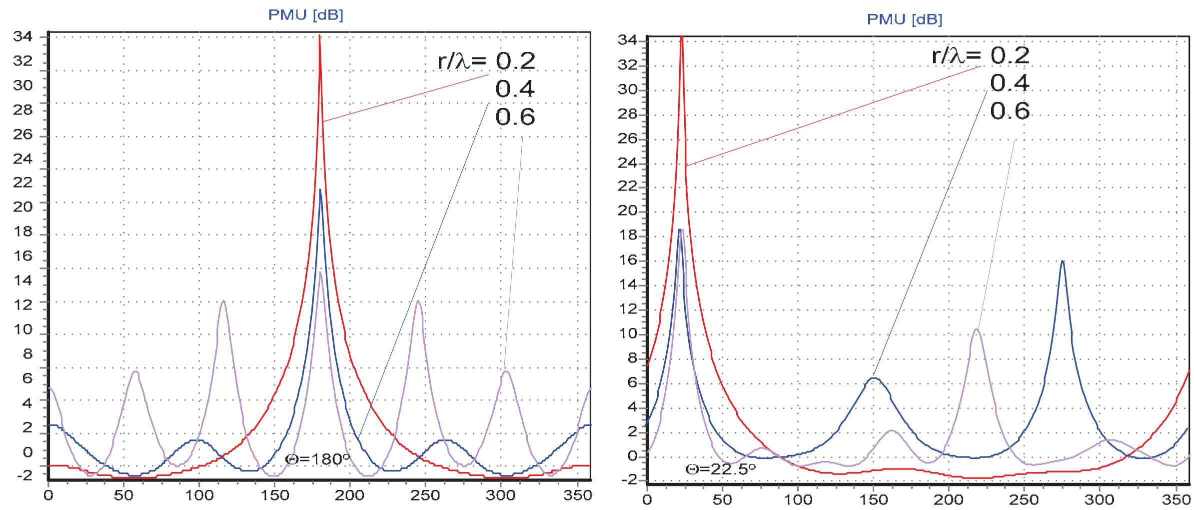

Fig.7. PMU criterion for 3-sensor antenna.

6. FOUR SENSOR ANTENNA (AMASS)

The antenna composed of four digital hydrophones of 1.6m diameter was manufactured by OBR CTM S.A. in scope of the 7FP. Initially it was several times tested in the shallow waters of the Baltic Sea, next it was transported to Gran Canaria Island and tested in the Atlantic Ocean. The results of the sea trials are presented in the figure below. The upper diagram presents the CTMEEKA boat route (dashed line). The circle with four hydrophones and compass direction are visible in the centre (the scale is not maintain). The four plots below correspond to recorded CETEEMKA position while encircling the antenna. Generally all results confirm the correct algorithm operation.

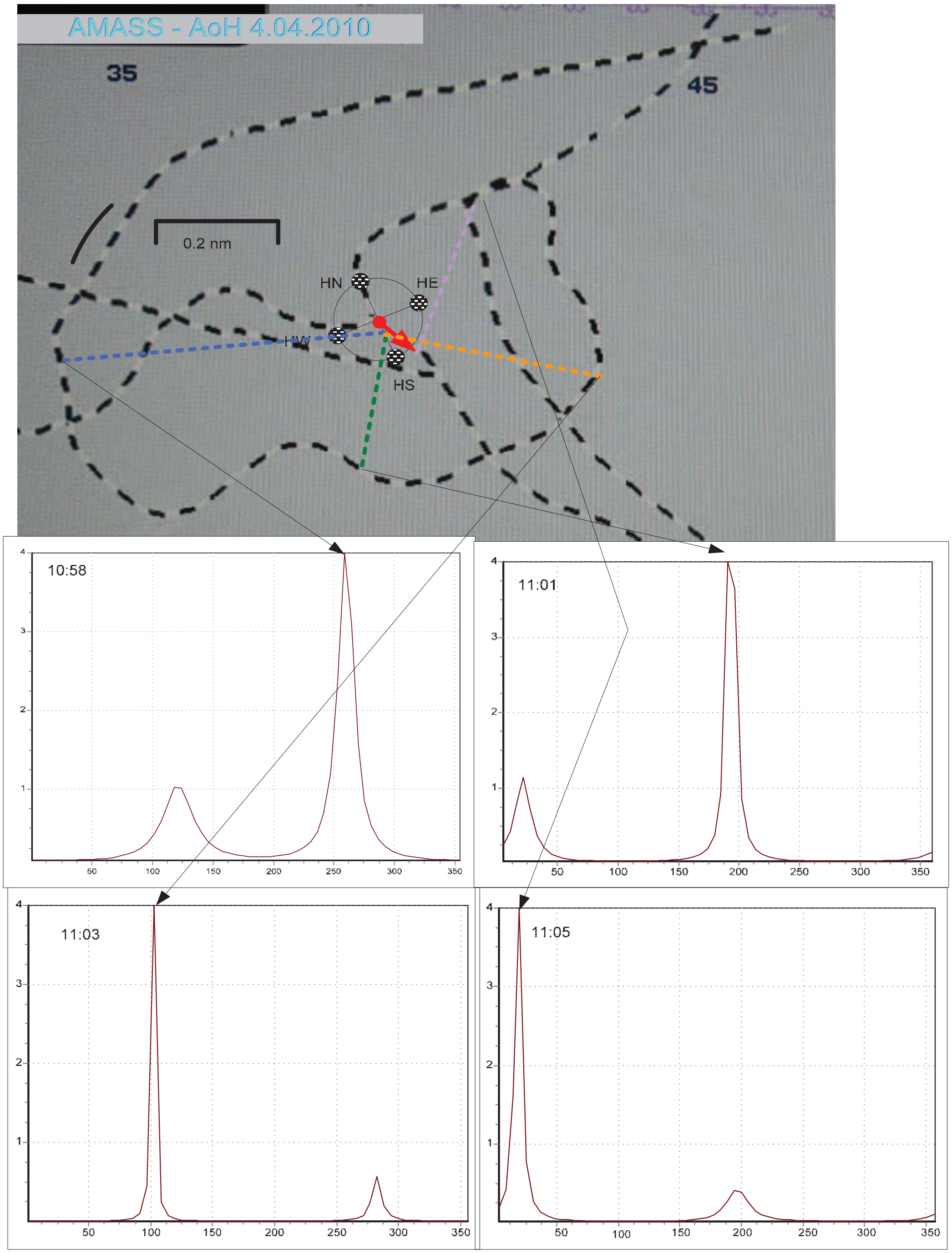

Fig.8. PMU criterion for 4-sensor antenna.

7. FIVE SENSOR ANTENNA

The simulation results are shown in the figure 9. In case of 5-sensor antenna the determination is omni-directionally possible even for r/λ>0.5 ratio.

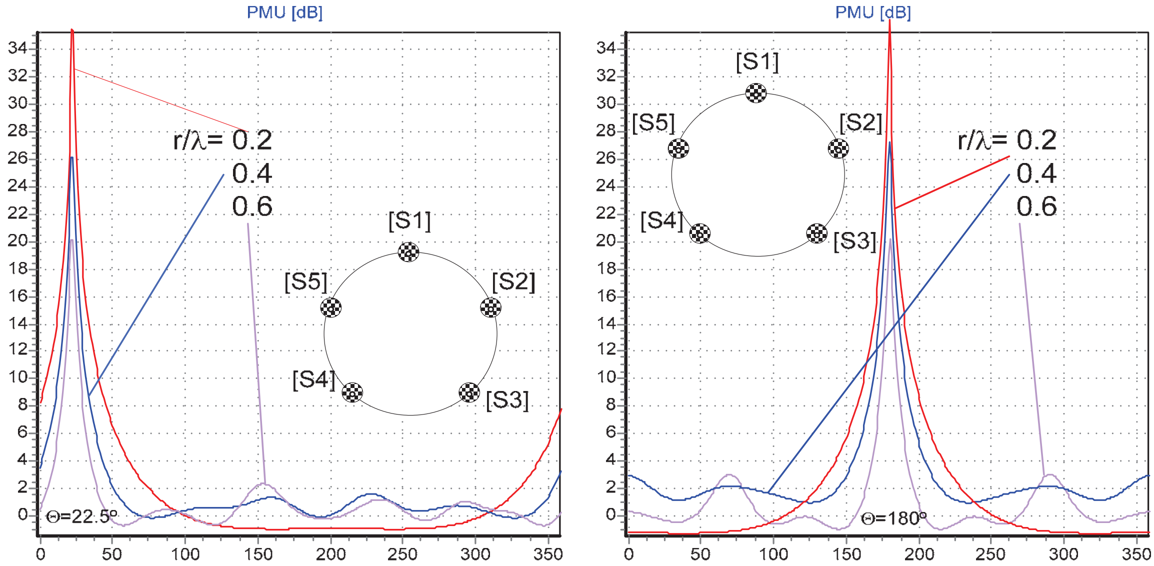

Fig.9. PMU criterion for 5-sensor antenna and single target.

Additional important feature of this antenna is the ability to simultaneously detect two targets.

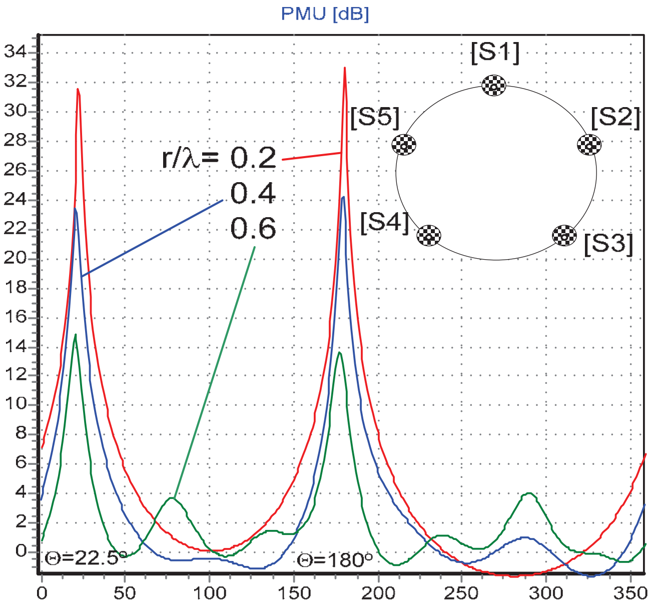

Fig.10. PMU criterion for 5-sensor antenna and two target.

Also in this case the maximums of PMU function are unique. The worst detection corresponds to r/λ>0.5 ratio.

8. CONCLUSSIONS

The 5-sensor circular array is the most efficient taking into account the equipment cost and detection propriety. It enables simultaneous detection of up to two targets additionally confirmed at two frequency modes corresponding adequately to r/λ=0.2 and r/λ=0.4 ratios. The compliance condition of two PMU maxims for both frequency modes decreases the false alarm probability. Such antenna can be applied in construction of various maritime surveillance systems.

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