SOME SEA BOTTOM BACKSCATTERING MODELS AND ESTIMATION OF THEIR USEFULNESS FOR EMPIRICAL DATA ANALYSIS

ŁUKASZ NOWAK

Polish Naval Academy in Gdynia
Śmidowicza 69, 91-103 Gdynia, Poland
ukasz333@wp.pl

The paper introduces two different methods of numerical computation of the time proceeding of low-frequency acoustic pulse reflected from layered sea bottom. The input of both mathematical models are time proceedings of known sampling pulses and parameters describing determined seabed structure. Parameters necessary for complete bottom description are number of sediment layers and mean grain size of specific type of sediment creating each layer. Two introduced approaches strongly differ from each other in complexity, computation time and domain, number of phenomenon taken into account in formal description of considered process. First of the presented methods is based on time analysis of sampling pulse propagation, second one treats the propagation media as a system characterized with given transmittance. The goal of introduced investigations is to estimate usefulness of results obtained with those two mathematical models for analysis empirical data gained using parametric echosounder for seabed structure examination.

INTRODUCTION

The main goal of performed mathematical modeling and computer simulations of acoustic pulses backscattering on seabed was to deliver data, that might have been used as a point of reference for empirical data analysis. Experimental examination of the sea bottom structure is conduct using parametric echosounder, which characterizes with very narrow main lobe and side lobes low enough to be skipped in described considerations. Two different methods of computations are considered. First approach – more general and detailed – is based on time domain analysis of the sampling pulse propagation in media characterized with determined structure and physical properties. Second method takes into account some further approximations consequent to specific features of measurements performed using parametric echosounder, that allows to treat the propagation media as a system described with given transmittance. The reflected pulse can be then determined as a response of this system for given excitation. Both algorithms require knowledge of the sampling pulse time proceeding and description of the propagation environment structure. Sampling pulses generated by the parametric echosounder have been recorded during experimental investigations performed under laboratory conditions, using hydrophones placed in front of the echosounder`s antenna. Complete description of the seabed structure consist of number of sediment layers and one parameter – mean grain size - describing each sediment layer. All of the necessary physical properties of the sediments are computed using empirical relations that connect them with this single input parameter.

Usefulness of the results obtained with described mathematical models depends mostly on possibility of matching them to experimental data – exact fit would suggest that seabed structure in the area where explorations took place is similar to structure assumed in computations. Some other, indirect simulations results can also be helpful for understanding processes under consideration and proper analysis of empirical data. Not only the accuracy of the theoretical echo pulse form forecasts is important, complexity and time of computations play vital role as well. Precise data matching requires performing lots of numerical simulations and if single process would take several hours or more, then the usefulness of such method would be limited.

1. SEABED ACOUSTIC BACKSCATTER MODEL

Source *Z* of acoustic pulses is considered. The source is situated under the water surface, distance to seabed consisting of sediment layers equals *a*. Sources directivity is described with given function $b(\theta_i)$. Duration time of single pulse equals τ_p. Sound propagation velocity in the water is v_w. Pulse, that incidents on the seafloor surface, partially reflects from the layers border and partially penetrates into sediments. Described situation is illustrated on Fig.1.

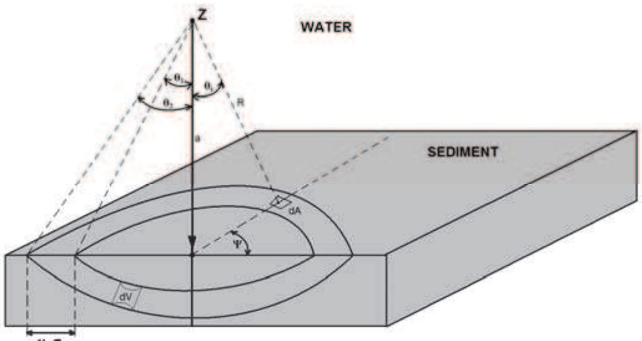

Fig.1. Acoustic pulse propagation geometry

Intensity of the backscattered echo pulse can be delineated as a sum of two components [4]:

$$I(t)=I_i(t)+I_v(t). \tag{1}$$

Component $I_i(t)$ delineates intensity of the pulse backscattered on rough seabed surface and component $I_v(t)$ describes intensity of signal backscattered in volume of inhomogeneous sediment layer.

Seafloor surface is not perfectly flat, it characterizes with large- and small scale roughness which causes incident wave backscattering. Either horizontal or vertical magnitude of the roughness are consistent with broad range of values from single sediment grain size up to several kilometers high and thousands of kilometers long oceanic ridges. There are many mathematical models that allows to compute parameters of surface backscattered wave. Most proper of them should be selected taking into account specification of the problem to be solved: parameters of the generated sampling pulses, incident angles, surface of the radiated area and seabed properties. For considered issues following equation delineating intensity of the wave backscattered on the seafloor surface can be used [2]:

$$I_i(t)=\int_{0}^{2\pi}\int_{\theta_i=\theta_1(t)}^{\theta_2(t)} I_x\left(t-\frac{2R}{v_w}\right)\frac{s_i(\theta_i)b^4(\theta_i)}{R^4 10^{\alpha_w R/5}}dA, \quad (2)$$

Where α_w is exponential attenuation coefficient and $s_i(\theta_i)$ is surface backscatter coefficient. Assuming that the beam generated by the source *Z* is narrow and the incident angle is less than 20° considered issue can be reduced to solving the Helmholz integral using Kirchhoff`s approximation. Proper equation has been given by Jackson [2]:

$$s_i(\theta_i)=\begin{cases} R^2\left[8\pi\cos^2(\theta_i)\sin^2(\theta_i)\right]^{-1}\int_0^{\infty}\exp\left(-qu^{2\alpha}\right)J_0(u)udu & \theta_i>0 \\ R^2\left[8\pi\alpha\right]^{-1}C_\xi^{2/\alpha}\left(2k^2\right)^{(\alpha-1)/\alpha}\Gamma(1/\alpha) & \theta_i=0 \end{cases}. \quad (3)$$

$J_0(u)$ is zeroth order Bessel`s function of the first kind, *k* is wavenumber of incidenting wave and $\Gamma$ denotes Euler`s gamma function. *α* coefficient is equal:

$$\alpha=\left(\frac{\gamma}{2}\right)-1, \quad (4)$$

Where *γ* is spectral exponent. For most types of prevalent seafloor surfaces it can be assumed that *γ=3,25* [5]. Coefficient C_ξ in equation (3) is structural constant, defined as [2]:

$$C_\xi^2=\frac{\left[2\pi w_2\Gamma\left(3-\frac{\gamma}{2}\right)2^{-(\gamma-2)}\right]}{\left[\left(\frac{\gamma}{2}-1\right)\left(2-\frac{\gamma}{2}\right)\Gamma\left(\frac{\gamma}{2}\right)\right]}, \quad (5)$$

where w_2 is estimated value of spectral strength of the seabed roughness magnitude. This value depends on type of sediment creating top seafloor layer and it can be computed using known empirical relations [5].

The *q* coefficient, present in integral core in equation (3) is delineated with following relation [2]:

$$q=\sin^2\theta_i\cos^{-2\alpha}\theta_i C_\xi^2 2^{1-2\alpha}k^{2(1-\alpha)}. \quad (6)$$

Computation of backscattering coefficient $s_i(\theta_i)$ for incident angles different than 0° is performed numerically. Calculations must take into account oscillation character of integral core and rapid changes of integrated function proceeding caused by small changes of input parameters describing seafloor parameters, signal frequency and incident angle. Described problem is illustrated on Fig.2. In case of slowly evanescent proceedings (i.e. Fig.2a)) integral is computed with determined precision for specified number of intervals defined by Bessel`s function zeros. If calculations require involving large number of intervals, spaces between further of them can be assumed to be equal π. In case of fast evanescent proceedings (i.e. Fig.2c)) it is important to make an estimation of integration limits. Because of very steep slopes of integrated function proceedings in such cases fair compromise between time and accuracy of numerical calculations has to be made.

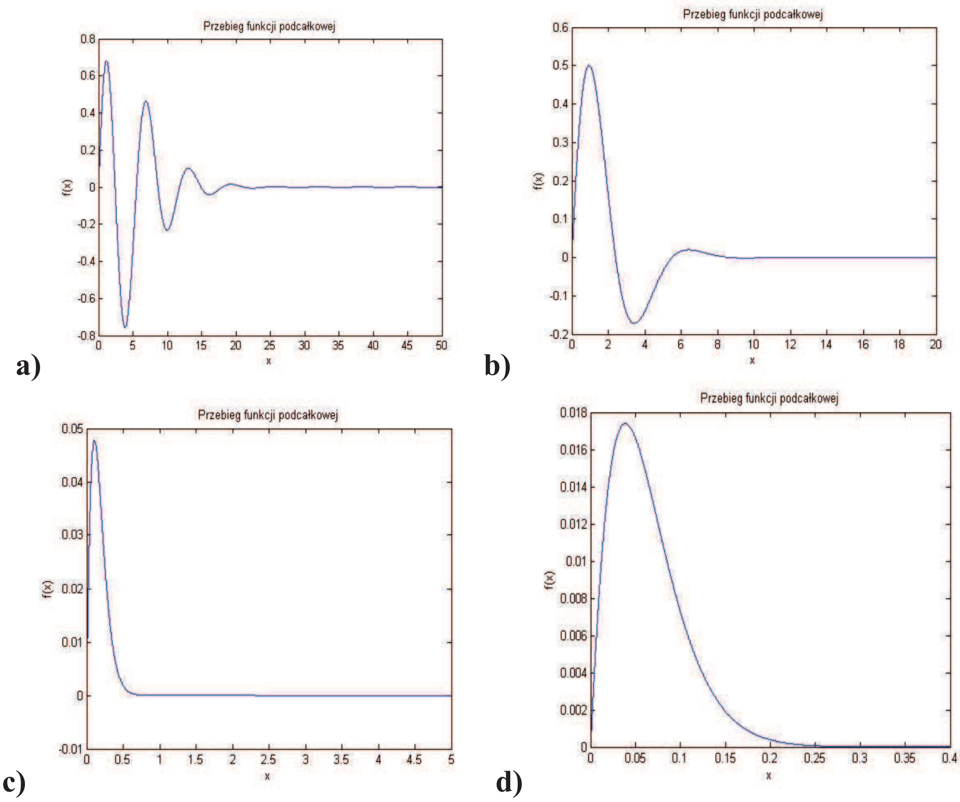

Fig.2. Proceedings of integral core from equation (3) for different values of signal frequency, incident angle and type of sediment creating top seafloor layer:
a) sediment: clay, frequency: 5 kHz, incident angle: 0,4 rad
b) sediment: clay, frequency: 5 kHz, incident angle: 0,1 rad
c) sediment: fine sand, frequency: 15 kHz, incident angle: 0,4 rad
d) sediment: gravel, frequency: 5 kHz, incident angle: 0,4 rad

To estimate integration limit following equation is solved using iteration method:

$$u = -\frac{1}{q}\log\left(\frac{d}{u'}\right)^{\frac{1}{2\gamma}}, \qquad (7)$$

where d is computation accuracy and u' is value obtained in previous iteration. Calculations are repeated until accuracy is less or equal desired value.

Presented algorithm asserts relatively short time of computations and freely chosen value of accuracy, independent of the input parameters. Some exemplary relations between incident angle and backscattering coefficient value for specified types of sediments are presented on Fig.3.

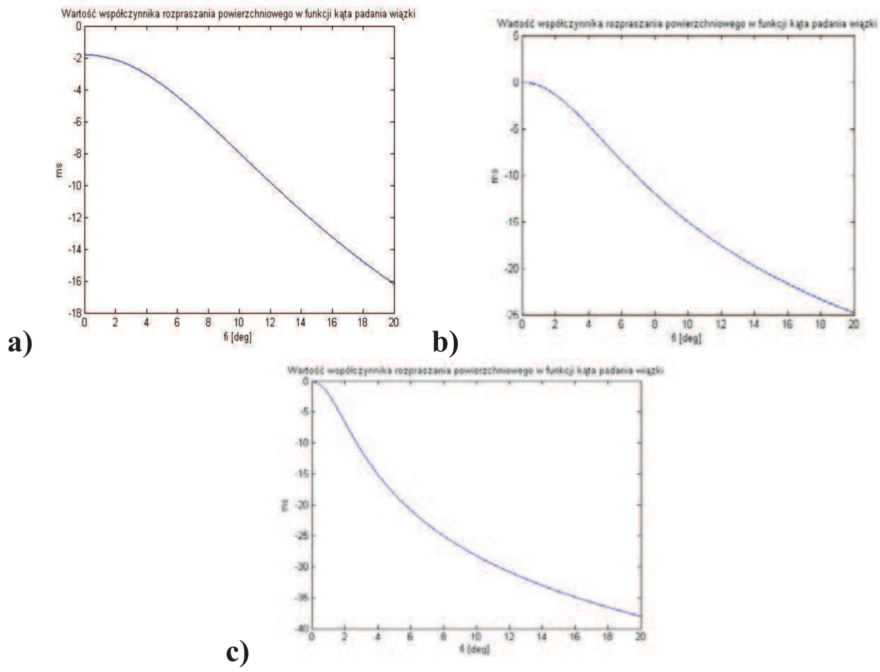

Fig.3. Values in dB of backscattering coefficient as a function of incident angle for different sediment types and signal frequency:
a) sediment: coarse sand, frequency: 15 kHz
a) sediment: fine sand, frequency: 15 kHz
a) sediment: silt, frequency: 15 kHz

Pulse component, that does not reflect from the seafloor interface penetrates into sediment layer. Time proceeding of intensity of signal backscattered on the volume inhomogeneities can be described with following relation [2]:

$$I_v(t)=\int_0^{2\pi}\int_{\theta_i=0}^{\theta_2(t)} I_x\left(t-\frac{2R}{v_w}\right)\frac{\sigma_v b^4(\theta_i)}{R^4 10^{\alpha_w R/5}} V_l(\theta_i)\times\left[\int_{l_1(t)}^{l_2(t)} I_x\left(\tau_p-\frac{l-l_1(t)}{v_b}\right)e^{-2\beta_e l}\,dl\right]dA \quad , \qquad (8)$$

where v_b is sound propagation velocity in the sediment, β_e is attenuation coefficient and σ_v is volume backscatter coefficient. All of those parameters can be computed using empirical relations connecting them with mean grain size of the sediment. Integration limits $l_1(t)$ and $l_2(t)$ denotes distance between beginning and end of the pulse of length l propagating in the sediments. $V_l(\theta_i)$ is transmission coefficient trough rough water-sediment interface, defined by equation [4]:

$$V(\theta_i) = \frac{1}{\sqrt{\pi}\varsigma} \int_{-\left(\frac{\pi}{2}-\theta_i\right)}^{\infty} V_f(\theta_i - \vartheta) \exp\left(-\frac{\vartheta^2}{\varsigma^2}\right) d\vartheta \quad , \tag{9}$$

where $V_f(\theta_i)$ is transmission coefficient defined by following relation [4]:

$$V_f(\theta_i) = \left[1 - R^2(\theta_i)\right]^2 \cos^2(\theta_i) \left[1 - (\nu \sin(\theta_i))^2\right]^{-1/2} . \tag{10}$$

$R(\theta_i)$ is reflection coefficient from a flat surface. All of the above relations concern wave incidenting on water-sediment interface. Since layered seafloor structure is under consideration, similar computations should be performed for each of the borders dividing sediment layers, which lay on the sampling pulses propagation path. If we assume, that interfaces between consequent seafloor layers are similar – in terms of roughness magnitude distribution – to water-sediment border, then we can modify equations above and, using proper values of physical parameters characterizing the propagation media, compute pulse backscattered in whole considered seabed structure.

Described algorithm has been implemented using Matlab. Sampling pulses recordings were obtain during laboratory measurements using parametric echosounder and hydrophones placed in front of the antenna. Example of recorded pulse envelope is illustrated on Fig.4.

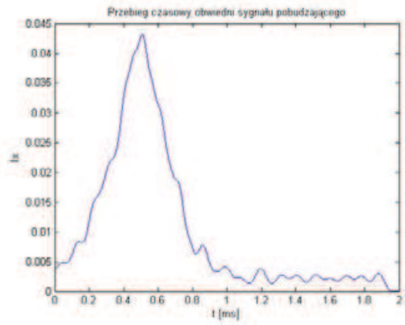

Fig.4. Envelope of the 5 kHz sampling pulse obtained using parametric echosounder

Result of example simulation of the pulse from Fig.4. backscattered on the seafloor created by two sediment layers: top, one meter thick, silt layer and bottom fine sand layer is presented on Fig.5. Two maximums correspond to two borders between different propagation media. Observed time extension of the echo pulse is caused by volume backscattering.

Effects of trials of matching the simulations results to true echo pulse recordings, obtained during experimental investigations proved poor effectiveness of considered method. Single computation, even in simple cases with very few sediment layers, can take even several hours on standard PC. Because exact matching requires many simulation steps and taking into account corrections resulting from comparison results of the previous step to empirical data, time is important factor in whole process. Except from described problem, assumed computation algorithm proved to be unable to deliver gratifying results that would fairly approximate the experimental data. That was probably caused by inability of assuring accurate enough information describing propagation environment: physical parameters of the sediment layers, volume inhomogeneities and properties of interfaces between layers.

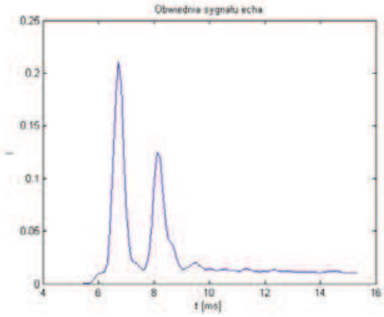

Fig.5. Computed envelope of the pulse backscattered on layered seabed

Nevertheless, described mathematical model can still be useful i.e. for recognizing some types of top sediment layers and helping to estimate the influence of some seafloor physical parameters (i.e. estimated value of surface backscattering coefficient for specified type of bottom and sampling signal properties). It can also complement in some cases results obtained with different simulation techniques.

2. TRANSMMITANCE MODEL

Another simulation algorithm, described widely in [6], takes into account more detailed and specific parameters describing measurement equipment that allows some further going simplifications and approximations. Parametric sampling pulse generated by echosounder is characterized by low (5 – 20 kHz) frequency, very wide main lobe with almost no side lobes [1]. During measurements the beam is pointed vertically. Due to those facts we simplify our earlier assumptions and consider plane wave perpendicular reflection from a stack of parallel layers with absolutely flat interfaces. Basing on empirical relations connecting specific type of sediment with its physical properties, such as sound propagation velocity, density and attenuation coefficient [5] we can compute the reflection coefficient on each border and transmission coefficient through each layer [3][6]. It is convenient to use the Fourier transform and conduct further calculations in frequency domain. Thereafter the propagation environment can be treated as a system described with given transmittance. Time proceeding of the echo pulse can be fixed as a inverse Fourier transform of response of this system to known excitation.

Described algorithm has been implemented using Matlab [6]. Many simulations were performed using this implementation to estimate usefulness of the method for considered purposes. Results of computations were compared to true echo pulses recordings obtained during experimental measurements. Theoretical and empirical reflected pulse time proceedings are presented on common chart, example of such collation of the results is presented on Fig.6.

Important advantage of described mathematical model is very short computation time – even for complex sea bottom structures consist of many sediment layers calculations take usually no more than several minutes on standard PC. This allows to perform many steps of simulations in short time period and inputting many corrections to assumed seafloor structure for exact match with empirical data.

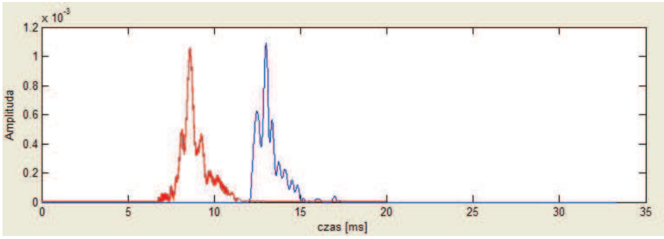

Fig.6. Theoretically (red) and experimentally (blue) obtained envelope of the echo pulse reflected from layered seafloor

Similarity of the shapes of signal envelopes presented on Fig.11. suggests, that seafloor structure in measurement area could consist of sediment layers of type and thickness close to assumed in theoretical considerations. Three maxima visible on both charts may be caused by the pulse reflection from three subsequent sediment layers. The middle layer is characterized with the greatest impedance contrast with surrounding media. Structure that would match sought pattern could be silt – coarse sand – fine sand, underlayed by layers of silt and gravel that cause observed further echo signal extension.

3. SUMMARY

Both of the presented simulation algorithms can be useful for empirical data analysis in specific cases, but none of them provide universal technique for determining reflected pulse for freely chosen environmental conditions and signal parameters. The more seafloor structure is close to ideal flat-parallel theoretical case, the better effects will bring the transmittance model, which allows performing large amount of computations in short time period. More detailed and complex analysis of signal backscattering on water-sediment interfaces and sound propagation in seafloor sediments may be often helpful for better understanding of proceeding processes and estimating error committed with more simplified simulation techniques.

ACKNOWLEDGMENTS

The investigation was supported by the Ministry of National Defence (Grant No DNiSW/U/13/BN/1.21/2008/848).

REFERENCES

[1] Innomar Technologie GmbH, *SES-2000 Narrow-beam parametric sub-bottom profilers*, Innomar Technologie GmbH, Rostock, 2008.

[2] Jackson D.R., Winebrenner D.P., Ishimaru A., *Application of the composite roughness model to high-frequency bottom backscattering*, Journal of Acoustical Society of America 79(5).

[3] Salomon R., *Systemy Hydrolokacyjne*, Gdańskie Towarzystwo Naukowe, Gdańsk 2006.

[4] Sternlicht D.D., de Moustier Ch.P., *Time-dependent seafloor acoustic backscatter (10-100 kHz)*, Journal of Acoustical Society of America 114(5), 2003.

[5] Tęgowski J., *Akustyczna klasyfikacja osadów dennych*, Instytut Oceanologii PAN, Sopot 2006.

[6] Nowak Ł., Szymczak W., *Investigation of the pulse reflected from bottom sediments*, Hydroacoustics vol.12, Gdańsk 2009.