MODELING OF SEA BOTTOM USING BÉZIER PIECES

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In the article the essence of creating Bézier piece, determination its points, degree elevation and division have been shown. These issues are essential during creation the digital terrain model DTM. For creation the sea bottom model bathymetric surveys of Slupsk Bank have been used. The visualization of sea bottom with detected underwater object recorded by multibeam echosounder using Bézier pieces has been presented.

INTRODUCTION

Modeling of surfaces in hydroacoustics and hydrography have many applications, e.g. for modeling of sea bottom and surface of constant sound speed in water [2, 10, 12]. There are used well known methods and developed new algorithms, which are used in computer graphics [1, 3, 4, 5, 8, 9, 11, 13, 14, 15]. Below another method has been presented.

1. DETERMINATION OF THE PIECE

Rectangular Bézier pieces (aka tensor Bézier pieces) of *n* – degree in relation to *u* variable and *m* – degree in relation to *v* variable ((*m,n*) – degree) are described by the equation [1]:

$$\boldsymbol{p}(u,v)=\sum_{i=0}^{n}\sum_{j=0}^{m}\boldsymbol{p}_{i,j}\boldsymbol{B}_i^n(u)\boldsymbol{B}_j^m(v) \tag{1}$$

For determination the piece of (*m,n*) – degree, there is necessary to give (*n*+1)(*m*+1) control points of $\boldsymbol{p}_{i,j}$. The set of segments connecting control points, which only one index differs from 1, is called control frame of the piece. In this control frame we distinguish rows, i.e. broken lines with $\boldsymbol{p}_{0j},\ldots,\boldsymbol{p}_{nj}$ points for established *j*, and columns, i.e. broken lines with $\boldsymbol{p}_{i0},\ldots,\boldsymbol{p}_{im}$ points for established *i*.

The method for determination the piece – using tensor base created from functions used for determination Bézier curves – makes possible to use for rectangular Bézier pieces all of theorems and algorithms connected with curves. We can observe, that:

- $\boldsymbol{B}_i^n(u)\boldsymbol{B}_j^m(v)$ functions for $i = 0,\ldots,n$, $j = 0,\ldots,m$ on the basis of relationship $\sum_{i=0}^{n}\boldsymbol{B}_i^n(t) = 1$ determine distribution of one. The picture of control frame in any affine transformation determines the picture of the piece in this transformation, because determination them is independent from control points' selection of the piece,
- because on the section [0,1] Bernstein polynomials are non-negative, so functions of tensor base are non-negative in the rectangle [0,1]×[0,1], therefore point $\boldsymbol{p}(u,v)$ for $u,v \in [0,1]$ is in convex border of control points set,
- because $\boldsymbol{B}_i^n(0)\boldsymbol{B}_j^m(0) = 0$ for $i \neq 0$ or $j \neq 0$, so $\boldsymbol{p}(0,0) = \boldsymbol{p}_{00}$. Similarly, in remaining corners: $\boldsymbol{p}(1,0) = \boldsymbol{p}_{n0}$, $\boldsymbol{p}(0,1) = \boldsymbol{p}_{0m}$, $\boldsymbol{p}(1,1) = \boldsymbol{p}_{nm}$. Extreme rows and columns of the control frame determine border curves of the piece.

2. DETERMINATION OF PIECE'S POINTS

Relationship (1) can be written in the form [1]:

$$\boldsymbol{p}(u,v) = \sum_{i=0}^{n}\left(\sum_{j=0}^{m}\boldsymbol{p}_{i,j}\boldsymbol{B}_j^m(v)\right)\boldsymbol{B}_i^n(u) = \sum_{i=0}^{n}\boldsymbol{q}_i\boldsymbol{B}_i^n(u) \tag{2}$$

or [1]:

$$\boldsymbol{p}(u,v) = \sum_{i=0}^{n}\sum_{j=0}^{m}\boldsymbol{p}_{i,j}\boldsymbol{B}_i^n(u)\boldsymbol{B}_j^m(v) \tag{3}$$

Determination of the point $\boldsymbol{p}(u,v)$ for established numbers (*u,v*) can be amounted to determination of points on curves in two ways. Points q_i on curves, which control broken lines are columns of the piece's frame, are control points of the curve with constant *v* parameter on the surface. In another case we change columns and rows. For calculation can be used any algorithm for determination the point on the curve – for example de Casteljau one.

De Casteljau algorithm for the piece application can be generalized. Let us assume, that one step of de Casteljau algorithm is simultaneously carried out on all rows of the frame, obtaining the frame with number fewer 1. Signing $p^{(kl)}$ the piece, which the frame arose as a result of *k* steps of de Casteljau algorithm on frame's rows, and next *l* steps on frame's columns. This same piece can be obtained performing these steps in free sequence. The surface $p^{(mn)}$ of 0 – degree in relation to *u* and *v* is reduced to one point $\boldsymbol{p}(u,v)$.

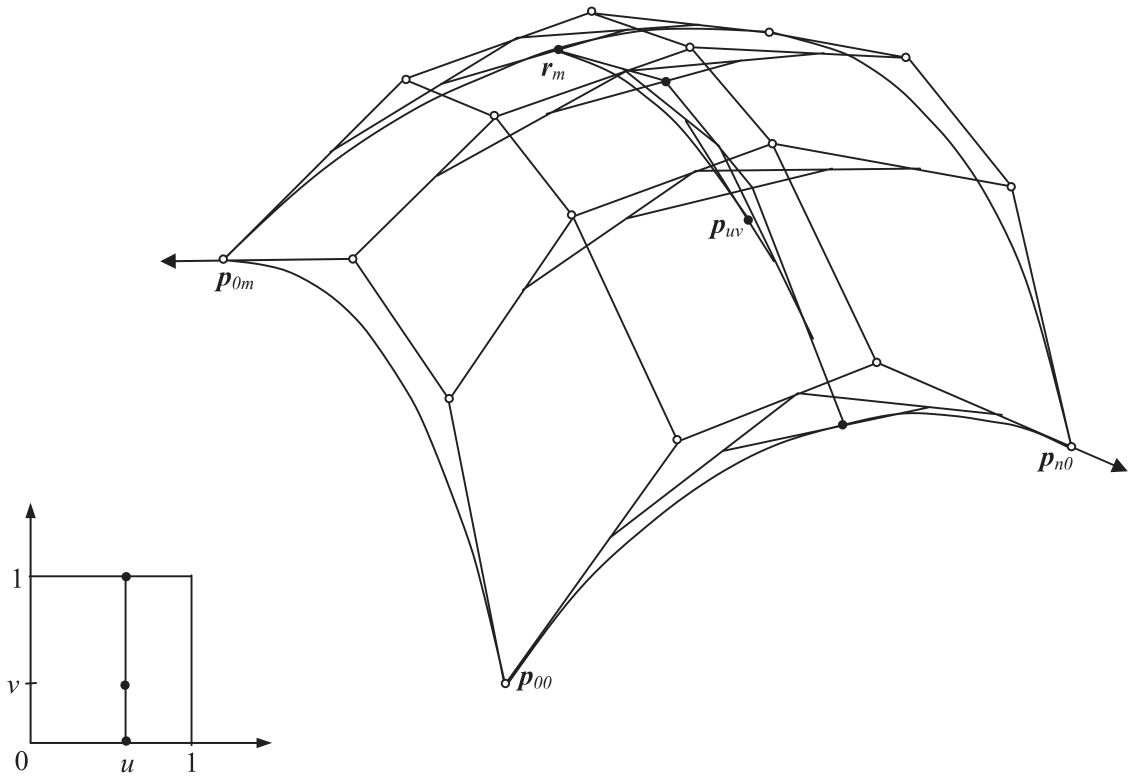

Fig.1. Determination the pieces's point as a result of determination of points on curves

3. DEGREE ELEVATION OF THE PIECE

For degree elevation in relation to *u* (for obtaining the representation of the piece in the tensor base $\{\boldsymbol{B}_i^{n+1}(u)\boldsymbol{B}_j^m(v): i=0,...,n+1, j=0,...,m\}$), the procedure of degree elevation can be used for all of rows in control frame of the piece. Similarly, for degree elevation in relation to *v*, the procedure can be used for all of columns. (Fig. 2)

Repeatedly degree elevation of the piece degree in relation to both variables give the sequence of control frames with points (control points corresponding to bases higher and higher degrees) closer to the piece.

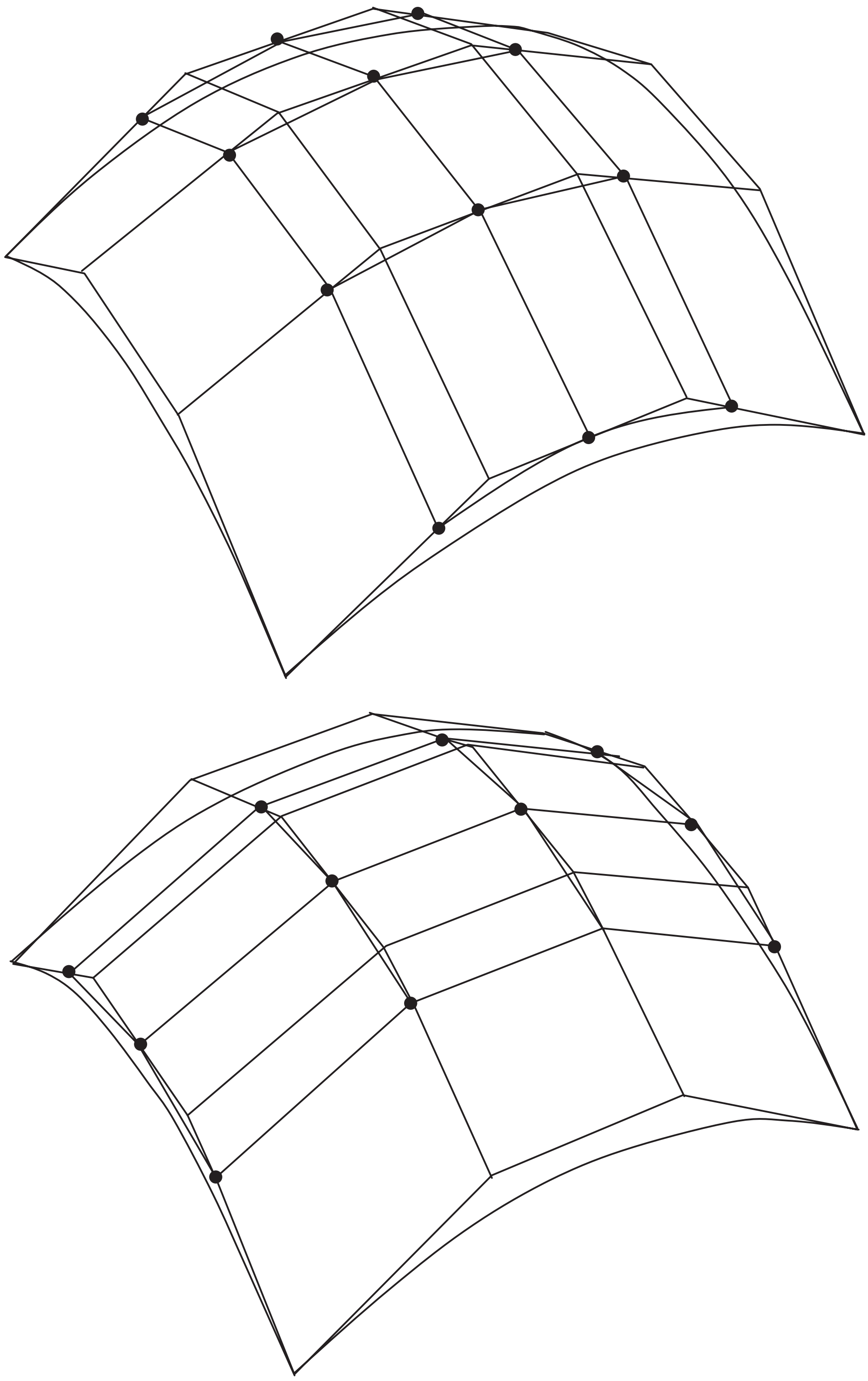

Fig.2. Degree elevation of the Bézier piece

4. DIVISION OF THE PIECE

Using de Casteljau algorithm for determination $\boldsymbol{q}_j$ or $\boldsymbol{r}_i$ control points of the curve with constant parameters: adequately v = const or u = const of the piece $\boldsymbol{p}$ (e.g. for determination the point $\boldsymbol{p}(u,v)$) we calculate points of broken lines, which are a result of division of rows or columns of the frame. These frames make control frames of piece's part corresponding to sets of parameters: [0, 1] × [0, v] and [0,1] × [v, 1] or [0, u] × [0, 1] and [u, 1] × [0, 1] (Fig. 3). It can be used for adaptive division of the piece for displaying it, similarly to adaptive, recurrent division of the curve. The piece can be divided until receiving "sufficiently flat" control frames (on the basis of convex property we obtain "sufficiently flat" piece's parts) and next – e.g. triangles – can be displayed.

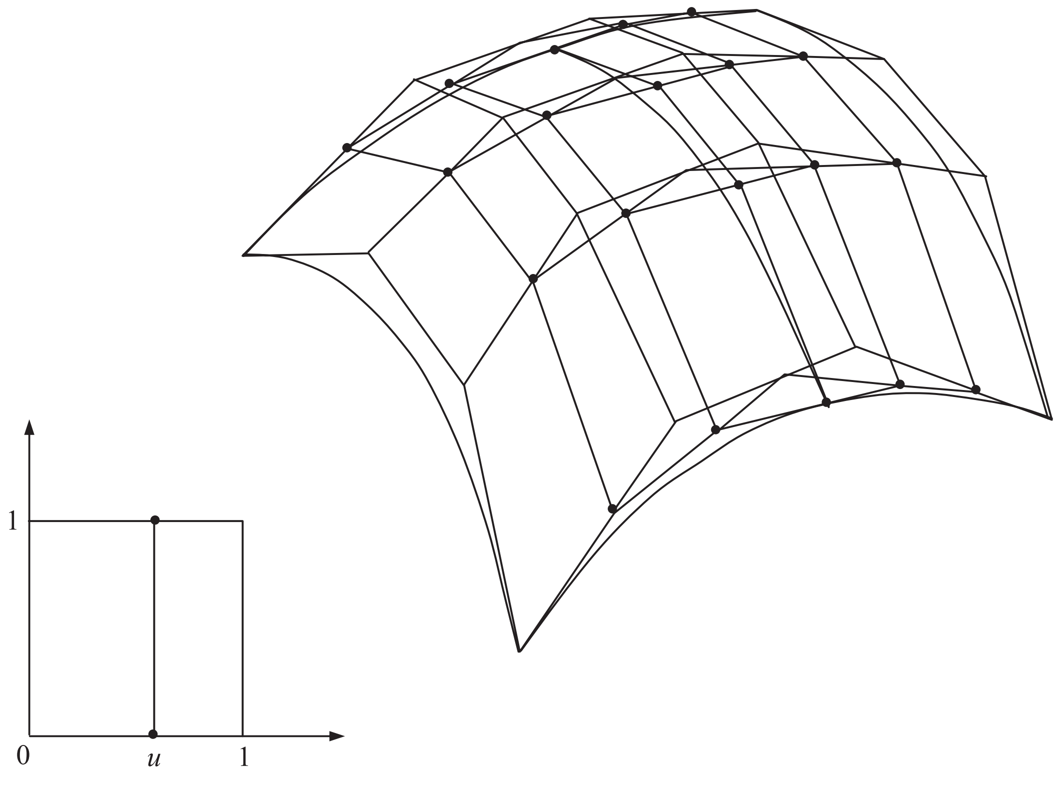

Fig.3. Division of the Bézier piece

4. RESULTS

Below the 2D and 3D visualizations have been presented. Results of hydrographic surveys of the Slupsk Bank realized by hydrographic vessel OH266 have been used. For depth measurements have been used hydrographic echosounders: multibeam echosounder Simrad EM3000D, singlebeam echosounders Simrad EA400 and Atlas Deso 25.

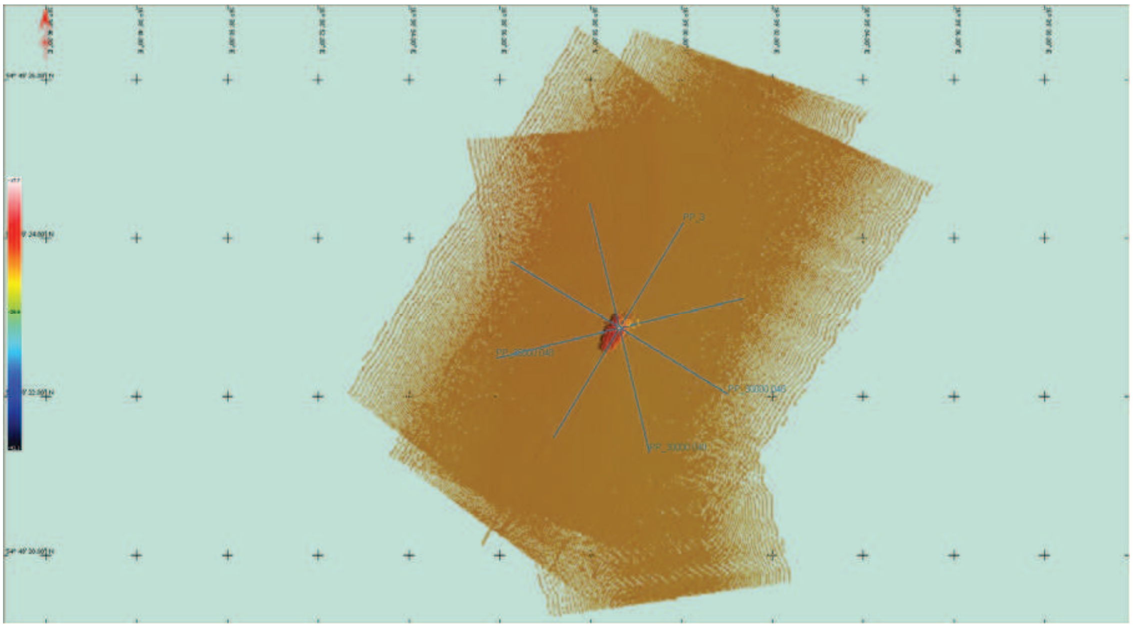

Fig.4. Visualization of detected object by multibeam echosounder using Qinsy application

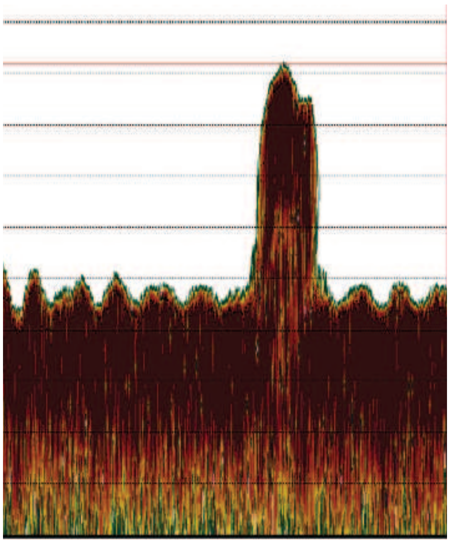

Fig.5. Echogram of the detected object

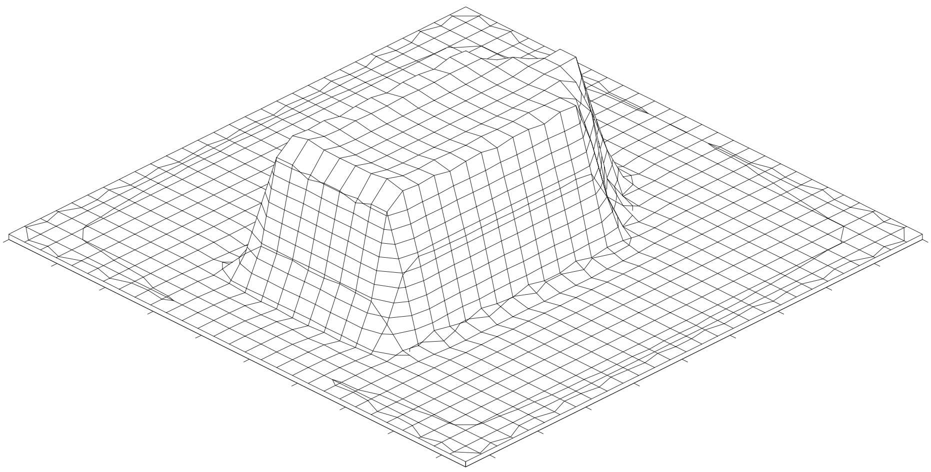

Fig.6. 3D visualization of the sea bottom and detected object

5. CONCLUSIONS

Bézier pieces are another method used in modeling of surfaces. Essential issue during modeling is piece's connecting for obtaining the continuity of selected degree and is direct generalization of the method of curves connecting. Rows or columns of frames are connected as curve's broken lines. Imposing of the condition of arising "from grater piece's division" on another column or row equally distant to control broken line of the border curve increasing the degree of connection continuity. This interpretation is direct analogy to connections continuity's conditions of Bézier curves.

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