JOURNAL OF THEORETICAL
AND APPLIED MECHANICS
4, **36**, 1998

DYNAMICS OF THERMALLY ACTIVATED SHAPE MEMORY ALLOY REINFORCED LAMINATED BEAMS

MAREK PIETRZAKOWSKI

Institute of Machine Design Fundamentals, Warsaw University of Technology
e-mail: mpi@simr.pw.edu.pl

A way of use of shape memory alloy (SMA) fibers for dynamic response modification of laminated beams is presented in the paper. The considered concept of adaptive control is based on the ability to change the Young modulus during a temperature activated, reversible, martensitic transition that is specific to SMAs. It is assumed that the laminate is midplane symmetric and the corresponding layers of the same fiber orientation are activated simultaneously, and the SMA fibers can freely elongate in the matrix. A cantilever laminated beam with mass distributed on its free edge is subject to numerical analysis. Due to the quasi-steady one-dimensional model of heat conduction, martensite fraction and natural frequency time relations for the phase transition of SMA have been obtained. The influence of temperature on natural frequencies of the system has been analysed for some different locations of activated plies and directions of SMA fibers. The frequency response functions varying with the material phase transformation for a harmonic load acting on the free end of the beam have also been presented.

Key words: laminate, thermal activation, natural frequencies

1. Introduction

Shape Memory Alloy (SMA) materials, because of their unique mechanical behaviour, like the Shape Memory Effect (SME) and pseudoelastic effect, have been utilised in many innovative engineering applications also in technology, robotics, microelectronics and biological implants. The ability to change and then recover large (apparently plastic) strain is a result of the martensite phase transition which can be reversed due to the temperature or stress. Experiments on SMA materials have proved that both the Young modulus and internal friction depend strongly on the martensite fraction (Jackson et al., 1972). Therefore, the SMA reinforced composites can be used in the active structural vibration control (Liang and Rogers, 1991) and buckling control (Rogers et al., 1989). In these cases, the SMA fibers within a composite are thermally driven to change the natural frequencies or alter the critical buckling load of the structure.

In this paper an application of SMA fibers to dynamic response modification of laminated beams is presented. The concept of active control is based on the ability of SMA material to change its stiffness when passed between austenite and martensite phases. A thermal response of the composite is obtained by considering an energy balance assuming the quasi-steady one-dimensional model of conduction (Wirtz et al., 1995) and the cosine phase transition model (Liang and Rogers, 1990).

The temperature and martensite fraction responses for the phase transition of SMA are calculated and applied to determination of time relations of the first natural frequency during the transition process. The influence of temperature on natural frequencies of the system has been analysed for some different locations of activated plies and directions of SMA fibers. The effect of material transformation from the martensite phase to the austenite one on the frequency response function (FRF) is also presented.

2. Model for the SMA transition process

The unique behaviour of SMA materials is caused by the reversible martensite phase transition. One of the most important characteristics of SMAs is the martensite volume fraction being a function of temperature. In the stress-free state, the martensite fraction changes as shown in Fig.1 (cf Liang and Rogers, 1990). The four temperature parameters are designated as martensite finish M_f, martensite start M_s, austenite start A_s, austenite finish A_f, respectively. Due to the effect of the stress induced phase transition, these parameters are related to the applied stress.

Based on the Liang and the Rogers model of the transition process (Liang and Rogers, 1990), the martensite fraction during the $M \to A$ and $A \to M$ transitions under stress-free conditions may be approximated by a cosine function of temperature as follows

$$\xi = \frac{1}{2}(1 + \cos \pi\theta_j) \qquad j = A, M \tag{2.1}$$

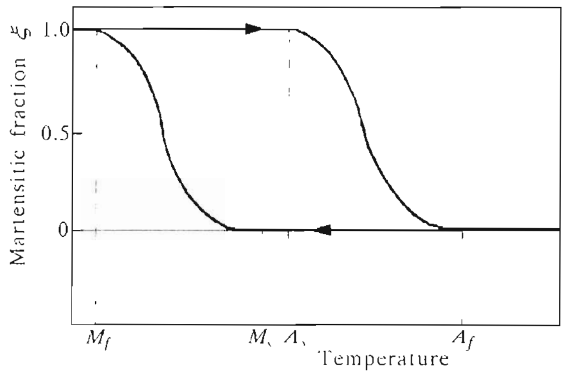

Fig. 1. Martensite fraction during transition versus temperature

where $\theta_j \in [0,1]$ is the nondimensional temperature of SMA defined separately for $M \to A$ (heating) and $A \to M$ (cooling) processes

$$\theta_A = \frac{T - A_s}{A_f - A_s} \qquad \theta_M = \frac{T - M_f}{M_s - M_f} \tag{2.2}$$

Herein, it was assumed that both the transition processes are complete and start from the state of purely martensite or austenite phase, respectively.

Experimental evidence shows that the elastic modulus of SMA depends strongly on the martensite fraction. For example, the Young modulus of nitinol increases by 3 to 4 times within the $M \to A$ transition temperature range (Jackson et al., 1972). It may be assumed that the Young modulus relates linearly to the martensite fraction

$$E = E_A + (E_M - E_A)\xi \tag{2.3}$$

where

E	–	Young modulus at temperature T
E_A, E_M	–	Young moduli in the austenite and martensite phases, respectively.

The SMA fibers can be heated by passing a current through them, and be cooled due to conduction by stopping current flow and by reducing the ambient temperature.

Assume that SMA fibers are initially kept in the martensite phase at the temperature A_s by steady application of the power P_0, and then at time $t \geqslant 0$ the additional power ΔP is supplied increasing the temperature T and changing the martensite fraction ξ from 1 to 0 during the phase transition.

The heat balance can be represented by the following equation (cf Wirtz et al., 1995)

$$(P_0 + \Delta P)dt = qdt - q_n \rho_n V_n d\xi + C_n dT \tag{2.4}$$

where

- q – time dependent rate of the heat lost via conduction through the laminate (the matrix with other fibers) to the heat sink at the ambient temperature T_0
- q_n – heat of SMA transition
- ρ_n – density
- V_n – volume of SMA fibers
- C_n – heat capacity with c_n the specific heat of the SMA.

The initial power P_0 is determined by the relation

$$P_0 = K(A_s - T_0) \tag{2.5}$$

where K is the conductance of the laminate.

Assuming the quasi-steady model of the heat conduction losses through the laminate and neglecting the thermal capacity of the matrix, $q(t)$ is obtained from the relation

$$q(t) = K[T(t) - T_0] \tag{2.6}$$

Substituting Eqs (2.1), $(2.2)_1$, (2.5) and (2.6) into Eq (2.4), the heat balance may be rewritten in dimensionless form

$$[1 - R_{MA}B(\theta_A)]\frac{d\theta_A}{d\tau} = R_p - \theta_A \tag{2.7}$$

where

- τ – nondimensional time, $\tau = Kt/C_n$
- R_{MA} – dimensionless heat of the $M \to A$ transition, $R_{MA} = q_n/[c_n(A_f - A_s)]$
- R_p – dimensionless heating power, $R_p = = \Delta P/[K(A_f - A_s)]$.

The function $B(\theta_A) = \partial\xi/\partial\theta_A$ relates to the phase transition model. In the considered case, since zero stress conditions are assumed the martensite fraction depends temperature only (see Eq (2.1)).

The cooling process starts from the purely austenite phase of SMA fibers which are maintained at the temperature M_s. When the ohmic heating is stopped the SMA fibers are cooled by conduction through the laminate material to the ambient temperature T_0. Neglecting other conduction processes, the heat balance analysis leads to the equation (cf Wirtz et al., 1995)

$$q_n \rho_n V_n d\xi - C_n dT - qdt = 0 \tag{2.8}$$

In the case of quasi-steady conduction cooling Eqs (2.1), $(2.2)_2$, (2.6) and (2.8) may be combined and written in the dimensionless form

$$[R_{AM}B(\theta_M) - 1]\frac{d\theta_M}{d\tau} = \theta_M + S \tag{2.9}$$

where

R_{AM} – dimensionless heat of the $A \to M$ transition, $R_{AM} = q_n/[c_n(M_s - M_f)]$

S – temperature ratio which is characteristic for conduction, $S = (M_f - T_0)/(M_s - M_f)$.

The solutions to Eqs (2.7) and (2.9) give the relation between the nondimensional time t and nondimensional temperature q_A or q_M for the $M \to A$ or $A \to M$ transitions, respectively.

3. Dynamical relations of laminated beam

Let us consider a thin, rectangular, symmetrically laminated one-dimensional plate with one edge clamped and mass uniformly distributed at the opposite edge as shown in Fig.2. The plate is composed of homogeneous orthotropic layers. Due to the control concept some of the layers are reinforced with SMA fibers. It is assumed that thermally activated SMA fibers can freely elongate in the matrix.

In most practical applications of thin plates an approximate plane stress state exists. A direct consequence of plane stress are the assumptions formulated by Kirchhoff that transverse shear and normal strains are negligible so the problem of flexural displacements can be reduced to a two dimensional study of the middle plane. According to the Kirchhoff hypothesis the flexural vibrations $w(x,y,t)$ of specially orthotropic plates are determined from the equation (cf Whitney, 1987)

$$D_{11}\frac{\partial^4 w}{\partial x^4} + 2(D_{12} + 2D_{66})\frac{\partial^4 w}{\partial x^2 \partial y^2} + D_{22}\frac{\partial^4 w}{\partial y^4} + \rho h\frac{\partial^2 w}{\partial t^2} = p \tag{3.1}$$

where D_{ij} are elements of the stiffness matrix, $p(x,y,t)$ represents an external load, $\rho = \frac{1}{h}\sum_k \rho_k h_k$ denotes the equivalent density, $h = \sum_k h_k$ is the total thickness while $h_k = z_k - z_{k-1}$ and ρ_k denote the thickness and density of the kth layer, respectively.

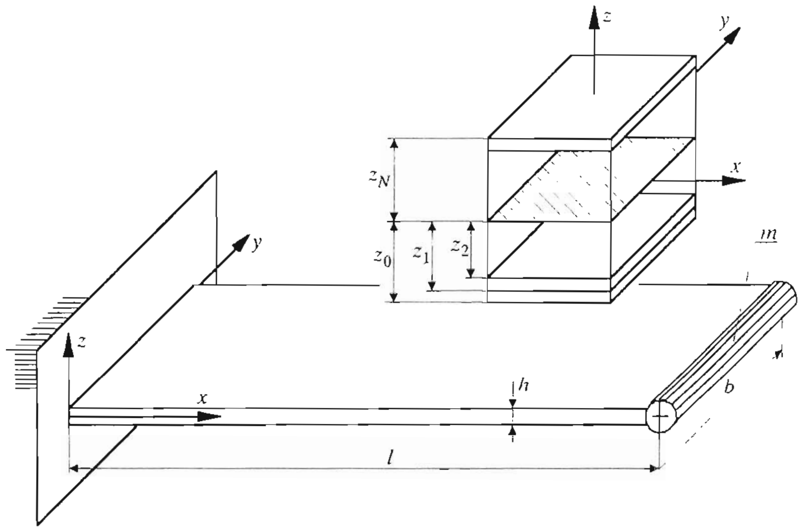

Fig. 2. Model of the plate with notation of layer co-ordinates

Assuming that the length l is much larger than the width b of the plate, and the intensity of load p does not change along the y direction, displacements w may be considered as independent of the width co-ordinate (y axis). Therefore, the equation of motion of a one-dimensional plate reduces to the simple form characteristic for laminated beams

$$D_{11}\frac{\partial^4 w}{\partial x^4} + \rho h\frac{\partial^2 w}{\partial t^2} = p \tag{3.2}$$

where

$$D_{11} = \frac{1}{3}\sum_{k=1}^{N} Q_{11}^k (z_k^3 - z_{k-1}^3) \tag{3.3}$$

The stiffness terms Q_{11}^k for the kth layer are referred to the plate axes and were given by Tsai and Pagano (1968) in the following form

$$Q_{11}^k = U_1 + U_2\cos 2\alpha + U_3\cos 4\alpha \tag{3.4}$$

where

$$U_1 = \frac{1}{8}(3q_{11} + 3q_{22} + 2q_{12} + 4q_{66}) \qquad U_2 = \frac{1}{2}(q_{11} - q_{22})$$
$$U_3 = \frac{1}{8}(q_{11} + q_{22} - 2q_{12} - 4q_{66}) \tag{3.5}$$

are the terms independent of the angle α between the lamina and the plate principal axes. The lamina stiffness coefficients q_{ij} for plane stress are of the form

$$\begin{aligned} q_{11} &= \frac{E_{11}}{1-\nu_{12}\nu_{21}} \qquad & q_{12} &= \frac{\nu_{21}E_{11}}{1-\nu_{12}\nu_{21}} = \frac{\nu_{12}E_{22}}{1-\nu_{12}\nu_{21}} \\ q_{22} &= \frac{E_{22}}{1-\nu_{12}\nu_{21}} \qquad & q_{66} &= G_{12} \end{aligned} \tag{3.6}$$

where E_{ij}, G_{ij}, n_{ij} denote the Young modulus, shear modulus and Poisson ratio of the kth layer, respectively.

In the considered case the equation of motion (3.2) must satisfy the folowing boundary conditions

$$\begin{aligned} & w(0,t) = 0 \qquad \frac{\partial w}{\partial x}\Big|_{x=0} = 0 \qquad \frac{\partial^2 w}{\partial x^2}\Big|_{x=l} = 0 \\ & \frac{\partial^3 w}{\partial x^3}\Big|_{x=l} = \frac{m}{D_{11}}\frac{\partial^2 w}{\partial x^2}\Big|_{x=l} \end{aligned} \tag{3.7}$$

where m is the uniform mass intensity along the plate edge.

According to the beam theory the solution to the boundary eigenvalue problem gives the shape of nth vibration mode

$$\begin{aligned} X_n &= (\sin k_n x - \sinh k_n x)(\cos k_n l + \cosh k_n l) + \\ &- (\cos k_n x - \cosh k_n x)(\sin k_n l + \sinh k_n l) \end{aligned} \tag{3.8}$$

The eigenvalues k_n, $(n = 1, 2, ...)$ are calculated from the characteristic equation

$$(kl)^3(1 + \cos kl \cosh kl) - \mu(kl)^4(\sin kl \cosh kl - \cos kl \sinh kl) = 0 \tag{3.9}$$

where $\mu = m/(\rho hl)$ denotes the ratio of the mass at the edge to the mass of the beam.

The steady-state responses are analysed so the system is harmonically loaded according to the relation

$$p(x,t) = p_0 e^{j\omega t}\delta(x-l) \tag{3.10}$$

where $\delta(x-l)$ indicates the Dirac function.

The flexural vibrations of the laminated beam can be expressed by the frequency function

$$H(x,\omega) = \frac{1}{\rho h}\sum_{n=1}^{\infty}\frac{X_n(l)X_n(x)}{\gamma_n^2(\omega_n^2 - \omega^2 + j\eta\omega_n^2\omega)} \tag{3.11}$$

where ω_n is the natural frequency which satisfies the relation

$$\omega_n = k_n^2\sqrt{\frac{D_{11}}{\rho h}} \qquad\qquad \gamma_n^2 = \int\limits_0^l X_n^2(x)\,dx + \mu l X_n^2(l) \tag{3.12}$$

In Equation (3.11) the internal damping is represented by the Kelvin-Voigt model with the equivalent constant η and serves to create a finite response at resonance.

Thermally activated SMA fibers, due to their material transition process, cause a change of the bending stiffness of the laminate so the natural frequency of the system can be modified.

4. Results

Calculations have been made for $[0°, -30°, 30°, 90°, 30°, -30°, 0°]$ the glass-epoxy laminated beam of dimensions $l = 0.25$ m, $b = 0.02$ m and $h = 0.0042$ m. The activated layers which are reinforced in 60% with SMA fibers are symmetrically located about the midplane.

The material properties of layers and thermophisical parameters of SMA fibers which are assumed to be made of nitinol are listed in Table 1. According to the SMA properties the nondimensional heat of transition are of the following values $R_{MA} = 1.1$ and $R_{AM} = 1.4$.

In Fig.3 and Fig.4 the dimensionless temperature and martensite fraction time responses for the $M \to A$ as well as $A \to M$ transitions are presented. The results are obtained for different power ratios R_p, of heating process and temperature ratios S, which characterise cooling of SMA fibers. Figures show that increase in both the dimensionless heating power and heat sink strength causes significant changes in the temperature and martensite fraction responses. In both cases, the complete transition process runs faster for greater values of R_p and S.

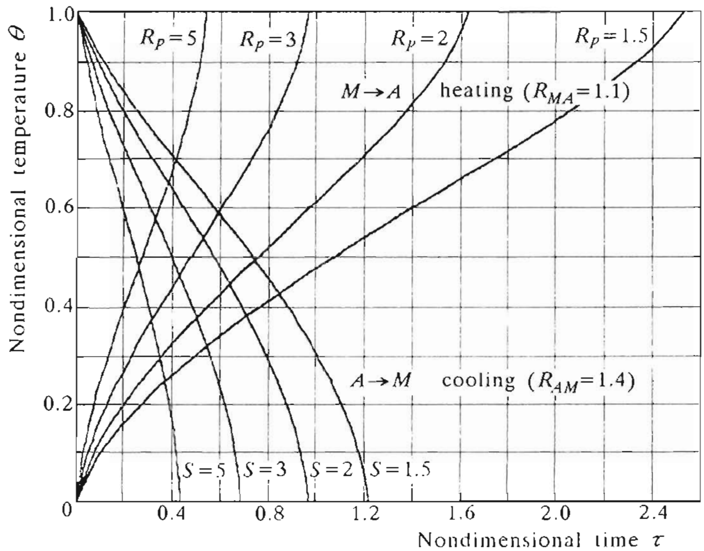

Fig. 3. Transition temperature response for different power ratios R_p ($M \to A$) and temperature ratios S ($A \to M$)

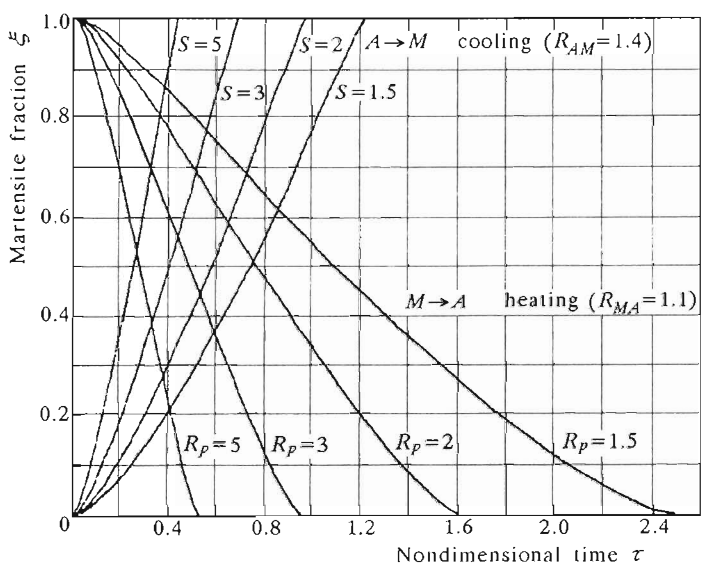

Fig. 4. Influence of power ratio R_p ($M \to A$) and temperature ratio S ($A \to M$) on the martensite fraction responses

Table 1. Material parameters used in calculations

Young modulus E_{11M}	N/m^2	$3.4 \cdot 10^{10}$
Young modulus E_{11A}	N/m^2	$7.0 \cdot 10^{10}$
Young modulus E_{11}	N/m^2	$5.5 \cdot 10^{10}$
Young modulus E_{22}	N/m^2	$1.8 \cdot 10^{10}$
Shear modulus G_{12}	N/m^2	$9.1 \cdot 10^9$
Poisson ratio ν_{12}		0.25
Equivalent density ρ	kg/m^3	$1.8 \cdot 10^3$
Martensite start temp. M_s	°C	15
Martensite finish temp. M_f	°C	5
Austenite start temp. A_s	°C	17
Austenite finish temp. A_f	°C	30
Specific heat of SMA c_n	J/kg°C	883
Heat of transition of SMA q_n	J/kg	12600

The ability of thermally activated SMA materials to change their stiffness is used for controlling of the laminate natural frequencies. The effects of variations in the power ratio R_p (heating) and temperature ratio S (cooling), respectively, on the first natural frequency is shown in Fig.5. The nondimensional form of frequency as follows

$$\Omega = \omega \frac{l^2}{h} \sqrt{\frac{\rho}{E_{22}}} \tag{4.1}$$

As it is expected the increase in the rates of power heating and heat sink decrease the time of both the transition processes. In Fig.6 it is shown how the activated fiber volume fraction μ which is defined as the ratio of the volume of activated SMA fibers to the total volume of SMA fibers in the layer influences the natural frequency responses. Results are obtained for the heat of transition parameters $R_p = S = 3$. We can notice a possibility of changing dynamical properties of the system by activation only a part of the SMA fibers.

Fig.7 shows the influence of the activated layer location on the first natural frequency being a function of temperature during the $M \rightarrow A$ transition. The activation of the layers which are placed near the midplane causes a rotation of the plots and significantly reduces the range of frequency changes. The efficiency of activated layers dramatically diminishes when the SMA fibers become perpendicular to the beam longitudinal axis. This effect can be observed by comparing the plots of the first natural frequency within the transition temperature range presented in Fig.8. These plots are obtained by changing the SMA fiber direction in the activated outer layers.

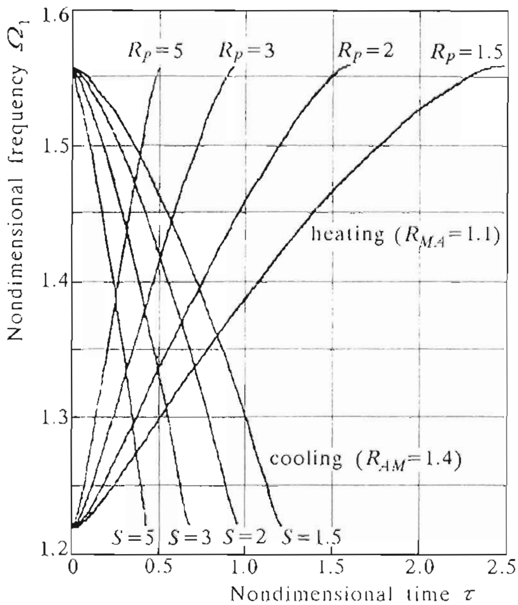

Fig. 5. Frequency Ω_1 versus time τ, effect of variations in power ratio R_p (heating), and temperature ratio S (cooling)

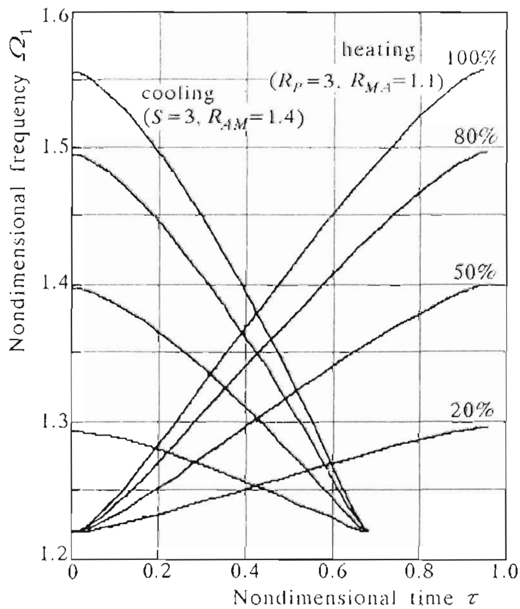

Fig. 6. Frequency Ω_1 versus time τ, effect of variations in active fiber volume fraction (heating $R_p = 3$, $R_{MA} = 1.1$; cooling $S = 3$, $R_{AM} = 1.4$)

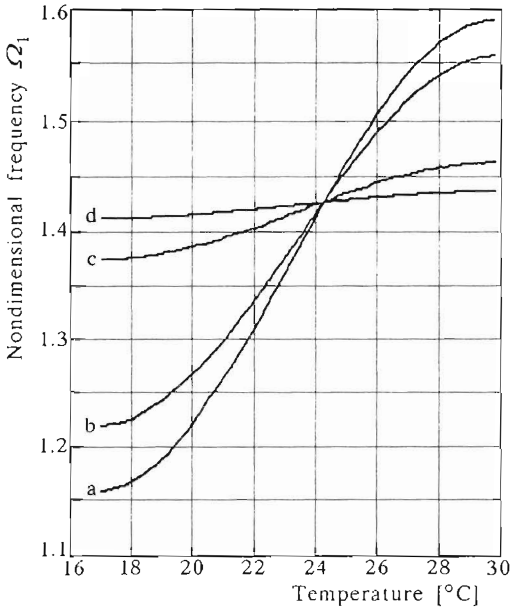

Fig. 7. Frequency Ω_1 versus temperature, activated layers: a - 1,2,6,7; b - 1,7; c - 2,6; d - 3,5

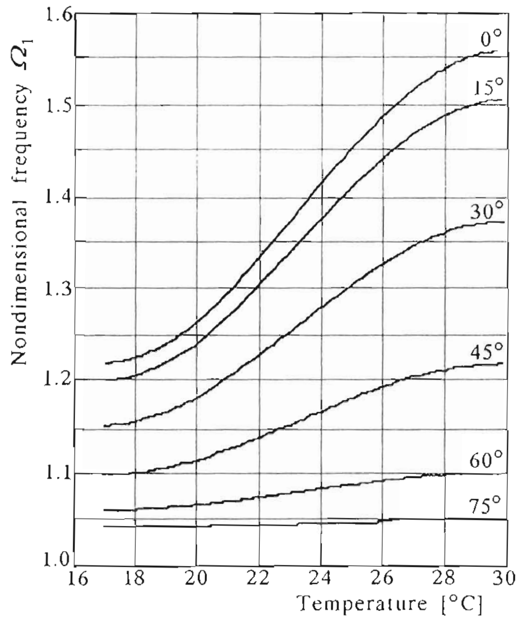

Fig. 8. Effect of SMA fiber direction on Ω_1 temperature relation, activated layers: 1,7

Fig.9 shows the effect of activation of the outer layers on the frequency response functions (FRF) calculated for the free edge of the beam ($x = l$). To obtain a finite response at resonance the calculations are made assuming the material damping parameter $\eta = 2 \cdot 10^{-6}$ s. Upon heating the SMA fibers, the $M \to A$ transition process causes an increase in the bending stiffness so the resonance frequencies become greater. The resonance amplitudes decrease slightly because of the applied model of internal damping.

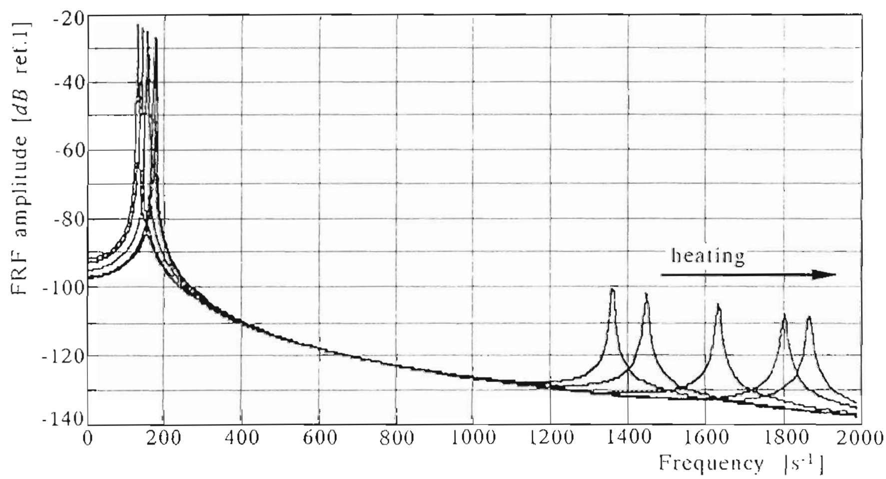

Fig. 9. Effect of heating on the frequency response function

5. Conclusions

Both the theoretical analysis and numerical results prove the utility of the adaptive control concept based on the thermal activation of SMA fibers within the laminate. Since the reversible martensitic transition of SMA material, the bending stiffness of the laminated beam may be modified and adapt to the external load conditions for vibration amplitudes to be reduced. Due to the quasi-steady 1D model of conduction the natural frequency time relations are obtained. It has been shown that the increase in both the heating power and heat sink strength significantly shortens the time of phase transition process as well as the time of change of the natural frequency values. The efficiency of the activated layer depends on its location (distance to the midplane) and direction of SMA fibers, and becomes extremely great for the activation of the outer layers reinforced with SMA fibers parallel to the beam longitudinal axis. In the paper the possibility of vibration control by a partial activation of SMA fibers has been also presented.

Acknowledgement

This study was supported by the State Committee for Scientific Research (KBN), grant No. 869/T07/95/09.

References

1. JACKSON C.M., WAGNER H.J., WASILEWSKI R.J., 1972, 55-Nitinol-The Alloy with a Memory: Its Physical Metallurgy, Properties, and Applications, NASA-SP-5110, Washington, D.C.
2. LIANG C., ROGERS C.A., 1990, One-Dimensional Thermomechanical Constitutive Relations for Shape Memory Materials, *J. of Intell. Mater. Syst. and Struct.*, **1**, 207-234
3. LIANG C., ROGERS C.A., 1991, Design of Shape Memory Alloys Coils and their Applications in Vibration Control, *Recent Advances in Active Control of Sound and Vibration*, Rogers C.A. edit., Technomic Publishing, Lancaster-Basel, 177-198
4. ROGERS C.A., LIANG C., JIA J., 1989, Behavior of Shape Memory Alloy Reinforced Composite Plates – Parts I and II, *Proceedings of the 30th Structures, Structural Dynamics and Material Conference*, Mobile, AL, 2011-2017
5. TSAI S.W., PAGANO N.J., 1968, Invariant Properties of Composite Materials, *Composite Materials Workshop*, Technomic Publishing Company
6. WHITNEY J.M., 1987, *Structural Analysis of Laminated Anisotropic Plates*, Technomic Publishing Co.Inc, Lancaster
7. WIRTZ R.A, GORDANINEJAD F., WU W., 1995, Free Response of a Thermally Driven, Composite Actuator, *J. of Intell. Mater. Syst. and Struct.*, **6**, 364-371

Dynamika termicznie uaktywnianej belki laminowanej wzmocnionej włóknami z pamięcią kształtu

Streszczenie

W pracy przedstawiono zastosowanie włókien ze stopu z pamięcią kształtu (SMA) do modyfikacji właściwości dynamicznych belek laminowanych. Koncepcję adaptacyjnego sterowania oparto na unikalnej zdolności stopów SMA do wyraźnej zmiany wartości modułu Younga w procesie odwracalnej przemiany martenzytycznej wywołanej temperaturą. Założono, że warstwy laminatu są ułożone i uaktywniane symetrycznie względem powierzchni środkowej, a włókna SMA mogą swobodnie odkształcać się w osnowie. Symulacja numeryczna dotyczy belki laminowanej utwierdzonej wzdłuż jednego brzegu, z masą równomiernie rozłożoną na swobodnym, przeciwległym brzegu. Zgodnie z przyjętym stacjonarnym, jednowymiarowym modelem przewodnictwa cieplnego, wyznaczono przebiegi czasowe temperatury i zawartości martenzytu we włóknach SMA, oraz przebiegi podstawowej częstości własnej w zakresie przemiany fazowej. Zbadano wpływ temperatury na częstości własne zmieniając położenie warstw uaktywnianych a także kierunek ułożenia włókien SMA. Zakładając wymuszenie harmoniczne działające na swobodny brzeg belki, wyznaczono charakterystyki amplitudowo-częstotliwościowe odpowiadające pełnemu zakresowi transformacji fazy materiałowej.

Manuscript received July 10, 1997; accepted for print October 31, 1997