Biuletyn WAT
Vol. LIX, Nr 2, 2010

Water capsule flight — a theoretical analysis and experimental verification

GRZEGORZ ŚMIGIELSKI[2], DAMIAN LEWANDOWSKI[1],
ROMAN S. DYGDAŁA[1,2], KRZYSZTOF STEFAŃSKI[3]

[1]Pawel Wlodkowic University College, 09-402 Płock, Al. Kilińskiego 12 Str.
[2]University of Economy, 85-229 Bydgoszcz, Garbary 2 Str.
[3]Collegium Medicum UMK, 85-067 Bydgoszcz, Jagiellońska 13-15 Str.

Abstract. The paper presents theoretical models of flight of water-filled bag dropped from an aircraft moving horizontally. Results of numerical computations based on this model are compared with parameters of trajectory of the bag dropped from a helicopter.
Keywords: drag influenced flight, water aerosol production, flight trajectory
Universal Decimal Classification: 533.6.013

1. Introduction

The models and experiments described in this paper are of importance for working-out a high-precision method of delivering, to a given point on the ground, a water capsule serving as source of explosion produced water-spray. It is a part of the research project whose final objective is to develop a water-spray based system of extinguishing and preventing large-scale fires (forest fires, oil plant fires etc.), when classical methods of fire damping are of limited use [1, 2]. Efficiency of water-spray in extinguishing fires can be used provided the spray cloud is produced at the desired points. Working-out such a method requires using good theoretical models, writing efficient numerical procedures allowing one to compute a trajectory quickly and precisely, and verifying their efficiency in experiments.

2. Purpose

The objective of research discussed in this paper consists in developing theoretical models of a capsule flight under the influence of drag in the air, in producing numerical procedures of computing its trajectory and in working-out techniques allowing one to measure parameters and to reconstruct shape of capsule's trajectory to verify the models and numerics.

3. Methods

In principle, the problem of delivering a water capsule to a given point on the ground is very similar to the problem of hitting a surface target with a bomb. There are, however, two problems that make direct application of the procedures used by military aviation difficult. The first follows from the fact that such procedures, as majority of procedures used by the military are either classified as a whole or comprise classified crucial components and non-military subjects are denied access to their details. The second problem is connected with other safety standards that must be observed in the case of placing water-capsule "in target". In particular, the water-capsule is by definition dropped on the "friendly" terrain and should neither cause injuries in the ground crew members nor damages in the ground equipment even located close (within several dozen meters) to the extinguished fire. Therefore, contrary to military projectiles, one has to use soft containers to prevent production of splinters during the explosion, which in turn forces the mode of delivering the water-bomb — hanging on a rope under the aircraft. Despite this, high precision of hitting the target is required. Taking the above into account it seems reasonable rather to develop procedures from the very beginning than to try to adopt non-classified elements of similar military procedures. A scheme of the process to be modeled is shown in Fig. 1.

Theoretical model is founded on the assumption that the water-capsule moves in the air under the influence of a constant and vertical gravitational force and of the Bernoulli drag (pressure drag) that acts against its motion with respect to the air and is proportional to the square of the velocity of this motion. After denoting the velocity by $\vec{v}$ one can write the following formula the drag force

$$\vec{Q} = -\frac{c\rho A}{2} v\vec{v}, \tag{1}$$

where $v = |\vec{v}| = \sqrt{v_1^2 + v_2^2}$, c is the drag coefficient depending on the shape of the moving body (in general being a tensor if the lift-type force were to be taken into account), ρ denotes the density of the air, and A is the frontal cross-section of the body.

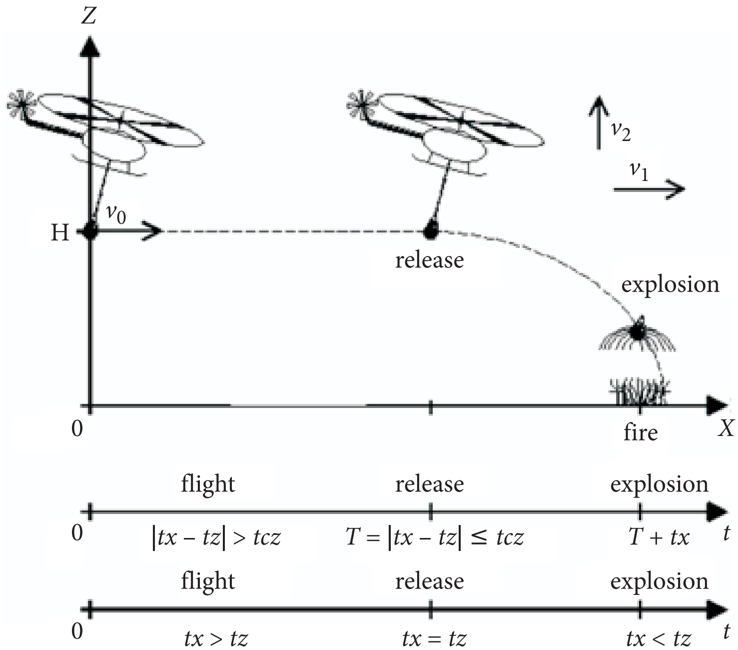

Fig. 1. Schematic view of the procedure of delivering water-capsule to a designed point

3.1. Equations describing flight of the water capsule

A water capsule dropped from a horizontally moving aircraft (e.g. helicopter) falls down under composite action of the drag force that has both vertical and horizontal components and the gravitational force that acts all time vertically. Introducing Cartesian coordinates: the horizontal one x_1 and the vertical one x_2, one can write equations of motion in the form

$$\dot{v}_1 = -\frac{c_1 \rho A_1}{2M}\sqrt{v_1^2 + v_2^2}\,v_1, \quad \dot{v}_2 = -\frac{c_2 \rho A_2}{2M}\sqrt{v_1^2 + v_2^2}\,v_2 - g, \tag{2}$$

where v_1 and v_2 are the horizontal and vertical coordinates of the capsule's velocity respectively, M is its mass and g denotes the gravitational acceleration. Various coefficients c_1 and c_2 would allow one to take into account deviation from the spherical shape of the bag.

Having these equations solved, one can obtain coordinates of the capsule by simple integration coordinates of velocity with respect to time. Unfortunately, equations (2) cannot be solved analytically without far going simplifications. It is so due to the coupling square root term. As such, to solve them, one has to apply numerical methods.

3.2. Numerical solutions

In this case the standard fourth order Runge-Kutta method [3] was used, and numerical computations were performed using the MATLAB computing environment. We will not describe technical details like optimization of the length of the step of integration.

Solution is obtained for standard initial conditions given by the equations

$$v_1(0) = v_0 \, , v_2(0) = 0, \tag{3}$$

which corresponds to horizontal motion of the water-capsule at the moment of release. Provided the value of the drag coefficients c_1 and c_2 are known, one can obtain both components of capsule's velocity as functions of time. Since the main objective consist in computing trajectory of the capsule, one has to compute its horizontal and vertical component using integrals

$$x_1(t) = \int_0^t v_1(\tau)d\tau + x_1(0), \tag{4a}$$

$$x_2(t) = \int_0^t v_2(\tau)d\tau + x_2(0), \tag{4b}$$

that, in general, have to be computed numerically since the explicit functional forms of v_1 and v_2 with respect to time are not known.

Numerical solution of equations for the components v_1 and v_2 of the capsule's velocity has one more advantage. After some modifications such a procedure can be applied to the problem of flight in the air moving with respect to the ground. In fact, Eqs. (2) describes velocity of the capsule with respect to the ground under the assumption that the air is static. If, however, velocities of wind and that of ascending or descending current are considerable, the equations have to be modified

$$\dot{v}_1 = -\frac{c_1 \rho A_1}{2M}\sqrt{\tilde{v}_1^2 + \tilde{v}_2^2}\,\tilde{v}_1, \quad \dot{v}_2 = -\frac{c_2 \rho A_2}{2M}\sqrt{\tilde{v}_1^2 + \tilde{v}_2^2}\,\tilde{v}_2 - g, \tag{5}$$

where

$$\tilde{v}_k = v_k - V_k, \, k = 1,2 \tag{6}$$

are coordinates of the capsule's velocity with respect to the air; V_1 denotes the velocity of wind and V_2 the velocity of vertical convective current (a further generalization, we will not discuss here, would be taking into account the fact that strong and random winds make the problem 3-dimensional instead of 2-dimensional planar problem of a flight in the static air).

Numerical solutions of equations of motion require inserting numerical data from the very beginning. Some of them like the mass M of the capsule or the density of the air ρ are at hand, but the drag coefficients $k_1 = c_1A_1$ and $k_2 = c_2A_2$, appearing in Eqs. (2) and (5) have to be determined from experimental data.

3.3. Reconstruction of a real trajectory of a capsule

A series of tests with dropping a bag from a horizontally flying helicopter served a double purpose. They were designed to both extracting data allowing one to estimate drag coefficients appearing in equations of motion, and to compare trajectories reconstructed from the videos registered during the actual flights with those obtained numerically for the values of parameters identical with those for the flight (horizontal velocity of the helicopter, and the height of the capsule above the ground at the moment of the release of the latter).

Trajectories of capsules dropped from the helicopter have been reconstructed from a set of frames obtained with a fast video-camera (registering pictures of the falling capsule at 250 fps). The camera was located in a considerable distance, and its optical axis was close to perpendicular with respect to the plane of trajectory. Thus, the parallax error was made as small as possible.

4. Results

Ultimate objective of the experimental tests consisted in comparing trajectories reconstructed from the recorded video frames with those computed for the corresponding values of parameters of the flight, i.e., the height of the capsule above the ground and its horizontal velocity.

Analysis of the capsule flight is based on equations (5) and their approximate counterparts for identical initial conditions. Simulations of the flight were performed for various values of the following parameters: the drag coefficients k_1 and k_2, the horizontal (v_1) and vertical (v_2) current velocities, the capsule mass M, the release height H, and the initial velocity v_0. Computations were carried on using MATLAB procedures based on the Runge-Kutta algorithm of the order 2 and 3 (*ode23*) and of the order 4 and 5 (*ode45*). For comparison of trajectory parameters obtained form the theoretical model with parameters of the actual trajectories, the equality $k_1 = k_2$ was assumed and the common drag coefficient was denoted k.

The helicopter tests allowed us to determine the values of the drag coefficient k and flight trajectories for a large number of the bag drops. The drag coefficient dependence on the mean drop velocity v fitted to the experimental data is shown in Fig. 2.

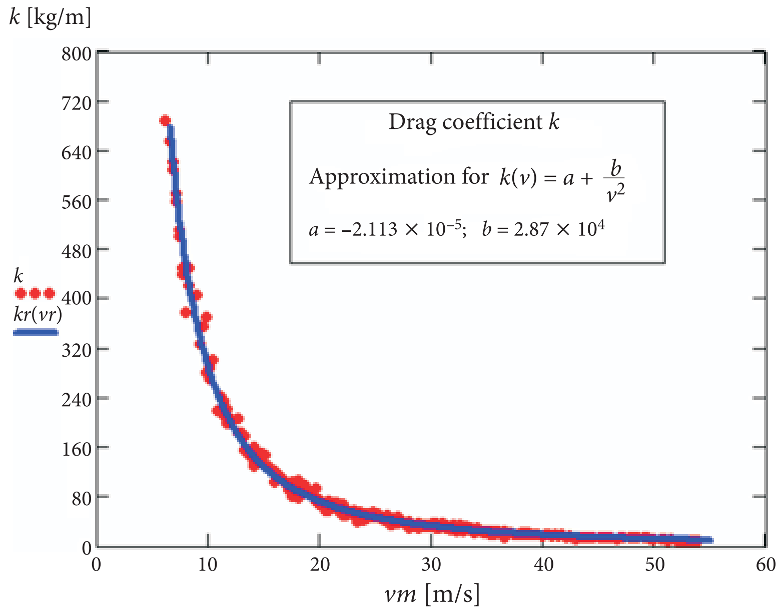

Fig. 2. Approximation of the drag coefficient $k(v)$ versus mean drop velocity for a number of helicopter tests

As it follows from Fig. 2, the best fit is obtained for the inverse power regression.

The obtained dependence allowed to insert suitable values of the drag coefficient k into the algorithm solving equations of motion (5) and to obtain a model trajectory of the capsule after being released from the helicopter moving horizontally at a given height and with given velocity. The model trajectory was then compared with the actual trajectory represented by a number of positions of the capsule registered with a fast video-camera. Result of such a comparison for a particular flight is shown in Fig. 3.

As it is clearly visible, the computed trajectory is very close to the real one, and the actual point the capsule touched ground is located about a meter from the point obtained from numerical computations. In almost all other tests, for which analogous comparison was made, discrepancies were of similar size.

It means that the theoretical model and numerical algorithm based on it, when experimentally determined values of the drag coefficients are used, allow one to compute the shape of trajectory of the falling capsule with high accuracy, and if the height of the capsule above the ground at the moment of release is precisely known, the position of the contact of the capsule with the ground can be obtained with accuracy of the order of a meter or two.

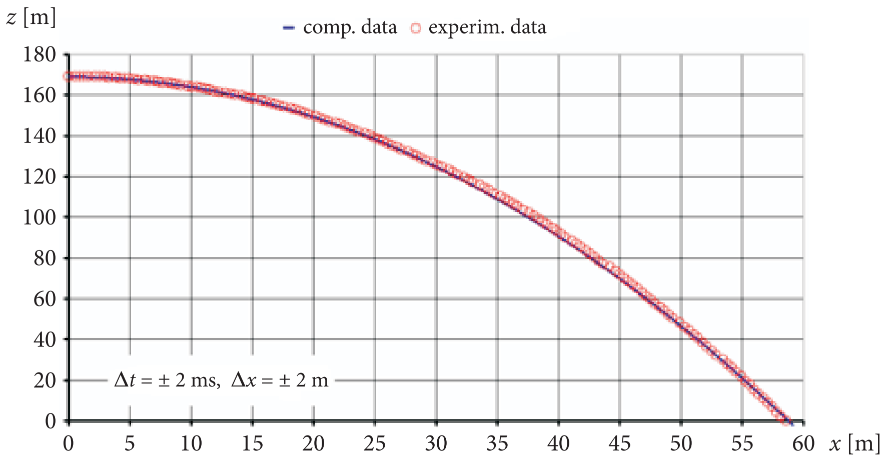

Fig. 3. Comparison of experimental and computed trajectory for one of the tests

5. Discussion

As it was mentioned in the introduction, the problem discussed in the present paper is closely related to the problem of dropping an unguided bomb by an aircraft. On the other hand, the problem of high-precision delivery of an object dropped from a moving aircraft to a given point on the ground up to now seemed to be of little importance for civilian applications (in typical airdrops for civilian purposes accuracy of the order of several dozen meters is quite tolerable). Therefore a direct comparison of the obtained results with those obtained by others is a difficult matter because majority of works on similar subjects are classified as applicable for military purposes.

The accuracy of reproduction of trajectories may look suspiciously good but one has to take into account that the tests had been performed under good weather conditions, without sudden high-velocity blows, and with no large scale fires generating strong ascending convective currents. In worse weather conditions, the model in the present form would be certainly less accurate. However, its improvement by taking into account strong air currents (that are usually far from the stationary) would be impossible without advanced equipment for monitoring air-flows and performing hundred or thousands of field tests, which is a task for a research project with budget considerably exceeding the present one.

6. Conclusions

The results of the experiments presented in this paper allow one to draw the following conclusions. The theoretical model of the flight of a water capsule under the drag and gravitational forces, when applied for numerical computations of trajectories of the flight under reasonable weather conditions turns is quite accurate. After supplementing with supporting systems like the capsule height measuring system etc. one will be able to compute the desired release point of the capsule almost instantaneously, thus assuring that the capsule will be delivered to and exploded at the point optimal for maximum efficiency of the produced water-spray cloud at least in the case of weak winds and vertical currents.

Further research will focus on developing such a supporting system. Generalization of the model and derived from it numerical programs as well as development of the technique of recording flight of the capsule and reconstructing its trajectories with taking into account their nonplanar character in the case of flight under the influence of stronger wind will depend on the finances. Its accomplishment would complete the whole system of delivering a capsule to a selected point in almost all but the roughest atmospheric conditions.

This work was partially supported by the Grant of the Polish Ministry of Science and Higher Education 8003/R/T00/2007/03.

Received December 12 2008, revised February 2010.

REFERENCES

[1] W. Teie, *Firefighter's handbook of wildland firefighting*, Dear Valley Press, Recue CA, 1994.

[2] E. Kuhrt, J. Knollenberg, V. Martens, *Annals of Burns and Fire Disasters*, 14, 2001, 151.

[3] A. Ralston, P. Rabinowicz, *First Course In Numerical Analysis*, Dover, 2001.

G. ŚMIGIELSKI , D. LEWANDOWSKI, R. S. DYGDAŁA, K. STEFAŃSKI

Modelowanie lotu kapsuły wodnej — analiza teoretyczna i weryfikacja eksperymentalna

Streszczenie. W pracy przedstawiono wyniki modelowania lotu pojemnika wypełnionego wodą, zrzucanego ze statku powietrznego, przemieszczającego się horyzontalnie. Wyniki obliczeń toru lotu pojemnika porównywane są z zarejestrowaną eksperymentalnie trajektorią kapsuły wodnej zrzucanej z helikoptera.

Słowa kluczowe: aerodynamiczny współczynnik oporu, wytwarzanie aerozolu wodnego, trajektoria lotu

Symbole UKD: 533.6.013