INTL JOURNAL OF ELECTRONICS AND TELECOMMUNICATIONS, 2012, VOL. 58, NO. 4, PP. 345–350
Manuscript received October 1, 2012; revised November, 2012. DOI: 10.2478/v10177-012-0047-x

Algorithm for Target Tracking Using Passive Radar

Mateusz Malanowski

***Abstract*—In the paper the problem of target tracking in passive radar is addressed. Passive radar measures bistatic parameters of a target: bistatic range and bistatic velocity. The aim of the tracking algorithm is to convert the bistatic measurements into Cartesian coordinates. In the paper a two-stage tracking algorithm is presented, using bistatic and Cartesian tracking. In addition, a target localization algorithm is applied to initialize Cartesian tracks from bistatic measurements. The tracking algorithm is tested using simulated and real data. The real data were obtained from an FM-based passive radar called PaRaDe, developed at Warsaw University of Technology.**

***Keywords*—Passive coherent location, passive bistatic radar, target tracking.**

I. Introduction

PASSIVE RADAR, or passive coherent location (PCL) radar, is a type of radar that utilizes existing transmitters as illuminators of opportunity. Various types of transmitters have been used for passive radar purposes: FM radio, DAB radio, DVB-T television, GMS and UMTS cellular telephones basestations, etc. [1], [2], [3], [4]. The aim of most passive systems developed so far is detection and tracking of airborne targets. The process of target tracking in passive radar is more complicated than in active radar, due to bistatic configuration and potential ambiguities (the so-called ghost targets). In this paper the problem of target tracking in Cartesian coordinates based on passive radar measurements is addressed.

In classical active monostatic surveillance radar, the transmit/receive antenna is rotating, illuminating periodically the surveillance area. The primary measurements are the range and azimuth of the target. Additionally, some radars measure also radial velocity and elevation of the target. Based on the range and azimuth measurements, it is easy to estimate the target position in Cartesian coordinates unambiguously.

The situation is different in passive radar. Due to the bistatic nature of passive radar, measurement of time delay allows the so called bistatic range of the target to be calculated. Bistatic range is the difference of transmitter-target-receiver and transmitter-receiver paths. Locus of constant bistatic ranges defines an ellipsoid with foci in the transmitter and receiver positions. Angle measurement in passive radar is possible, however, not very common. If it is used, however, it is based not on rotating antenna, but usually on phase comparison principle [1]. In many systems, the angle measurement is not provided, and this situation is assumed in the paper. In such a case, to localize a target in Cartesian coordinates, measurements from multiple transmitter-receiver pairs are necessary. The localization is based on calculating intersections of bistatic ellipsoids corresponding to different transmitter-receiver pairs. One of the most important problems in this case is solving the ambiguities, which arise from the fact that the source of bistatic measurements are unknown. Additionally, the localization should be combined with the tracking algorithm. In this paper, an algorithm is presented, which uses bistatic measurements to localize and track the targets in 3D space. The method is tested on simulated and measured data. The measured data were acquired with an experimental passive radar, called PaRaDe (Passive Radar Demonstrator) developed at Warsaw University of Technology.

Mateusz Malanowski is with the Warsaw University of Technology, Institute of Electronic Systems, Nowowiejska 15/19, 00-665 Warsaw, Poland (e-mail: m.malanowski@elka.pw.edu.pl).

II. Passive Radar Principles

A typical geometry of passive radar is shown in Fig. 1. Without loss of generality, the receiver located in position (0, 0, 0) is assumed. The parameters estimated by the radar are the bistatic range R and bistatic velocity V, defined for the i-th transmitter as:

$$R_i = \sqrt{x_t^2 + y_t^2 + z_t^2} + \sqrt{(x_i - x_t)^2 + (y_i - y_t)^2 + (z_i - z_t)^2} - \sqrt{x_i^2 + y_i^2 + z_i^2} \quad (1)$$

$$V_i = \frac{x_t v_x + y_t v_y + z_t v_z}{\sqrt{x_t^2 + y_t^2 + z_t^2}} + \frac{(x_i - x_t)v_x + (y_i - y_t)v_y + (z_i - z_t)v_z}{\sqrt{(x_i - x_t)^2 + (y_i - y_t)^2 + (z_i - z_t)^2}}, \quad (2)$$

where (x_t, y_t, z_t) is the target position, (v_x, v_y, v_z) is the target velocity vector, and (x_i, y_i, z_i) is the i-th transmitter position.

Passive radar is equipped with at least two receiving channels. One channel is used as a source of the reference signal, received directly from the transmitter. The other channel(s), called surveillance, is used to receive the echoes from the observed targets. The detection and parameter estimation is carried out by correlating the reference signal with the surveillance signal. Based on this, the bistatic range R and bistatic velocity V of the target can be estimated.

III. Tracking Algorithm

The localization and tracking algorithm proposed in this paper is based on a combination of bistatic and Cartesian tracking [5], [6]. The block diagram of the proposed method is shown in Fig. 2. The bistatic plots, containing measurements of bistatic range and velocity, are first correlated with established Cartesian tracks. Unassigned plots are passed to the bistatic trackers. The confirmed bistatic tracks are used by the localization algorithm. The algorithm finds the intersection of bistatic ellipsoids, thus calculating the position of a target in Cartesian coordinates. Additionally, 3D velocity vector is

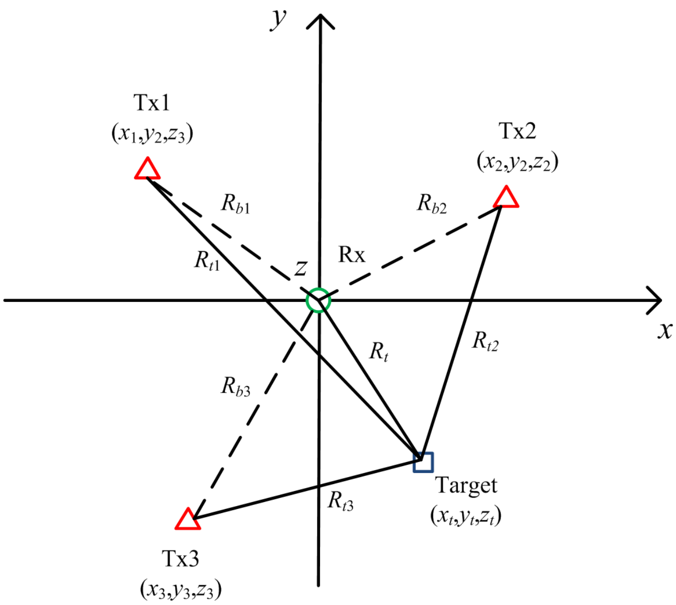

Fig. 1. Typical geometry of passive radar.

found. The results of the localization procedure are used as tentative tracks for the Cartesian tracker.

The proposed two-stage approach is advantageous from the point of view of localization accuracy and ghost target phenomenon mitigation. The reason for this is that the localization algorithm operates on confirmed bistatic tracks. Therefore, the estimation accuracy of bistatic parameters is higher than for raw bistatic plots. Moreover, in the input of the localization algorithm there are only true targets (with a high probability), so the probability of ghost targets originating from false alarms is reduced.

A. Bistatic Tracker

In the bistatic tracker the measurements from consecutive blocks of data are processed to associate the measurements and estimate the state of the target. The measurement vector at time k is composed of bistatic range and bistatic velocity $\mathbf{z}(k) = [R(k), V(k)]'$. The state vector is assumed to have the following form: $\mathbf{x}_b(k) = [R(k), V(k), A(k)]'$, where $A(k)$ is the bistatic acceleration. Because the relationship between the vector of measurements and state vector is linear, a standard linear Kalman filter can be applied [7], [8]. Prediction of the state vector and state covariance are as follows:

$$\hat{\mathbf{x}}_b(k+1|k) = \mathbf{F_b}\hat{\mathbf{x}}_b(k|k), \tag{3}$$

$$\mathbf{P_b}(k+1|k) = \mathbf{F_b}\mathbf{P_b}(k|k)\mathbf{F_b}' + \mathbf{Q_b}, \tag{4}$$

where $\hat{\mathbf{x}}_b(k|k)$ and $\hat{\mathbf{x}}_b(k+1|k)$ are *a posteriori* and *a priori* state estimates, respectively, $\mathbf{P_b}(k|k)$ and $\mathbf{P_b}(k+1|k)$ are *a posteriori* and *a priori* state covariances, respectively,

$$\mathbf{F_b} = \begin{bmatrix} 1 & T & T^2/2 \\ 0 & 1 & T \\ 0 & 0 & 1 \end{bmatrix}, \tag{5}$$

$\mathbf{Q_b}$ is the process noise covariance matrix, and T is the refresh rate.

The state estimate and its covariance are updated as follows:

$$\hat{\mathbf{x}}_b(k+1|k+1) = \hat{\mathbf{x}}_b(k+1|k) + \mathbf{K_b}(k)\mathbf{v}_b(k), \tag{6}$$

$$\mathbf{P_b}(k+1|k+1) = (\mathbf{I} - \mathbf{K_b}(k+1)\mathbf{H_b})\,\mathbf{P_b}(k+1|k), \tag{7}$$

where $\mathbf{K_b}(k)$ is the Kalman gain, $\mathbf{H_b}$ is the stat-to-measurement matrix, and $\mathbf{v}_b(k) = \mathbf{z}(k) - \mathbf{H_b}\hat{\mathbf{x}}_b(k|k-1)$ is the process innovation.

B. Cartesian Tracker

The state vector in the Cartesian tracker has the form: $\mathbf{x}_c(k) = [x_t(k), v_x(k), y_t(k), v_y(k), z_t(k), v_z(k)]'$. The measurement vector corresponding to i-th transmitter is $\mathbf{z}_i(k) = [R_i(k), V_i(k)]'$. Because the relationship between the state vector and measurement vector is nonlinear (see (1) and (2)), classical Kalman filter cannot be used. Instead, extended Kalman filter can be applied [7], [8].

The prediction of the state vector and state covariance is performed as follows:

$$\hat{\mathbf{x}}_c(k+1|k) = \mathbf{F_c}\hat{\mathbf{x}}_c(k|k), \tag{8}$$

$$\mathbf{P_c}(k+1|k) = \mathbf{F_c}\mathbf{P_c}(k|k)\mathbf{F_c}' + \mathbf{Q_c}, \tag{9}$$

where the matrices are denoted analogously as in the bistatic tracker.

In the extended Kalman filter, the nonlinear relationships are approximated by calculating a matrix of derivatives and using it in standard Kalman filter equations. Let us define a function $h_i(\cdot)$ converting the parameters of a target from the Cartesian to bistatic coordinates, for the i-th transmitter:

$$\begin{bmatrix} R_i \\ V_i \end{bmatrix} = h_i\left(\begin{bmatrix} x_t \\ v_x \\ y_t \\ v_y \\ z_t \\ v_z \end{bmatrix}\right), \tag{10}$$

according to (1) and (2).

The nonlinear function $h(\cdot)$ is linearized in the EKF by calculating the Jacobian matrix:

$$\mathbf{H}_{ci}(k) = \left.\frac{\partial h_i(\mathbf{x}_c)}{\partial \mathbf{x}_c}\right|_{\hat{\mathbf{x}}_c(k|k-1)}, \tag{11}$$

which corresponds to calculating the value of the partial derivatives for the predicted state vector $\hat{\mathbf{x}}_c(k|k-1)$. The $\mathbf{H}_{ci}(k)$ matrix is then used in the standard Kalman filtering equations.

Having bistatic measurements form multiple transmitters at our disposal, two approaches can be used when updating state vector $\mathbf{x}_c$: parallel and sequential update [5]. In the parallel approach, the bistatic measurement vectors are stacked, and they are used to update the state vector. In the sequential method, the measurements corresponding to each transmitter are used separately for updating the state vector. In this paper the latter approach is used, since it involves calculations on smaller matrices, and it is possible to update the state vector in the case when not all measurements corresponding to all transmitters are available [5].

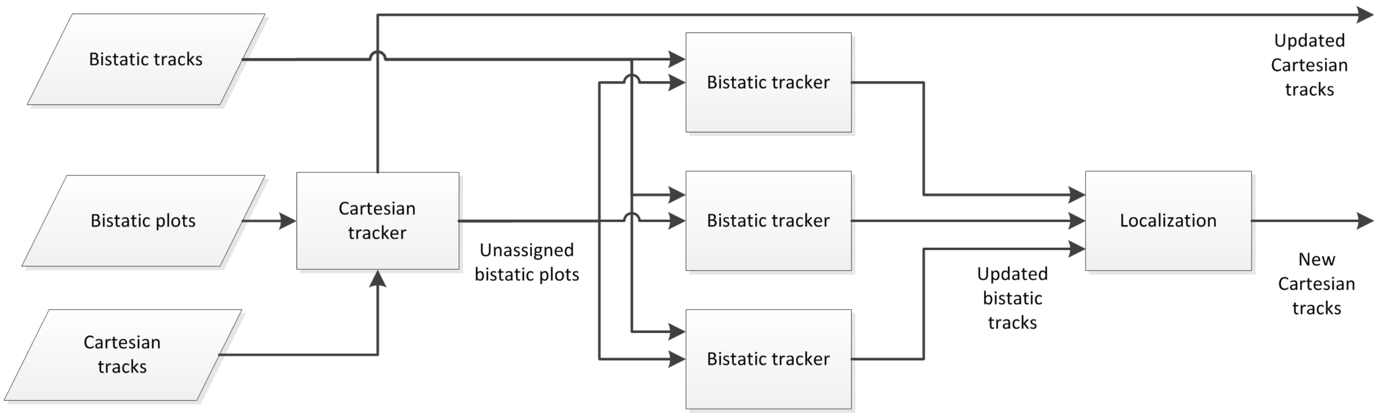

Fig. 2. Block diagram of the tracking algorithm.

In such case, the state update procedure is started by assigning:

$$\hat{\mathbf{x}}_{c0}(k+1|k) = \hat{\mathbf{x}}_c(k+1|k), \tag{12}$$

$$\mathbf{P}_{c0}(k+1|k) = \mathbf{P}_c(k+1|k). \tag{13}$$

Next, the updating is performed separately, for each transmitter according to:

for $i = 1, \ldots, N_{\mathrm{Tx}}$

$$\mathbf{S}_{ci}(k+1) = \mathbf{H}_{ci}(k+1)\mathbf{P}_{c(i-1)}(k+1|k)\mathbf{H}_{ci}{}'(k+1) + \tilde{\mathbf{R}}_{ci}, \tag{14}$$

$$\mathbf{K}_{ci}(k+1) = \mathbf{P}_{c(i-1)}(k+1|k)\mathbf{H}_{ci}{}'(k+1)\left(\mathbf{S}_{ci}(k+1)\right)^{-1}, \tag{15}$$

$$\hat{\mathbf{x}}_{ci}(k+1|k) = \hat{\mathbf{x}}_{c(i-1)}(k+1|k) + \mathbf{K}_{ci}(k+1)\mathbf{v}_{ci}(k+1), \tag{16}$$

$$\mathbf{P}_{ci}(k+1|k) = \mathbf{P}_{c(i-1)}(k+1|k) - \mathbf{K}_c(k+1)\mathbf{H}_{ci}(k+1)\mathbf{P}_{c(i-1)}(k+1|k), \tag{17}$$

end

where N_{Tx} is the number of used transmitters, and $\tilde{\mathbf{R}}_{ci}$ is the measurement error covariance matrix corresponding to the i-th transmitter. The final state vector and state covariance matrix are obtained by assigning:

$$\hat{\mathbf{x}}_c(k+1|k+1) = \hat{\mathbf{x}}_{cN_{\mathrm{Tx}}}(k+1|k), \tag{18}$$

$$\mathbf{P}_c(k+1|k+1) = \mathbf{P}_{cN_{\mathrm{Tx}}}(k+1|k). \tag{19}$$

C. Target Localization

The localization is the process of estimating the Cartesian parameters of the target (position and velocity vector) from bistatic measurements. Constant bistatic range defines an ellipsoid in Cartesian space with foci in transmitter and receiver positions. Assuming that single receiver and multiple transmitters are used, multiple ellipsoids can be formed with one common focus. By calculating the intersection points of the ellipsoids, the position of the target can be found. The concept of calculating target position in Cartesian coordinates based on bistatic ellipsoids is shown in Fig. 3, where three bistatic ellipsoids are shown, intersecting in the position of the target. After the position is calculated, the bistatic velocity measurements can be used to find the velocity vector in Cartesian coordinates.

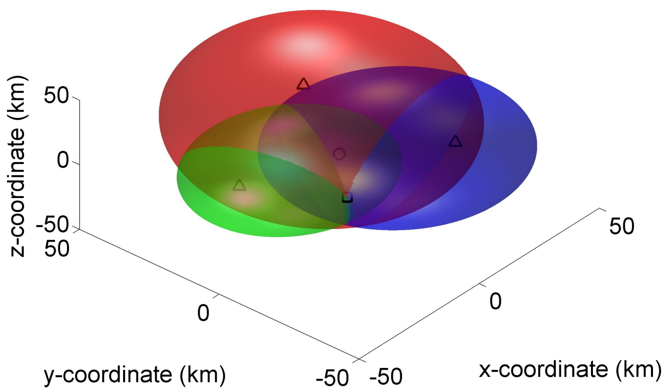

Fig. 3. The concept of finding target position in Cartesian coordinates based on bistatic measurements (triangles – transmitters, circle – receiver, square – target).

Several approaches can be used to find the intersection points of bistatic ellipsoids. Here, an algorithm developed by the author is used [9], [10]. The algorithm is based on similar method for Time-Difference-of-Arrival systems [11], [12]. It is worth to mention that the algorithm operates in three dimensions, i.e. x, y, z coordinates are calculated, as well as thee-component velocity vector [9].

The localization is performed in three steps. First, the range between the target and receiver is estimated by solving a quadratic equation. Next, the range to the target is used to calculate the position by solving a matrix equation. The last step is calculation of the velocity vector. The full deriviation of this algorithm is shown in [9], [10]. In this paper, only the most important equations are presented.

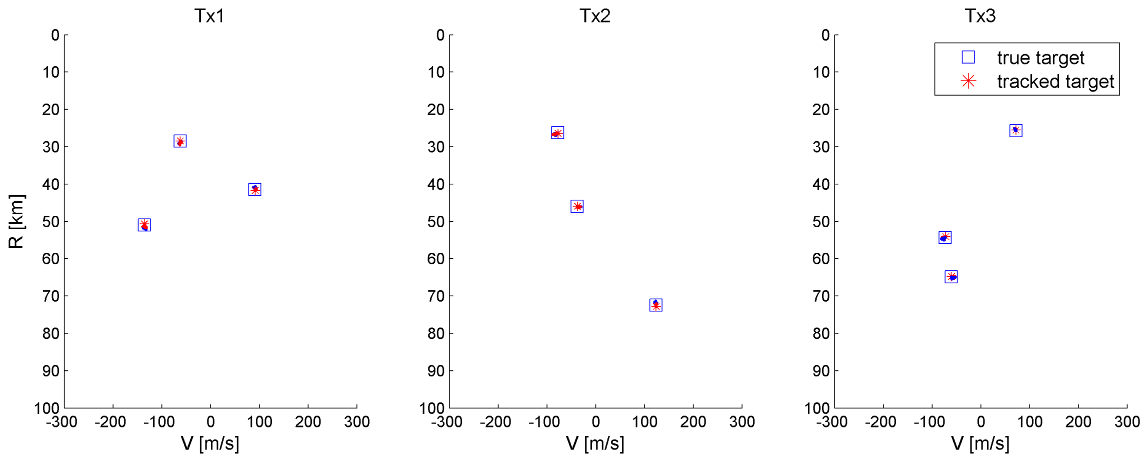

Fig. 4. Results of the tracking in bistatic coordinates (simulated data).

To simplify notation, a matrix of transmitter positions is introduced

$$\mathbf{S} = \begin{bmatrix} x_1 & y_1 & z_1 \\ x_2 & y_2 & z_2 \\ x_3 & y_3 & z_3 \end{bmatrix}, \tag{20}$$

and an additional vector defined as

$$\mathbf{z} = \frac{1}{2} \begin{bmatrix} x_1^2 + y_1^2 + z_1^2 - R_{s1}^2 \\ x_2^2 + y_2^2 + z_2^2 - R_{s2}^2 \\ x_3^2 + y_3^2 + z_3^2 - R_{s3}^2 \end{bmatrix}, \tag{21}$$

where

$$R_{si} = \sqrt{x_t^2 + y_t^2 + z_t^2} + \sqrt{(x_i - x_t)^2 + (y_i - y_t)^2 + (z_i - z_t)^2}. \tag{22}$$

Additional two vectors are defined as:

$$\mathbf{a} = (\mathbf{S}'\mathbf{S})^{-1} \mathbf{S}'\mathbf{z}, \tag{23}$$

and

$$\mathbf{b} = (\mathbf{S}'\mathbf{S})^{-1} \mathbf{S}'\mathbf{r}, \tag{24}$$

where

$$\mathbf{r} = \begin{bmatrix} R_{s1} \\ R_{s2} \\ R_{s3} \end{bmatrix}. \tag{25}$$

First, the range to the target is calculated by solving a quadratic equation:

$$\hat{R}_t = \frac{-2\mathbf{a}'\mathbf{b} \pm \sqrt{4(\mathbf{a}'\mathbf{b})^2 - 4(\mathbf{b}'\mathbf{b} - 1)\mathbf{a}'\mathbf{a}}}{2(\mathbf{b}'\mathbf{b} - 1)}. \tag{26}$$

Next, the position of a target is found as:

$$\hat{\mathbf{x}}_t = (\mathbf{S}'\mathbf{S})^{-1} \mathbf{S}'\mathbf{z} + (\mathbf{S}'\mathbf{S})^{-1} \mathbf{S}'\mathbf{r} R_t. \tag{27}$$

At the end, the velocity vector is calculated [9]:

$$\hat{\mathbf{v}}_{xyz} = (\mathbf{C}'\mathbf{C})^{-1}\mathbf{C}'\mathbf{v}, \tag{28}$$

where

$$\mathbf{v}_{xyz} = \begin{bmatrix} v_x \\ v_y \\ v_z \end{bmatrix}, \tag{29}$$

$$\mathbf{v} = \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix}, \tag{30}$$

and

$$\mathbf{C} = \begin{bmatrix} \frac{x_t}{R_t} + \frac{x_1 - x_t}{R_{t1}} & \frac{y_t}{R_t} + \frac{y_1 - y_t}{R_{t1}} & \frac{z_t}{R_t} + \frac{z_1 - z_t}{R_{t1}} \\ \frac{x_t}{R_t} + \frac{x_2 - x_t}{R_{t2}} & \frac{y_t}{R_t} + \frac{y_2 - y_t}{R_{t2}} & \frac{z_t}{R_t} + \frac{z_2 - z_t}{R_{t2}} \\ \frac{x_t}{R_t} + \frac{x_3 - x_t}{R_{t3}} & \frac{y_t}{R_t} + \frac{y_3 - y_t}{R_{t3}} & \frac{z_t}{R_t} + \frac{z_3 - z_t}{R_{t3}} \end{bmatrix}, \tag{31}$$

where:

$$R_t = \sqrt{x_t^2 + y_t^2 + z_t^2}, \tag{32}$$

$$R_{ti} = \sqrt{(x_i - x_t)^2 + (y_i - y_t)^2 + (z_i - z_t)^2}. \tag{33}$$

IV. Numerical Results

A. Simulated Data

To test the performance of the presented algorithm, a series of simulations was performed. In the simulation, a scenario was analyzed with three transmitters, one receiver and three targets. In Fig. 4 three bistatic graphs corresponding to three transmitters are shown. On each of the graphs, the tracks are shown, corresponding to the three simulated targets. The graphs show the phase of the tracking, after the bistatic tracks were established, target localization was performed, and tracking was taken over by the Cartesian tracker. Therefore, the data in the bistatic coordinates are actually Cartesian tracks mapped to the bistatic coordinates. In Fig. 5 the results of the Cartesian tracking are shown. It can be seen that all three targets are tracked successfully, and no ghost targets are present.

B. Real Data

The proposed tracking algorithm was tested also on real-life data. For this purpose, real-life signals acquired with PaRaDe demonstrator, developed at Warsaw University of Technology, were used [2], [13], [14], [15]. The demonstrator uses FM radio transmission. The reference and surveillance beams are created by means of digital beamforming. The system is capable of recording four FM radio channels simultaneously. In the

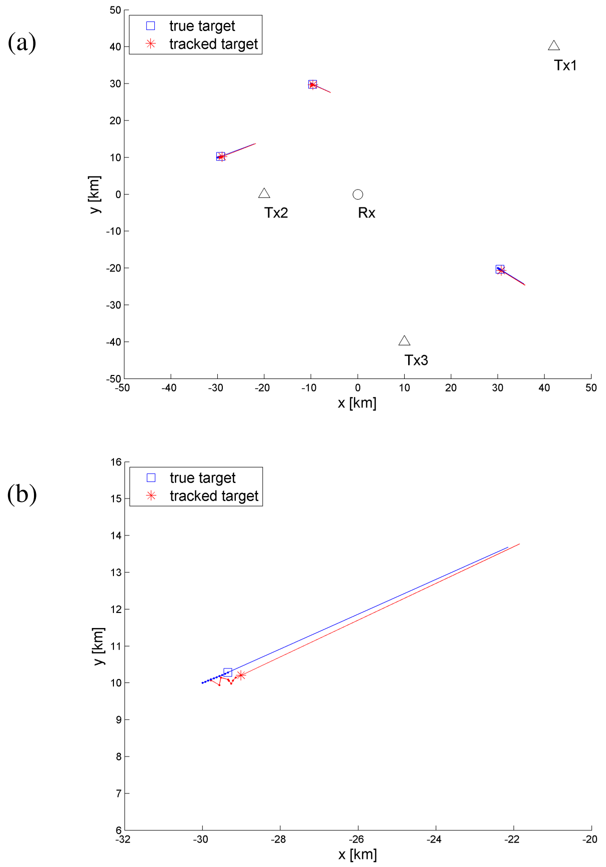

Fig. 5. Results of the tracking in Cartesian coordinates (simulated data), (a) full view, (b) zoom.

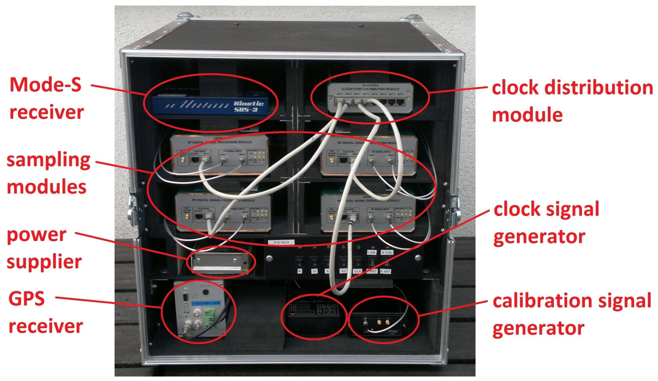

Fig. 6. Digital part of the receiver of the PaRaDe system.

considered case, only three channels were used, corresponding to three geographically separated transmitters.

In Fig. 6 the digital part of the receiver is shown. It consists of four sampling modules (each equipped with two signal inputs), clock signal generator, clock distribution module, Mode-S receiver (for reference data), GPS receiver (for synchronization) and power supplier.

The receiver is connected to the antenna array consisting of 8 dipoles. The array is placed on a 12m height mast. The array and the mast are shown in Fig. 7.

In Figs. 8 and 9 the real-life results are shown. In the graphs

Fig. 7. Deployable mast (12m height) of the PaRaDe system with 8-element antenna array.

the track is seen, as well as data obtained from a commercial Mode-S receiver, which are used as a reference. In Fig. 8 the tracks in the bistatic coordinates are shown. Figure 9 shows the tracks in the Cartesian coordinates. It can be seen that from two targets only one is tracked. The reason for this is that the second target was not detected in all three channels, which prevents successful target localization.

V. Conclusions

In the paper an algorithm for target tracking in passive radar was presented. The localization and tracking in the Cartesian coordinates in passive radar is a challenging task. The reason for this is that the transformation from the bistatic measurements to Cartesian parameters is relatively complicated. Moreover, the ghost target phenomenon occurs, which can generate false targets.

The algorithm shown in the paper can be successfully used in practical applications. The combination of bistatic and Cartesian tracker enhances the localization accuracy and mitigates the ghost target phenomenon. The algorithm was successfully tested on simulated and real-life data. The future research will include using the information about the direction of arrival, and using more than three transmitters and multiple receivers.

References

[1] P. Howland, D. Maksimiuk, and G. Reitsma, “FM radio based bistatic radar,” *IEE Proc. Radar, Sonar and Navigation*, vol. 152, no. 3, pp. 107–115, June 2005.
[2] M. Malanowski, K. Kulpa, and J. Misiurewicz, “PaRaDe - Passive Radar Demonstrator family development at Warsaw University of Technology,” in *Microwaves, Radar and Remote Sensing*, Kiev, Ukraine, 22-24 September 2008, p. CD.
[3] P. Howland, “Target tracking using television-based bistatic radar,” *IEE Proc. Radar, Sonar and Navigation*, vol. 146, no. 3, pp. 166–174, June 1999.

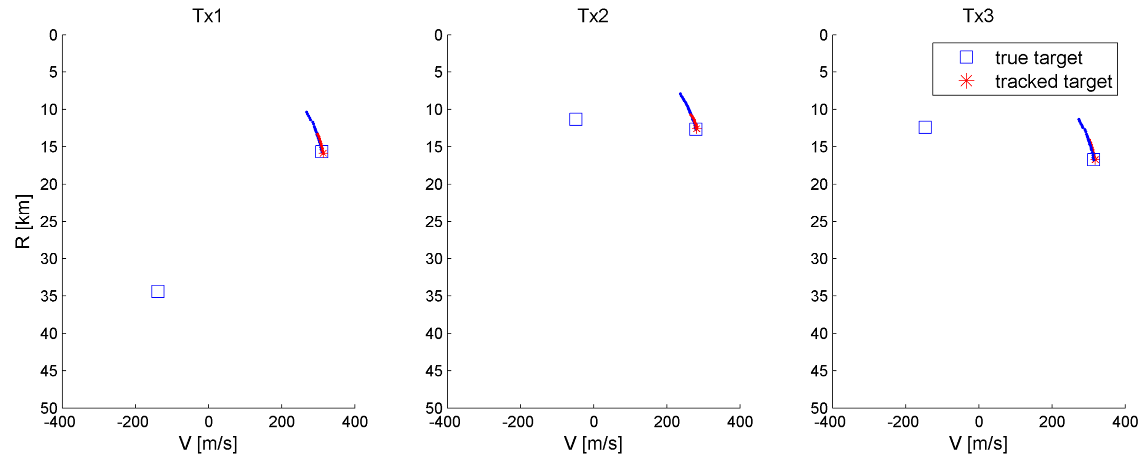

Fig. 8. Results of the tracking in bistatic coordinates (real-life data).

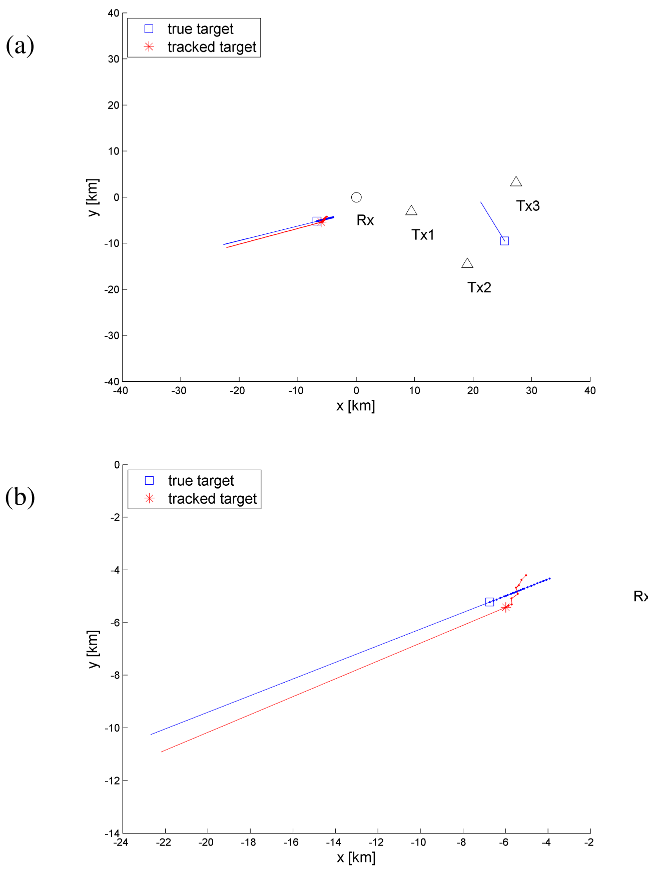

Fig. 9. Results of the tracking in Cartesian coordinates (real-life data), (a) full view, (b) zoom.

[4] D. Tan, H. Sun, Y. Lu, M. Lesturgie, and H. Chan, "Passive radar using Global System for Mobile communication signal: theory, implementation and measurements," *IEE Proc. Radar, Sonar and Navigation*, vol. 152, no. 3, pp. 116–123, June 2005.

[5] M. Malanowski, K. Kulpa, and R. Suchozebrski, "Two-stage tracking algorithm for passive radar," in *Proc. 12th International Conference on Information Fusion – FUSION*, Seattle, Washington, USA, 6–9 July 2009.

[6] M. Malanowski, "Target tracking using FM-based passive radar," in *Signal Processing Symposium 2011*, Jachranka, Poland, June 2011, pp. 1–6.

[7] Y. Bar-Shalom and X. R. Li, *Multitarget-Multisensor Tracking: Principles and Techniques*. Storrs, USA: Yaakov Bar-Shalom, 1995.

[8] ——, *Estimation and Tracking: Principles, Techniques, and Software*. Boston, USA: Artech House, 1993.

[9] M. Malanowski, "An algorithm for 3D target localization from passive radar measurements," in *Proc. SPIE – Photonics Applications in Astronomy, Communications, Industry, and High-Energy Physics Experiments 2009*, Wilga, Poland, 25–31 May 2009 2009, pp. 75 021B–1–75 021B–6, vol. 7502.

[10] M. Malanowski and K. Kulpa, "Two methods for target localization in multistatic passive radar," *IEEE Trans. Aerosp. Electron. Syst.*, vol. 48, no. 1, pp. 572–580, January 2012.

[11] G. Mellen II, M. Pachter, and J. Raquet, "Closed-form solution for determining emitter location using time difference of arrival measurements," *IEEE Trans. Aerosp. Electron. Syst.*, vol. 39, no. 3, pp. 1056–1058, July 2003.

[12] J. Smith and J. Abel, "Closed-form least-squares source location estimation from range-difference measurements," *IEEE Trans. Acoust., Speech, Sign. Process.*, vol. ASSP-35, no. 12, pp. 1661–1669, December 1987.

[13] M. Malanowski, K. Kulpa, M. Mordzonek, and P. Samczyski, "PaRaDe – reconfigurable software defined passive radar," in *Proc. NATO Specialist Meeting SET–136*, Lisbon, Portugal, 23–25 June 2009, p. CD.

[14] K. Kulpa and M. Malanowski, "Simple COTS PCL demonstrator," in *Proc. 5th Multi-National Passive Covert Radar Conference*, Shrivenham, UK, 13–15 November 2006, p. CD.

[15] M. Malanowski, G. Mazurek, K. Kulpa, and J. Misiurewicz, "FM based PCL radar demonstrator," Cologne, Germany, 5–7 September 2007, pp. 431–435.