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Criticism of linear form of burning rate law with reference to conventional fine-grained propellants

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Abstract. Linear form of the burning rate law $r = r_1 p$, describing changes (with the pressure p) in burning rate of propellants, is very popular in East European ballistics laboratories for analysis and computer simulations of propellant gun systems regardless of propellant type and dimensions of propellant grains. The coefficient r_1 of the linear form of burning rate law is usually calculated on the basis of average dimensions of a grain (layer of burnt propellant) and integrated experimental pressure-time curve. A recorded picture of pressure of propellant gas mixture is an effect of closed vessel test. It is assumed that a value of the coefficient r_1 is constant (for given type of propellant) regardless of a value of a propellant gas pressure. Different single-base propellants were fired in closed vessel tests to determine their burning rate behaviour. In order to determine the burning rate law coefficient, the variations in mass of igniter material (black powder) at the same value of loading density were used. The results of experimental tests and calculations presented in this paper show significant influence of the used type of ignition system (mass of black powder) on burning rate (the coefficient r_1) of propellant. Differences in burning rate calculations may be the reason of considerable errors in theoretical calculations of pressure-travel and velocity-travel curves during internal ballistic computer simulations of a gun propellant system.

Keywords: smokeless propellants, burning rate, closed vessel tests
Universal Decimal Classification: 662.1/.4

1. Introduction

The characteristic pressure-travel curve of a gun system is dependent upon many factors: variation in chemical composition of a propellant, ignition characteristics, propellant grain characteristics, loading conditions (charge weight variations), variations in burning rate of a propellant etc. The burning time of a propellant grain can be controlled by several means: the size and shape of the propellant grain, the number of perforations in each grain, the web thickness and the rate of burning of the grain.

The linear burning rate vs. pressure behaviour of a gun propellant (known as burning rate law) is a characteristic of the propellant composition. The burning rate of a propellant (the rate of reduction in size of the propellant with time) varies with pressure. Accurate knowledge on the form of propellant burn law $r(p)$ and values of its coefficients plays a fundamental role in internal ballistics calculations. In many East European ballistics laboratories, according to national students teaching books and technical documents [1, 5, 6, 7], the burning rate of a propellant is still approximated by the linear form of burning rate law

$$r = r_1 \cdot p. \tag{1.1}$$

In the case of geometric, regular shape of propellant grains with smooth surface, the burning rate coefficient r_1 of propellant may be calculated from the experimental pressure-time curve (Fig. 1) of the closed chamber firings, average properties (e.g. length, radius, etc.) of grains and the following equations using integrated pressure-time curve

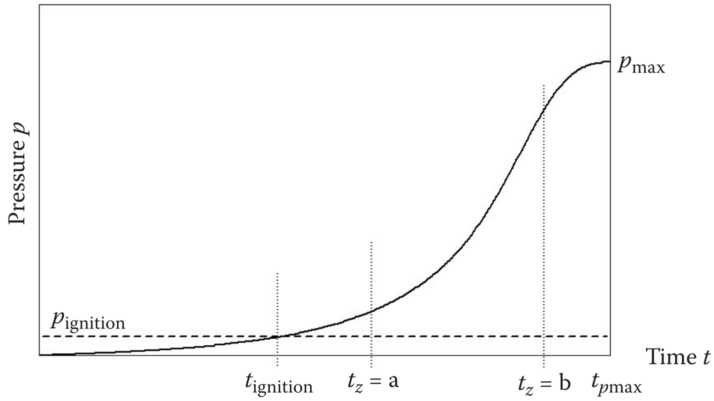

Fig. 1. Pressure-time curve from closed vessel test and borders of integration

$$r_1 = \frac{e_1}{I_{p_t}} = \frac{e_1}{\int\limits_{t_{p_{ign}}}^{t_{p_{\max}}} p\,dt}, \tag{1.2}$$

where: e_1 — the total layer of burnt propellant (half of web size);
I_{pt} — the total impulse of pressure of propellant gases calculated from the ignition of propellant (p_{ign}) to the end of propellant combustion ($p_{\max}$);

or

$$r_1 = \frac{e_{a-b}}{I_{p_{a-b}}} = \frac{e_{a-b}}{\int_{t_a}^{t_b} p dt}, \tag{1.3}$$

where: e_{a-b} — the limited layer of burnt propellant (regression distance e_{a-b} of propellant granule);

$I_{p_{a-b}}$ — the limited (for limited layer of burnt propellant e_{a-b} and limited fraction of burnt mass z_{a-b}) impulse of pressure of propellant gases (limited process of gas creation).

From the experimental pressure-time curve, limited layer $e_{a-b} = e_b - e_a$ of burnt propellant can be calculated from the burnt mass fraction (z_a and z_b) using transformed Noble-Abel's equation of state of propellant gases

$$z_{a(b)} = \frac{1}{\left[1 + \left(\frac{1 - \eta \cdot \Delta}{1 - \frac{\Delta}{\delta}}\right) \cdot \left(\frac{p_{\max} - p_{z_{a(b)}}}{p_{z_{a(b)}}}\right)\right]}, \tag{1.4}$$

where: Δ — the loading density (mass of propellant and volume of closed vessel ratio);

η — the covolume;

δ — the propellant density;

p_{max} — the maximum experimental pressure;

$p_{za(b)}$ — the pressure corresponding to time $t_{z=a}$ ($t_{z=b}$)

— and geometrical dependence [5] between calculated value of burnt mass fraction (z_a or z_b from eq. 1.4) and relative layer ($\varepsilon_{a(b)}$) of burnt propellant, defined by equation

$$z_{a(b)} = \kappa \cdot \varepsilon_{a(b)} \cdot (1 + \lambda \cdot \varepsilon_{a(b)}), \tag{1.5}$$

where $\varepsilon_{a(b)}$ — the relative layer of burnt propellant

$$\varepsilon_{a(b)} = \frac{e_{a(b)}}{e_1}, \tag{1.6}$$

κ, λ — the geometrical characteristic of propellant grain.

In the Eq. 1.4, the pressures $p_{\max}$ and $p_{za(b)}$ take into account the pressure coming from ignition system. The pressure p_{za} or p_{zb} should be chosen from middle part of experimental pressure-time curve lying between ignition period (p_{ign}) and the end of combustion of propellant ($p_{\max}$).

The aim of this work is to investigate closed vessel tests, which permit to verify the view on legitimacy of linear form of burning rate law using in internal ballistic analysis. For this purpose different ignition systems were used.

2. Method and grain geometry

Closed vessel tests are a good method of obtaining burning rate information for propellants. These experiments measure gas generation rates and therefore the burning rate information can only be accurately deduced if conditions of tests precisely meet the major assumptions of internal ballistic analysis. Experimental pirostatic investigations were carried out in a vessel of 200 cm^3. Technical parameters of the used closed vessel, a pressure measurement system consisting of a piezoelectric transducer (HPI 5QP 6000M), a data acquisition chain (amplifier, A/D converter, computer), and also the methodology of investigation were the same as described in Ref. 8. An ignition system consisted of a power source and an ignition material. Black powder was the ignition material. Adequate formal standards and regulations [5, 8, 9] recommend different conditions of ignition. Cotton bag containing 0.5÷2 g of black powder (the mass depends on loading density) is the rule, but it was decided to carry out closed vessel tests in which the ignition systems with various masses of black powder (only electric match without black powder or electric match and 0.5 g, 2 g, 4 g, 6 g or 8 g mass of black powder) were used. The major assumptions [2, 5] used in burning rate analysis are:

1. The propellant gas mixture is described by the Noble-Abel's equation of state.
2. Propellant grains are all of the same size and configuration.
3. All propellant grains are ignited uniformly, with all exposed surface areas of the grains having recessed by a small distance.
4. All exposed burning surfaces recede at a uniform rate, implying that all grains shrink symmetrically.
5. Decomposition of a unit mass of propellant always liberates the same amount of energy, which heats product gases to the same temperatures.

This paper uses some conventional fine-grained propellants (A and B — Table 1) to demonstrate any peculiarities of burning rate determination. Conventional single base propellants C and D with large tube grains were comparative propellants.

TABLE 1

Average dimensions (in mm) and geometrical characteristics of investigated propellant grains

Average dimensions of grain (producer's declaration)	Single-base propellant (tube)			Single-base propellant D (seven-tubed)
	A	B	C	
total layer — e_1	0.1625	0.185	0.76	0.5
inside diameter	0.15	0.25	1.91	0.47
length ($2c$)	1.9	6.2	75	12.3
$\kappa = 1 + e_1/c$	1.1757	1.0597	1.0203	0.7178*
$\lambda = -(e_1/c)/(1 + e_1/c)$	-0.1494	-0.0563	-0.0199	0.1937*

* — before breaking into slivers

Producer's declaration on average properties of very small, tube grains does not correspond with the facts. Tolerances in propellant manufacture can result in variation of dimension and shape of propellant grains throughout a charge. First of all it is shown for propellant A and B (Figures 2 and 3).

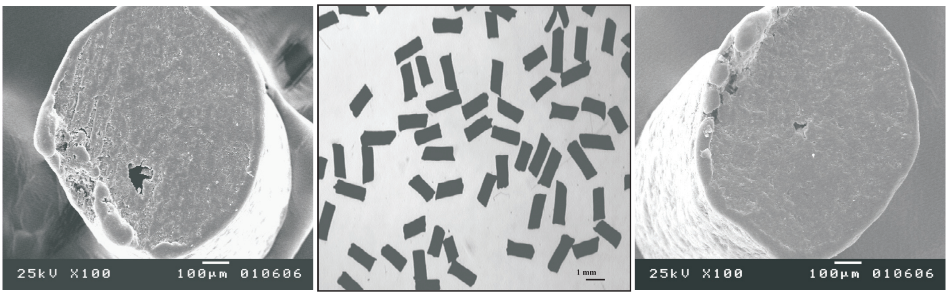

Fig. 2. Examples of real shapes of analysed fine-grained, tube propellant A (in the middle) and front surface of grain: partially blocked entrance (on the right) or displacement of the inside hole towards the outside surface of grain (on the left)

Fig. 3. Examples of real shapes of analysed fine-grained, tube propellant B (on the left), tube propellant C (in the middle) and 7-hole propellant D (on the right)

The results of the comparison between experimental investigations of propellants (A, B, C and D) using different ignition methods are presented in this paper.

3. Closed vessel tests — results and discussion

Figures 4-7 show comparison of values of linear burning rate law coefficient r_1 for different masses of black powder and for two calculating methods (from total or limited impulse of pressure — Eqs. 1.2 and 1.3).

The both calculation methods, although treated as equivalent, give different values of coefficient r_1 for all investigated fine-grained propellants and only propellant C (with larger and more regular geometrically grains) gives similar values of coefficient r_1.

Large tube grains of propellant C are more regular geometrically, without the imperfections typical of fine-grained propellants, and products of combustion for black powder have easier access to all surface of grain. It creates, for propellant C, better conditions of heat transfer between hot ignition gases and propellant surface and in consequence better conditions to meet theoretical assumptions used in burning rate analysis. Therefore the values of coefficient r_1 are similar for two different calculating methods.

Figures 4-7 present also changes of coefficient r_1 with growing mass of the igniter charge. In comparison with ignition system without black powder (only electric match) coefficient r_1 growing up about 20÷40% with mass of the igniting charge for

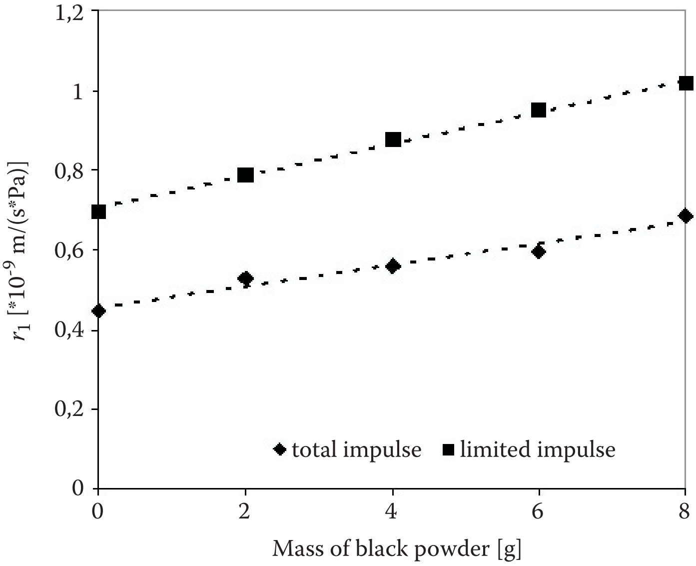

Fig. 4. Coefficient r_1 changes (with mass of igniter) for propellant A

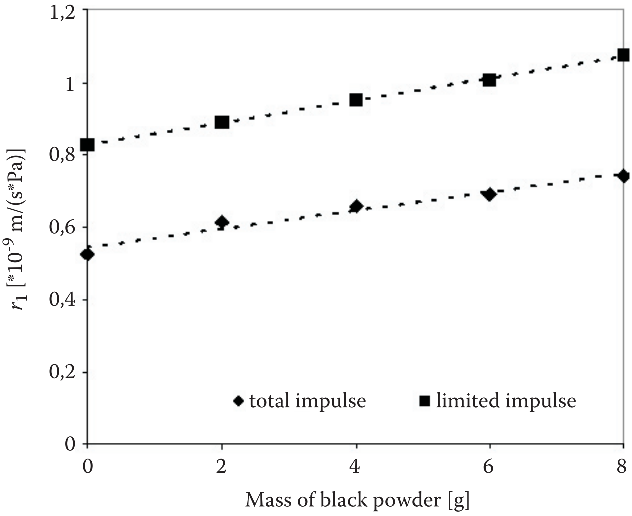

Fig. 5. Coefficient r_1 changes (with mass of igniter) for propellant B

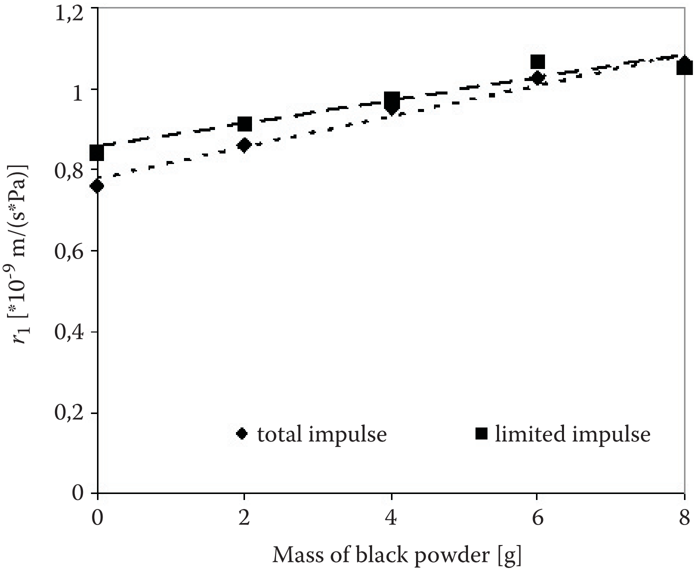

Fig. 6. Coefficient r_1 changes (with mass of igniter) for propellant C

ignition system with 8 g of black powder. The differences in burning rate calculations may be a reason of significant errors in theoretical calculations of pressure-travel curves and values of maximum pressure in internal ballistic simulations. Presented above results are consequence of used method of pressure-time curve integration, that considers burning process entirely and gives averaging in time.

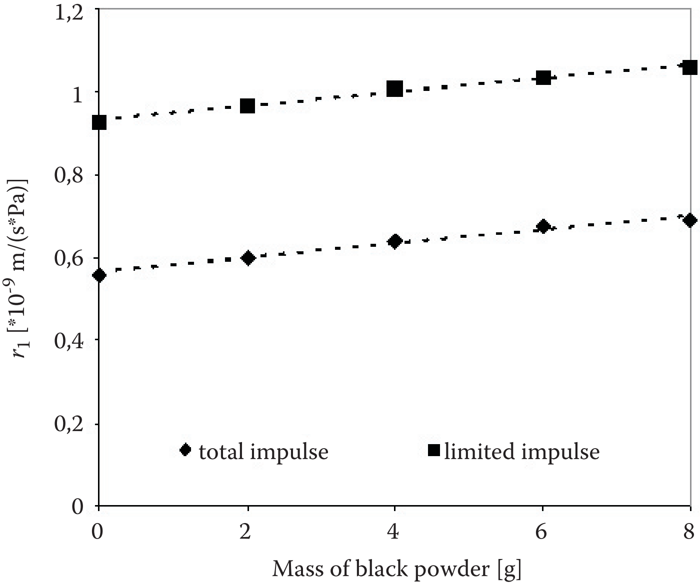

Fig. 7. Coefficient r_1 changes (with mass of igniter) for propellant D

The coefficient r_1 behaviour throughout the all process of burning propellant ($0 \le z \le 1$) may be obtained using differentiated experimental pressure-time curve and mass fraction burning rate equation [5]

$$\omega \cdot \frac{dz}{dt} = \delta \cdot S_1 \cdot \Phi(z) \cdot r_1 \cdot p \tag{3.1}$$

transformed to the below equation

$$r_1 = \frac{dz}{dt} \cdot \frac{V_1}{S_1} \cdot \frac{1}{\Phi(z) \cdot p_z} = \frac{dz}{dp} \cdot \frac{dp}{dt} \cdot \frac{V_1}{S_1} \cdot \frac{1}{\Phi(z) \cdot p_z}, \tag{3.2}$$

where: dz/dp — the burnt mass change with pressure calculated from the Noble-Abel's equation of state;
dp/dt — the rate of change of pressure obtained from the experimental pressure-time curve;
V_1, S_1 — the initial values of volume and exposed surface area of the grain;
$\Phi(z)$ — the form function

$$\Phi(z) = \sqrt{1 + 4\frac{\lambda}{\kappa} \cdot z}, \tag{3.3}$$

ω — the mass of propellant charge;
δ — the density of propellant.

In this case, the linear burning rate law $r = r_1 p$ is assumed. Figures 8-11 present changes of coefficient r_1 behaviour throughout the all process of burning. The figures inform that value of coefficient r_1 changes and only for propellant C may

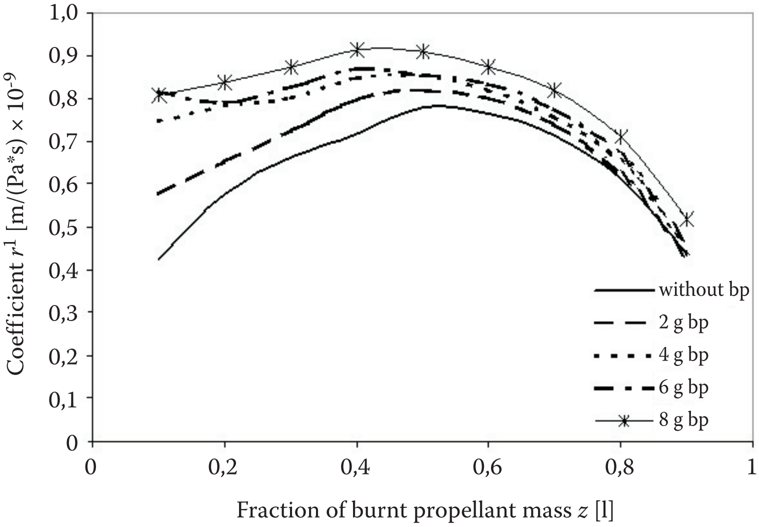

Fig. 8. Coefficient r_1 changes during burning process of propellant A

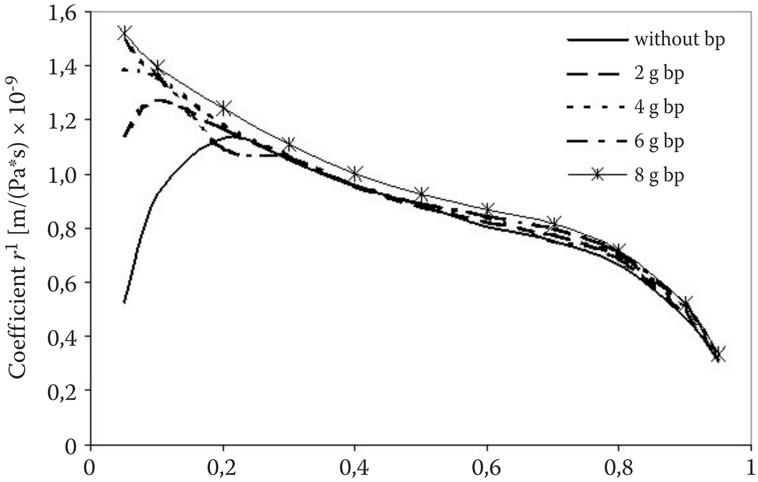

Fig. 9. Coefficient r_1 changes during burning process of propellant B

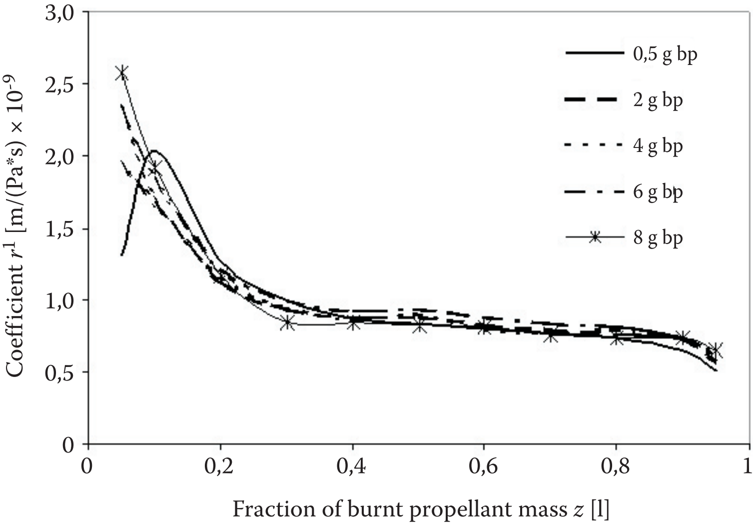

Fig. 10. Coefficient r_1 changes during burning process of propellant C

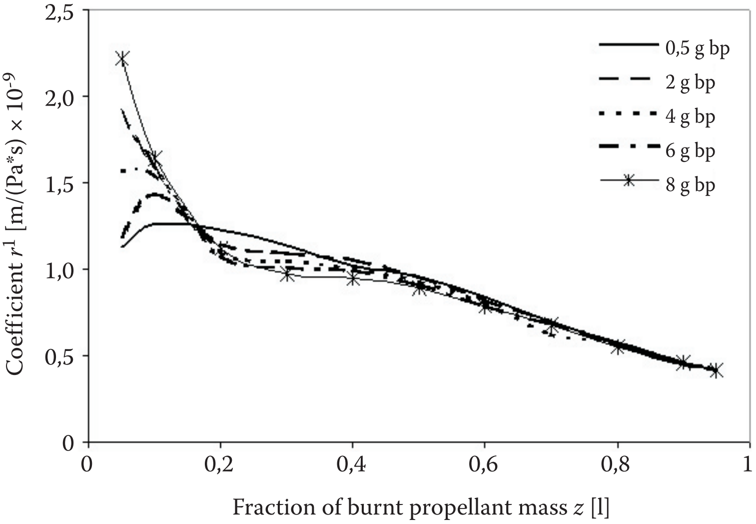

Fig. 11. Coefficient r_1 changes during burning process of propellant D

be treated as nearly constant but only in fundamental period of combustion. The smaller grains the larger deviations from constancy and irregularity (particularly for propellant A). Performed calculations show similar coefficient r_1 behaviour to the experimental vivacity behaviour also for other initial temperatures of propellant [3]. It means that value of coefficient r_1 cannot be treated as constant during internal ballistic simulations of shooting.

Large tube grains (propellants C and D) are more regular geometrically, without the imperfections typical of fine-grained tube propellant (A and B) and ignition gases have easier (in contrast to propellant A) access to all surface of grain [4]. Real ignition process of propellant A (in the case of small mass of black powder) does not meet theoretical assumptions but significant increase of black powder mass during closed vessel firings creates better and better conditions of ignition process becoming close to the theoretical model of propellant burning.

4. Conclusion

In the paper the linear form of burning rate law $r = r_1 p$ was reviewed. So far the coefficient r_1 has been treated (for a given propellant) as constant for total combustion process of all conventional propellants and the linear form of burning rate law has been used in interior ballistics governing equations. The main conclusions following from this work are:

1. Closed vessel tests are often conducted under conditions that are far different from those encountered in a gun system. Therefore extrapolation of formulated burning rate laws to a gun system usually yields poor results unless some adjustments are made.
2. Real ignition process of fine-grained propellant does not meet theoretical assumptions but significant increase in black powder mass during closed vessel firings creates better and better conditions of ignition process becoming close to the theoretical model of propellant burning.
3. The differences in calculations of propellant burning rate indicate that there are limitations to the validity of the linear approach of burning rate law. The linear form of burning rate law with constant value of coefficient r_1 should not be particularly used in ballistic calculations of gun systems with fine-grained propellants.
4. The linear form of burning rate law may be used rather for propellants with large, more regular geometrically grains and such ignition system that permit uniformly propellant ignition with all exposed surface areas of the grains. In this case the values of coefficient r_1 may be similar for two different calculating methods (from total or limited impulse of pressure). Linearity of this form should be tested in advance (according to Eq. 3.2).

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Z. K. LECIEJEWSKI

Krytyczna analiza liniowej postaci prawa szybkości spalania w odniesieniu do konwencjonalnych prochów drobnoziarnistych

Streszczenie. Bardzo popularna w laboratoriach balistycznych Europy Wschodniej liniowa postać $r = r_1 p$ prawa szybkości spalania prochów wykorzystywana jest w analizach i symulacjach komputerowych pracy prochowych układów miotających niezależnie od typu prochu i wymiarów ziaren prochowych. Wartość współczynnika r_1 liniowej postaci tego prawa zgodnie z przyjętą metodyką jest obliczana na podstawie średnich wymiarów ziaren prochu (grubości spalonej warstwy prochu) oraz impulsu ciśnienia gazów prochowych z badań pirostatycznych. Zakłada się, że wartość współczynnika r_1 jest stała (dla danego typu prochu) i nie zależy od wartości ciśnienia gazów prochowych. W ramach niniejszej pracy — w celu określenia wartości współczynnika prawa szybkości spalania — przeprowadzono, dla jednej określonej gęstości ładowania, badania pirostatyczne kilku prochów jednobazowych o różnych kształtach ziaren prochowych. W trakcie badań zastosowano różne masy zapłonników z prochu czarnego. Zaprezentowane w artykule wyniki badań eksperymentalnych i obliczeń pokazują istotny wpływ zastosowanego układu zapłonowego na szybkość spalania (współczynnik r_1 prawa szybkości spalania) prochu. Ukazane różnice w wynikach obliczeń współczynnika r_1 mogą być przyczyną błędów w kalkulacjach krzywych balistycznych charakteryzujących pracę prochowych układów miotających.

Słowa kluczowe: prochy jednobazowe**,** szybkość spalania, badania pirostatyczne
Symbole UKD: 662.1/.4