TRANSPORT AND GENERATION OF THE EXCESS CARRIERS IN SEMICONDUCTORS IN THE PRESENCE OF A TEMPERATURE GRADIENT AND A MAGNETIC FIELD

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ABSTRACT

In the Sections 1 and 2 of the paper the theory and a quantitative analysis of ambipolar thermodiffusion are presented. Formulas are derived describing the internal and total current densities and the excess carrier density distribution in a semiconductor in the presence of a temperature gradient perpendicular to the surface of the semiconductor plate. The Sections 3 and 4 describe the interaction of thermodiffusion current and perpendicular magnetic field resulting with *emf* and voltage between sample electrodes. We have called this phenomenon as thermomagnetoelectric effect (TME). The theory gives the formula that defines the TME voltage, which depends on the carrier lifetime and the surface recombination velocity of the semiconductor sample.

1. Introduction

The longest known electrical phenomenon associated with a temperature gradient is the Seebeck thermoelectric effect, i.e. the flow of electric current due to this gradient.

A temperature gradient may also induce an electrically neutral flow of holes and electrons due to the Soret thermodiffusion. This effect was described by Price and named the ambipolar diffusion [1]. An analysis of the transport equations has shown that the Price theory is strictly correct where the volume and/or surface recombination are sufficiently extensive, since the electrically neutral flux of hole-electron pairs results in the carrier concentration in the individual regions of the semiconductor sample being increased or decreased. This was discussed in [2] where we also draw the reader's attention to the split of the Fermi level and to the measuring experimental possibilities that result from this fact. This problem has been discussed also by Gurevich et al. in [3] and [4].

The magnetic field acting in an irradiated plate placed between the poles of a magnet splits the electron and hole fluxes, as a result of which electric charges and a voltage appear on the sample walls parallel to the fluxes and to the magnetic field. This phenomenon is widely known as the photoelectromagnetic effect.

The magnetic field will act in an analogous way also when the electron-hole pair flux is induced by the Price ambipolar thermodiffusion. The present paper describes and analyses this effect, which we term as the thermomagnetoelectric effect (TME).

The first part of the paper contains the derivation of the formula that defines the internal current density J^*. This current defines quantitatively the ambipolar thermodiffusion in the absence of the magnetic field, under the assumption that the values of the lifetime and the velocities of surface recombination are finite. The paper also describes the distribution of the excess carrier concentration in a cuboidal semiconductor plate within which a temperature gradient perpendicular to the plate surface occurs.

The second part of the paper gives the derivation of the formulae that describe the action of a magnetic field on the thermodiffusion carrier flux. Formulae are given for the current densities and the distribution of the electric field in a semiconductor plate, under the assumption that the temperature gradient is

perpendicular to its surface whereas the magnetic field acts is parallel to it.

This has led us to a quantitative description of the thermomagnetoelectric effect. For the sake of experiments, we also give the formula for the voltage induced by this effect.

2. Internal and total current densities and the distribution of excess carriers in a semiconductor plate with the temperature gradient acting within it

Figure 1 shows a cross-section of an infinite plate of thickness w made of inhomogeneous semiconductor. The axis z of the co-ordinate system is perpendicular to the free surfaces of the plate. We shall consider the case in which the absolute temperature T only depends on the co-ordinate z, which means that *grad T* is perpendicular to the sample surface and that all the other parameters that describe the state of the material are also dependent on z alone.
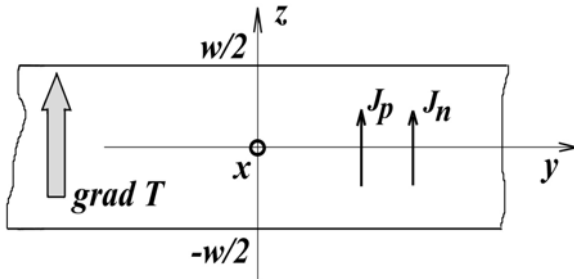

Fig. 1. Currents and temperature gradient directions in an infinite homogeneous semiconductor plate.

According to Ref. [5], the densities of the hole current J_p and the electron current J_n in this case are given by:

$$J_p = -\sigma_p \frac{dV}{dz} - \mu_p k_B T \frac{dp}{dz} + \sigma_p \frac{k_B T}{q} \frac{d \ln n_i}{dz} -$$

$$\sigma_p \left(\varepsilon_p - \ln \frac{n_i}{N_V} \right) \frac{d}{dz} \left(\frac{k_B T}{q} \right), \qquad (1)$$

$$J_n = -\sigma_n \frac{dV}{dz} + \mu_n k_B T \frac{dn}{dz} - \sigma_n \frac{k_B T}{q} \frac{d \ln n_i}{dz} -$$

$$\sigma_n \left(\varepsilon_n - \ln \frac{n_i}{N_C} \right) \frac{d}{dz} \left(\frac{k_B T}{q} \right), \qquad (2)$$

where σ_n, σ_p are the hole and electron conductivities, k_B is the Boltzmann constant, μ_p, μ_n are the hole and electron mobilities, respectively, ε_p, ε_n are coefficients dependent on the current carrier scattering model, N_V, N_C are the state densities of the basic and conductance bands, q is the electron charge, V is the electric potential, and n_i is the intrinsic concentration at a given temperature.

Equations (1) and (2) have a general character and do not reflect the role played by the excess carriers, although the quantities n and p include both the equilibrium concentrations n_o, p_o and the concentration of excess carriers.

To demonstrate how the excess carrier concentration affects the phenomena that occur in the semiconductor, we should use the quasi-neutrality principle and express the concentrations in the forms:

$$n = p + N, \qquad (3)$$

$$n_o = p_o + N \qquad (4)$$

which can be referred to both complete concentrations n and p, and the equilibrium concentrations n_o, p_o. It is also assumed that the concentration N is equal to the difference between the concentrations of ionised donors and ionised acceptors, $N_D^+ - N_A^-$, and does not depend on the excess concentrations or on the temperature.

Subtracting Eqs. (3) and (4), we obtain

$$n - n_o = p - p_o = \Delta p \qquad (5)$$

and hence

$$n = n_o + \Delta p, \; p = p_o + \Delta p. \qquad (6)$$

Let the current densities be expressed as

$$J_p = \frac{\sigma_p}{\sigma} J + J^*,$$

$$J_n = \frac{\sigma_n}{\sigma} J - J^*, \qquad (7)$$

where the density of the current which we will denote with the term "internal current" is

$$J^* = \frac{\sigma_n}{\sigma} J_p - \frac{\sigma_p}{\sigma} J_n. \qquad (8)$$

The current components, which are fractions of the total current J, are expressed in a simple form; what is to be determined is the internal current J^* that is involved in both the currents but with the opposite signs.

When the total current $J = 0$, the hole and electron currents cancel each other, but nevertheless they carry hole and electrons fluxes that flow in the same directions. This results from the fact that $J_p = J^*$ and $J_n = -J^*$ on the assumption of zero total current. This motivates the term "internal current".

If a temperature gradient occurs, these currents and the fluxes they carry play a particular role, which will be discussed in this Section. The internal current J^* is the sum of the three components

$$J^* = J_{th} + J_1^* + J_2^*, \qquad (9)$$

where

$$J_{th} = -\frac{\sigma_p \sigma_n}{\sigma} \left(\varepsilon_p + \varepsilon_n + \frac{E_G}{k_B T} \right) \frac{d}{dz} \left(\frac{k_B T}{q} \right). \quad (10)$$

J_{th} is the basic component of the electrothermal current (this is why we use the index '*th*') It is consistent with the Price equation for ambipolar thermodiffusion [1] in the derivation of which Price took no account of excess carriers, an assumption which would be justified if a very strong bulk and/or surface recombination took place in the semiconductor sample. The aim of the present Section is to include the effect of a finite value recombination.

Let us express J_1^* in the form

$$J_1^* = \frac{\sigma_n \sigma_p}{\sigma} \left(\frac{n_o}{p} + \frac{p_o}{n} \right) \cdot$$

$$\left(3 - \frac{1}{k_B} \frac{dE_G}{dT} + \frac{E_G}{k_B T} \right) \frac{\Delta p}{n_o + p_o} \frac{d}{dz} \left(\frac{k_B T}{q} \right) \quad (11)$$

and J_2^* in the form

$$J_2^* = -qD \frac{d\Delta p}{dx} = -\mu k_B T \frac{d\Delta p}{dx}. \quad (12)$$

Equations (10), (11) and (12) are derived in Appendix 1, which also gives the definitions of the ambipolar diffusion coefficient D and the ambipolar mobility μ.

Introducing the quantity (with dimension ms^{-1})

$$V_{th} = \left(3 - \frac{1}{k_B} \frac{dE_G}{dT} + \frac{E_G}{k_B T} \right) \frac{1}{n_o + p_o} \frac{d}{dz} \left(\frac{k_B T}{q} \right) \quad (13)$$

we can write, using Eqs. (9), (10), (11) and (12), that

$$J^* = J_{th} + qV_{th} \Delta p - qD \frac{d\Delta p}{dz}. \quad (14)$$

This equation is one of the transport equations. The second equation is the continuity equation. In a stationary case, assuming the simplest recombination model and taking into account that no excess carrier generation sources are present, we have

$$\frac{dJ_p}{dz} = -\frac{dJ_n}{dz} = -\frac{\Delta p}{\tau}, \quad (15)$$

where τ is the lifetime of the excess carriers.

Since no electrodes are installed on the sample surfaces, we should take $J = 0$. Then using Eq. (7) we obtain

$$\frac{dJ^*}{dz} = -\frac{q\Delta p}{\tau}. \quad (16)$$

The second continuity equation is derived from Eqs. (13) and (14) and has the form:

$$\frac{d\Delta p}{dz} = \frac{V_{th}}{D} \Delta p - \frac{J^*}{qD} + \frac{J_{th}}{qD}. \quad (17)$$

The last two equations constitute a system of differential transport equations with the two unknown functions $J^*(z)$ and $\Delta p(z)$. Both equations are linear equations with variable coefficient V_{th} and J_{th} according to Eqs. (10) and (13). Hence the solutions

should be found by numerical methods. It is, however, advisable that the solutions should be sought with the approximately constant values of V_{th} and J_{th} assumed *a priori*.

In order to obtain an equation with the single unknown J^*, we use Eq. (16) for determining the quantities

$$\Delta p = -\frac{\tau}{q} \frac{dJ^*}{dz}, \quad \frac{d\Delta p}{dz} = -\frac{\tau}{q} \frac{d^2 J^*}{dz^2}. \quad (18)$$

Then substituting these quantities into Eq. (14) we obtain

$$J^* = J_{th} - V_{th} \tau \frac{dJ^*}{dz} + D \tau \frac{d^2 J^*}{dz^2}. \quad (19)$$

Now denoting

$$L_u = \frac{V_{th} \tau}{2}, \qquad L_D^2 = D \tau \quad (20)$$

and using this notation in Eq. (19) we obtain

$$L_D^2 \frac{d^2 J^*}{dz^2} - 2 L_u \frac{dJ^*}{dz} - J^* = -J_{th}. \quad (21)$$

The solution of this equation is the function

$$J^* = A e^{\frac{z}{L_1}} + B e^{-\frac{z}{L_2}} + J_{th}, \quad (22)$$

where A and B are arbitrary constants, whereas L_1 and L_2 are given by

$$L_1 = \sqrt{L_u^2 + L_D^2} - L_u ;$$

$$L_2 = \sqrt{L_u^2 + L_D^2} + L_u, \quad (23)$$

where L_u and L_D are given by (20).

The derivative of J^* with respect to z is

$$\frac{dJ^*}{dz} = \frac{A}{L_1} e^{\frac{z}{L_1}} - \frac{B}{L_2} e^{-\frac{z}{L_2}} \quad (24)$$

since, by virtue of the initial assumption, $J_{th} = $ const.

The constants A and B should be determined from the boundary conditions

$$J^* \left(-\frac{w}{2} \right) = -qS_1 \Delta p \left(-\frac{w}{2} \right),$$

$$J^* \left(\frac{w}{2} \right) = qS_2 \Delta p \left(\frac{w}{2} \right), \quad (25)$$

where $-w/2$ and $w/2$ are the co-ordinates of the bottom and top surfaces of the sample, respectively, and S_1, S_2 are the surface recombination velocities on the bottom and top sample surfaces, respectively.

Since we seek J^*, we substitute Δp as defined by Eq. (18) and obtain

$$J^* \left(-\frac{w}{2} \right) = S_1 \tau \left(\frac{dJ^*}{dz} \right)_{z = -w/2},$$

$$J^*\left(\frac{w}{2}\right) = -S_2\tau\left(\frac{dJ^*}{dz}\right)_{z=w/2}. \quad (26)$$

Then, inserting the values of $J^*(\pm w/2)$ and $(dJ^*/dz)_{z=\pm w/2}$ as given by Eqs. (22) and (25) into Eq. (26), we obtain

$$Ae^{-\frac{w}{2L_1}} + Be^{\frac{w}{2L_2}} + J_{th} =$$

$$S_1\tau\left(\frac{A}{L_1}e^{\frac{w}{2L_1}} - \frac{B}{L_2}e^{-\frac{w}{2L_2}}\right), \quad (27)$$

$$Ae^{\frac{w}{2L_1}} + Be^{-\frac{w}{2L_2}} + J_{th} =$$

$$-S_2\tau\left(\frac{A}{L_1}e^{\frac{w}{2L_1}} - \frac{B}{L_2}e^{-\frac{w}{2L_2}}\right). \quad (28)$$

Ordering these equations we obtain

$$Ae^{\frac{-w}{2L_1}}\left(1 - \frac{S_1\tau}{L_1}\right) + Be^{\frac{w}{2L_2}}\left(1 + \frac{S_1\tau}{L_2}\right) = -J_{th}, \quad (29)$$

$$Ae^{\frac{w}{2L_1}}\left(1 + \frac{S_2\tau}{L_1}\right) + Be^{\frac{-w}{2L_2}}\left(1 - \frac{S_2\tau}{L_2}\right) = -J_{th}. \quad (30)$$

Hence we can determine the constants A and B as

$$A = -J_{th}\frac{Q_2 - R_2}{\Delta},$$

$$B = -J_{th}\frac{Q_1 - R_1}{\Delta}, \quad (31)$$

where

$$Q_1 = \left(1 - \frac{S_1\tau}{L_1}\right)e^{-\frac{w}{2L_1}}, Q_2 = \left(1 - \frac{S_2\tau}{L_2}\right)e^{-\frac{w}{2L_2}},$$

(32)

$$R_1 = \left(1 + \frac{S_2\tau}{L_1}\right)e^{\frac{w}{2L_1}}; R_2 = \left(1 + \frac{S_1\tau}{L_1}\right)e^{\frac{w}{2L_2}}, \quad (33)$$

$$\Delta = Q_1Q_2 - R_1R_2. \quad (34)$$

Substituting these into Eq. (22) gives

$$J^*(z) = J_{th}\left(1 - \frac{Q_2 - R_2}{\Delta}e^{\frac{z}{L_1}} - \frac{Q_1 - R_1}{\Delta}e^{-\frac{z}{L_2}}\right). \quad (35)$$

Now by differentiating $J^*(z)$ of Eq. (35) and using the first formula in (18) we obtain the equation for the distribution $\Delta p(z)$ in the form

$$\Delta p(z) = \frac{\tau}{q\Delta}J_{th}\left(\frac{Q_2 - R_2}{L_1}e^{\frac{z}{L_1}} + \frac{Q_1 - R_1}{L_2}e^{-\frac{z}{L_2}}\right).$$

(36)

For further calculations we need a formula that defines the average value of the current density, J^*_{av}, which can be obtained from Eq. (35) to be

$$J^*_{av} = \frac{1}{w}\int_{-w/2}^{w/2}J^*(z)dz =$$

$$J_{th}\left(\begin{array}{c}1 - \frac{Q_2 - R_2}{\Delta}\frac{2L_1}{w}sh\frac{w}{2L_1} - \\ \frac{Q_1 - R_1}{\Delta}\frac{2L_2}{w}sh\frac{w}{2L_2}\end{array}\right). \quad (37)$$

Let us consider the two cases:
1) If the carrier lifetime τ is great enough for the inequality

$$\sqrt{L_u^2 + L_D^2} - |L_u| \gg \max\left(\frac{w}{2}, \frac{D}{S_1}, \frac{D}{S_2}\right), \quad (38)$$

to be fulfilled, we can take

$$\frac{2L_1}{w}sh\frac{w}{2L_1} \cong 1 \cong \frac{2L_2}{w}sh\frac{w}{2L_2} \quad (39)$$

and, then, we have

$$J^*_{av} \cong J_{th}\left(1 - \frac{Q_2 - R_2 + Q_1 - R_1}{\Delta}\right). \quad (40)$$

The bracketed members can further be simplified by leaving, in the brackets of Eqs. (32) and (33), only the members that contain τ. If this is done, we have

$$Q_2 + Q_1 - R_2 - R_1 \cong -\frac{S_1\tau}{L_1} - \frac{S_2\tau}{L_2} - \frac{S_1\tau}{L_2} - \frac{S_2\tau}{L_1} =$$

$$-\frac{(S_1 + S_2)(L_1 + L_2)}{D} \quad (41)$$

and

$$\Delta = \frac{S_1 S_2 \tau^2}{L_1 L_2} e^{-\frac{w}{2}\left(\frac{1}{L_1}+\frac{1}{L_2}\right)} - \frac{S_1 S_2 \tau^2}{L_1 L_2} e^{\frac{w}{2}\left(\frac{1}{L_1}+\frac{1}{L_2}\right)} \cong$$

$$-\frac{S_1 S_2 \tau^2}{L_1 L_2} 2sh\frac{w}{2}\left(\frac{1}{L_1}+\frac{1}{L_2}\right) \cong$$

$$-\frac{S_1 S_2 \tau^2}{L_1 L_2} w\left(\frac{1}{L_1}+\frac{1}{L_2}\right) =$$

$$\frac{S_1 S_2 \tau^2}{L_D^4}(L_1+L_2)w = -\frac{S_1 S_2}{D^2}(L_1+L_2)w$$

(42)

where Eqs. (23) are used.

It follows from these equations that

$$\frac{-(S_1+S_2)(L_1+L_2)}{-\frac{S_1 S_2}{D}(L_1+L_2)w} = \frac{D}{w}\left(\frac{1}{S_1}+\frac{1}{S_2}\right). \quad (43)$$

Hence, if the condition (38) is fulfilled, we obtain J^*_{av} from equation (40) as

$$J^*_{av} \cong J_{th}\left[1 - \frac{D}{w}\left(\frac{1}{S_1}+\frac{1}{S_2}\right)\right]. \quad (44)$$

2) If $S_1 = S_2 = 0$, we have

$$Q_2 - R_2 = -2 sh\frac{w}{2L_2},$$

$$Q_1 - R_1 = -2 sh\frac{w}{2L_1}, \quad (45)$$

$$\Delta = Q_1 Q_2 - R_1 R_2 = e^{-\frac{w}{2L_1}} e^{-\frac{w}{2L_2}} -$$
$$e^{\frac{w}{2L_1}} e^{\frac{w}{2L_2}} = -2 sh\frac{w}{2}\left(\frac{1}{L_1}+\frac{1}{L_2}\right) \quad (46)$$

Substituting these results in Eq. (37) we obtain

$$J^*_{av} = J_{th}\left(\begin{array}{c} 1 - 2\frac{L_1}{w}\frac{sh\frac{w}{2L_2}sh\frac{w}{2L_1}}{sh\frac{w}{2}\left(\frac{1}{L_1}+\frac{1}{L_2}\right)} - \\ 2\frac{L_2}{w}\frac{sh\frac{w}{2L_1}sh\frac{w}{2L_2}}{sh\frac{w}{2}\left(\frac{1}{L_1}+\frac{1}{L_2}\right)} \end{array} \right) =$$

$$J_{th}\left(1 - 2\frac{L_1+L_2}{w}\frac{sh\frac{w}{2L_2}sh\frac{w}{2L_1}}{sh\frac{w}{2}\left(\frac{1}{L_1}+\frac{1}{L_2}\right)} \right). \quad (47)$$

If, in addition, min $(w/L_1, w/L_2) \gg 1$, we can replace the hyperbolic sinuses by exponential functions which gives

$$J^*_{av} \cong J_{th}\left[1 - \frac{L_1+L_2}{w}\right] = J_{th}\left[1 - 2\frac{\sqrt{L_u^2+L_D^2}}{w}\right]. \quad (48)$$

In the opposite case when the inequality max$(w/L_1, w/L_2) \ll 1$, the hyperbolic sinuses are equal to their arguments and we have

$$J^*_{av} = J_{th}\left[1 - 2\frac{L_1+L_2}{w}\frac{w^2}{4L_1 L_2}\frac{2}{w}\left(\frac{1}{L_1}+\frac{1}{L_2}\right)^{-1}\right] = 0 \cdot\cdot$$

(49)

These results show that Eq. (48) holds true when the sample thickness is greater than the diffusion length L_D and, thus, when the carrier lifetime is short. Equation (49) indicates that if J^*_{av} is close to zero, the diffusion length exceeds the sample thickness which corresponds to a sufficiently long carrier lifetime.

Further calculations require considering the total current density J. We start from Eqs. (1) and (2) which enable us to determine the sum of the hole and electron currents. A general case of this problem, was discussed in [6] where Eqs. (16) to (24) were involved. Here we shall present certain properties of the components of the thermodynamic force $\mathcal{E}_{th}$ that occur when the excess carriers with a concentration Δp are generated thermally without excitation by light.

The thermodynamic force $\mathcal{E}_{th}$ can be expressed in the form

$$\mathbf{E}_{th} = \frac{\mathbf{J}}{\sigma} =$$
$$- grad\ \mathit{\Psi} + \Gamma\ \Delta p grad\ T + S_{th-el}\ grad\ T . \quad (50)$$

The potential $\mathit{\Psi}$ is has been defined in [6] by Eq. (17) as

$$\mathit{\Psi} = V - \frac{k_B T}{q}\frac{\mu_n-\mu_p}{\mu_n+\mu_p}\ln\frac{\sigma}{\sigma_0}. \quad (51)$$

The components of the function Γ are

$$\Gamma = \Gamma_{bulk} + \Gamma_\mu + \Gamma_{th-p}, \quad (52)$$

where Γ_{bulk} is associated with the bulk field (Eqs. (18) and (19) in [6]), Γ_μ results from the variation of the mobility (Eq. (19)), and Γ_{th-p} is analogous to the function that occurs in the thermophotovoltaic effect (Eq. (20) in [6]). These three effects are proportional to *grad T* and to Δp. This latter quantity is here also proportional to *grad T* which results in its proportionality to *grad T*. The last term (containing the coefficient S_{th-el}) in Eq. (50) is also proportional to *grad T* as a result of the Seebeck effect.

Using Eq. (50) we can write

$$J = -\sigma \ grad \ \Psi + \sigma \boldsymbol{E}_V \ , \qquad (53)$$

where $\mathcal{E}_V$ denotes the two last terms in right-hand side of Eq. (50).

3. Currents and the electric field in the presence of a temperature gradient and a magnetic field. Thermomagnetoelectric effect

It is commonly known that, when the Hall effect occurs, the total current density J^B possesses not only the basic current density component J, which is independent of the magnetic field B, but also a components perpendicular to J and proportional to the Hall angle Θ_H. If the internal current $J*$ is not equal to zero, we have still another component perpendicular to $J*$. In effect, in the presence of magnetic field the total current J^B is equal to

$$\boldsymbol{J}^{\,B} = \boldsymbol{J} + \Theta_H \boldsymbol{J} \times \boldsymbol{b} + \Theta \boldsymbol{J}^{\,*} \times \boldsymbol{b} \ , \quad (54)$$

where

$$\Theta_H = \frac{\sigma_p}{\sigma}\Theta_p + \frac{\sigma_n}{\sigma}\Theta_n \ , \qquad (55)$$

$$\Theta = \Theta_p - \Theta_n \ , \qquad (56)$$

$$\Theta_p = \mu_{pH}\left|\boldsymbol{B}\right| , \ \Theta_n = -\mu_{nH}\left|\boldsymbol{B}\right| ,$$

$$b = \frac{\boldsymbol{B}}{\left|\boldsymbol{B}\right|} \ , \qquad (57)$$

where μ_{pH} and μ_{nH} are the Hall mobilities of the holes and electrons, respectively.

Equation (54) forms the basis of the theory of the photomagnetoelectric phenomena. The derivation of this equation can be found in [7].

Let us now consider the case of a cuboidal plate (Fig. 2) of thickness w, placed in a magnetic field that acts along the x-axis of the plate, in which a temperature gradient perpendicular to the plate surface, and thus parallel to the z-axis, has been
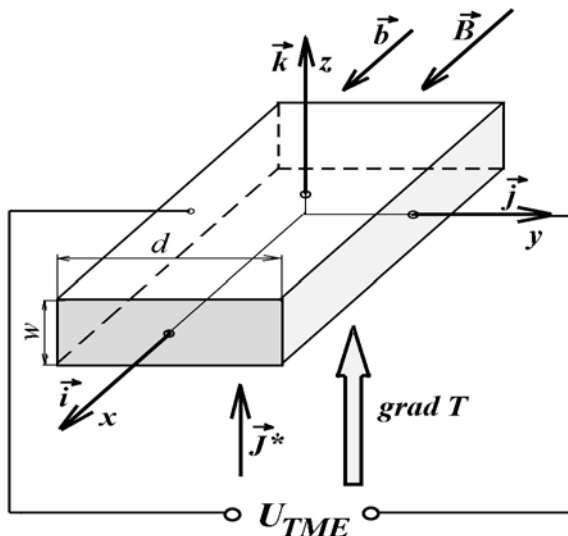

Fig. 2. Temperature gradient and magnetic field directions during the measurement of the thermomagnetoelectric voltage in a semiconductor cuboidol sample.

generated. The length and width d of the plate exceed considerably its thickness w.

The plate is equipped with point electrodes installed on its faces perpendicular to the y-axis. We assume that the measurement of the voltage at the electrodes does not induce a current flow and that the electrodes do not affect the distributions of the carriers and electric potential inside the sample. Furthermore no electrodes are installed on the remaining faces of the sample.

These circumstances determine the boundary conditions concerning the currents J^B and J. Thanks to the simple geometry of the sample and the suitable directions of the magnetic field B and *grad T* we can anticipate so that J^B has only the y-component so that

$$\boldsymbol{J}^B = J_y^B \boldsymbol{j} \qquad (58)$$

and J has the y- and z-components

$$J = J_y \boldsymbol{j} + J_z \boldsymbol{k} \ . \qquad (59)$$

The internal current $J*$, defined by Eq. (35), has the same direction as *grad T*. Thus we can write

$$\boldsymbol{J}* = J_z* \boldsymbol{k} \ . \qquad (60)$$

The direction b of the magnetic field coincides with the x-axis, and thus

$$\boldsymbol{b} = \boldsymbol{i}. \qquad (61)$$

Now substituting the results (58) to (61) into Eq. (54) we obtain

$$\boldsymbol{J} = J_x^B \boldsymbol{i} + J_y^B \boldsymbol{j} =$$

$$J_y \boldsymbol{j} + J_z \boldsymbol{k} + \Theta_H \left(J_y \boldsymbol{j} + J_z \boldsymbol{k}\right)\times \boldsymbol{i} + \Theta \, J_z^* \boldsymbol{k} \times \boldsymbol{i} =$$

$$\left(J_y + \Theta_H J_z + \Theta \, J_z^*\right)\boldsymbol{j} + \left(J_z - \Theta_H J_y\right)\boldsymbol{k} \qquad (62)$$

and hence

$$J_y^{\,B} = J_y + \Theta_H J_z + \Theta \, J_z^{\,*} \ , \quad (63)$$

$$0 = J_z - \Theta_H J_y \ . \qquad (64)$$

Let us determine the component J_z from Eq. (64) and substitute it to Eq. (63). Then we obtain

$$J_y^{\,B} = J_y + \Theta_H^2 J_y + \Theta \, J_z^{\,*} \ . \quad (65)$$

The term $\Theta_H^2 J_y$ should be neglected since our theory is restricted to the phenomena linear with respect to the magnetic field. Hence we have

$$J_y^{\,B} \cong J_y + \Theta \, J_z^{\,*} \ . \qquad (66)$$

To determine the z-component of J we use Eqs. (63) and (64), which only involve currents, and relate them with the potential Ψ by using Eq. (53).

When this is done we obtain

$$J_z = \sigma \frac{\partial \Psi}{\partial z} + \sigma \mathsf{E}_v \ . \qquad (67)$$

Taking J_z from Eq. (64) and substituting it in the above equation we obtain

$$\Theta_H J_y = -\sigma \frac{\partial \Psi}{\partial y} + \sigma \mathsf{E}_v . \quad (68)$$

Hence we have

$$\frac{\partial \Psi}{\partial z} = \frac{\Theta_H}{\sigma} J_y + E_v . \quad (69)$$

If the width of the sample fulfils the condition $d \gg w$ we can assume that J_y does not depend on y. This means that the edges parallel to x-axis is do not influence the distribution of $\boldsymbol{J}$ in the bulk of the sample. Moreover, the conductivity σ and $\mathcal{E}_v$ depend only on z because the temperature T is a function of the co-ordinate z because the temperature T is a function of the co-ordinate z.

Why the right-hand side of Eq. (69) depends only on z and differentiation of Eq. (69) with respect to the co-ordinate y gives

$$\frac{\partial}{\partial y}\left(\frac{\partial \Psi}{\partial z} \right) = 0 \quad (70)$$

and consequently

$$\frac{\partial}{\partial z}\left(\frac{\partial \Psi}{\partial y} \right) = 0 \quad (71)$$

which means that

$$\frac{\partial \Psi}{\partial y} = const . \quad (72)$$

Now we can define how the current density J_y^B depends on $\partial \Psi / \partial y$. From Eq. (53) we have

$$J_y^B = -\sigma \frac{\partial \Psi}{\partial y}$$

$$(73)$$

and substituting this in Eq. (66) we obtain

$$J_y^B = -\sigma \frac{\partial \Psi}{\partial y} + \Theta J_z^* . \quad (74)$$

In the right-hand side of the above equation the values of $\partial \Psi / \partial y$ is constant but the conductivity σ and J_z are not, and, thus J^B is not constant. Especially it is impossible to get $J_y^B = 0$ independently of z. To fulfil this boundary condition, even approximately, it is advisable to accept the condition

$$\int_{-w/2}^{+w/2} J_y^B \, dz = 0 . \quad (75)$$

Integrating both sides of Eq. (74) we obtain

$$\int_{-w/2}^{+w/2} J_y^B \, dz = 0 = -\frac{\partial \Psi}{\partial y} \int_{-w/2}^{w/2} \sigma dz + \Theta \int_{-w/2}^{w/2} J_z^*$$

$$(76)$$

and hence we can calculate $\partial \Psi / \partial y$

$$\frac{\partial \Psi}{\partial y} = \Theta \frac{\displaystyle\int_{-w/2}^{w/2} J_z^*}{\displaystyle\int_{-w/2}^{w/2} \sigma dz} = \Theta \frac{J_{av}^*}{\sigma_{av}} , \quad (77)$$

where the average current density J_{av}^* is defined by (37).

The formula (76) enables us to calculate the voltage U_{TME} generated between the electrodes shown in Fig. 2. This voltage appears to be

$$U_{TEM} = \Theta \frac{J_{av}^*}{\sigma_{av}} d , \quad (78)$$

where d is the sample thickness.

The voltage U_{TEM} is a measure of the thermomagnetoelectric effect. Measuring it is equivalent with the measurement of the electrothermal current composed of the Price current and the components dependent on the temperature gradient and the bulk and surface recombination properties.

4. Numerical calculations

These numerical calculations permitted us to determine the magnitudes of the thermomagnetoelectric voltage that occurred between the contact electrodes of a semiconductor sample placed in a magnetic filed when a temperature gradient is imposed along the sample thickness (see Fig. 2). The principal aim of the calculations was to find how the carrier lifetime and the surface recombination velocity affected this voltage and to determine the recombination parameters at which the contribution of the excess carriers to this effect is small and the magnitude of the thermomagnetoelectric voltage only depends on the Price thermodiffusion current − a current which depends on the variation of the forbidden band gap with temperature, and on the magnitude of the temperature gradient established in the sample.

The semiconductor sample chosen for the calculations was a cuboidal *n*-type germanium plate (of a cross-section of 1x1 cm) with a donor concentrations of $5 \cdot 10^{19}$ m^{-3} and $1 \cdot 10^{20}$ m^{-3}, within which a temperature gradient of 4000 K/m was imposed, whereas the average temperature was 300 K. The assumption underlying the calculations was that the carrier mobilities, intrinsic concentration, effective state concentrations and the bandgap one a corresponding to the the average temperature. The intensity of the magnetic field was taken to be 0.5 T.

Figure 3a shows the variation of the voltage U_{TME}, given by Eq. (78), as a function of the bulk carrier lifetime for the two donor concentrations: $5 \cdot 10^{19}$ m^{-3} (thick line) and $1 \cdot 10^{20}$ m^{-3} (thin line).

The difference between the density of the Price thermodiffusion current and the density of the generalised Price current, given by Eq. (39), can be seen in Fig. 3b. This plot indicates that, as the lifetime becomes shorter, the difference between these two densities quickly decreases to zero. The depth distribution of the excess carrier concentration is shown in Figs. 3c for donor concentrations of $5 \cdot 10^{19}$ m^{-3} and the carrier lifetimes of $5 \cdot 10^{-4}$ s (thin line) and $1 \cdot 10^{-5}$ s (thick line), respectively. The calculation results shown in Fig. 3 were obtained under the assumption that the surface recombination velocities are $S_1 = S_2 = 1$ cm/s.
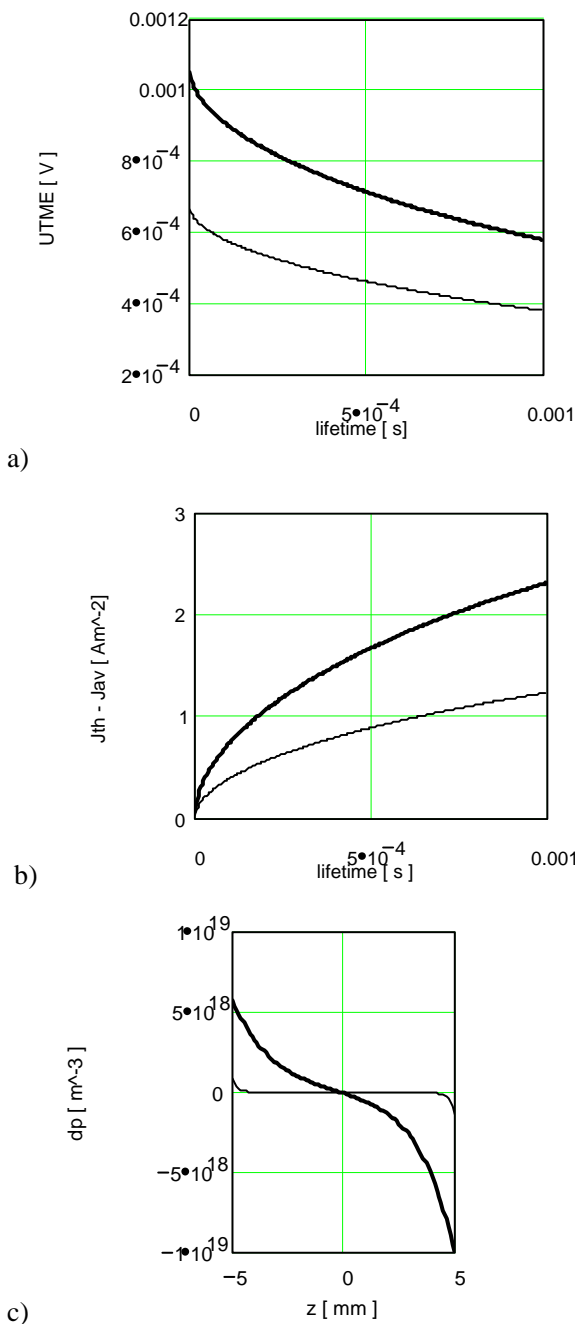
Fig. 3. a) Thermomagnetoelectric voltage as a function of the bulk lifetime for donor concentration $5 \cdot 10^{19}$ m^{-3} (thick line) and $1 \cdot 10^{20}$ m^{-3} (thin line); b) difference between the density of the Price thermodiffusion current and the density of generalised current for donor concentration $5 \cdot 10^{19}$ m^{-3} (top line) and $1 \cdot 10^{20}$ m^{-3} (down line); c) the depth distribution of the excess carrier concentration in sample with two carriers lifetime $5 \cdot 10^{-4}$ s (thin line) and $1 \cdot 10^{-5}$ s (thick line), donor concentration $5 \cdot 10^{19}$ m^{-3}. The recombination velocity $S_1 = S_2 = 1$ cm/s.

Figure 4 shows the relationships corresponding to those of Fig. 3, but with the surface recombination velocities taken to be $S_1 = S_2 = 10000$ cm/s. Figure 4a shows magnitude of U_{TME} for $N = 5 \cdot 10^{19}$ m^{-3} (thick line) and for $N = 1 \cdot 10^{20}$ m^{-3} (thin line). The plots show that, in this case, the thermomagnetoelectric voltage depends on the carrier lifetime in a very small degree.

Even though the lifetime is relatively short (one microsecond), the magnitude of U_{TME} is chiefly determined by the Price thermodiffusion current (Fig. 4b).
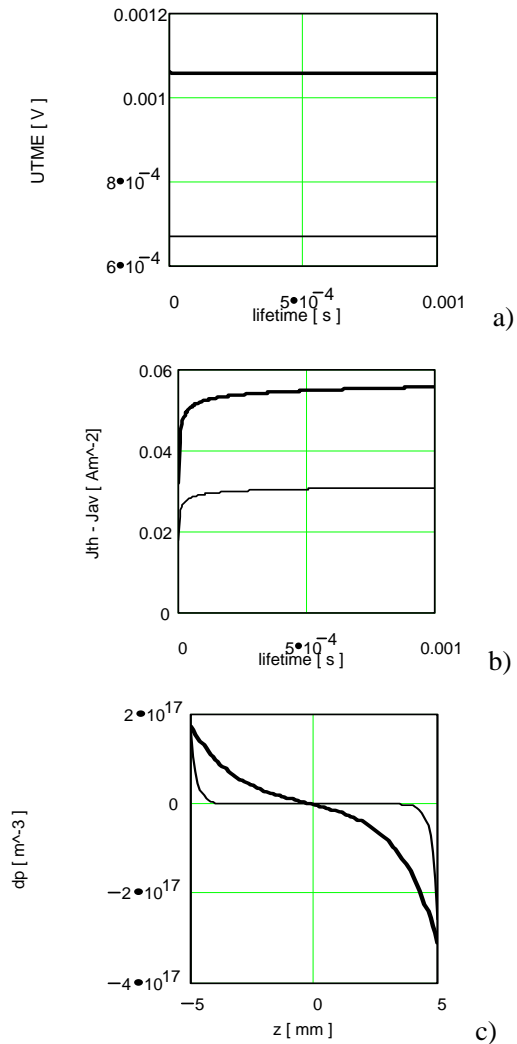

a)

b)

c)

a)

b)

c)

Fig. 4. The same like in Fig. 3 for $S_1 = S_2 = 10^4$ cm/s.

The depth distributions of the excess carrier concentration shown in Figs. 4c ($N = 5 \cdot 10^{19}$ m^{-3}) are

similar to those of Figs. 3c, but the concentrations of the excess carriers are largely smaller, especially at a lifetime of $5 \cdot 10^{-4}$ s (thin line). The solid thick lines represent these distributions for a lifetime of $1 \cdot 10^{-5}$ s.

The effect of the surface recombination velocity upon the variation of U_{TME} is demonstrated in Fig. 5 (lifetime $5 \cdot 10^{-4}$ s) and Fig. 6 (lifetime $1 \cdot 10^{-6}$ s) . Both plots were calculated taking the donor concentration in the sample to be $N = 5 \cdot 10^{19}\,\text{m}^{-3}$.

It appears that with increasing surface recombination velocity, the voltage U_{TME} increases, and so it does as the carrier lifetime decreases. This means that U_{TME} increases when the bulk and surface recombination velocities are high. The variation of the voltage U_{TME} is shown in Fig. 5a. The difference between the density of the Price thermodiffusion current and the density of the Price generalised current is demonstrated in Fig. 5b.
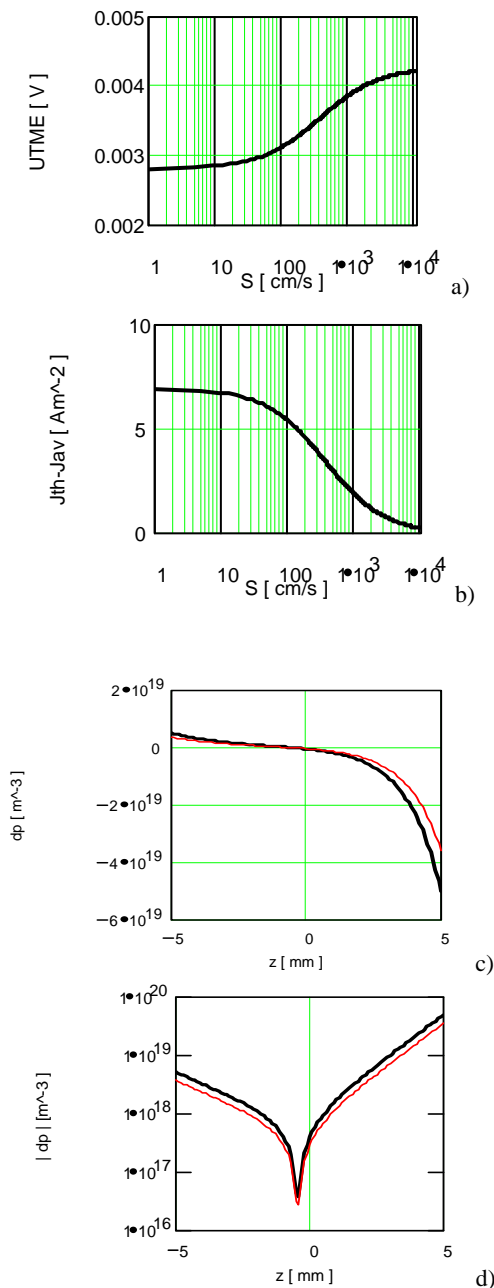
Fig. 5. a) Thermomagnetoelectric voltage as a function of the recombination velocity for donor concentration $5 \cdot 10^{19}$ m^{-3}; b) difference between the density of the Price thermodiffusion current and the density of generalised current; c) the depth distribution of the excess carrier concentration Δp in sample with two recombination velocities of 1 cm/s (thick line) and $1 \cdot 10^4$ cm/s (solid thin line) in linear scale; d) the absolute the same dependencies in logarithmic scale. The lifetime $5 \cdot 10^{-4}$ s.

At the surface recombination velocities exceeding $1 \cdot 10^4$ cm/s, the effect of excess carriers on U_{TME} is minute.

The depth distribution of the excess carrier concentration is shown in Figs. 5c and 5d. The thin lines correspond to a surface recombination velocity of 1 cm/s, and the thick line − to a value of $1 \cdot 10^4$ cm/s. In order to better visualise the depth distribution of the carrier concentration Δp, its absolute values are shown in Fig. 5d in a logarithmic scale.
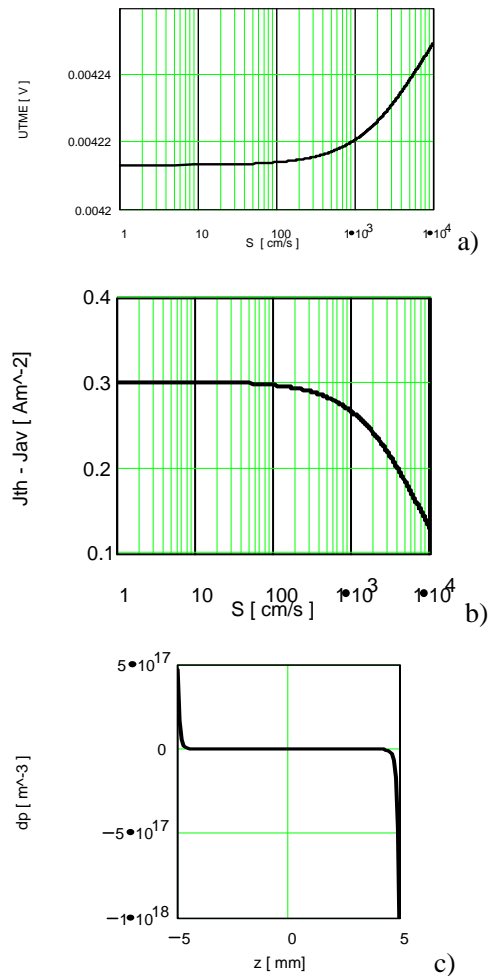

Fig. 6. The same like in Fig. 5 for lifetime $1 \cdot 10^{-6}$ s.

Figure 6a shows the U_{TME} values calculated with the lifetime taken to be $1 \cdot 10^{-6}$ s. In this case, with a minute error, the Price thermodiffusion current determines the magnitude of U_{TME} for all the values of the surface recombination velocity (Fig .6b).

The depth distribution of the excess carrier concentration is shown in Fig. 6c. Practically, the lines

obtained for $S = 1$ cm/s coincide with the lines drawn for $S = 1 \cdot 10^4$ cm/s.

5. Conclusions

Equations generalising the concept of the Price thermodiffusion current have been derived with allowance made for the generation of excess carriers within the region of the temperature gradient and for the effect of the recombination parameters of the semiconductor upon this current.

The theory of interaction of thermodiffusion current and a magnetic field showed the existence of thermomagnetoelectric effect (TME).

Due to TME the voltage between electrodes of semiconductor sample is observed. It depends on thermodiffusion current and on recombination parameters of semiconductor.

Numerical calculations have shown that the difference between the density of the Price thermodiffusion current and the density of the Price generalised current, discussed in this study, becomes evident when the carrier lifetime is long and the surface recombination velocity is small, and this effect is appears particularly in weakly-doped semiconductors.

The calculations shown in the figures give the TME voltages of the order of 10^{-3} V. This points the significance of TME as a tool to prove experimentally the existence and features of thermo-diffusion current.

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Appendix

J^* can be found by multiplying Eq. (1) by σ_n/σ_p and then subtracting Eq. (2), multiplied by σ_p/σ, from it. As a result, the terms containing dV/dz reduce themselves, which can be seen from the formula

$$J^* = -\frac{\sigma_n}{\sigma}\mu_p k_B T grad\ p - \frac{\sigma_p}{\sigma}\mu_n k_B T grad\ n + 2\frac{\sigma_n\sigma_p}{\sigma}\frac{k_B T}{q} grad\ \ln n_i -$$

$$\frac{\sigma_n\sigma_p}{\sigma}\left(\varepsilon_p - \ln\frac{n_i}{N_V}\right)grad\ \frac{k_B T}{q} - \frac{\sigma_p\sigma_n}{\sigma}\left(\varepsilon_n - \ln\frac{n_i}{N_C}\right)grad\ \frac{k_B T}{q} =$$

$$-\frac{\sigma_n}{\sigma}\mu_p k_B T grad\ p - \frac{\sigma_p}{\sigma}\mu_n k_B T grad\ n + \frac{\sigma_n\sigma_p}{\sigma}\frac{k_B T}{q} grad\ \ln n_i^2 -$$

$$\frac{\sigma_n\sigma_p}{\sigma}\left(\varepsilon_p + \varepsilon_n - \ln\frac{n_i^2}{N_C N_V}\right)grad\ \frac{k_B T}{q}, \tag{A1}$$

where we have used the operator *grad*, since it is a more general symbol than the derivative d/dz.

In order to identify the terms that hide Δp and *grad* Δp, let us transform Eq. (9) according to formula (6). When this is done we obtain

$$J^* = -\frac{\sigma_n\sigma_p}{\sigma}\left(\varepsilon_p + \varepsilon_n - \ln\frac{n_i^2}{N_C N_V}\right)grad\ \frac{k_B T}{q} -$$

$$\frac{\sigma_n}{\sigma}\mu_p k_B T grad\ p_o - \frac{\sigma_p}{\sigma}\mu_n k_B T grad\ n_0 + \frac{\sigma_n\sigma_p}{\sigma}\frac{k_B T}{q} grad\ \ln n_i^2 -$$

$$\frac{\sigma_n}{\sigma}\mu_p k_B T grad \quad \Delta p - \frac{\sigma_p}{\sigma}\mu_n k_B T grad \quad \Delta p \ . \tag{A2}$$

Next, we separate and transform the three components of the internal current denoted by the symbols J_{th}, J_1^* and J_2^*. Then, J_{th} is

$$\boldsymbol{J}_{th} = -\frac{\sigma_n \sigma_p}{\sigma}\left(\varepsilon_p + \varepsilon_n - \ln \frac{n_i^2}{N_C N_V}\right) grad \frac{k_B T}{q} = -\frac{\sigma_n \sigma_p}{\sigma}\left(\varepsilon_p + \varepsilon_n + \ln \frac{E_G}{k_B T}\right) grad \ \frac{k_B T}{q} \tag{A3}$$

and J_1^* is

$$\boldsymbol{J}_1^* = -\frac{\sigma_n}{\sigma}\mu_p k_B T grad \quad p_o - \frac{\sigma_p}{\sigma}\mu_n k_B T grad \quad n_o + \frac{\sigma_n \sigma_p}{\sigma}\frac{k_B T}{q} grad \ \ln n_i^2 =$$

$$-\frac{\sigma_n \sigma_p}{\sigma}\frac{k_B T}{q}\frac{grad \quad p_0}{p} - \frac{\sigma_n \sigma_p}{\sigma}\frac{k_B T}{q}\frac{grad \quad n_o}{n} + \frac{\sigma_n \sigma_p}{\sigma}\frac{k_B T}{q} grad \ \ln n_i^2 =$$

$$-\frac{\sigma_n \sigma_p}{\sigma}\frac{k_B T}{q}\left(\frac{grad \quad p_o}{p} + \frac{grad \quad n_0}{n} - grad \ \ln n_i^2\right) =$$

$$-\frac{\sigma_n \sigma_p}{\sigma}\frac{k_B T}{q}\left(\frac{grad \quad p_o}{p} + \frac{grad \quad n_o}{n} - grad \ \ln n_o p_o\right) =$$

$$-\frac{\sigma_n \sigma_p}{\sigma}\frac{k_B T}{q}\left(\frac{grad \quad p_o}{p} + \frac{grad \quad n_o}{n} - \frac{grad \quad p_o}{p_o} - \frac{grad \quad n_o}{n_o}\right) =$$

$$-\frac{\sigma_n \sigma_p}{\sigma}\frac{k_B T}{q}\left[\left(\frac{1}{p} - \frac{1}{p_o}\right) grad \quad p_0 + \left(\frac{1}{n} - \frac{1}{n_o}\right) grad \quad n_o\right] =$$

$$-\frac{\sigma_n \sigma_p}{\sigma}\frac{k_B T}{q}\left[\frac{p_o - p}{p p_o} grad \quad p_o + \frac{n_o - n}{n n_o} grad \quad n_o\right] =$$

$$-\frac{\sigma_n \sigma_p}{\sigma}\frac{k_B T}{q}\left[\frac{grad \quad p_o}{p p_o} + \frac{grad \quad n_o}{n n_o}\right]\Delta p \ , \tag{A4}$$

where we have used the identity

$$n_i^2 = n_o \ p_o \ . \tag{A5}$$

Taking a gradient of the two sides of Eq. (4) we obtain

$$grad \quad n_o = grad \quad p_o \tag{A6}$$

since $N =$ const.

To calculate *grad* p_o or *grad* n_o versus *grad* $\ln n_i^2$, we should use identities (12) and (13)

$$grad \ \ln n_i^2 = grad \ \ln(p_o n_o) = grad \ \ln n_o + grad \ \ln p_o =$$

$$\frac{grad \quad n_o}{n_o} + \frac{grad \quad p_o}{p_o} = \left(\frac{1}{n_o} + \frac{1}{p_o}\right) grad \quad p_o \tag{A7}$$

and, hence

$$grad \quad p_o = \left(\frac{1}{n_o} + \frac{1}{p_o}\right)^{-1} grad \ \ln n_i^2 = \frac{n_o p_o}{n_o + p_o} grad \ \ln n_i^2 \ . \tag{A8}$$

Substituting this equation in (12) and using (14) we obtain

$$\boldsymbol{J}_1^* = \frac{\sigma_n \sigma_p}{\sigma} \frac{k_B T}{q} \left[\frac{1}{pp_o} + \frac{1}{nn_o} \right] \Delta p \, grad \quad p_o =$$

$$\frac{\sigma_n \sigma_p}{\sigma} \frac{k_B T}{q} \left[\frac{1}{pp_o} + \frac{1}{nn_o} \right] \frac{n_o p_o}{n_o + p_o} \Delta p \, grad \quad \ln n_i^2 =$$

$$\frac{\sigma_n \sigma_p}{\sigma} \frac{k_B T}{q} \left[\frac{n_o}{p} + \frac{p_o}{n} \right] \frac{\Delta p}{n_o + p_o} grad \quad \ln n_i^2 . \tag{A9}$$

For the purposes of these calculations it is useful to utilise the equation that describes the coefficient $k_B T/q$ $grad \ln n_i^2$, derived as (A13). When this is done we obtain

$$\boldsymbol{J}_1^* = \frac{\sigma_n \sigma_p}{\sigma} \frac{k_B T}{q} \left[\frac{n_o}{p} + \frac{p_o}{n} \right] \left(\frac{3}{T} - \frac{1}{k_B T} \frac{dE_G}{dT} + \frac{E_G}{k_B T^2} \right) \frac{\Delta p}{n_o + p_o} grad \quad T =$$

$$\frac{\sigma_n \sigma_p}{\sigma} \left[\frac{n_0}{p} + \frac{p_0}{n} \right] \left(3 - \frac{1}{k_B} \frac{dE_G}{dT} + \frac{E_G}{k_B T} \right) \frac{\Delta p}{n_0 + p_0} grad \quad \frac{k_B T}{q} . \tag{A10}$$

The last member J_2^* in Eq. (10) is

$$\boldsymbol{J}_2^* = - \left(\frac{\sigma_n}{\sigma} \mu_p k_B T + \frac{\sigma_p}{\sigma} \mu_n k_B T \right) grad \, \Delta p =$$

$$- \left(\frac{\sigma_n}{\sigma} \mu_p + \frac{\sigma_p}{\sigma} \mu_n \right) k_B T \, grad \, \Delta p = - \mu k_B T \, grad \, \Delta p = - q D \, grad \, \Delta p , \tag{A11}$$

where we introduced the ambipolar mobility μ and the ambipolar diffusion coefficient D.

Therefore, finally, the internal current may be expressed in the form

$$\boldsymbol{J}^* = \boldsymbol{J}_{th} + \boldsymbol{J}_1^* + \boldsymbol{J}_2^* , \tag{A12}$$

where the individual components are given by Eqs. (11), (17) and (18).

According to the Maxwell-Boltzmann statistics we have

$$n_i^2 = N_C N_V \exp \left(- \frac{E_G}{k_B T} \right) \tag{A13}$$

and thus,

$$\frac{k_B T}{q} grad \quad \ln n_i^2 = \frac{k_B T}{q} grad \quad \ln N_C N_V - \frac{k_B T}{q} grad \left(\frac{E_G}{k_B T} \right) . \tag{A14}$$

Assuming that the effective masses of holes and electrons are constant, we obtain

$$\frac{k_B T}{q} grad \quad \ln N_C N_V = 3 \, grad \quad \frac{k_B T}{q} \tag{A15}$$

and then

$$\frac{k_B T}{q} grad \frac{E_G}{k_B T} = \frac{k_B T}{q} \left(\frac{1}{k_B T} grad E_G - \frac{E_G}{T^2} grad T \right) = \left(\frac{1}{k_B T} \frac{dE_G}{dT} - \frac{E_G}{k_B T} \right) grad \quad T . \tag{A16}$$

Subtracting (A16) from (A15) gives

$$\frac{k_B T}{q} grad \quad \ln n_i^2 = \left(3 - \frac{1}{k_B} \frac{dE_G}{dT} - + \frac{E_G}{k_B T} \right) grad \quad T \tag{A17}$$

which after substituting in Eq. (A9) becomes Eq. (A10).