

# Calculation of the Areas Most Likely to Appear Suitable for Communication Meteor Trails

Helen V. Kharchenko

**Abstract**—The method of calculation of the areas most likely to appear meteor trails with the electromagnetic waves specular reflection point for the radio links of different lengths is proposed. The linear electron density of the meteor trail, depending on the velocity and mass of the meteoroid, as well as zenith angles, defined by radiants of sporadic meteors and meteor showers are taken into account during calculating.

**Keywords**—meteor trail, specular reflection, linear electron density, radiant, zenith angle.

## I. INTRODUCTION

IN order to design meteor communication system it is necessary to have an ability to calculate signal intensity in the receiving point and the areas most likely to appear in the zone of radio visibility of the meteor trails with the specular reflection point [1]–[3].

The aim of this work is to calculate the areas most likely to appear suitable for communication meteor trails for radio lines of different lengths, taking into account the basic radio physical parameters of the meteor trail [4]. Antenna orientation to these areas provides increasing of meteor radio links bandwidth.

The capacity of the meteor radio system is determined by the number of suitable for communication meteor trails and their duration. Suitable for communication meteor trail is ionized meteor trail with the linear electron density sufficient to ensure the required signal-to-noise ratio for given parameters of transmitting and receiving equipment, and on the meteor trail there is a radio waves specular reflection point [5]. The length of the trail, with the electron density is sufficient to information transfer, is called “effective length of the meteor trail.”

The electron density of the trail depends on the parameters of the meteoroid (mass, velocity, density and chemical composition) and is a function of height [1]. The condition of sufficiency level of the electron density determines the upper and lower limit of the meteor zone, within which the condition for communication. These boundaries and orientation of the meteor trail in space determine its effective length.

Specular point on the trail is formed, if the trail has a point of contact with the ellipsoid with focuses at the transmitting and receiving points [5]. Since the radiants of meteor showers and sporadic meteors has appropriate probability distributions [1], zenith angles and reduced azimuths (angle measured relative to the axis of the ellipsoid, but not in the direction “north”) of the meteor trail are random. Location and size of

the areas most likely to appear in the radio visibility zone of meteor trails with a specular reflection point (“hot areas”) will depend on the length of the radio, as well as time of day and year [5].

Location and size of the “hot areas” for meteor showers and sporadic meteors allows to select the right orientation and beam-width of transmitting and receiving antennas for different lengths of radio, and thereby to provide maximum bandwidth of meteor radio systems.

## II. ESTIMATOR OF EFFECTIVE METEOR TRAIL LENGTH

### A. Calculation of the Linear Electron Density

During entering into the Earth atmosphere meteoroid ablation process occurs. Atoms and molecules left the surface of meteoric object continue their movement, and coming into collision with atmosphere molecules give rise to free electrons and ions. The number of free electrons per length unit of meteor trail defines linear electron density  $\alpha$ . Its variation along meteor trail defined by the expression [1]–[3]

$$\alpha(h) = 4.03 \cdot 10^{14} \frac{m(v - 8.15)^3}{H} \cdot \cos \gamma \cdot z(t), \quad (1)$$

where  $m$  and  $v$  are mass and velocity of meteoroid during entering in the meteor region

$$z(t) = \frac{9}{4} e^{-t} \left( 1 - \frac{1}{3} e^{-t} \right) \quad \text{when} \quad -\ln 3 \leq t \leq 1.7, \quad (2)$$

else  $z(t) = 0$ .

$t$  defines as  $t = (h - h_{max})/H$ ,

where  $h$  is the atmosphere height,

$h_{max}$  is the height with maximum linear electron density  $\alpha_{max}$ ,

$H$  is reduced atmosphere height [3]  $H = 6.4 + 0.09(h - 95)$ .

The height of the ionization maximum is given by the relation [3]

$$h_{max} = 47.4 + 12.76 \ln v. \quad (3)$$

Maximal linear electron density  $\alpha_{max}$  is defined by empirical expression [2]

$$\alpha_{max} = 4.03 \cdot 10^{14} \frac{m(v - 8.15)^3}{H} \quad (4)$$

Figure 1 shows the graphs of linear electron density along the meteor trail, formed by the meteoroid with mass 0.5 g, entering the atmosphere at velocities of 40–70 km/s, calculated by the formula (1).

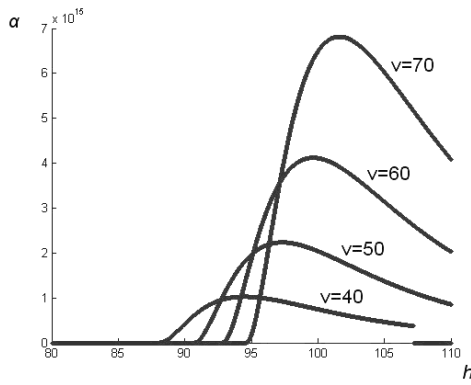


Fig. 1. Linear electron density versus height.

### B. Calculation of the Effective Length of the Meteor Trail

Based on the obtained dependence of the electron density versus height can be calculated the effective width of the meteor region at the given parameters of radio and, according to zenith angle of meteor trail, the effective length of the meteor trail.

The effective length of the meteor trail is determined by the linear electron density sufficient to provide the required signal-to-noise ratio for given parameters of transmitting and receiving equipment and zenith angle of the meteor trail.

The probability of signal detection scattered by the meteor trail depends on power of the receiving signal and the noise level on the given frequency. Define minimum value of the linear electron density of the ionized meteor trail which provides required signal-to-noise ratio depending on meteor communication system parameters.

During reflection from underdense meteor trail the signal power in the input of the receiver has the form [5]

$$P_{rv} = \frac{P_{tr} G_{rv} G_{tr} \sigma \lambda^3 \alpha^2 \cos^2 \mu \exp\left(-\frac{8\pi r_0}{\lambda \sec^2 \theta}\right)}{(4\pi^3) r_{tr} r_{rv} (r_{tr} + r_{rv}) (1 + \sin^2 \theta \cos^2 \gamma)} \quad (5)$$

where  $P_{tr}$  is transmitter power;

$G_{tr}, G_{rv}$  are transmitter and receiver antennas directivity gains;

$\sigma = 10^{-28} \text{ m}^2$  is an absolute cross-section of electron;

$\lambda$  wavelength;

$\alpha$  is linear electron density;

$\mu$  is the angle between electric field vector of incident wave and trail axis;

$\theta$  is transmitted wave trail input-phase angle;  $r_0$  is the initial radius of meteor trail;

$\gamma$  is the angle between the meteor trail and the plane traced across receiving, transmitting points and reflecting point of the meteor trail;

$r_{tr}, r_{rv}$  are distances between transmitting antenna and reflecting field, receiving antenna and reflecting field.

The initial radius of meteor trail could be found by the following empirical formula [2]

$$r_0 = 1.65 \sqrt{\frac{v}{40} \cdot \exp\left(\frac{h-95}{2H}\right)} \quad (6)$$

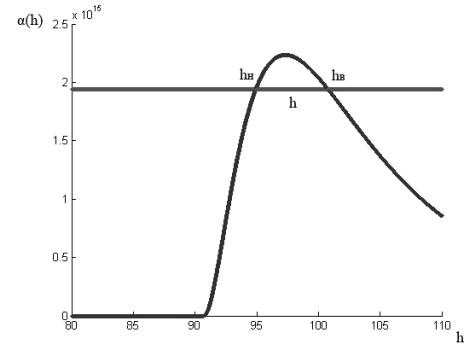


Fig. 2. Effective width of the meteor region.

The noise power depends on interferences field intensity on the given frequency  $E_n$ , which may be defined on the basis of empirical data, and effective height of receiving antenna  $h_a$

$$P_n = \frac{(E_n h_a)^2}{R}, \quad (7)$$

where  $R$  is the input resistance of the receiver, and

$$h_a = \frac{\lambda}{\pi} \sqrt{\frac{\eta G_{rv} R}{120}}, \quad (8)$$

where  $\eta$  is antenna radiation efficiency.

The minimum electron density depends on the required ration  $P_{rv}/P_n$  and calculates by the formula (5).

Obviously, the demanded signal-to-noise ratio in the input of the receiving system could be obtained for values  $\alpha$  exceeding values  $\alpha_{min}$ . Thus, the line crossing points  $\alpha_{min}$  and the function  $\alpha(h)$  define the points of the effective meteor region beginning  $h_b$  and its end  $h_e$ , and effective meteor region where the electron density of meteor trail is enough for communication in accordance with a given signal-to-noise ratio is defined by the expression

$$h = h_e - h_b. \quad (9)$$

And the effective length of the meteor trail defines by following expression

$$l_{ij} = \rho \cos \gamma_z - \sqrt{\rho^2 \cos^2 \gamma_z + h^2 - 2h\rho}, \quad (10)$$

where

$$\rho = R_E + h_e + h, \quad (11)$$

$R_E = 6372.795 \text{ km}$  – the Earth's radius.

Figure 2 shows a relation of the linear electron density versus height, obtained on the basis of expression (1) for the meteoroid of mass 1 gram and velocity 40 km/sec when the maximum value of the linear electron density  $\alpha_{max} = 2.1 \cdot 10^{14}$ ,  $\alpha_{min}$  has been calculated according to formulas (5)–(8) for the meteor communication system with following parameters:  $P_{tr} = 500$ ;  $G_{tr} = G_{rv} = 20\text{dB}$ ;  $\lambda = 7.5 \text{ m}$ ; length of the radio link  $c = 1000 \text{ km}$ ,  $P_{tr}/P_n = 10$ .

Table 1 shows the maximum and minimum electron density, and the effective width of the meteor region, for different parameters of the meteoroid.

TABLE I  
ELECTRON DENSITY AND EFFECTIVE WIDTH OF THE METEOR REGION

m,[g]	v,[km/sec]	$\alpha_{max},[el/m]$	$\alpha_{min},[el/m]$	h,[km]
0.25	30	$1.7553 \cdot 10^{14}$	$1.9305 \cdot 10^{15}$	0
0.25	50	$1.1174 \cdot 10^{15}$	$1.9374 \cdot 10^{15}$	0
0.25	70	$3.4078 \cdot 10^{15}$	$1.9422 \cdot 10^{15}$	12.66
0.5	30	$3.4905 \cdot 10^{14}$	$1.9305 \cdot 10^{15}$	0
0.5	50	$2.2349 \cdot 10^{15}$	$1.9374 \cdot 10^{15}$	5.9
0.5	70	$6.8157 \cdot 10^{15}$	$1.9422 \cdot 10^{15}$	13.8
1	30	$6.9811 \cdot 10^{14}$	$1.9305 \cdot 10^{15}$	0
1	50	$4.4698 \cdot 10^{15}$	$1.9374 \cdot 10^{15}$	15.94
1	70	$1.3631 \cdot 10^{16}$	$1.9422 \cdot 10^{15}$	14.37

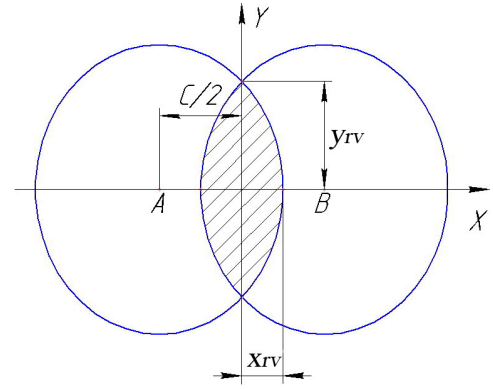


Fig. 3. Radio visibility zone.

TABLE II  
EXTREME POINTS OF THE RADIO VISIBILITY ZONE

c, [km]	$x_{rv}$ , [km]	$y_{rv}$ , [km]
500	882.4	1105.4
1000	629.8	1016.3
2000	119.3	509.7

### III. ESTIMATOR OF THE METEOR TRAILS SUITABLE FOR COMMUNICATION

#### A. Radio Visibility Zone

During checking of the suitable for communication meteor trails, it is necessary to take into account the subspace which is simultaneously seen from transmitting and receiving points (radio visibility zone), and also to set the coordinates of entry points of the meteoroid in the meteor region. The radio visibility zone is a part of sphere bounded by the planes tangent to the Earth's surface at the points of transmission and reception. The intersection of these planes with the sphere, form a circle with a radius  $R_k$ . The projection of the intersection on the XY plane is shown in Fig. 3, where  $x_{rv}$  – the extreme points of X and Y, respectively.

For radio lines AB with a length of  $c$ , the extreme points of this area can be calculated by the following relations

$$\begin{aligned} x_{rv} &= \frac{R_k - l}{R_E} \sqrt{R_E^2 - \frac{c^2}{4}}, \\ y_{rv} &= \sqrt{R_k - l}, \end{aligned} \quad (12)$$

where  $R_k$   $l$  are given by

$$R_k = \sqrt{(R_E + H + h)^2 - R_E^2}, \quad (13)$$

$$l = \frac{cR_E}{2\sqrt{R_E^2 - \frac{c^2}{4}}}, \quad (14)$$

In Fig. 3 the extreme points of the radio visibility zone is shown.

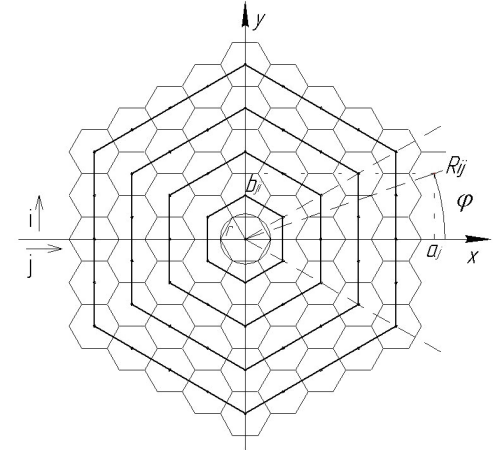


Fig. 4. High density packing in the XY plane as a set of regular hexagons.

Table 2 shows the extreme points values of the radio visibility zone for the radio links of different lengths.

#### B. Orientation of the Meteor Trail in Space

To determine an entry point of the meteoroid in the meteor region we should a spherical surface, defined by a zone of radio visibility, divide into surface elements of equal area. To minimize the number of surface elements it is necessary to represent a high density packing on the sphere.

Figure 4 illustrates the high density packing in the XY plane as a set of regular hexagons.

This construction allows for each  $ij$ -th center of the hexagon, round which can be described by a circle of radius  $r$ , determine the distance  $R$  from the origin and the angle  $\phi$  in polar coordinates

$$R_{ji} = \sqrt{a_j^2 + b_{ji}^2}, \quad (15)$$

$$a_j = j \frac{3}{2} r, \quad j = 1, 2, \dots, N \quad (16)$$

$$b_{ji} = r\sqrt{3} \left( i - \frac{j}{2} \right), \quad j = 0, 1, \dots, j \quad (17)$$

- projections of  $R_{ji}$  respectively on the axis X and Y.

Coordinates of the center of any hexagon can be described by the distance  $R$  from the origin and the angle  $\psi_r$  in the polar coordinates system

$$\psi_r = \frac{\pi}{2} \left( 1 - \frac{R}{\rho} \right). \quad (18)$$

Thus, on the basis of the expressions (5,8) the coordinates of each  $ji$ -th hexagon center on the surface of a sphere defined by

$$\rho = R_E + H + h, \quad (19)$$

$$\phi_{ji} = \arctan \frac{(i - \frac{j}{2})}{j\sqrt{3}} + k \frac{\pi}{3}, \quad k = 0, 1, \dots, 5 \quad (20)$$

$$\psi_{ji} = \frac{\pi}{2} \left( 1 - \frac{R_{ji}}{\rho} \right), \quad (21)$$

where  $H$  – the lower boundary of the meteor region;  
 $h$  – width of the meteor region.

It should be taken into account that meteor trails are formed only in the specified range of heights, where the conditions are being performed for the sufficiently long presence of plasma formation caused by collision ionization and meteoric object evaporation.

Figure 5 shows the meteor trail location in the effective meteor region, where linear electron density exceeds value  $\alpha_{min}$ , and the field lower boundary is located on  $H = h_e$ . The angles  $\psi$  and  $\phi$ , and, also, polar radius  $\rho$  gives input point coordinates into the layer, and zenith angle  $\gamma_z$ , and reduced azimuth  $\alpha$  define the orientation of the meteor trail in space.

The fixed point of the meteor enter given in the Cartesian coordinates is converted into polar coordinates using relations [6]

$$\begin{aligned} x_{in\ ji} &= \rho \cos(\psi_{ji}) \cdot \cos(\phi_{ji}), \\ y_{in\ ji} &= \rho \cos(\psi_{ji}) \cdot \sin(\phi_{ji}), \\ z_{in\ ji} &= \rho \sin(\psi_{ji}). \end{aligned} \quad (22)$$

The coordinates of output point  $M_{out}(x_{out}, y_{out}, z_{out})$  are defined by the following relations

$$\begin{aligned} x_{out\ ji} &= m(\cos^2 \phi_{ji} \sin \psi_{ji} + \sin^2 \phi_{ji}) + \\ &+ n \sin \phi_{ji} \cos \phi_{ji} (1 - \sin \psi_{ji}) + p \cos \psi_{ji} \cos \phi_{ji}, \\ y_{out\ ji} &= m \cos \phi_{ji} \sin \phi_{ji} (1 - \sin \psi_{ji}) + \\ &+ n(\sin^2 \phi_{ji} \sin \psi_{ji} + \cos^2 \phi_{ji}) + p \cos \psi_{ji} \sin \phi_{ji}, \\ z_{out\ ji} &= -m \cos \phi_{ji} \cos \psi_{ji} - n \sin \phi_{ji} \cos \psi_{ji} + p \sin \psi_{ji}, \end{aligned} \quad (23)$$

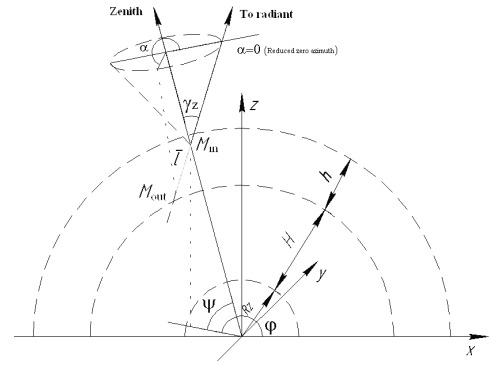


Fig. 5. Location of the meteor trail in the meteor region.

where  $m, n, p$  are guiding vectors of the meteor trails

$$m = l \sin \gamma_z \cos \alpha, \quad (24)$$

$$n = l \sin \gamma_z \sin \alpha,$$

$$p = -l \cos \gamma_z,$$

$l$  is meteor trail length,

$\alpha$  is reduced azimuth of radiant [5].

The straight line, which defines the trail orientation in space, is given by the equation

$$\frac{x - x_0}{m} = \frac{y - y_0}{n} = \frac{z - z_0}{p}, \quad (25)$$

where  $x_0, y_0, z_0$  are point coordinates on the trail.

### C. Check Availability of the Specular Reflection Point on the Meteor Trail

The specular reflection points have trails which tangential to the one of the ellipsoid families with focuses in the receiving and transmitting points. The indication of the osculation is the equality of the cosines of the angle between the direction to radiant and the perpendicular to tangential plane during variation of the minor axis value of ellipsoid  $b_n$  from the bottom to upper boundary of the meteor zone [7], in the limits of one the conditions  $\alpha(h) > \alpha_{min}$

$$\cos \gamma = \frac{\frac{m x_0}{b_n^2 + c_e} + \frac{n y_0}{b_n^2} + \frac{p z_0}{b_n^2}}{(m^2 + n^2 + p^2) \left[ \left( \frac{x_0}{b_n^2 + c_e} \right)^2 + \left( \frac{y_0}{b_n^2} \right)^2 + \left( \frac{z_0}{b_n^2} \right)^2 \right]} \quad (26)$$

where  $c_e$  is a half of focus distance.

Having found the  $\cos \gamma$  for the entry point of meteor into the meteor zone and point of output from the zone it is checked whether the sign of the cosines changes or not for these points. If the sign changes, then the condition of the osculation for this trail is performed and this meteor trail could be used for communication, otherwise the trail with this radiant could not be used for the communication providing.

#### IV. CALCULATION OF THE AREAS MOST LIKELY TO APPEAR SUITABLE FOR COMMUNICATION METEOR TRAILS

##### A. The Probability of the Meteor Trail with the Specular Reflection Point Formation

The probability of forming a meteor trail with specular reflection point, for the meteoroid with zenith angle  $\gamma_z$  and azimuth  $\alpha$ , that appear in the neighborhood of the given point  $M_{in}(x_{in}, y_{in}, z_{in})$ , defined by the expression

$$k \cdot P(x_{in\ ji}, y_{in\ ji}, z_{in\ ji}) = \int_{\gamma_{min}}^{\gamma_{max}} w(\gamma_z) \int_{\alpha_{min}(\gamma_z)}^{\alpha_{max}(\gamma_z)} w(\alpha/\gamma_z) d\alpha d\gamma_z \quad (27)$$

where  $k$  is probability coefficient determining that a meteor get into surface element,

$w(\gamma_z)$  is the zenith angle density of probability of the sporadic meteors,

$[\gamma_{min}, \gamma_{max}]$  is the interval of the zenith angles during one, for the given point of meteor entering into the meteor region the specular reflection point is,

$w(\alpha/\gamma_z)$  is conditional probability of the case when the meteor trail with zenith angle  $\gamma_z$  has reduced azimuth  $\alpha$ ,

$[\alpha_{min}(\gamma_z), \alpha_{max}(\gamma_z)]$  is the interval of the zenith angles during one, for the given  $\gamma_z$  the specular reflection point is.

Supposing that the probable angles  $\gamma_z$  and  $\alpha$  are independent, and  $\alpha$  has a uniform distribution in the interval  $[0, 2\pi]$  then the expression (27) is being simplified

$$k \cdot P(x_{in\ ji}, y_{in\ ji}, z_{in\ ji}) = \int_{\gamma_{min}}^{\gamma_{max}} w(\gamma_z) \Delta\alpha(\gamma_z) d\gamma_z \quad (28)$$

where  $\Delta\alpha(\gamma_z) = \alpha_{max}(\gamma_z) - \alpha_{min}(\gamma_z)$ .

To estimate the probability of suitable for communication trail appear in the surrounding of the entry point it could be assumed that  $\gamma_z$  is uniformly distributed in the interval  $[0, 2\pi]$ . Then the estimate of the probability has the form

$$k \cdot P(x_{in\ ji}, y_{in\ ji}, z_{in\ ji}) = \frac{2}{\pi^2 \cdot N} \sum_{i=1}^N \gamma_{z\ i} \cdot \Delta\alpha_i(\gamma_{z\ i}) \quad (29)$$

where  $\gamma_{z\ i} = \gamma_z + i\Delta\gamma$ , and  $\Delta\gamma = \frac{1}{N}(\gamma_{max} - \gamma_{min})$ .

##### B. Areas Most likely to Appear Suitable for Communication Meteor Trails

To build ‘‘hot zones’’ where the occurrence probability of meteor trails with a specular reflection point exceeds a predetermined value necessary to make the calculation by formula (29) for all coordinates of entry points linked to the centres of the high density packing hexagons that fall into the radio visibility zone.

As an example, the calculation of areas most likely to appear meteor trails with the specular reflection point, take the following initial data: the lengths of the radio links are 2000, 1000, 500 km; the meteor communication system parameters:  $R_{tr} = 500W$ ;  $G_{tr} = G_{rv} = 20dB$ ;  $\lambda = 7.5m$ ; parameters of the meteoroid: mass  $m = 1g$ , velocity

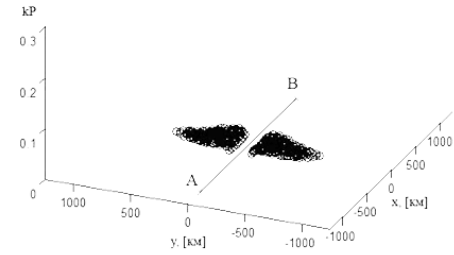
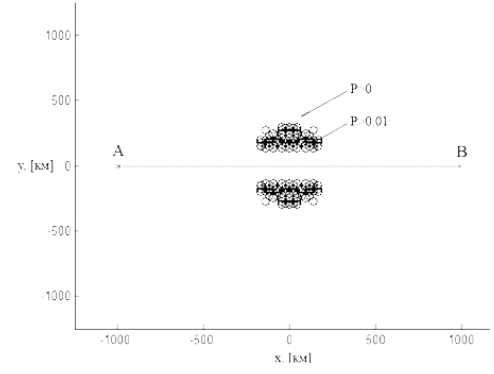


Fig. 6. The areas most likely to appear meteor trails with the specular reflection point for the radio link length 2000 km.

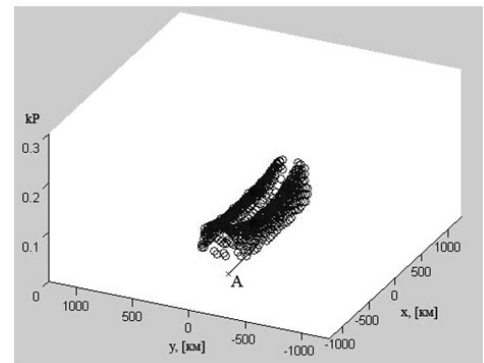
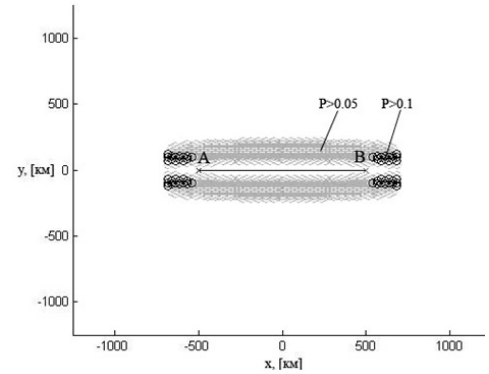


Fig. 7. The areas most likely to appear meteor trails with the specular reflection point for the radio link length 1000 km.

$v = 40km/sec$ ; characteristics of the meteor trail: the interval of zenith angles  $[\gamma_{min}, \gamma_{max}] = [\frac{\pi}{6}, \frac{\pi}{3}]$ ,  $\alpha$  is uniformly dis-

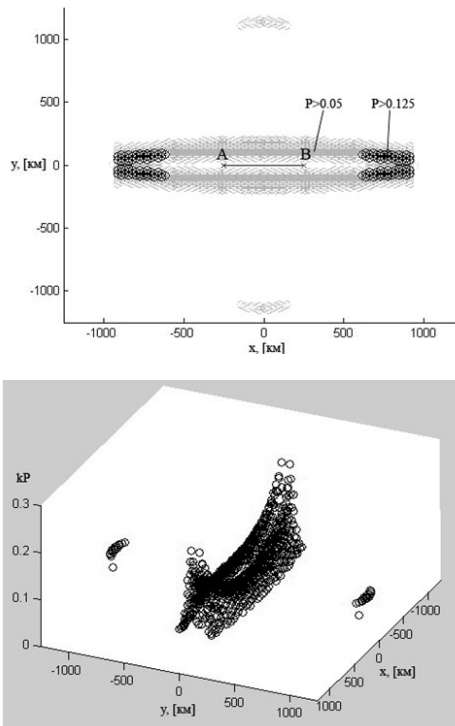


Fig. 8. The areas most likely to appear meteor trails with the specular reflection point for the radio link length 500 km.

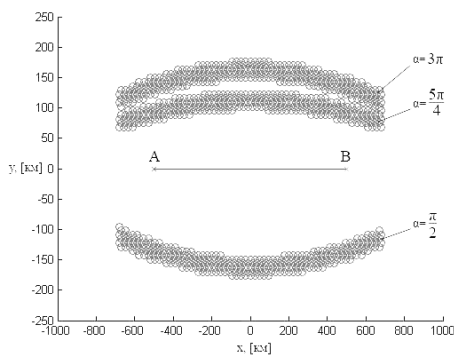


Fig. 9. The areas most likely to appear meteor trails with the specular reflection point for the radio link length 1000 km.

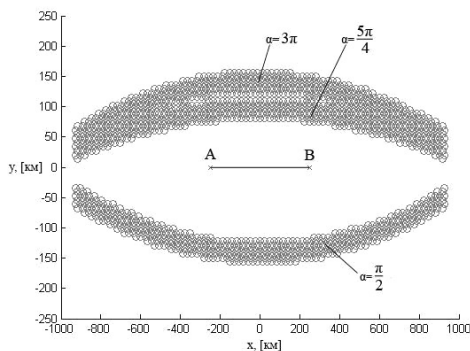


Fig. 10. The areas most likely to appear meteor trails with the specular reflection point for the radio link length 500 km.

tributed in the interval  $[0, 2\pi]$ . The results of area calculations are presented in Fig. 6-8 for the lengths of radio links 2000, 1000, 500 km respectively.

For the meteor showers with the fixed zenith angle  $\gamma_z = \frac{\pi}{6}$  and reduced azimuth  $\alpha = \frac{\pi}{2}, \frac{5\pi}{4}, 3\pi$  the areas most likely to appear suitable for communication meteor trails are shown in the Fig. 9, 10 for the radio link lengths 1000 and 500 km respectively.

### V. CONCLUSION

As opposed to famous estimates of meteor trails suitable for communication, the proposed method allows to calculate the probability of occurrence in a given area of meteor trails, providing conditions for communications. In the calculations determined the effective length of the meteor trail, where the linear electron density provides the possibility of communications for given parameters of the radio link. The method allows not only determine the probability of a suitable trail, but also to assess the probabilistic contribution of each point of the radio visibility zone. This provides the possibility of building areas most likely to appear trails with the specular reflection point for a given length of the radio link. Definition of the "hot zones" allows to set the basic parameters of antennas and direction of their orientation.

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