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EXCESS CURRENT CARRIER DISTRIBUTION IN THE BASE REGION OF THE SEMICONDUCTOR MULTI-JUNCTION STRUCTURE

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ABSTRACT

A new approach to the theory of excess current carrier distribution in the homogeneous base region of the semiconductor multi-junction structure is proposed. Numerical analysis of this structure is performed taking into consideration an assumption that concentrations of excess electrons and holes in the semiconductor are equal (the neutrality principle). To obtain excess carrier distributions in this structure it is necessary to solve continuity equations of electron J_n and hole J_p current densities. A general solution is obtained and numerically calculated distributions of excess carriers and electrical potential for cases interesting from the point of view of their application in injection modulated thermal radiation structures destined for dynamic scene projectors are presented.

1. Introduction

A basic structure which is able to produce excess current carriers in the semiconductor structure is the *p-n* junction. Amongst many works published in early stage of the development of semiconductor science and technology one can distinguish publications of Sosnowski et al. [1, 2] concerning PbS rectifying structure.

In 1948 Shockley published his crucial one-dimensional (1D) theory of the *p-n* junction [3] and this theory is now generally accepted as a basis of all works in which *p-n* junction appears. All ideas presented in his theory still hold true due to his invention and description of phenomena appearing in this junction.

In our work we consider three-dimensional (3D) distribution of carriers injected to the base by the *p-n* junction polarized in the forward direction. To formulate proper and generally right current carrier transport equations it is necessary to use W. van Roosbroeck formalism [4] where he introduced a quasi-neutrality principle as well as potential Ψ which become a basis for the three-dimensional theory of the *p-n* junction.

In the paper there is presented a theory enabling to calculate a distribution of excess carrier concentration $\Delta p(x,y,z)$ in the arbitrary formed block of the *n*-type homogeneous semiconductor supplied with the arbitrary distributed p^+ or n^+ electrodes placed at its surface. For all particular cases boundary conditions are formulated. Transport equations based on W. van Roosbroeck formalism make possible to use time non consuming numerical methods of calculations. Infrared radiation emitting devices made of germanium cover the transmission band of 8 – 12μm wavelength and are described in publications [5–8]. Emitters of injection modulated thermal radiation can also be made of silicon which is an advantageous semiconductor because of its very well developed technology, however it is necessary to remember that the free carrier absorption coefficient in silicon is smaller than in germanium [9].

A very important question in the development of the semiconductor p^+-n-n^+ structures, especially emitting IR injection modulated radiation, is determination of an exact excess carrier distribution in the base region. This can help to obtain an efficient radiation from the structure as well as to avoid so called cross-talk effect when structures are integrated and form an array. These problems are also a subject of the presented paper.

2. Distributions of excess current carriers Δp and electrical potential ψ in an homogeneous base of the p^+-n-n^+ junction structure

In the work a semiconductor structure of a general form presented in Fig. 1 is considered. The structure

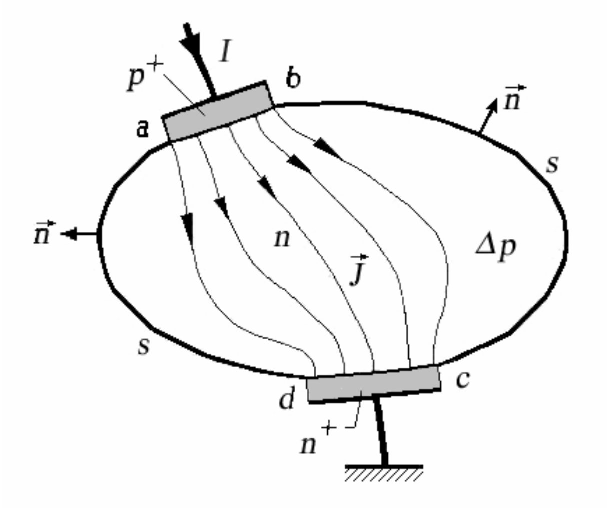

Fig. 1. General scheme of the p^+-n-n^+ structure.

is formed of an homogeneous n-type semiconductor supplied with the p^+-n and n-n^+ junctions.

Excess current carrier transport equation in the n-region of the structure

Let us consider a block of an homogeneous n-type semiconductor with equilibrium concentrations of electrons n_0 = const and holes p_0 = const. In the *ab* and *cd* parts of the surface there are situated p^+-type and n^+-type layers, respectively. In general it is possible to consider more of such electrodes. When the current flows in the volume between *ab* and *cd* electrodes there are excess carriers (holes and electrons) “created” due to the work of the p-n junction (*ab*).

Transport equations of excess carriers in the homogeneous semiconductor were derived by W. van Roosbroeck in the work [4]. In this work he used the quasi-neutrality principle which postulate that excess concentrations of holes Δp and electrons Δn are equal

$$\Delta p = \Delta n. \tag{1}$$

For the simplicity of further calculations we will use only the symbol Δp. Thus we can state that total concentrations of electrons n and holes p are

$$n = n_0 + \Delta p, \tag{2}$$

$$p = p_0 + \Delta p \tag{3}$$

and the total conductivity is:

$$\sigma = \sigma_0 + q\,(\mu_n + \mu_p)\,\Delta p. \tag{4}$$

The current carrier transport is determined by continuity equations of electron $\vec{J}_n$ and hole $\vec{J}_p$ currents.

Since in the stationary case a density of the space charge is equal to zero, then the total current fulfils the equation

$$div\,\vec{J} = 0. \tag{5}$$

If there is no external generation of current carriers, then excess current carriers density Δp is connected with $div\,\vec{J}_p$ (or $div\,\vec{J}_n$) and with the lifetime τ of excess carriers by:

$$div\,\vec{J}_p = -div\,\vec{J}_n = -(\Delta p/\tau)\{1 + [\Delta p/(n_0 + p_0)]\} \tag{6}$$

It is necessary to mention that the recombination rate is characterized here by one parameter τ only (lifetime of excess carriers) which is reasonable in case of direct recombination, but in case of indirect recombination (through recombination centers) can be only an approximation.

To formulate a set of two equations, which will be a base for numerical calculations, it is necessary to describe more closely currents $\vec{J}_n$ and $\vec{J}_p$. For this reason we have to choose proper of unknown values. These values are excess carrier concentrations Δp and potential Ψ, which will be discussed later.

When the principle of quasi-neutrality is fulfilled, we can introduce potential Ψ in the following form given by W. van Roosbroeck [4]

$$\Psi = V - \frac{kT}{q}\,\frac{\mu_n - \mu_p}{\mu_n + \mu_p}\ln\frac{\sigma}{\sigma_0} \tag{7}$$

where: V – electrostatic potential; k – Boltzmann constant; T – absolute temperature, q – elementary charge; μ_n, μ_p – electron and hole velocities; σ – – total conductivity; σ_0 – equilibrium conductivity, when $\Delta p = 0$.

Applying potential Ψ we obtain a very convenient formula for the total current

$$\vec{J} = -\sigma\, grad\psi \tag{8}$$

while the hole current is expressed by the formula

$$\vec{J}_p = -\sigma\, grad\psi - qD\, grad\Delta p \tag{9}$$

which evidently shows ohmic and diffusion components due to introduction of the ambipolar diffusion coefficient

$$D = \frac{\sigma_n}{\sigma}D_p + \frac{\sigma_p}{\sigma}D_n \tag{10}$$

where: D_p, D_n – hole and electron diffusion coefficients.

The obtained till now results permit to formulate an equation which fulfils potential $\Psi(x,y,z)$ and concentration $\Delta p(x,y,z)$. Combining Eqs. (5) and (8) we get a following shape of the first continuity equation

$$div(\sigma\, grad\psi) = 0 \tag{11}$$

where σ is given by Eq. (4).

The second continuity equation is obtained after substituting Eqs. (9) to (6)

$$div\,(qD\, grad\Delta p + \sigma_p\, grad\psi) = \frac{\Delta p}{\tau}\left(1 + \frac{\Delta p}{n_0 + p_0}\right) \tag{12}$$

where:

$$\sigma_p = \sigma_{p0} + q\mu_p \Delta p \qquad (13)$$

while D is given by (10).

3. Boundary conditions

Basic Eqs. (11) and (12) fulfilling unknowns $\Delta p(x,y,z)$ and $\Psi(x,y,z)$ demand formulation of boundary conditions which should be satisfied by these functions or their derivatives.

3.1. Boundary conditions on the free surface bc, ad

a) *A general condition concerning* Δp

This condition is following:

$$-D\vec{n}\,grad\Delta p = s\Delta p \qquad (14)$$

where $\vec{n}$ – unit normal vector, s (ms^{-1}) – surface recombination velocity.

The above formula is derived in App. A1.

To evaluate an influence of the surface recombination on the investigated construction performance it is necessary to consider two extreme conditions: $s = 0$ and $s = \infty$.

In the first case

$$\vec{n}\,grad\Delta p = 0,\ \ s = 0, \qquad (15)$$

that means that normal component of $grad\,\Delta p$ is equal to zero.

In the second case

$$\Delta p = 0,\ s = \infty\,. \qquad (16)$$

It is clear that particular parts of the surface could have different recombination properties.

b) *A boundary condition concerning potential* ψ

A total current density must be parallel to the surface (free surface without electrodes), namely $\vec{J}\vec{n} = 0$. Thus taking into consideration Eq. (8) we can write

$$\vec{n}\,grad\,\psi = 0 \qquad (17)$$

which means that normal component of $grad\ \psi$ must be equal to 0, not depending on s.

3.2. Boundary conditions concerning electrode n$^+$ (presented separately in Fig. 2)

a) *A boundary condition concerning* Δp

This condition is identical to the repeated here recombination condition (14)

$$-D\vec{n}\,grad\Delta p = s\Delta p \qquad (18)$$

It should be reasonable to discuss also such cases as given by formulae (15) and (16).

Rewriting them we have

$$\vec{n}\,grad\Delta p = 0 \quad \text{when } s = 0 \qquad (19)$$

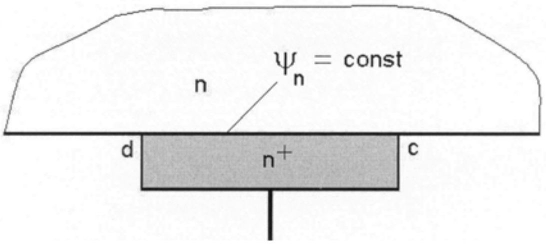

Fig. 2. n-n^+ base junction

and

$$\Delta p = 0 \quad \text{when} \quad s = \infty\,. \qquad (20)$$

b) *A boundary condition concerning* ψ

It is necessary to decide if potential of the electrode n^+ is here a reference potential. If it is, then it means that

$$\psi_{n+} = const. \qquad (21)$$

The simplest assumption is $const = 0$. In this case potential of the electrode p^+ will have simultaneously a value of the voltage applied to the structure if we admit that the potential drop in the layer p^+ is negligible.

3.3. Boundary conditions for the electrode p$^+$ (surface ab)

a) *A boundary condition concerning potential* ψ

Let us assume that the surface ab of the p^+-n junction (Fig. 3) is perpendicular to the z-axis in the point

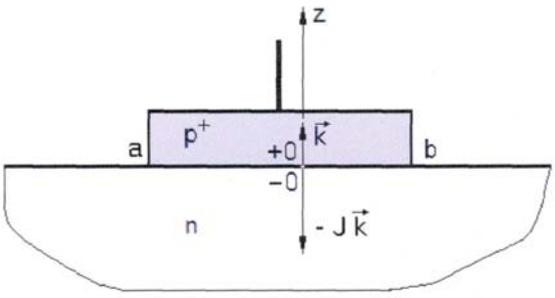

Fig. 3. p^+-n junction

z=0. Through all this surface flows in to the structure a total current of a constant density $-J\vec{k}$ ($\vec{k}$ – unit vector of axis z). Then according to (8) potential ψ in the vicinity of the surface (z = – 0) will fulfil a condition

$$\vec{k} \times grad\ \psi = 0. \qquad (22)$$

Notice that conditions (21) and (22) concern both electrodes having constant potential. First of them determines a value of the potential and the second its derivative, because a value of the potential can be imposed in one case only. If necessary above conditions can be exchanged.

b) *A boundary condition concerning carrier concentration $\Delta p(-0)$*

A determination of the carrier concentration under the surface *ab* demands longer calculations as it is shown in App. A2. The result is following:

$$\Delta p(-0) = \frac{J}{q\left(\dfrac{D(-0)}{L_n} + \dfrac{D(+0)}{L_{p^+ef}}\dfrac{n_0(-0)}{p_0(+0)}\right)} \qquad (23)$$

where: L_n – diffusion length of excess carriers in n-region, L_{p^+ef} – effective diffusion length of excess carriers in p^+-region, n_0 – equilibrium electron concentration in n-region, p_0 – equilibrium hole concentration in p^+-region, $D(-0)$ – diffusion coefficient of excess carriers in n-region, $D(+0)$ – diffusion coefficient of excess carriers in p^+-region.

Both values of diffusion lengths are taken in the vicinity of the *p-n* junction.

An effective diffusion length L_{p^+ef} means that there is taken into account an influence of the surface recombination at *cd* (Fig. 3). If the value of L_{p^+ef} is not to small and the ratio of $n_0(-0)/p_0(+0)$ is small from the assumption, formula (23) can be simplified

$$\Delta p(-0) = \frac{1}{q} J \frac{L_n}{D(-0)}. \qquad (24)$$

4. Calculations of charge carrier and potential distributions

Using the above presented theory there were considered following structures:

a) a structure with p^+-n and n-n^+ junctions in the form of stripes situated on the top and bottom surface of the silicon wafer (Fig. 4),

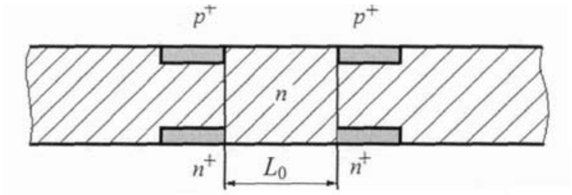

Fig. 4. Structure with p^+-n and n-n^+ junctions in the form of stripes situated on the top and bottom surfaces of the silicon wafer.

b) a structure formed of p^+-n junctions in the form of stripes situated on the top surface of the wafer and the n-n^+ contact covering all the bottom surface of the wafer (Fig. 5).

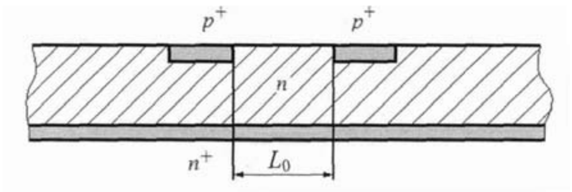

Fig. 5. Structure with p-n^+ junctions in the form of stripes situated on the top surface of the wafer and the n-n^+ junction covering all the bottom surface of the wafer.

These structures correspond to our running works concerning a development of the construction and technology of silicon edge radiating sources (radiating in the direction parallel to the surface of the wafer).

Calculations were performed for the structure made of n-type silicon with the equilibrium concentration of electrons $n_0 = 5 \cdot 10^{11}$ cm^{-3} (resistivity $\rho \approx 9$ kΩ·cm), the lifetime of excess carriers $\tau = 300$ µs and the temperature $T = 300$ K.

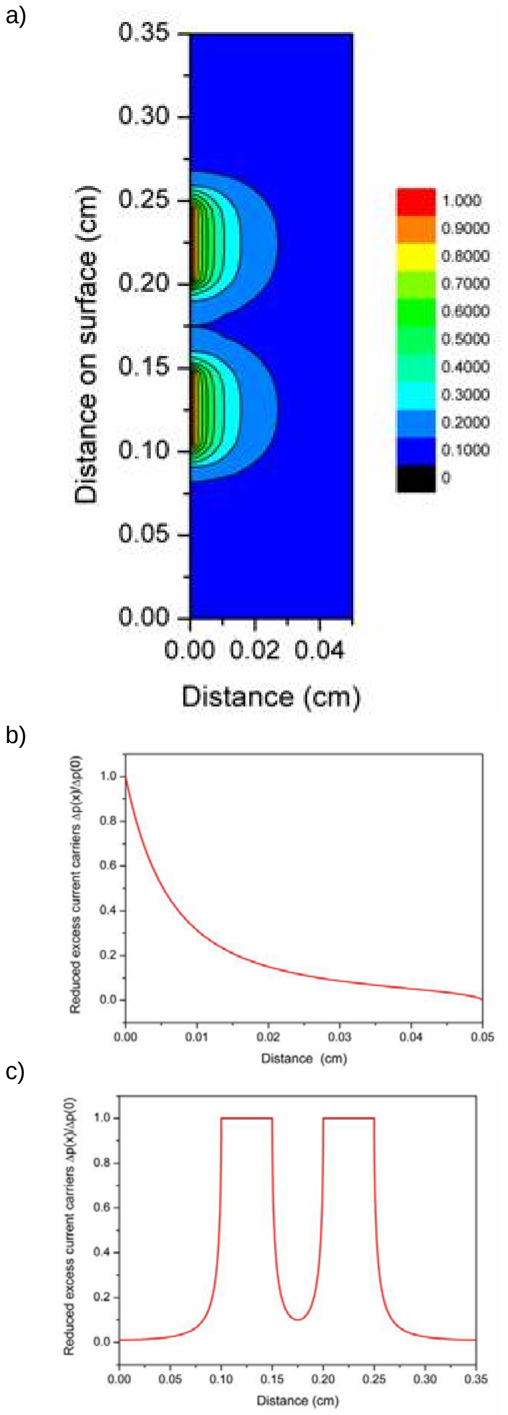

Fig. 6 Contour distribution of excess current carriers in the base (a), distribution of excess current carriers under the p^+-n junction along the base thickness (b) and on the base top surface (c) of the structure shown in Fig. 4.

Numerical calculations were performed solving partial Eqs. (11) and (12) by the method of finite elements with the system FlexPDE, version 4. In these calculations were used complete boundary conditions for the concentration of excess current carriers and electrical potential in forms given by Eq. (14), (21) and (22). Results of these calculations for considered structures are presented in Figs. 6–9 where distributions of excess carriers and of the potential in their bases are shown.

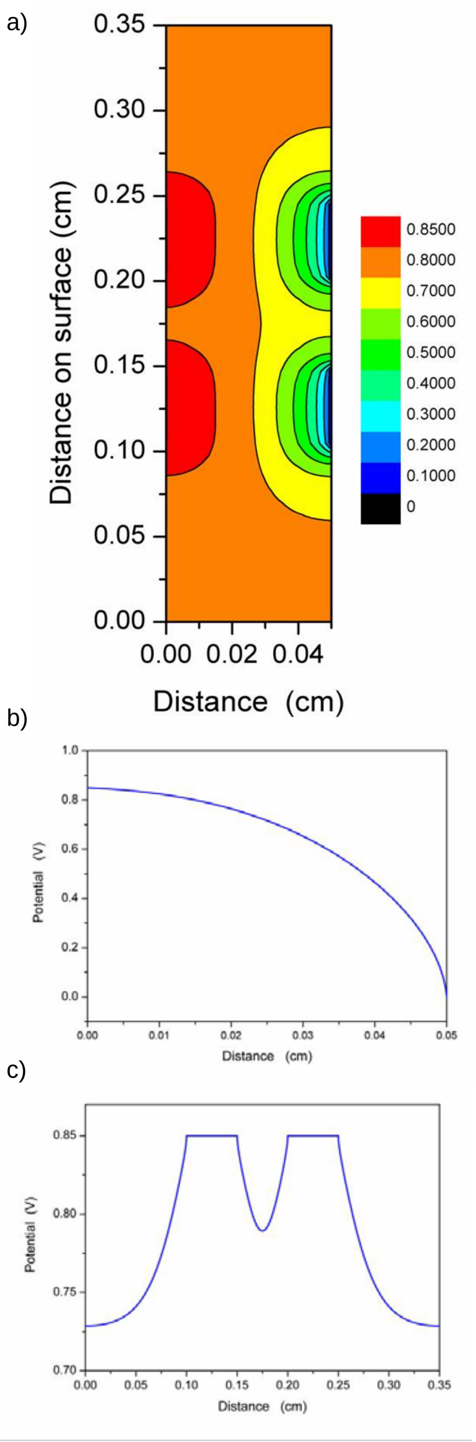

Fig. 7. Contour distribution of the electrical potential in the base (a), distribution of the electrical potential along the base thickness (b) and on the top surface (c) of the structure shown in Fig. 4.

In the integration of IR radiating sources there arise a problem of their approaching. To determine an optimal configuration of neighbouring junctions from the point of view of the cross-talk there were calculated integrals of excess carrier distributions determined along the line in the half of the distance between these junctions. Results of these calculations for considered structures are shown in Fig. 10. From this picture it is evident that the shape of the bottom n^+ electrode has a very small influence on the distribution

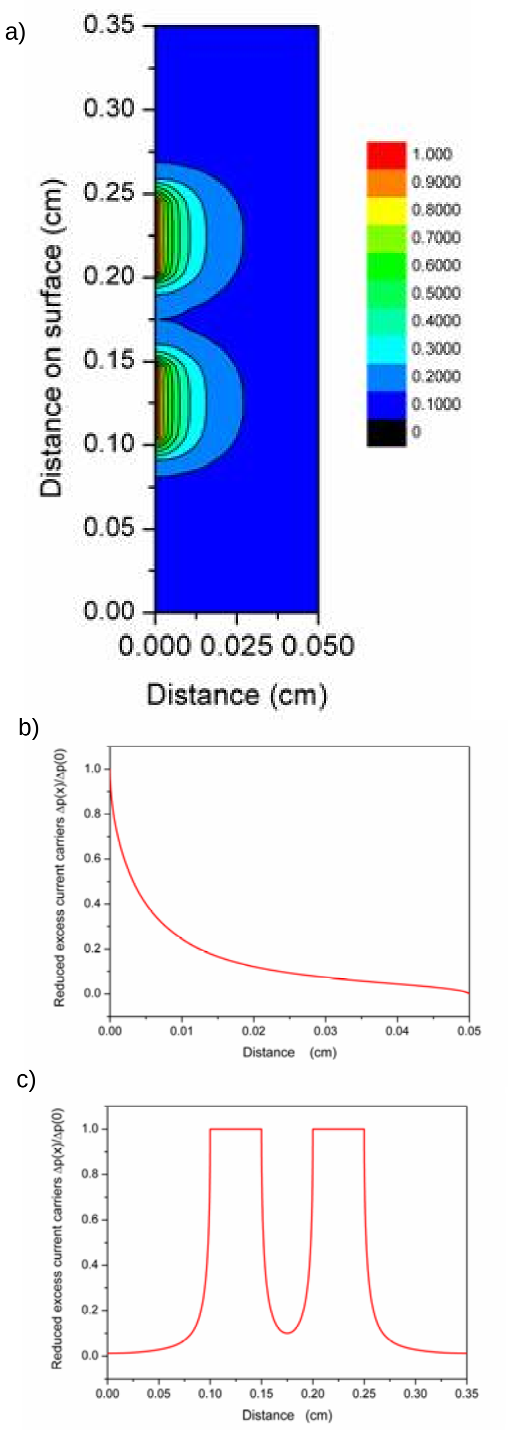

Fig. 8. Contour distribution of excess current carriers in the base (a), distribution of excess current carriers under the p^+-n junction along the base thickness (b) and on the base top surface (c) of the structure shown in Fig. 5.

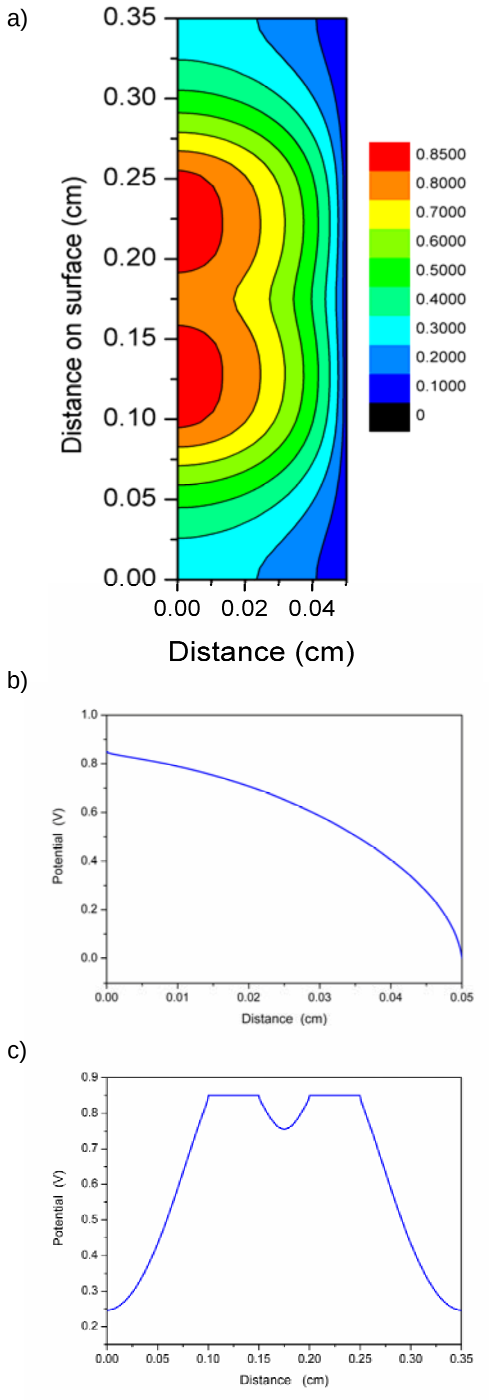

Fig. 9. Contour distribution of the electrical potential in the base (a), distribution of the electrical potential along the base thickness (b) and on the top surface of the structure (c) shown in Fig. 5.

of excess carriers in the base region and thus on the cross-talk of the signal. In this situation it is reasonable to apply the configuration of electrodes shown in Fig. 5 which is much simpler from the technological point of view.

5. Conclusions

The presented theory enables to determine excess current carriers and electrical potential distributions for the arbitrary configuration of p^+-n and n-n^+ junctions as well as for the arbitrary level of injection and surface recombination velocity.

Normalized dependences of the cross-talk enable to choose a proper construction of the developed structure.

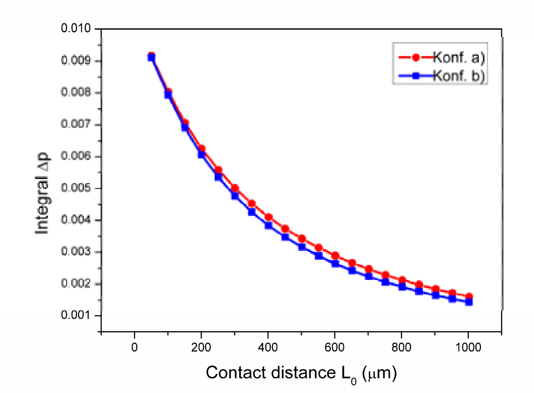

Fig. 10. Calculated integrals of excess current carrier distributions determined along the line perpendicular to the surface in the half of the distance between p^+-n junctions.

References

1. L. Sosnowski, *Excess-Defect Semiconductor Contacts, Phys. Rev*., 1947, **72**, 461–462.
2. L. Sosnowski, B. W. Soole, S. Starkiewicz, *Occurrence of Random Photo-Voltaic Barriers in Photo-Conductive Layers,* 1947, Nature, **160,** 471–472**.**
3. W. Shockley, *The theory of p-n Junctions in Semiconductors and p-n Junction Transistors*, BSTJ, 1948, **28,** 435.
4. W. van Roosbroeck, *The Transport of Added Current Carriers in a Homogeneous Semiconductor*, Phys. Rev., 1955, **91,** 282.
5. V. K. Malyutenko, A. Melnik, O. Malyutenko, *High Temperature (T > 300 K) Light Emitting Diodes for 8-12 μm Spectral Range*, Infrared Phys. & Technol., 2000, **41**, 373.
6. V. K. Malyutenko, V. V. Bogatyrenko, O. Yu. Malyutenko, D. R. Snyder, A. Huber A., J. Norman, *Semiconductor Screen Dynamic Visible to Infrared Scene Converter*, Proc. SPIE, 2002, **4818,** 147–156.
7. V. K. Malyutenko, S. S. Bolgov, O. Yu. Malyutenko, *Un-Cooled Infrared (8-12 μm) Emitters of Positive and Negative Contrast*, Infrared Phys. & Technol., 2004, **45,** 217–222.
8. V. K. Malyutenko, K. V. Michailovskaya, O. Yu. Malyutenko, V. V. Bogatyrenko, R. R. Snyder, *Infrared Dynamic Scene Simulating Device Based on Light Down-Conversion*, IEE Proc. - Optoelectron. 2003, **150**, 391–394.
9. R. A. Smith, *Semiconductors*, Cambridge University Press, 1978.

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Appendix A1 – Deriving the formula (14)

A recombination of holes and electrons in the vicinity of the surface are

$$\vec{J}_p\vec{n} = qs\Delta p\,, \tag{A1.1}$$

$$\vec{J}_n\,\vec{n} = -\,qs\Delta p\,. \tag{A1.2}$$

Multiplying first equation by $\sigma_n / q\sigma$ and second one by $\sigma_p / q\sigma$, then subtracting them by sides we obtain

$$\left(\frac{\sigma_n}{q\sigma}\vec{J}_p - \frac{\sigma_p}{q\sigma}\vec{J}_n\right)\vec{n} = -D\vec{n}grad\Delta p = s\Delta p \tag{A1.3}$$

where according to Eq. (9) and analogical equation for $\vec{J}_n$, the component containing $grad\psi$ abolish. The second and third elements of Eq. (A.1.3) form Eq. (14).

Appendix A2 – Deriving the formula (23)

Due to the continuity of Fermi levels in the *p-n* junction there is fulfilled a mass-action law

$$\begin{aligned}&[p_0(+0) + \Delta p(+0)][n_0(+0) + \Delta p(+0)] = \\ &= [p_0(-0) + \Delta p(-0)]\;[n_0(-0) + \Delta p(-0)]\end{aligned} \tag{A2.1}$$

where: $p_0(+0)$, $n_0(+0)$, $p_0(-0)$, $n_0(-0)$ – equilibrium concentrations in p^+- and *n*-regions.

The mass effect law concerning equilibrium concentrations is described by

$$p_0(+0)n_0(+0) = p_0(-0)n_0(-0)\,. \tag{A2.2}$$

Subtracting above equations by sides and neglecting expressions of Δp^2 order we obtain

$$\begin{aligned}&[p_0(+0)+n_0(+0)]\Delta p(+0) = \\ &= [p_0(-0)+n_0(-0)]\Delta p(-0)\end{aligned} \tag{A2.3}$$

where from

$$\begin{aligned}\Delta p(-0) &= \frac{p_0(+0)+n_0(+0)}{p_0(-0)+n_0(-0)}\Delta p(+0) \cong \\ &\cong \frac{p_0(+0)}{n_0(-0)}\Delta p(+0).\end{aligned} \tag{A2.4}$$

Since $p_0(+0)$ in the region p^+ is much bigger than $n_0(-0)$ in the region *n* we can state that

$$\Delta p(-0) >> \Delta p(+0) \tag{A2.5}$$

and we will use this inequality in the discussion of obtained results from continuity equations of currents at the *p-n* boundary.

In the *p-n* junction theory [3] it is assumed that in the space charge region do not exist recombination states and in a consequence currents J_n and J_p are continuous. Thus in a stationary case total current density *J* is also continuous and it is sufficient to take into account in further considerations one current only, e.g. J_p:

$$J_p(+0) = J_p(-0)\,. \tag{A2.6}$$

To make use of the above equation let us express J_p in the form

$$J_p = -\frac{\sigma_p}{\sigma}J - qD\frac{d\Delta p}{dz} \tag{A2.7}$$

which is easy to derive joining formulae (8) and (9).

Using above expression and taking into consideration a continuity of currents J_p and *J*

$$J_p(+0) = J_p(-0),\;\; J(+0) = J(-0) \tag{A2.8}$$

we obtain

$$\begin{aligned}&-\frac{\sigma_p}{\sigma}(+0)\;J - qD(+0)\frac{d\Delta p}{dz}(+0) = \\ &= -\frac{\sigma_p}{\sigma}(-0)\,J - qD(-0)\frac{d\Delta p}{dz}(-0)\end{aligned} \tag{A2.9}$$

where from

$$\begin{aligned}&-qD(+0)\frac{d\Delta p}{dz}(+0) + qD(-0)\frac{d\Delta p}{dz}(-0) = \\ &= \left[\frac{\sigma_p}{\sigma}(+0) - \frac{\sigma_p}{\sigma}(-0)\right]J.\end{aligned} \tag{A2.10}$$

Since $\sigma_p(+0)$ in the region p^+ is almost equal to and $\sigma_n(+0)$ and $\sigma_p(-0)$is much smaller than $\sigma(-0)$, we can accept that the value of square parenthesis in the right side of Eq. (A2.10) is equal to 1, and finally we obtain

$$-D(+0)\frac{d\Delta p}{dz}(+0) + D(-0)\frac{d\Delta p}{dz} = \frac{1}{q}J\,. \tag{A2.11}$$

Considering an influence of the surface recombination velocity at *cd* (Fig. 3), then introducing an effective diffusion length L_{p^+ef} smaller than the real one and taking into account that a derivative of $\Delta p(z)$ is negative when the *p-n* junction is polarised in the forward direction, we obtain

$$\frac{d\Delta p}{dz}(+0) = -\frac{\Delta p(+0)}{L_{p^+ef}}\,. \tag{A2.12}$$

A similar formula in the region *n* is

$$\frac{d\Delta p}{dz}(-0) = +\frac{\Delta p(-0)}{L_n}\,. \tag{A2.13}$$

Introducing above formula to (A2.11) we obtain

$$D(+0)\frac{\Delta p(+0)}{L_{p^+ef}} + D(-0)\frac{\Delta p(-0)}{L_n} = \frac{1}{q}J \tag{A2.14}$$

From Eq. (A2.4) we have:

$$\Delta p(+0) = \frac{n_0(-0)}{p_0(+0)}\Delta p(-0) \tag{A2.15}$$

and substituting it to (A2.14) we obtain

$$D(+0)\frac{n_0(-0)}{p_0(+0)}\frac{\Delta p(-0)}{L_{p^+ef}} + D(-0)\frac{\Delta p(-0)}{L_n} = \frac{1}{q}J\,. \tag{A2.16}$$

And from the above equation we get finally

$$\Delta p(-0) = \frac{J}{q\left(\dfrac{D(-0)}{L_n} + \dfrac{D(+0)}{L_{p^+ef}}\dfrac{n_0(-0)}{p_0(+0)}\right)}\,. \tag{A2.17}$$

Appendix A3 - Deriving the boundary condition for *n*-*n*$^+$ junction

There are two conditions concerning *n*-*n*$^+$ junction:

a) *Continuity condition for the total current*

$$\left[\frac{\sigma_p}{\sigma}(+0) - \frac{\sigma_p}{\sigma}(-0)\right]J = \\ = q\left[D(+0)\frac{\Delta p}{L}(+0) + D(-0)\frac{\Delta p}{L}(-0)\right] \tag{A3.1}$$

b) *Mass action law*

$$[p_0(+0) + \Delta p(+0)][n_0(+0) + \Delta p(+0)] = \\ = [p_0(-0) + \Delta p(-0)][n_0(-0) + \Delta p(-0)] \tag{A3.2}$$

where +0 and -0 mean coordinates in the vicinity of n^+-side and *n*-side of the junction, respectively.

Taking into account inequality

$$\frac{\sigma_p}{\sigma}((0) << \frac{\sigma_p}{\sigma}(-0) \tag{A3.3}$$

which takes place even if $p(+0) << p(-0)$ one can neglect $\sigma_p(+0)/\sigma(0)$ in the LHS of (A3.1).

Transforming the mass action law (A3.2) and neglecting $\Delta p^2(+)$ and $\Delta p^2(-0)$ one obtains

$$\Delta p(+0) = \Delta p(-0)\frac{n_0(-0)}{n_0(+0)}. \tag{A3.4}$$

The *n*-*n*$^+$ junction is effective if the diffusion length $L(+0)$ in n^+-region is too small, so it is reasonable to accept a moderate condition

$$L(+0) >> \frac{n_0(-0)}{n_0(+0)}L(-0). \tag{A3.5}$$

Dividing (A3.4) by (A3.5) gives

$$\frac{\Delta p}{L}(+0) << \frac{\Delta p}{L}(-0). \tag{A3.6}$$

Due to inequalities (A3.3) and (A3.6) one can simplify Eq. (A3.1) and finally to get an effective boundary condition for the *n*-*n*$^+$ junction in the form:

$$\Delta p(-0) = -\frac{1}{q}\frac{\sigma_p}{\sigma}(-0)\frac{L}{D}(-0)\,J\,. \tag{A3.7}$$