

INVERSE PROBLEM FOR LOOPED RIVER NETWORKS – LOWER ODER RIVER CASE STUDY

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Keywords: Open channels, looped networks, inverse problem, roughness coefficient, lower Oder River.

Abstract: Identification of coefficients determining flow resistance, in particular Manning's roughness coefficients, is one of the possible inverse problems of mathematical modeling of flow distribution in looped river networks. The paper presents the solution of this problem for the lower Oder River network consisting of 78 branches connected by 62 nodes. Using results of six sets of flow measurements at particular network branches it was demonstrated that the application of iterative algorithm for roughness coefficients identification on the basis of the sensitivity-equation method leads to the explicit solution for all network branches, independent from initial values of identified coefficients.

INTRODUCTION

River networks are frequently occurring natural objects that have significant economic influence as fresh water supply sources and sewage recipients. Additionally, river networks may create serious problems due to flooding. Therefore, the recognition of their behavior, including principles of flow distribution between particular riverbeds as well as development of research methods and computational techniques for river networks are important tasks of great practical significance.

Looped networks are a separate type of river networks and a topologic alternative to dendritic networks. Looped networks require special attention due to the fact that any localized activity, such as construction of hydrotechnical structures, new intakes and/or discharges, dredging etc., may result in changes of flows in other remote regions of the network. These networks may also create unexpected, although serious numerical problems [14]. Thus, the flow in looped networks should be analyzed comprehensively by appropriate mathematical tools. This need is most likely one of the reasons for the limited number of relevant scientific sources concerning looped networks. Nevertheless, dendritic networks also cannot be perceived as well-known objects; some computational concepts are still being examined [22].

The inverse problem is usually defined as identification of parameters for mathematical model of a given phenomenon or object, performed on the basis of recorded values of modeled variables, under the assumption that the form of operator transforming vector of input values to output is fixed and known. The inverse problem can be presented either as discrete (at finite number of parameters invariable in space and time), also called the parameters' estimation, or as a continuous one, where parameters vary in space and/or time. On the other hand, the determination of the transformation operator using a set of input/output vectors is known as the system identification problem [2]. In the case of river network flow modeling, the set of possible inverse problems contains, among others, the determination of Manning roughness coefficients or absolute roughness for particular network branches.

If the complete data set is available, i.e. real water surface elevations at all the nodes and flows at all the branches are given, the inverse problem for the entire network is equivalent to a set of independent inverse problems for each network branch. The network structure is then inessential and if the problem concerns only one of the flow parameters, its solution is uniquely determined and requires no additional assumptions. In an incomplete data set there exists an infinite number of solutions in general; thus, additional conditions are necessary to obtain finite and acceptable number of solutions. The problem of solvability of those cases was analyzed among others by Altman and Boulos [1]. Practically, the uniquely determined solution can be obtained by the formulation of the inverse problem as an optimization problem with constraints resulting from measured data values and e.g. from analysis of the network sensitivity to roughness coefficients of particular branches (equality constraints) and from the fact that identified coefficients should vary within reasonable, practical domains (inequality constraints).

Research carried out hitherto on inverse problems for river networks is practically limited to the estimation of the Manning roughness coefficients only. In the majority of cases the records for stages and/or flows at unsteady motion have been used. Becker and Yeh [4, 5] adopted as a solution the optimization problem referring to the minimization of an objective function being a sum of squares of differences between measured and modeled stages. Similar assumptions, but related to dendritic networks only, were further applied by some researchers, e.g. [10, 18, 19, 20]. The concept of inverse problem formulation as an optimization problem was generally accepted and developed in several papers, e.g. [7, 13, 24]. Ding *et al.* [9] distinguish three methods for this problem solving: the influence-coefficient method, the adjoint-equation method and the sensitivity-equation method.

The influence-coefficient method consists of seeking an objective function extreme by changing the roughness of particular network branches in turn; therefore, some of the direct search algorithms, like Hooke and Jeeves' or Rosenbrock's methods can be applied. The influence-coefficient method was also used by Becker and Yeh [4, 5].

The adjoint-equation method consists of the formulation of the inverse problem as a variational one. Liggett and Chen [17] applied this method to the looped water-supply network while Atanov *et al.* [3] used it to the roughness identification of trapezoidal channels.

The sensitivity-equation method transforms the inverse problem to the solution of relevant optimization problem, whilst constraints can be determined using the so-called

sensitivity matrix where the elements are the reactions of stages/flows within the network to variations of optimized values.

It is worthy of notice that the above methods are not the only possibilities of the inverse problem solution; some hopes can be set on genetic algorithms as relatively new approach [21].

Despite some systematization of inverse problem solution methods for networks one should notice that the issue still does not belong to well-recognized ones. As recently as 2000 Ramesh *et al.* formulated an opinion about insufficiencies concerning those methods and the situation has not been improved since. Many papers discuss networks with simple structure, e.g. consisting of two [24] or three loops [7]. Han [12] applied the variant of the influence-coefficient method (in a version of “trial-and-error procedure”) to the looped network of channels consisting of 145 branches and 92 nodes, but assumed arbitrarily the possibility of roughness changes within a set of four values only, which has simplified the optimization problem considerably. In addition, the National Center for Hydroscience and Engineering, Mississippi, developing mathematical modeling of river networks for many years, among others CCHE1D and CCHE2D models [8, 23], deals with dendritic networks only.

This paper discusses the solution of the inverse problem, defined as Manning’s roughness coefficients identification, for the lower Oder River looped network consisting of 78 branches connected by 62 nodes.

THE LOWER ORDER RIVER NETWORK

The Oder River ranks second in Poland and 12–14th in Europe with regard to length and basin area. Mean flow at the river outlet into the Szczecin Lagoon connected to the Baltic Sea exceeds $500 \text{ m}^3\text{s}^{-1}$. The lower Oder network is located at the 60 km long stretch between water gauge stations at Widuchowa and Trzebież (Fig. 1). Downstream from Widuchowa the riverbed splits into two parallel branches – the East Oder and the West Oder, connected transversely in a few places and forming a looped network. Both main branches run through municipality of Szczecin and join again some kilometers upstream from the outlet to the Szczecin Lagoon. The network also comprises Dąbie Lake which has an average depth of about 2.6 m and with an area of about 54 sq. km ranks 4th in Poland in terms of the area.

The Oder River’s channel bathymetry is stable; a comparison of cross-sections for particular branches measured in the period 2009–2010 with the archival data originating from the seventies of the 20th century does not show any significant differences in the bed elevations. However, Dąbie Lake is permanently getting more shallow and silty. Cross-sections of the network main branches are regular, rectangular or parabolic as a rule, with mean depths of particular branches varying from 2–3 m to 8–9 m. Depths may reach approximately 14 m locally.

Riverbeds of the East and the West Oder are sandy with fine and medium fractions prevailing. Bottoms of the remaining branches are muddy with organic silts of different thickness, characterized by thixotropic structure [16], with spatially differentiated additions of unputrefied organic matters (shells, parts of plants). These additions may affect the parameters of the flow shear stresses (absolute roughness, Manning’s roughness coefficient).

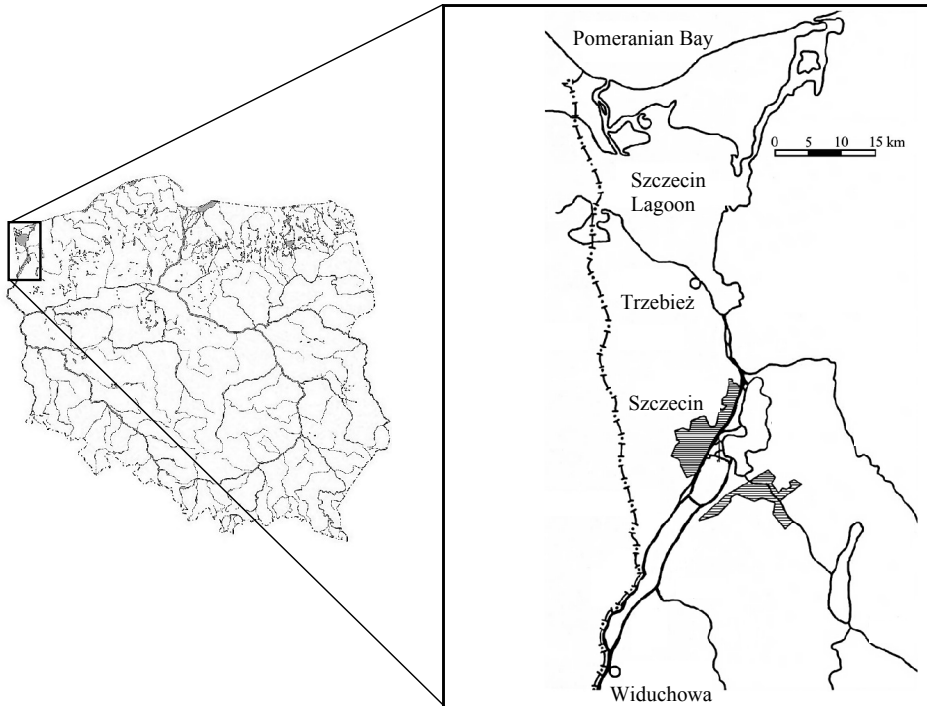


Fig. 1. The lower Oder River location

HYDROMETRIC MEASUREMENTS WITHIN THE LOWER ODER NETWORK

The Lower Oder River network is located entirely within the zone of non-uniform flow (static backwater) caused by the Szczecin Lagoon interference. Thus, any of water gauge cross-sections within the network has no stage-flow relation determined and the standard hydrologic data records of this area contain water stages only, whilst flows have not been recorded. An additional problem is created by the very low water surface slopes in the network channels. Mean slope between the Widuchowa and Trzebież gauges at average flows can be estimated as about $4 \cdot 10^{-6}$ whilst in the neighborhood of Dąbie Lake and downstream (so in the region of the highest network density) this slope is reduced to the order of magnitude 10^{-7} . This fact makes calibration of a mathematical model using stages recorded at gauges located within the area (e.g. the Szczecin water gauge) impossible, because of the practically indistinguishable stage values at neighbouring gauges. Therefore, hitherto all existing mathematical models of flow distribution within the network had to be identified using stages at border gauges (Widuchowa and Trzebież) only, which allowed the calibration of the so-called global (averaged) roughness coefficient for the entire network [15], but did not differentiate this coefficient for particular network branches.

In the 2009–2010 time frame of research project No. N N525 168435 managed by the author and financed from research funds of the Polish Ministry of Science and Higher

Education several sets of flow measurements using ADCP technique were taken within the lower Oder network. Measurements were aimed towards identification of roughness coefficients for particular branches and carried out with the four-beam equipment Workhorse Rio Grande 1200 kHz device made by Teledyne Technologies Co., with work capability ranging from 0.7 m to over 20 m with velocity resolution of 0.001 ms⁻¹. Fig. 2 presents the scheme of the lower Oder, used in the mathematical modeling of flows within the network [15] with conventionally assumed positive flow directions for each branch. The scheme contains 78 branches connected by 62 nodes, which forms a looped structure with 17 independent loops. Branches, in which flows were measured in each set are marked in bold. Their positions allow flow determination at almost every network branch (except some simple, three-branched loops having no essential influence on the whole network, e.g. the loop consisting of branches Nos. 23, 25 and 26). Flow values at each branch can be determined either by direct measurements or from flow balances in relevant nodes. The balance equation for a network node has the form:

$$\sum_{i=1}^{k_i} \eta_i |Q_i| + Q_0 = 0 \tag{1}$$

...where:

- k – number of branches belonging to a node ($i = 1, 2, \dots, k$),
- $\eta_i = 1$ for a branch i assumed conventionally as inflowing to the node,
- $\eta_i = -1$ for a branch i assumed conventionally as outflowing from the node,
- Q_i – flow at branch i [m³s⁻¹],
- Q_0 – algebraic sum of additional water inflows and outflows (intakes and discharges) for a node [m³s⁻¹].

From the total measurements taken, the group of six sets that may be regarded as representing steady flow cases were selected. These measurement sets were carried out under nearly windless conditions and are characterized by small errors of flow balance closure at each node. Next, the measured flows were corrected in accordance with the principle of flow balance at each node, which was performed by solving the optimization problem, with the objective function being a sum of squares of relative flow corrections, using the formula:

$$\min \sum_{i=1}^M \left(\frac{Q_i^* - Q_i}{Q_i^*} \right)^2 \tag{2}$$

...where:

- M – number of network branches submitted to correction,
- Q_i^* – measured flow at branch i ,
- Q_i – corrected (balanced) flow at branch i ,

with constraints given by the equation (1). Flows in the East Oder and the West Oder were assumed as the invariants due to their insensibility to the short term wind influence and

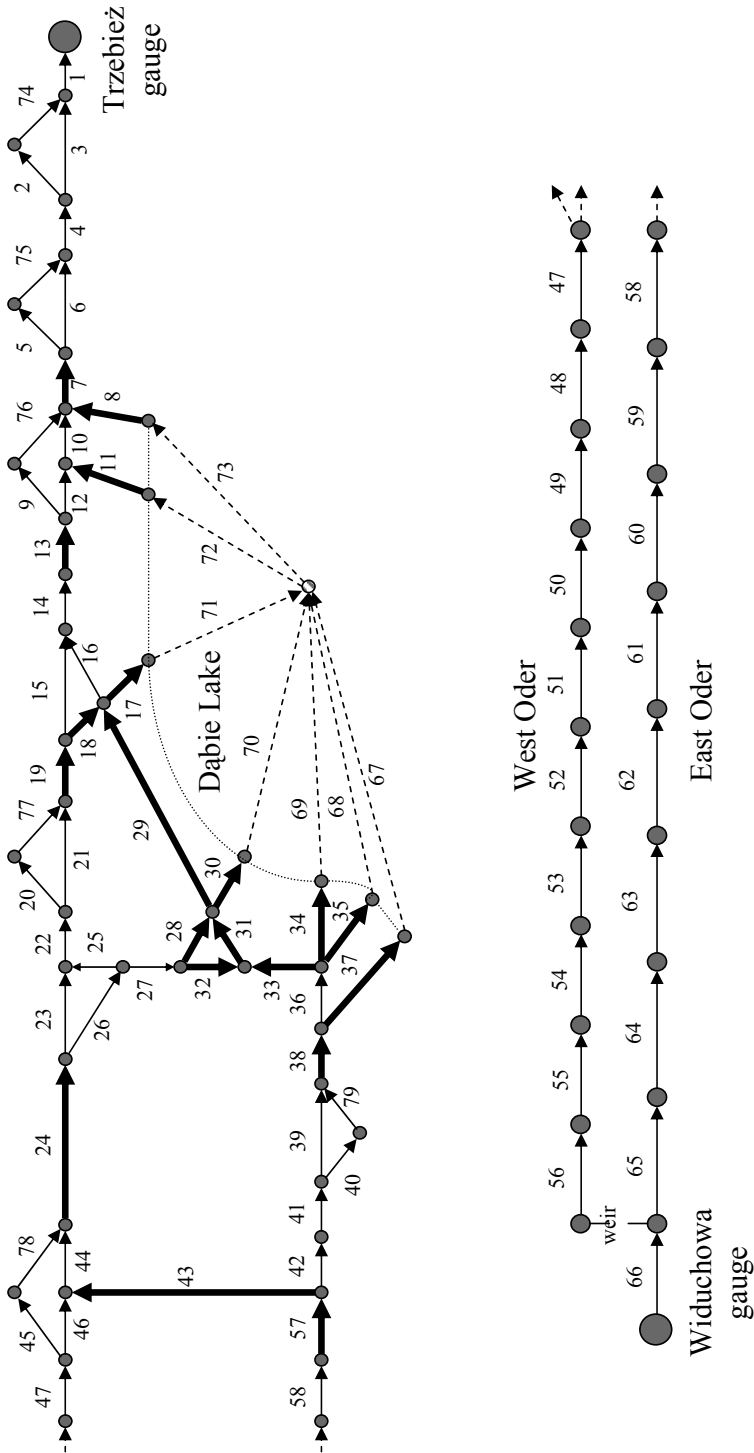


Fig. 2. Lower Oder River network scheme

relatively high variability of the stage at the Trzebież gauge affecting flow fluctuations in northern branches of the network. Next, corrected flows at each branch Q were compared with values of flows Q_m obtained by mathematical modeling of flow distribution within the network at one roughness coefficient (global roughness) common for the whole network [15]. Global roughness was selected in such a way to assure the modeled difference of the water table elevations at the Widuchowa and the Trzebież gauges to be equal to the measured values, as taken on a given day of measurements.

The results of comparison of Q and Q_m values are given in Table 1. Large discrepancies between measures and modeled flows are noticeable. As a rule, the nature of those discrepancies is similar for all measurements, e.g. for Przekop Mieleński (branches Nos. 16, 29 and 31) modeled flows are underestimated significantly in relation to the measured ones whilst modeled flows in branches of Duńczyca (No. 28), East Duńczyca (No. 30), Dąbski Nurt (No. 35) and Dąbska Struga (No. 37) are overestimated. Therefore, the solution of the inverse problem by appropriate differentiation of the roughness coefficients for particular branches is necessary.

INVERSE PROBLEM FOR THE LOWER ODER NETWORK

Identification of roughness coefficients for the lower Oder network was performed using the sensitivity method and the formulating of the appropriate optimization problem. Formulation of the problem required making an assumption, which would determine the form of the minimized objective function. Such an assumption was necessary to obtain the set of branches' roughness coefficients, with relative values variance as minimal as possible. This leads to the following problem formulation:

For the network consisting of M branches and for any initial roughness vector $\mathbf{n} = [n_1, n_2, \dots, n_M]$ at given weighting coefficients vector $\mathbf{r} = [r_1, r_2, \dots, r_M]$ find the vector of roughness corrections $\Delta \mathbf{n} = [\Delta n_1, \Delta n_2, \dots, \Delta n_M]$ that at given linear constraints satisfies the condition:

$$\min \sum_{i=1}^M r_i \left[(n_i + \Delta n_i) - \bar{n} \right]^2 \tag{3}$$

that may be rearranged as follows:

$$\min \sum_{i=1}^M r_i \left[\Delta n_i^2 + 2\Delta n_i (n_i - \bar{n}) \right] \tag{4}$$

where \bar{n} is an average network roughness (e.g. global one). Coefficients r_i theoretically can be assumed arbitrarily as of any value; nevertheless, e.g. the assumption $r_i = 1$ for any i leads to the situation where branches with relatively small section factors and high real roughness coefficients will produce an unnatural decrease of identified values for these coefficients, also compensated by unjustified increase of roughness for smooth, large riverbeds with high flow capacities. Hence, the following weights coefficients were assumed:

$$r_i = Q_i^2 \quad (5)$$

This relation results from the fact that at a given section factor the product $n^2 Q^2$ is proportional to the water slope; next, the global roughness of the lower Oder River is basically determined by conditions at branches with high flows (the East Oder, Regalica) whilst branches with smaller flows (e.g. Dąbski Nurt, Dąbska Struga, Duńczycza) would not affect this value significantly even at these branches full cut-off. Since the global roughness is identified on the base of conformity of measured and calculated water level slopes, assumption (5) ensures small deviations of large channels' roughness from the global value.

Linear constraints of the problem (4) can be formulated using sensitivity matrix \mathbf{D} :

$$\mathbf{D} = \begin{bmatrix} \frac{\partial Q_{p1}}{\partial n_1} & \dots & \frac{\partial Q_{p1}}{\partial n_M} \\ \vdots & \ddots & \vdots \\ \frac{\partial Q_{ps}}{\partial n_1} & \dots & \frac{\partial Q_{ps}}{\partial n_M} \\ \frac{\partial Z_W}{\partial n_1} & \dots & \frac{\partial Z_W}{\partial n_M} \end{bmatrix} \quad (6)$$

where $\{p1, \dots, ps\}$ is a set of numbers of s branches with measured flows and Z_W denotes water surface elevation at the Widuchowa gauge. The gauge of Trzebież is a reference gauge in this case. Therefore, the constraints have the form of a linear set of equations:

$$\mathbf{D} \cdot \Delta \mathbf{n}^T = \mathbf{P}^T \quad (7)$$

...where:

$$\begin{aligned} \Delta \mathbf{n} &= \{\Delta n_1, \dots, \Delta n_M\} \\ \mathbf{P} &= \{Q_{p1}^0 - Q_{p1}^m, \dots, Q_{ps}^0 - Q_{ps}^m, Z_W^0 - Z_W^m\} \end{aligned} \quad (8)$$

whilst upper indices denote: „0” – measured values, “m” – current calculated values. Particular elements of matrix \mathbf{D} are functions of vector \mathbf{n} components and require updating at each change of this vector.

The optimization problem defined above is a quadratic programming problem where objective function is a sum of quadratic and linear forms with linear constraints. Quadratic programming is one of the standard optimization problems and can be solved by many methods [11].

The solution algorithm for the problem of roughness coefficients identification for the lower Oder network appears as follows:

- Step 1. Calculate the network average (global) roughness and assume any initial components of roughness coefficients vector.
- Step 2. For given boundary conditions: water surface elevation at the Trzebież gauge, global flow Q_G and wind direction/velocity calculate flows at particular network branches and water surface elevation at the Widuchowa gauge.

Table 1. Steady motion in the main branches of the lower Oder network: measured (balanced) flows Q [m^3s^{-1}] versus modeled flows Q_m [m^3s^{-1}] at network constant roughness and their proportions

No.	Branch name	1 – Apr. 06, 2009			2 – Jul. 02, 2009			3 – Oct. 27, 2009			4 – Apr. 09, 2009			5 – Apr. 09, 2010			6 – Jun. 23, 2010		
		Q	Q_m	Q_m/Q	Q	Q_m	Q_m/Q	Q	Q_m	Q_m/Q	Q	Q_m	Q_m/Q	Q	Q_m	Q_m/Q	Q	Q_m	Q_m/Q
7	Domiąża	875.0	875.0	1.00	594.0	594.0	1.00	444.0	444.0	1.00	916.1	916.1	1.00	899.0	899.0	1.00	1244.0	1244.0	1.00
8	Inski Nurt	360.5	468.5	1.30	198.3	228.1	1.15	168.0	302.7	1.80	370.3	542.9	1.47	303.9	461.5	1.52	478.1	652.3	1.36
11	Babina and Czapina	150.2	133.6	0.89	72.8	109.2	1.50	94.1	58.0	0.62	148.8	148.2	1.00	194.8	139.7	0.72	202.8	195.7	0.96
13	Odra Czajca	364.3	272.9	0.75	323.0	256.7	0.79	181.9	83.3	0.46	397.0	225.0	0.57	400.3	297.8	0.74	563.1	396.1	0.70
15	Odra Gryfia	138.6	135.5	0.98	101.3	123.0	1.21	71.7	36.9	0.51	152.6	114.0	0.75	142.0	144.7	1.02	220.6	196.8	0.89
16	Przekop Mielenski North	225.7	137.4	0.61	221.7	133.7	0.60	110.2	46.4	0.42	244.4	111.1	0.45	258.3	153.0	0.59	342.5	199.2	0.58
17	Orli Przesmyk	61.7	72.4	1.17	8.5	46.9	5.51	36.4	61.8	1.70	67.8	83.1	1.22	69.9	80.5	1.15	74.9	109.8	1.47
18	Kanał Grabowski	130.3	133.5	1.02	76.3	65.2	0.85	66.2	83.6	1.26	122.9	165.2	1.34	121.7	124.4	1.02	163.0	188.9	1.16
19	Odra Zbożowa	268.9	269.0	1.00	177.6	188.2	1.06	137.9	120.5	0.87	275.5	279.2	1.01	263.7	269.1	1.02	383.6	385.7	1.01
24	Odra Pucka	415.0	413.0	1.00	278.4	275.9	0.99	212.3	206.0	0.97	426.9	434.5	1.02	409.3	418.0	1.02	581.2	591.0	1.02
27	West Parnica	146.1	144.0	0.99	100.8	87.7	0.87	74.4	85.5	1.15	151.4	155.3	1.03	145.7	148.9	1.02	197.6	205.4	1.04
28	Duńczyca	10.1	36.1	3.58	11.9	27.0	2.26	5.9	26.4	4.46	12.3	34.6	2.80	14.1	35.8	2.53	21.1	53.0	2.51
29	Przekop Mielenski Centr.	157.0	76.3	0.49	153.9	115.3	0.75	80.4	24.6	0.31	189.3	28.9	0.15	206.5	109.2	0.53	254.4	120.2	0.47
30	East Duńczyca	45.4	80.6	1.78	13.4	43.8	3.28	24.4	55.9	2.29	8.6	93.7	10.87	41.2	82.2	1.99	52.2	115.8	2.22
31	Przekop Mielenski South	192.3	120.9	0.63	155.4	132.1	0.85	98.8	54.1	0.55	185.6	88.0	0.47	233.6	155.5	0.67	285.5	182.9	0.64
32	Central Parnica	136.0	108.0	0.79	88.9	60.7	0.68	68.5	59.1	0.86	139.1	120.7	0.87	131.5	113.1	0.86	176.5	152.4	0.86
33	East Parnica	56.3	12.9	0.23	66.5	71.4	1.07	30.3	-5.0	-0.17	46.5	-32.7	-0.70	102.0	42.4	0.42	109.0	30.6	0.28
34	Mienia	344.0	330.3	0.96	230.7	189.7	0.82	172.2	169.2	0.98	375.5	371.0	0.99	324.4	322.3	0.99	466.4	453.8	0.97
35	Dąbski Nurt	43.0	85.1	1.98	13.8	37.3	2.70	27.0	51.1	1.89	47.1	103.1	2.19	42.2	81.6	1.94	60.3	118.7	1.97
37	Dąbska Struga	16.7	33.6	2.01	4.6	19.6	4.26	2.2	22.7	10.32	20.0	40.2	2.01	21.0	34.7	1.65	27.1	49.9	1.84
38	Regalica	460.0	462.0	1.00	315.6	318.1	1.01	231.7	238.0	1.03	489.2	481.6	0.98	489.7	481.0	0.98	662.8	653.0	0.99
43	Skosmica	64.5	62.5	0.97	57.4	54.9	0.96	36.3	30.0	0.83	63.8	71.4	1.12	63.3	72.0	1.14	84.2	94.0	1.12
47-56	West Oder	350.5	350.5	1.00	221.0	221.0	1.00	176.0	176.0	1.00	363.1	363.1	1.00	346.0	346.0	1.00	497.0	497.0	1.00
57-65	East Oder	524.5	524.5	1.00	373.0	373.0	1.00	268.0	268.0	1.00	553.0	553.0	1.00	553.0	553.0	1.00	747.0	747.0	1.00

- Step 3. Verify the conditions of conformity of measured and calculated flows and conformity of calculated and measured water surface elevation at Widuchowa. If fulfilled, go to Step 7, otherwise go to Step 4.
- Step 4. Determine elements of sensitivity matrix **D** changing the roughness coefficients for consecutive branches.
- Step 5. Solve quadratic programming problem (4) with constraints (7).
- Step 6. Introduce corrections of roughness vector components being a solution of quadratic programming problem and go to Step 2.
- Step 7. End of calculations.

This method of calculations leads to the identical final roughness vector for the whole network independently of the initial values.

Table 2 shows the results of roughness coefficient identification for main branches as relative values (in proportion to the global roughness). The last column contains values averaged excluding extremely different elements (emphasized by italic).

The course and the final results of the calculations lead to the following remarks:

1. For each investigated measurement set the satisfying consistency between measured and calculated flow values was obtained after introducing the sensitivity matrix **D** consisting of ten rows (nine flow measurements and the Widuchowa water stage).
2. For the majority of network branches the relative roughness values were similar for all measurement sets. This fact allows to assume mean values as representative for the whole investigated flow range. The exceptions are:
 - a) branches directly connected with Dąbie Lake (Dąbski Nurt No. 35, Dąbska Struga No. 37, the East Duńczyca No. 30, Orli Przesmyk No. 17). The roughness values for these branches are significantly differentiated. This results from temporary variations of flow parameters caused by unsteady wind actions on the lake, that is noticeable particularly at low global flows;
 - b) Skońnica (No. 43), which indicates permanent and statistically significant tendency of relative roughness increase with the global flow, that excludes possibility of mean value (in brackets) acceptance as a sufficiently precise generalization. Since the velocities in Skońnica are generally low (a few cm/s), it is possible that this variability results from various flow regimes including hydraulically smooth flow regime at low global network flows.
3. In general, the obtained roughness values are consistent with the ones estimated on the basis of morphological features of riverbeds [6]. Worthy of special notice are:
 - a) very high roughness values of Orli Przesmyk (No. 17), Dąbski Nurt (No. 35), Dąbska Struga (No. 37), the East Duńczyca (No. 30) and particularly of Duńczyca between Parnica and Przekop Mieleński (No. 28) with the averaged roughness about 0.090 (after Chow [6], the roughness coefficient about 0.100 is typical for very weedy natural streams). Indeed, the specified branches are relatively shallow, with very rich riparian vegetation and dense reeds;
 - b) low roughness values at part of the Szczecin – Świnoujście waterway (Przekop Mieleński as a whole, Odra Czajcza – Nos. 13 and 14) varying between 73% and 95% of the global roughness. These branches are distinguished from other branches by maintenance works carried on along the waterway and by intensive

ship motion; therefore, the hypothesis about all riverbed form destruction as an effect of motion of vessels with large draft cannot be neglected. The branch of the East Parnica (No. 33) located in the close vicinity of the Szczecin – Świnoujście waterway and considered an element of the upstream Oder River inland waterway, with the roughness of about 85% of the global value can be included into this group as well. On the other hand, it should be noted that navigable riverbeds with large widths (Domiąza, Regalica, the East Oder) do not differ significantly from the global roughness conditions;

- c) Mienia (No. 34) being a spatial extension of Regalica, with roughness coefficient about 20% higher than global value and the roughness of Regalica itself. Analysis of bed material samples at this site reveals exceptionally high content of unputrefied organic matter including lignified plant stems, what is likely the reason of the relatively high roughness;
- d) Odra Gryfia (No. 15), also with roughness about 20% higher than global value. Contrary to Mienia, the bed of Odra Gryfia is clean, bottom silty sediments are homogenous, without any additions that may increase the roughness [16]; however, this branch is a place of permanent mooring of big vessels and the floating docks of the Szczecin Ship Repair Yard “Gryfia” S.A. whose hulls narrow the bed cross-section, what seems like an increase in the roughness.

Identified roughness coefficients for all network branches are shown in Fig. 3 as values belonging to relevant ranges of relative roughness.

CONCLUSIONS

The course of the calculations presented above proves that the solution of the inverse problem perceived as the identification of roughness coefficients for particular branches of a looped river network can be performed effectively using the sensitivity method. It is applicable even for networks with complicated structure, having several independent branch cycles. This solution does not require introduction of any additional conditions regarding roughness variability, whilst the obtained values of roughness coefficients do not contradict the standard values for the given riverbed types. Application of an iterative algorithm leads to the explicit solution regardless of the initial estimation of the roughness coefficients. Minimized objective function should include weight coefficients as determined by the flow quantities attributed to particular branches.

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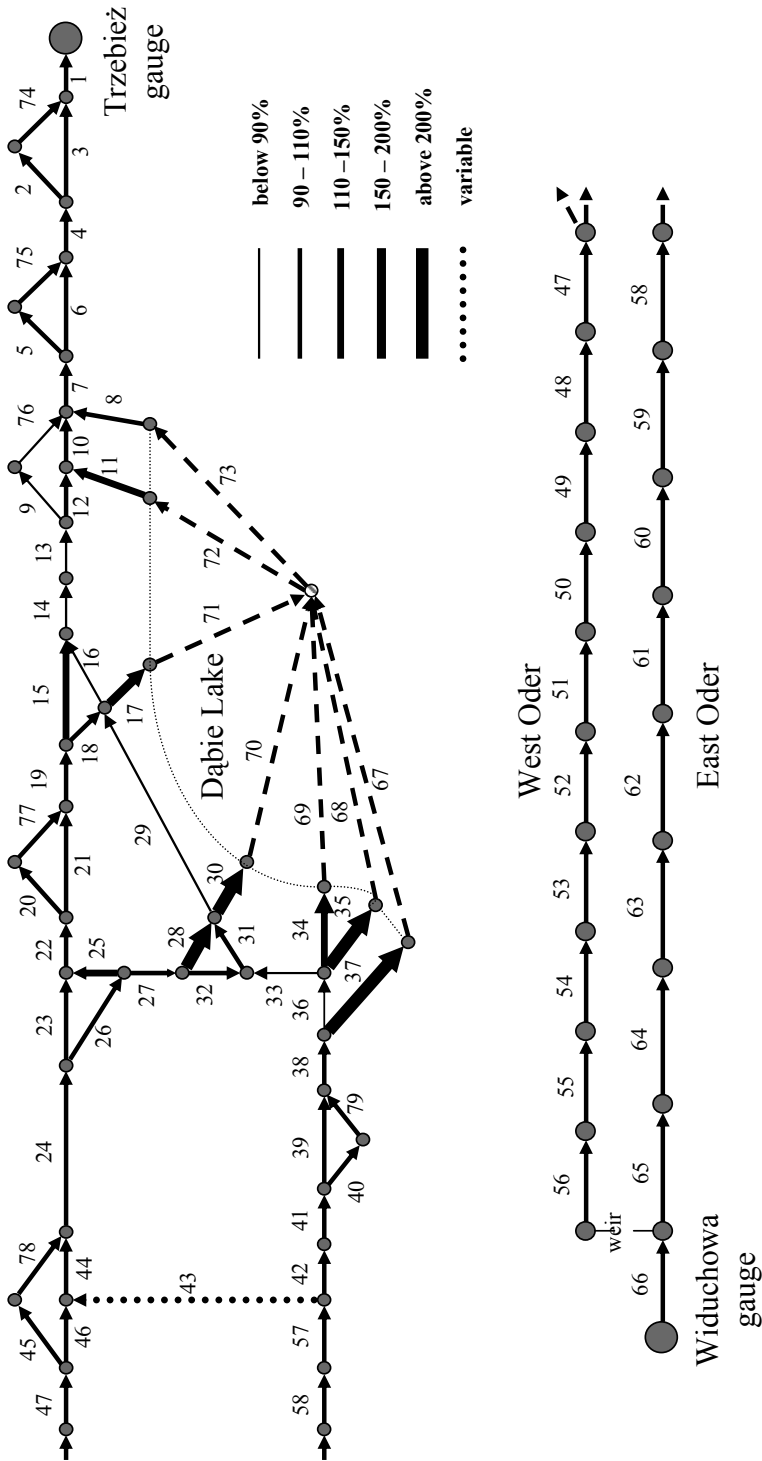


Fig. 3. Relative roughness values for particular branches of the lower Oder River network

Table 2. Identified roughness coefficients for main branches of lower Oder network

Measurement set	3	2	1	5	4	6	Mean	
Q_G [m^3s^{-1}]	444.0	594.0	875.0	899.0	916.1	1244.0		
Global roughness	0.0208	0.0248	0.0258	0.0253	0.0267	0.0247		
No.	Branch name	Relative roughness coefficients [%]						
7	Domiąża	101.0	100.8	100.8	100.4	100.4	100.8	100.7
8	Iński Nurt	101.9	102.0	101.6	101.6	101.9	102.0	101.8
11	Babina and Czapina	108.2	134.3	107.0	112.6	108.2	112.1	113.7
13	Odra Czajca	83.2	78.2	88.8	81.4	83.1	81.8	82.8
15	Odra Gryfia	109.6	136.3	119.4	124.5	118.0	113.0	120.1
16	Przekop Mieleński North	88.0	77.8	86.8	81.8	84.6	85.8	84.2
17	Orli Przesmyk	176.9	<i>831.9</i>	137.6	134.4	152.4	187.0	157.7
18	Kanał Grabowski	99.0	96.4	98.4	97.2	98.1	99.2	98.1
19	Odra Zbożowa	100.5	101.2	101.2	100.4	100.7	100.8	100.8
24	Odra Pucka	100.0	100.0	100.8	101.6	101.9	102.0	101.0
27	West Parnica	104.8	87.1	95.3	96.0	95.5	99.2	96.3
28	Duńczyca	474.0	265.7	423.6	317.0	357.3	304.0	357.0
29	Przekop Mieleński Centr.	81.3	73.4	85.7	76.3	78.3	77.7	78.8
30	Duńczyca Wschodnia	240.4	<i>586.3</i>	213.6	250.2	<i>1333.7</i>	291.9	249.0
31	Przekop Mieleński South	93.3	90.7	95.0	91.3	92.5	92.3	92.5
32	Central Parnica	95.7	92.3	93.4	94.1	93.6	95.1	94.0
33	East Parnica	85.6	81.9	90.3	81.8	85.0	84.2	84.8
34	Mienia	122.1	132.3	110.9	128.5	118.7	123.1	122.6
35	Dąbski Nurt	211.5	<i>608.9</i>	245.0	272.7	257.7	266.0	250.6
37	Dąbska Struga	<i>1037.5</i>	<i>658.5</i>	233.3	204.0	223.2	227.1	221.9
38	Regalica	100.5	100.8	100.8	100.0	100.0	100.4	100.4
43	Skośnica	84.6	90.3	100.4	133.6	130.7	129.1	(111.5)
47–56	West Odra	101.0	100.8	100.8	100.4	100.4	100.8	100.7
57–65	East Odra	98.8	99.1	99.6	99.3	98.4	99.1	99.1

Italic – extremely different values, have not contributed to mean values

Value in brackets – mean value for the set of values with statistically significant growth tendence

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