Acoustic Power Radiated by a System of Two Vibrating Circular Membranes Located at the Boundary of Three-Wall Corner Spatial Region

Krzysztof SZEMELA, Wojciech P. RDZANEK, Witold J. RDZANEK

Department of Acoustics, Institute of Physics, University of Rzeszów
al. Rejtana 16A, 35-310 Rzeszów, Poland; e-mail: alpha@univ.rzeszow.pl

Two vibrating circular membranes radiate acoustic waves into the region bounded by three infinite baffles arranged perpendicularly to one another. The Neumann boundary value problem has been investigated in the case when both sources are embedded in the same baffle. The analyzed processes are time harmonic. The membranes vibrate asymmetrically. External excitations of different surface distributions and different phases have been applied to the sound sources' surfaces. The influence of the radiated acoustic waves on the membranes' vibrations has been included. The acoustic power of the sound sources system has been calculated by using a complete eigenfunctions system.

Keywords: three-wall corner region, Green function, Neumann boundary value problem, modal analysis, asymmetric vibrations, vibration of an excited circular membrane, acoustic power, mutual interaction of sound sources.

1. Introduction

Sound surface sources are very common acoustic radiators. Therefore, an analysis of their sound radiation is essential from the practical point of view. It enables predicting acoustic properties of systems containing such sources. Moreover, this knowledge can be used for active noise control. There are many studies devoted to the sound radiation of surface sources embedded in a single flat baffle (LEE, SINGH, 1994; LI, LI, 2008; TAKAHAGI *et al.*, 1995; ZAGRAI, DONSKOY, 2005; ZHANG, LI, 2010; ZOU, CROCKER, 2009b). Noise control has also been analyzed for these radiators (BRAŃSKI, SZELA, 2011; HUANG, HUNG, 2011; KOZIEŃ, KOŁTOWSKI, 2011; KOZUPA, WICIAK, 2010; 2011; LENIOWSKA, 2009; MAZUR, PAWEŁCZYK, 2011; TROJANOWSKI, WICIAK, 2010; ZOU, CROCKER, 2009a). However, surface sources very often radiate acoustic waves into spatial regions bounded by more than one flat baffle. This causes an appearance of acoustic waves reflected from baffles located near the sound source. The additional waves modify the distribution of the acoustic pressure into the region. Moreover, interacting with the vibrating surface of the plates and membranes they can also influence their vibrations' velocity and, consequently, change their acoustic properties. The problem of sound radiation

into the region bounded by flat baffles can be solved by introducing some simplifying assumptions. In many practical cases baffles can be considered as perfectly rigid. It can also be assumed for simplicity that baffles are infinite, which is valid for high frequencies.

The sound radiation of a single source has been analyzed for the regions of a two-wall and three-wall corners. These regions are bounded by three and two baffles arranged perpendicularly to one another. The acoustic radiation of the piston source has been examined for the regions of two-wall and three-wall corners (RDZANEK *et al.*, 2006b; 2007). It has been assumed that the piston is located at one of the baffles bounding the considered regions. The acoustic pressure distribution and the acoustic power have been analyzed in the case of the vibrating circular membrane located at the boundary of the two-wall corner region (RDZANEK *et al.*, 2009; 2011). The acoustic power of the vibrating circular plate has been presented for the three-wall corner region (SZEMELA *et al.*, 2011). In the paper (HASHEMINEJAD, AZARPEYVAND, 2004), the radiation of the vibrating sphere into quarter-space has been analyzed.

In many practical cases, the sound waves are radiated by an acoustic system containing more than one source. The analysis of the sound radiation of such systems requires additionally including interactions of

mutual sources. The sound radiation has been analyzed for different sources' systems embedded in a flat baffle (ARASE, 1964; PRITCHARD, 1960; WITKOWSKI, 1997; ZAWIESKA, RDZANEK, 2007). The analysis of sound radiation by sources' system located near transverse baffles is more complicated. The solution to this problem has to include interactions among mutual sources, as well as interactions between sources and waves reflected from baffles. The acoustic power of two pistons located at different baffles of a three-wall corner region has been analyzed (RDZANEK, SZEMELA, 2007). So far, the sound radiation by the vibroacoustic system containing sources of deformable surfaces has not been analyzed for regions bounded by more than one flat baffle.

In this paper, the sound radiation by two circular vibrating membranes has been investigated for the three-wall corner region. It has been assumed that both sources are located at the same baffle. The formulas describing the acoustic power have been obtained for the considered vibroacoustic system. They include the influence of radiated acoustic waves on the vibrations of both membranes. The influence of acoustic waves reflected from the transverse baffles have been included by using an appropriate form of the Green function.

2. Analysis assumptions

The vibroacoustic system consists of two membranes of radii a_1 and a_2. They are made of homogeneous and isotropic materials. It has been assumed that the membrane of radius a_1 is the first source and the membrane of radius a_2 is treated as the second source. The membranes radiate acoustic waves into the three-wall corner region $\Omega = \{0 \leq x < \infty, 0 \leq y < \infty, 0 \leq z < \infty\}$ bounded by three baffles that are arranged perpendicularly to one another (Fig. 1). These baffles are perfectly rigid and infinite. Additionally, it has been assumed
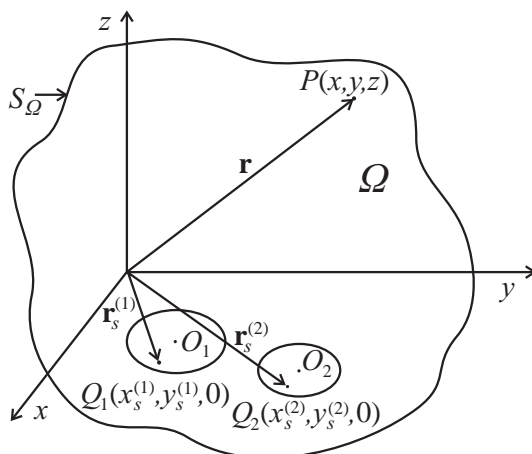

Fig. 1. Considered vibroacoustic system: the region Ω, the field point P, the sources' points Q_1, Q_2, the leading vector of field point $\mathbf{r}$, the leading vectors of sources' points $\mathbf{r}_s^{(1)}$, $\mathbf{r}_s^{(2)}$.

that both membranes are located at the same baffle. The considered region is filled with a homogeneous, lossless, gaseous medium of the rest density ρ_0. The propagation velocity in the medium is equal to c. The amplitude of radiated acoustic waves is small enough to apply the linear theory of the acoustic field. Moreover, all the analyzed processes are time harmonic with time dependence described by the following function: $\exp(-i\omega t)$, where ω is the vibrations' circular frequency. The membranes' vibrations are asymmetric. External factors derived from outside of Ω region do not influence vibrations of the sound sources. The field point location in the global Cartesian coordinates system is determined by the leading vector $\mathbf{r} = (x, y, z)$. The vectors $\mathbf{r}_s^{(1)} = (x_s^{(1)}, y_s^{(1)}, 0)$ and $\mathbf{r}_s^{(2)} = (x_s^{(2)}, y_s^{(2)}, 0)$ indicate the points located at the surface of the first and the second membrane, respectively. The location of the membranes' central points in the global Cartesian coordinates system is defined by the vectors $\mathbf{l}^{(1)} = (l_x^{(1)}, l_y^{(1)}, 0)$, $\mathbf{l}^{(2)} = (l_x^{(2)}, l_y^{(2)}, 0)$ (Fig. 2). Additionally, to describe the locations of the sources' points, two local polar coordinates systems $(r_0^{(1)}, \varphi_0^{(1)})$ and $(r_0^{(2)}, \varphi_0^{(2)})$ have been introduced. The origins of the local coordinates systems are related to the membranes' central points, and their radial axes are parallel to the x axis (Fig. 2). The surfaces of both membranes are forced to vibrations by external asymmetric excitations. Generally, the excitations' distributions, as well as excitations' phases, can be different.
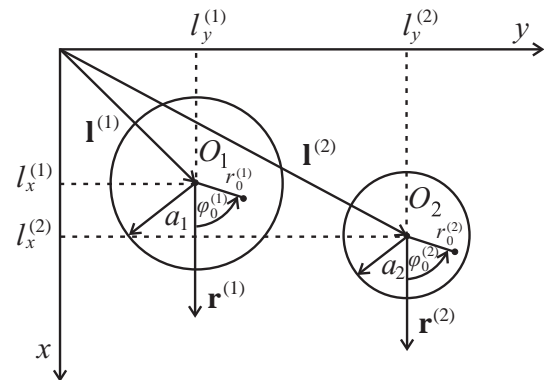

Fig. 2. Locations of the membranes on the horizontal baffle of the three-wall corner region: vectors describing the locations of sources' central points $\mathbf{l}^{(1)}$, $\mathbf{l}^{(2)}$, local polar coordinates system $(r_0^{(1)}, \varphi_0^{(1)})$ and $(r_0^{(2)}, \varphi_0^{(2)})$.

The equations of motion for the excited membranes (cf. RDZANEK *et al.*, 2009; WITKOWSKI, 1997) can be formulated as

$$\left(k_T^{(i)} - 2\nabla^2 + 1\right) W_i\left(r_0^{(i)}, \varphi_0^{(i)}\right) = -\frac{1}{\omega^2 \sigma_i}$$
$$\cdot \left[f_i\left(r_0^{(i)}, \varphi_0^{(i)}\right) + p_1\left(r_0^{(i)}, \varphi_0^{(i)}\right) + p_2\left(r_0^{(i)}, \varphi_0^{(i)}\right)\right] \quad (1)$$

for $i \in \{1, 2\}$, where the case of $i = 1$ is related to the first membrane, the case of $i = 2$ is related

to the second membrane, $W_i(r_0^{(i)}, \varphi_0^{(i)})$ are the amplitudes of transverse deflections for both sources, $k_T^{(i)} = \sqrt{\sigma_i/T_i}\omega$, σ_i denote the surface densities of the membranes, T_i are the uniform tensions applied along the membranes' edges, $f_i(r_0^{(i)}, \varphi_0^{(i)})$ are the functions describing the distributions of external excitations. The components $p_1(r_0^{(1)}, \varphi_0^{(1)})$ and $p_2(r_0^{(1)}, \varphi_0^{(1)})$ that appear in Eqs. (1) for $i = 1$ present the acoustic pressures on the surface of the first membrane resulting from the sound radiation by the first and second sources, respectively. Analogously, the components $p_2(r_0^{(2)}, \varphi_0^{(2)})$ and $p_1(r_0^{(2)}, \varphi_0^{(2)})$ appearing in Eqs. (1) for $i = 2$ present the acoustic pressures on the surface of the second source resulting from the sound radiation of the second and first source, respectively. The solutions of Eqs. (1) can be written in the form of an eigenfunctions series (MEIROVITCH, 1967; RDZANEK et al., 2009):

$$W_i\left(r_0^{(i)}, \varphi_0^{(i)}\right) = \sum_{m=0}^{\infty} \sum_{n=1}^{\infty} \Big\{ c_{m,n}^{(c,i)} W_{m,n}^{(c,i)}\left(r_0^{(i)}, \varphi_0^{(i)}\right)$$
$$+ c_{m,n}^{(s,i)} W_{m,n}^{(s,i)}\left(r_0^{(i)}, \varphi_0^{(i)}\right) \Big\},$$
$$i \in \{1,2\}, \quad (2)$$

where $W_{m,n}^{(c,i)}(r_0^{(i)}, \varphi_0^{(i)})$, $W_{m,n}^{(s,i)}(r_0^{(i)}, \varphi_0^{(i)})$ denote the eigenfunctions and $c_{m,n}^{(c,i)}$, $c_{m,n}^{(s,i)} \in \mathbf{C}$ are the complex coefficients which have to be calculated. The eigenfunctions can be expressed as (cf. MCLACHLAN, 1955; RDZANEK et al., 2009)

$$\begin{Bmatrix} W_{m,n}^{(c,i)}\left(r_0^{(i)}, \varphi_0^{(i)}\right) \\ W_{m,n}^{(s,i)}\left(r_0^{(i)}, \varphi_0^{(i)}\right) \end{Bmatrix} = W_{m,n}^{(i)}\left(r_0^{(i)}\right) \begin{Bmatrix} \cos m\varphi_0^{(i)} \\ \sin m\varphi_0^{(i)} \end{Bmatrix},$$
$$(3)$$
$$W_{m,n}^{(i)}\left(r_0^{(i)}\right) = \sqrt{\varepsilon_m} \frac{J_m\left(k_{m,n}^{(i)} r_0^{(i)}\right)}{J_{m+1}\left(\beta_{m,n}\right)}, \quad i \in \{1,2\},$$

where $r_0^{(i)} \in [0, a_i]$, $\varphi_0^{(i)} \in [0, 2\pi)$, $k_{m,n}^{(i)} = \omega_{m,n}^{(i)}/C_M^{(i)}$, $\omega_{m,n}^{(i)} = \beta_{m,n} C_M^{(i)}/a_i$ are the circular eigenfrequencies of the membranes, $C_M^{(i)} = \sqrt{T_i/\sigma_i}$, T_i are the tensions uniformly distributed along the membranes' edges, σ_i denote the surface densities of the membranes' materials, $J_m(\cdot)$ denotes the Bessel function of the m-th order, $\beta_{m,n} = k_{m,n}^{(i)} a_i$ are the eigenvalues satisfying the following frequency equation: $J_m(\beta_{m,n}) = 0$, $\varepsilon_m = 1$ for $m = 0$, $\varepsilon_m = 2$ for $m \geq 1$. Based on Eqs. (2), the vibration velocity amplitudes of both sound sources can be expressed as

$$v_i\left(r_0^{(i)}, \varphi_0^{(i)}\right) = \sum_{m=0}^{\infty} \sum_{n=1}^{\infty} \frac{\omega}{\omega_{m,n}^{(i)}}$$
$$\cdot \Big\{ c_{m,n}^{(c,i)} v_{m,n}^{(c,i)}\left(r_0^{(i)}, \varphi_0^{(i)}\right)$$
$$+ c_{m,n}^{(s,i)} v_{m,n}^{(s,i)}\left(r_0^{(i)}, \varphi_0^{(i)}\right) \Big\},$$
$$i \in \{1,2\}, \quad (4)$$

where

$$\begin{Bmatrix} v_{m,n}^{(c,i)}\left(r_0^{(i)}, \varphi_0^{(i)}\right) \\ v_{m,n}^{(s,i)}\left(r_0^{(i)}, \varphi_0^{(i)}\right) \end{Bmatrix} = -i\omega_{m,n}^{(i)} \begin{Bmatrix} W_{m,n}^{(c,i)}\left(r_0^{(i)}, \varphi_0^{(i)}\right) \\ W_{m,n}^{(s,i)}\left(r_0^{(i)}, \varphi_0^{(i)}\right) \end{Bmatrix},$$
$$i \in \{1,2\} \quad (5)$$

are the modal coefficients related to the vibration velocity amplitudes. The calculation of constants $c_{m,n}^{(c,i)}$, $c_{m,n}^{(s,i)}$ appearing in Eq. (2) is necessary to solve the equations of motion of both membranes. These constants can be calculated from the equations system which will be presented later.

3. Acoustic pressure – modal quantities

The acoustic pressure amplitude for the analyzed Neumann boundary value problem can be formulated in the form of (MORSE, INGARD, 1968):

$$p(\mathbf{r}) = -ik_0\rho_0 c \int_S v(\mathbf{r}_s) G(\mathbf{r} \,|\, \mathbf{r}_s) \, \mathrm{d}S, \qquad (6)$$

where $k_0 = 2\pi/\lambda$ is the acoustic wavenumber, λ is the acoustic wavelength, $v(\mathbf{r}_s)$ is the vibration velocity of the sources' points, $S = S_1 \cup S_2$, S_1, S_2 are the surfaces of the first and second source, respectively, $\mathbf{r}_s = (x_s, y_s, z_s)$ is the vector describing the locations of sources' points in the global Cartesian coordinates system, $G(\mathbf{r}|\mathbf{r}_s)$ is the Green function of the Neumann boundary value problem. The Green function for analyzed problem can be written as (RDZANEK, RDZANEK, 2006a)

$$G(\mathbf{r}|\mathbf{r}_s) = \frac{4i}{\pi} \int_0^{\infty} \int_0^{\infty} \cos\xi x \cos\xi x_s \cos\eta y \cos\eta y_s$$
$$\cdot \exp(i\gamma z) \frac{\mathrm{d}\xi \, \mathrm{d}\eta}{\gamma}, \qquad (7)$$

where $\gamma^2 = k_0^2 - \xi^2 - \eta^2$. The vibration velocity of the sources' points can be presented in the form of

$$v(\mathbf{r}_s) = \begin{cases} v_1(\mathbf{r}_s^{(1)}) & \text{for} \quad S_1, \\ v_2(\mathbf{r}_s^{(2)}) & \text{for} \quad S_2. \end{cases} \qquad (8)$$

Inserting Eq. (8) into Eq. (6) leads to

$$p(\mathbf{r}) = p_1(\mathbf{r}) + p_2(\mathbf{r}), \qquad (9)$$

where

$$p_i(\mathbf{r}) = -ik_0\rho_0 c \int_{S_i} v_i\left(r_0^{(i)}, \varphi_0^{(i)}\right) G\left(\mathbf{r}|\mathbf{r}_s^{(i)}\right) \mathrm{d}S_i,$$
$$i \in \{1,2\}. \quad (10)$$

The quantities $p_1(\mathbf{r})$ and $p_2(\mathbf{r})$ are the components of the acoustic pressure resulting from the sound radiation by the first and second membrane, respectively.

After making use of Eqs. (4), these components can be written as follows (cf. RDZANEK *et al.*, 2009; SZEMELA *et al.*, 2011):

$$p_i(\mathbf{r}) = \sum_{m=0}^{\infty} \sum_{n=1}^{\infty} \frac{\omega}{\omega_{m,n}^{(i)}} \left\{ c_{m,n}^{(c,i)} p_{m,n}^{(c,i)}(\mathbf{r}) + c_{m,n}^{(s,i)} p_{m,n}^{(s,i)}(\mathbf{r}) \right\},$$
$$i \in \{1,2\}, \quad (11)$$

where

$$\left\{ \begin{matrix} p_{m,n}^{(c,i)}(\mathbf{r}) \\ p_{m,n}^{(s,i)}(\mathbf{r}) \end{matrix} \right\} = -ik_0\rho_0 c \int_{S_i} \left\{ \begin{matrix} v_{m,n}^{(c,i)}\left(r_0^{(i)}, \varphi_0^{(i)}\right) \\ v_{m,n}^{(s,i)}\left(r_0^{(i)}, \varphi_0^{(i)}\right) \end{matrix} \right\}$$
$$\cdot\, G\left(\mathbf{r}|\mathbf{r}_s^{(i)}\right) \mathrm{d}S_i, \qquad i \in \{1,2\} \quad (12)$$

are the modal coefficients of the acoustic pressure. The global coordinates of the sources' points can be expressed by means of their local polar counterparts:

$$x_s^{(i)} = l_x^{(i)} + r_0^{(i)} \cos\varphi_0^{(i)},$$
$$y_s^{(i)} = l_y^{(i)} + r_0^{(i)} \sin\varphi_0^{(i)}, \qquad i \in \{1,2\}. \quad (13)$$

Taking into account Eqs. (7), (12), and (13), the modal coefficients of the acoustic pressure can be presented as

$$\left\{ \begin{matrix} p_{m,n}^{(c,i)}(\mathbf{r}) \\ p_{m,n}^{(s,i)}(\mathbf{r}) \end{matrix} \right\} = \frac{4k_0\rho_0 ca_i^2}{\pi} \int_0^{\infty} \int_0^{\infty} \cos\xi x \cos\eta y \exp(i\gamma z)$$
$$\cdot \left\{ \begin{matrix} M_{m,n}^{(c,i)}(\xi,\eta) \\ M_{m,n}^{(s,i)}(\xi,\eta) \end{matrix} \right\} \frac{\mathrm{d}\xi\,\mathrm{d}\eta}{\gamma},$$
$$i \in \{1,2\}, \quad (14)$$

where

$$\left\{ \begin{matrix} M_{m,n}^{(c,i)}(\xi,\eta) \\ M_{m,n}^{(s,i)}(\xi,\eta) \end{matrix} \right\} = \frac{-i\omega_{m,n}^{(i)}}{S_i} \int_0^{a_i} W_{m,n}^{(i)}\left(r_0^{(i)}\right)$$
$$\cdot \left\{ \begin{matrix} F_m^{(c,i)}(\xi,\eta) \\ F_m^{(s,i)}(\xi,\eta) \end{matrix} \right\} r_0^{(i)}\,\mathrm{d}r_0^{(i)}, \quad (15)$$

$$\left\{ \begin{matrix} F_m^{(c,i)}(\xi,\eta) \\ F_m^{(s,i)}(\xi,\eta) \end{matrix} \right\} = \int_0^{2\pi} \cos\left(\xi x_s^{(i)}\right) \cos\left(\eta y_s^{(i)}\right)$$
$$\cdot \left\{ \begin{matrix} \cos m\varphi_0^{(i)} \\ \sin m\varphi_0^{(i)} \end{matrix} \right\} \mathrm{d}\varphi_0^{(i)}. \quad (16)$$

Using the following coordinates transformation

$$\xi = k_0\tau\cos\alpha, \qquad \eta = k_0\tau\sin\alpha \quad (17)$$

and performing integration in Eq. (16) yields

$$\left\{ \begin{matrix} F_m^{(c,i)}(\xi,\eta) \\ F_m^{(s,i)}(\xi,\eta) \end{matrix} \right\} =$$
$$2\pi \left\{ \begin{matrix} \cos\eta l_y^{(i)} \cos\left(\xi l_x^{(i)} + m\pi/2\right)\cos\alpha \\ -\sin\eta l_y^{(i)} \sin\left(\xi l_x^{(i)} + m\pi/2\right)\sin\alpha \end{matrix} \right\}$$
$$\cdot\, J_m\left(k_0\tau\, r_0^{(i)}\right). \quad (18)$$

Finally, inserting Eq. (18) into Eq. (15) and carrying out integration over the radial variable results in

$$\left\{ \begin{matrix} M_{m,n}^{(c,i)}(\xi,\eta) \\ M_{m,n}^{(s,i)}(\xi,\eta) \end{matrix} \right\} = -2i\omega_{m,n}^{(i)}\psi_{m,n}^{(i)}(\tau)$$
$$\cdot \left\{ \begin{matrix} \cos\eta l_y^{(i)} \cos\left(\xi l_x^{(i)} + m\pi/2\right)\cos\alpha \\ -\sin\eta l_y^{(i)} \sin\left(\xi l_x^{(i)} + m\pi/2\right)\sin\alpha \end{matrix} \right\},$$
$$i \in \{1,2\}, \quad (19)$$

where

$$\psi_{m,n}^{(i)}(\tau) = \sqrt{\varepsilon_m}\,\frac{J_m(\beta s_i\tau)}{\beta_{m,n}^2 - (\beta s_i\tau)^2},$$
$$s_i = a_i/a_1, \qquad \beta = k_0a_1. \quad (20)$$

After changing variable according to Eqs. (17) the modal quantities from Eqs. (14) can be expressed as

$$\left\{ \begin{matrix} p_{m,n}^{(c,i)}(\mathbf{r}) \\ p_{m,n}^{(s,i)}(\mathbf{r}) \end{matrix} \right\} =$$
$$\frac{4k_0^2\rho_0 ca_i^2}{\pi} \int_0^{\infty} \int_0^{\pi/2} \cos\xi x \cos\eta y \exp(i\gamma z)$$
$$\left\{ \begin{matrix} M_{m,n}^{(c,i)}(\tau,\alpha) \\ M_{m,n}^{(s,i)}(\tau,\alpha) \end{matrix} \right\} \frac{\tau\,\mathrm{d}\tau\,\mathrm{d}\eta}{\sqrt{1-\tau^2}}, \qquad i \in \{1,2\}. \quad (21)$$

Based on Eqs. (9), (11) and (21), the distribution of the acoustic pressure amplitude can be determined for the analyzed vibroacoustic system. The obtained formulas describing the modal quantities of the sound pressure will be used to calculate the acoustic power radiated by the membranes' system.

4. Acoustic power – modal quantities

The acoustic power of the considered vibroacoustic system can be calculated based on the following formula:

$$\Pi = \frac{1}{2} \int_{S'} p(\mathbf{r}_s)v^*(\mathbf{r}_s)\,\mathrm{d}S', \quad (22)$$

where $S' = S_1' \cup S_2'$, S_1', S_2' are surfaces directly adjoining the surfaces of the first and second source, respectively, $v^*(\mathbf{r}_s)$ is the conjugate value of the vibration

velocity amplitude of the sources' points (MORSE, INGARD, 1968). Applying Eqs. (8) and (9), the acoustic power can be written as (cf. WITKOWSKI, 1997)

$$\Pi = \Pi_{1,1} + \Pi_{2,2} + \Pi_{1,2} + \Pi_{2,1}, \qquad (23)$$

where

$$\Pi_{i,j} = \frac{1}{2} \int\limits_{S'_j} p_i(\mathbf{r}) v_j^*(\mathbf{r}_s^{(j)}) \, \mathrm{d}S'_j, \quad i,j \in \{1,2\}. \quad (24)$$

The components $\Pi_{1,1}$ and $\Pi_{2,2}$ can be interpreted as the self acoustic powers resulting from interactions between the vibrating source surface and acoustic waves radiated by this source. The components $\Pi_{1,2}$ and $\Pi_{2,1}$ present the mutual acoustic powers related to the interactions between the vibrating source surface and acoustic waves radiated by the other source. Using Eqs. (4) and (11) gives Eqs. (24) in the form of

$$\Pi_{i,j} = \frac{1}{2}\rho_0 c\omega^2 \pi a_i a_j \sum_{m,k=0}^{\infty} \sum_{n,l=1}^{\infty} \Big[c_{m,n}^{(c,i)} c_{k,l}^{(c,j)*} \zeta_{m,n,k,l}^{(c,i,c,j)}$$
$$+ c_{m,n}^{(c,i)} c_{k,l}^{(s,j)*} \zeta_{m,n,k,l}^{(c,i,s,j)} + c_{m,n}^{(s,i)} c_{k,l}^{(c,j)*} \zeta_{m,n,k,l}^{(s,i,c,j)}$$
$$+ c_{m,n}^{(s,i)} c_{k,l}^{(s,j)*} \zeta_{m,n,k,l}^{(s,i,s,j)} \Big], \qquad i,j \in \{1,2\}, \quad (25)$$

where

$$\begin{Bmatrix} \zeta_{m,n,k,l}^{(c,i,c,j)} \\ \zeta_{m,n,k,l}^{(c,i,s,j)} \\ \zeta_{m,n,k,l}^{(s,i,c,j)} \\ \zeta_{m,n,k,l}^{(s,i,s,j)} \end{Bmatrix} = \frac{1}{2\Pi_{m,n,k,l}^{(i,j)}}$$

$$\cdot \int\limits_{S'_j} \begin{Bmatrix} p_{m,n}^{(c,i)}(\mathbf{r}) v_{k,l}^{(c,j)*}\left(\mathbf{r}_s^{(j)}\right) \\ p_{m,n}^{(c,i)}(\mathbf{r}) v_{k,l}^{(s,j)*}\left(\mathbf{r}_s^{(j)}\right) \\ p_{m,n}^{(s,i)}(\mathbf{r}) v_{k,l}^{(c,j)*}\left(\mathbf{r}_s^{(j)}\right) \\ p_{m,n}^{(s,i)}(\mathbf{r}) v_{k,l}^{(s,j)*}\left(\mathbf{r}_s^{(j)}\right) \end{Bmatrix} \mathrm{d}S'_j \quad (26)$$

$\Pi_{m,n,k,l}^{(i,j)} = 1/2\rho_0 c\omega_{m,n}^{(i)}\omega_{k,l}^{(j)}\sqrt{S_i S_j}$ is the normalizing value. The quantities given by Eqs. (26) are the modal components of the acoustic power. Inserting Eqs. (21) into Eqs. (26) yields

$$\begin{Bmatrix} \zeta_{m,n,k,l}^{(c,i,c,j)} \\ \zeta_{m,n,k,l}^{(c,i,s,j)} \\ \zeta_{m,n,k,l}^{(s,i,c,j)} \\ \zeta_{m,n,k,l}^{(s,i,s,j)} \end{Bmatrix} = \frac{-4k_0^2 a_i a_j}{\omega_{m,n}^{(i)}\omega_{k,l}^{(j)}\pi}$$

$$\cdot \int\limits_0^{\infty} \int\limits_0^{\pi/2} \begin{Bmatrix} M_{m,n}^{(c,i)}(\tau,\alpha) M_{k,l}^{(c,j)}(\tau,\alpha) \\ M_{m,n}^{(c,i)}(\tau,\alpha) M_{k,l}^{(s,j)}(\tau,\alpha) \\ M_{m,n}^{(s,i)}(\tau,\alpha) M_{k,l}^{(c,j)}(\tau,\alpha) \\ M_{m,n}^{(s,i)}(\tau,\alpha) M_{k,l}^{(s,j)}(\tau,\alpha) \end{Bmatrix} \frac{\tau\,\mathrm{d}\tau\,\mathrm{d}\alpha}{\sqrt{1-\tau^2}},$$
$$i,j \in \{1,2\}. \quad (27)$$

Then, applying Eqs. (19) gives the modal components of the acoustic power as follows:

$$\begin{Bmatrix} \zeta_{m,n,k,l}^{(c,i,c,j)} \\ \zeta_{m,n,k,l}^{(c,i,s,j)} \\ \zeta_{m,n,k,l}^{(s,i,c,j)} \\ \zeta_{m,n,k,l}^{(s,i,s,j)} \end{Bmatrix} = \beta^2 s_i s_j \int\limits_0^{\infty} \begin{Bmatrix} \phi_{m,k}^{(c,i,c,j)}(\tau) \\ \phi_{m,k}^{(c,i,s,j)}(\tau) \\ \phi_{m,k}^{(s,i,c,j)}(\tau) \\ \phi_{m,k}^{(s,i,s,j)}(\tau) \end{Bmatrix}$$

$$\cdot \frac{\psi_{m,n}^{(i)}(\tau)\,\psi_{k,l}^{(j)}(\tau)\tau\,\mathrm{d}\tau\,\mathrm{d}\alpha}{\sqrt{1-\tau^2}},$$
$$i,j \in \{1,2\}, \qquad (28)$$

where

$$\begin{Bmatrix} \phi_{m,k}^{(c,i,c,j)}(\tau) \\ \phi_{m,k}^{(c,i,s,j)}(\tau) \\ \phi_{m,k}^{(s,i,c,j)}(\tau) \\ \phi_{m,k}^{(s,i,s,j)}(\tau) \end{Bmatrix} =$$

$$\frac{16}{\pi} \int\limits_0^{\pi/2} \begin{Bmatrix} \cos\eta l_y^{(i)} \cos\left(\xi l_x^{(i)}+m\pi/2\right)\cos m\alpha \\ \cos\eta l_y^{(j)} \cos\left(\xi l_x^{(j)}+k\pi/2\right)\cos k\alpha \\ -\cos\eta l_y^{(i)} \cos\left(\xi l_x^{(i)}+m\pi/2\right)\cos m\alpha \\ \sin\eta l_y^{(j)} \sin\left(\xi l_x^{(j)}+k\pi/2\right)\sin k\alpha \\ -\sin\eta l_y^{(i)} \sin\left(\xi l_x^{(i)}+m\pi/2\right)\sin m\alpha \\ \cos\eta l_y^{(j)} \cos\left(\xi l_x^{(j)}+k\pi/2\right)\cos k\alpha \\ \sin\eta l_y^{(i)} \sin\left(\xi l_x^{(i)}+m\pi/2\right)\sin m\alpha \\ \cos\eta l_y^{(j)} \sin\left(\xi l_x^{(j)}+k\pi/2\right)\sin k\alpha \end{Bmatrix} \mathrm{d}\alpha \quad (29)$$

are the factors resulting form the existence of acoustic waves reflected from the transverse baffles. It is obvious that

$$\zeta_{0,n,k,l}^{(s,i,s,j)} = \zeta_{m,n,0,l}^{(s,i,s,j)} = 0 \quad \text{and} \quad \zeta_{m,n,0,l}^{(c,i,s,j)} = \zeta_{0,n,k,l}^{(s,i,c,j)} = 0.$$

Moreover, based on Eqs. (28) and (29) it can be noticed that

$$\zeta_{m,n,k,l}^{(s,i,c,j)} = \zeta_{k,l,m,n}^{(c,j,s,i)}, \qquad \zeta_{m,n,k,l}^{(c,i,c,j)} = \zeta_{k,l,m,n}^{(c,j,c,i)},$$
$$\zeta_{m,n,k,l}^{(s,i,s,j)} = \zeta_{k,l,m,n}^{(s,j,s,i)}, \qquad i \in \{1,2\}. \quad (30)$$

The above relations can be used to reduce the computational complexity of the acoustic power calculations. Changing the variables according to Eqs. (17) and applying the following series (MORSE, FESHBACH, 1953):

$$\cos\left(\overline{z}\begin{Bmatrix} \cos\gamma \\ \sin\gamma \end{Bmatrix}\right) = \sum_{k=0}^{\infty} \varepsilon_k(\mp 1)^k J_{2k}(\overline{z})\cos(2k\gamma),$$

$$\sin\left(\overline{z}\begin{Bmatrix} \cos\gamma \\ \sin\gamma \end{Bmatrix}\right) = 2\sum_{k=0}^{\infty} (\mp 1)^k J_{2k+1}(\overline{z})$$
$$\cdot \begin{Bmatrix} \cos\left[(2k+1)\gamma\right] \\ \sin\left[(2k+1)\gamma\right] \end{Bmatrix}$$

enables us to perform the integration in Eqs. (29). Finally, the factors from Eqs. (29) can be written as

$$\phi_{m,k}^{(c,i,c,j)}(\tau) = (-1)^k \left(Z_{k-m}^{(+)} + Z_{k+m}^{(-)} + (-1)^m \right.$$
$$\left. \cdot \left(Z_{k+m}^{(+)} + Z_{k-m}^{(-)} \right) \right),$$

$$\phi_{m,k}^{(c,i,s,j)}(\tau) = (-1)^k \left(R_{k-m}^{(+)} + R_{k+m}^{(-)} \right.$$
$$\left. + (-1)^m \left(R_{k+m}^{(+)} + R_{k-m}^{(-)} \right) \right), \qquad (31)$$

$$\phi_{m,k}^{(s,i,s,j)}(\tau) = (-1)^k \left(V_{k-m}^{(+)} + V_{k+m}^{(-)} \right.$$
$$\left. - (-1)^m \left(V_{k+m}^{(+)} + V_{k-m}^{(-)} \right) \right),$$

$$\phi_{m,k}^{(s,i,c,j)}(\tau) = \phi_{k,m}^{(c,j,s,i)}(\tau),$$

where

$$Z_b^{(\mu)} = J_b \left(\beta\tau A^{(\mu,+)} \right) \cos\left(bw^{(\mu,+)} \right)$$
$$+ J_b \left(\beta\tau A^{(\mu,-)} \right) \cos\left(bw^{(\mu,-)} \right),$$

$$R_b^{(\mu)} = J_b \left(\beta\tau A^{(\mu,+)} \right) \sin\left(bw^{(\mu,+)} \right)$$
$$+ J_b \left(\beta\tau A^{(\mu,-)} \right) \sin\left(bw^{(\mu,-)} \right),$$

$$\qquad (32)$$

$$V_b^{(\mu)} = J_b \left(\beta\tau A^{(\mu,+)} \right) \cos\left(bw^{(\mu,+)} \right)$$
$$- J_b \left(\beta\tau A^{(\mu,-)} \right) \cos\left(bw^{(\mu,-)} \right),$$

$$\begin{cases} 2L_x^{(\mu)} = A^{(\mu,\nu)} \cos\left(w^{(\mu,\nu)} \right) \\ 2L_y^{(\nu)} = A^{(\mu,\nu)} \sin\left(w^{(\mu,\nu)} \right) \end{cases}, \quad \mu, \nu \in \{+, -\}$$

and

$$L_x^{(+)} = \left(L_x^{(1)} + L_x^{(2)} \right)/2, \qquad L_x^{(-)} = \left(L_x^{(1)} - L_x^{(2)} \right)/2,$$

$$L_y^{(+)} = \left(L_y^{(1)} + L_y^{(2)} \right)/2, \qquad L_y^{(-)} = \left(L_y^{(1)} - L_y^{(2)} \right)/2,$$

$$L_x^{(i)} = l_x^{(i)}/a_1, \qquad\qquad L_y^{(i)} = l_y^{(i)}/a_1.$$

The modal components of the acoustic power have been expressed by the formulas containing only the single integrals, which simplifies numerical calculations.

5. Solution of the equations of motion for the excited membranes

The eigenfunctions series given by Eqs. (2) present the solutions of the equations of motion for the considered vibroacoustic system. These series contain the constants $c_{m,n}^{(c,i)}$, $c_{m,n}^{(s,i)}$ which must be calculated. Inserting Eqs. (2) into Eqs. (1) and using the orthogonality property of the eigenfunctions yields (cf. RDZANEK *et al.*, 2009; WITKOWSKI, 1997)

$$\begin{Bmatrix} c_{m,n}^{(c,1)} \\ c_{m,n}^{(s,1)} \end{Bmatrix} \left(\frac{k_{m,n}^{(1)2}}{k_T^{(1)2}} - 1 \right) = \frac{1}{\omega^2\sigma_1}$$
$$\cdot \left[\begin{Bmatrix} f_{m,n}^{(c,1)} \\ f_{m,n}^{(s,1)} \end{Bmatrix} + \begin{Bmatrix} P_{m,n}^{(c,1,1)} \\ P_{m,n}^{(s,1,1)} \end{Bmatrix} + \begin{Bmatrix} P_{m,n}^{(c,2,1)} \\ P_{m,n}^{(c,1,2)} \end{Bmatrix} \right],$$

$$\begin{Bmatrix} c_{m,n}^{(c,2)} \\ c_{m,n}^{(s,2)} \end{Bmatrix} \left(\frac{k_{m,n}^{(2)2}}{k_T^{(2)2}} - 1 \right) = \frac{1}{\omega^2\sigma_2}$$
$$\cdot \left[\begin{Bmatrix} f_{m,n}^{(c,2)} \\ f_{m,n}^{(s,2)} \end{Bmatrix} + \begin{Bmatrix} P_{m,n}^{(c,2,2)} \\ P_{m,n}^{(s,2,2)} \end{Bmatrix} + \begin{Bmatrix} P_{m,n}^{(c,1,2)} \\ P_{m,n}^{(c,2,1)} \end{Bmatrix} \right],$$

$$(33)$$

where

$$\begin{Bmatrix} f_{m,n}^{(c,i)} \\ f_{m,n}^{(s,i)} \end{Bmatrix} = \frac{1}{S_i} \int_{S_i} \begin{Bmatrix} W_{m,n}^{(c,i)} \left(r_0^{(i)}, \varphi_0^{(i)} \right) \\ W_{m,n}^{(s,i)} \left(r_0^{(i)}, \varphi_0^{(i)} \right) \end{Bmatrix}$$
$$\cdot f^{(i)} \left(r_0^{(i)}, \varphi_0^{(i)} \right) \, dS_i, \quad i, \in \{1, 2\}, \qquad (34)$$

$$\begin{Bmatrix} P_{m,n}^{(c,i,j)} \\ P_{m,n}^{(s,i,j)} \end{Bmatrix} = \frac{1}{S_j} \int_{S_i} \begin{Bmatrix} W_{m,n}^{(c,j)} \left(r_0^{(j)}, \varphi_0^{(j)} \right) \\ W_{m,n}^{(s,j)} \left(r_0^{(j)}, \varphi_0^{(j)} \right) \end{Bmatrix}$$
$$\cdot p_i \left(r_0^{(j)}, \varphi_0^{(j)} \right) \, dS_j, \quad i, j \in \{1, 2\}. \quad (35)$$

Taking into account Eqs. (11) and (26), the coefficients described by Eq. (35) can be formulated as an infinite series of the modal quantities of the acoustic power. Then, Eqs. (33) can be rewritten in the form of

$$\begin{Bmatrix} c_{m,n}^{(c,1)} \\ c_{m,n}^{(s,1)} \end{Bmatrix} \left(\frac{k_{m,n}^{(1)2}}{k_T^{(1)2}} - 1 \right) + i\frac{\omega_{0,1}^{(1)}}{\omega}\varepsilon_1 \sum_{k=0,\,l=1}^{\infty}$$
$$\cdot \left[c_{k,l}^{(c,1)} \begin{Bmatrix} \xi_{k,l,m,n}^{(c,1,c,1)} \\ \xi_{k,l,m,n}^{(c,1,s,1)} \end{Bmatrix} + c_{k,l}^{(s,1)} \begin{Bmatrix} \xi_{k,l,m,n}^{(s,1,c,1)} \\ \xi_{k,l,m,n}^{(s,1,s,1)} \end{Bmatrix} \right]$$
$$+ i\frac{\omega_{0,1}^{(1)}}{\omega}\varepsilon_1 s_2 \sum_{k=0,\,l=1}^{\infty} \left[c_{k,l}^{(c,2)} \begin{Bmatrix} \xi_{k,l,m,n}^{(c,2,c,1)} \\ \xi_{k,l,m,n}^{(c,2,s,1)} \end{Bmatrix} \right.$$
$$\left. + c_{k,l}^{(s,2)} \begin{Bmatrix} \xi_{k,l,m,n}^{(s,2,c,1)} \\ \xi_{k,l,m,n}^{(s,2,s,1)} \end{Bmatrix} \right] = \frac{1}{\omega^2\sigma_1} \begin{Bmatrix} f_{m,n}^{(c,1)} \\ f_{m,n}^{(s,1)} \end{Bmatrix},$$

$$(36)$$

$$\begin{Bmatrix} c_{m,n}^{(c,2)} \\ c_{m,n}^{(s,2)} \end{Bmatrix} \left(\frac{k_{m,n}^{(2)2}}{k_T^{(2)2}} - 1 \right) + i\frac{\omega_{0,1}^{(2)}}{\omega}\varepsilon_2 \sum_{k=0,\,l=1}^{\infty}$$
$$\cdot \left[c_{k,l}^{(c,2)} \begin{Bmatrix} \xi_{k,l,m,n}^{(c,2,c,2)} \\ \xi_{k,l,m,n}^{(c,2,s,2)} \end{Bmatrix} + c_{k,l}^{(s,2)} \begin{Bmatrix} \xi_{k,l,m,n}^{(s,2,c,2)} \\ \xi_{k,l,m,n}^{(s,2,s,2)} \end{Bmatrix} \right]$$
$$+ i\frac{\omega_{0,1}^{(2)}}{\omega s_2}\varepsilon_2 \sum_{k=0,\,l=1}^{\infty} \left[c_{k,l}^{(c,1)} \begin{Bmatrix} \xi_{k,l,m,n}^{(c,1,c,2)} \\ \xi_{k,l,m,n}^{(c,1,s,2)} \end{Bmatrix} \right.$$
$$\left. + c_{k,l}^{(s,1)} \begin{Bmatrix} \xi_{k,l,m,n}^{(s,1,c,2)} \\ \xi_{k,l,m,n}^{(s,1,s,2)} \end{Bmatrix} \right] = \frac{1}{\omega^2\sigma_2} \begin{Bmatrix} f_{m,n}^{(c,2)} \\ f_{m,n}^{(s,2)} \end{Bmatrix},$$

where $\varepsilon_1 = \rho_0 c/\left(\omega_{0,1}^{(1)}\sigma_1 \right)$, $\varepsilon_2 = \rho_0 c/\left(\omega_{0,1}^{(2)}\sigma_2 \right)$ are the dimensionless coefficients describing the influence

of the acoustic attenuation. After finding all the constants $c_{m,n}^{(c,i)}$, $c_{m,n}^{(s,i)}$ based on Eqs. (36), the acoustic power of the analyzed vibroacoustic system can be calculated based on Eqs. (23) and (25).

6. Numerical analysis

It has been assumed that the surfaces of both membranes are excited by the point excitations of the following surface distributions:

$$f_i\left(r_0^{(i)}, \varphi_0^{(i)}\right) = \frac{F_i \exp\left(\theta_i\right)}{r_0^{(i)}} \delta\left(r_0^{(i)} - \overline{r}_0^{(i)}\right)$$
$$\delta\left(\varphi_0^{(i)} - \overline{\varphi}_0^{(i)}\right), \qquad i \in \{1, 2\}, \quad (37)$$

where $\delta(\cdot)$ is the Dirac delta, F_i are the amplitudes of the exciting forces, θ_i denote the excitations' phases, $\left(\overline{r}_0^{(i)}, \overline{\varphi}_0^{(i)}\right)$ are the local polar coordinates of the excited membranes' points. Making use of Eqs. (37), the coefficients given by Eqs. (34) can be written as

$$\begin{Bmatrix} f_{m,n}^{(c,i)} \\ f_{m,n}^{(s,i)} \end{Bmatrix} = \frac{F_i}{S_i} \begin{Bmatrix} W_{m,n}^{(c,i)}\left(\overline{r}_0^{(i)}, \overline{\varphi}_0^{(i)}\right) \\ W_{m,n}^{(s,i)}\left(\overline{r}_0^{(i)}, \overline{\varphi}_0^{(i)}\right) \end{Bmatrix} \exp\left(i\theta_i\right),$$
$$i \in \{1, 2\}. \quad (38)$$

The excitation phase of the first sound source has been assumed as equal to zero and the difference between excitations' phases has been denoted by $\Delta\theta$. The numerical calculations have been performed for some sample values of the parameters describing the analyzed vibroacoustic system. These parameters' values have been presented in Table 1. It has been assumed that the medium is air. The acoustic power has been calculated based on Eqs. (23) and (25). For the practical reasons, the acoustic power calculations can be performed by including only a finite number of vibrating modes. An appropriate accuracy of obtained results can be achieved by including in Eq. (25) all the modes of the circular eigenfrequencies smaller than the circular frequency of the excitations. Using the successive modes of the higher eigenfrequencies increases the accuracy. However, this causes an increase in the computational complexity and consequently limits the range of analyzed circular frequencies, as well as the number of performed analysis. The acoustic power has been calculated for the circular frequencies $\omega < 500$ rad/s. The following vibrating modes of both membranes: $(0,1)$, $(0,2)$, $(1,1)$, $(2,1)$, $(3,1)$ have been included in the numerical calculations. The values of the circular eigenfrequencies $\omega_{m,n}^{(1)}$, $\omega_{m,n}^{(2)}$ have been presented in Table 2 for the included modes. The formula describing the acoustic power contains 256 modal components for the assumed number of included vibrating modes. The modal components are given by the integral formulas. However, using Eqs. (30) limits the number of modal

components to 136, which causes almost a twofold decrease in the computational complexity.

Table 1. Parameters values of the analyzed vibroacoustic system. It has been assumed that the medium is air.

Parameter	Value
membranes' radii	$a_1 = 0.15$ m, $a_2 = 0.1$ m
sound velocity in the medium	$c = 340$ m/s
amplitudes of the exciting forces	$F_1 = 1$ N, $F_2 = 1$ N
relative location of the first membrane central point	$L_x^{(1)} = 1.5$, $L_y^{(1)} = 1.5$
relative location of the second membrane central point	$L_x^{(2)} = 2$, $L_y^{(2)} = 3.5$
polar coordinates of the excited point of the first membrane	$\overline{r}_0^{(1)} = 0.5\,a_1$, $\overline{\varphi}_0^{(1)} = \pi/4$
polar coordinates of the excited point of the second membrane	$\overline{r}_0^{(2)} = 0.5\,a_2$, $\overline{\varphi}_0^{(2)} = \pi/4$
tensions uniformly distributed along the membranes' edges	$T_1 = 340$ N/m, $T_2 = 300$ N/m
difference between the excitations' phases	$\Delta\theta = 0$
surface densities of the membranes	$\sigma_1 = 2$ kg/m^2, $\sigma_2 = 3$ kg/m^2
medium density	$\rho_0 = 1.293$ kg/m^3

Table 2. Circular eigenfrequencies of both membranes related to the modes included in the numerical calculations. The quantities have been expressed in rad/s. The parameters values of the vibroacoustic system have been assumed based on Table 1.

(m,n)	$(0,1)$	$(0,2)$	$(1,1)$	$(2,1)$	$(3,1)$
$\omega_{m,n}^{(1)}$	209	479.8	333	446.4	554.6
$\omega_{m,n}^{(2)}$	240.5	552	383.2	513.6	638

It is essential to examine the influence of the parameters $r_0^{(1)}$, $r_0^{(2)}$ on the acoustic power. The normalized modulus and phase cosine of the acoustic power have been presented in Fig. 3 as functions of the circular frequency. Some sample values of the parameters $r_0^{(1)}$, $r_0^{(2)}$ have been discussed. It has been assumed that the normalized value of the acoustic power modulus is equal to $\Pi_0 = c\rho_0 F_1^2 / \left(\omega_{0,1}^{(1)^2} \sigma_1^2 S_1\right)$. Additionally, changes in the acoustic power modulus have been presented in Fig. 3b within a narrow range of the circular frequency containing the lowest maximum associated with the asymmetric vibrating mode. It allows to visualize the influence of the analyzed parameters in the vicinity of the maxima. Figure 3a shows that the parameters $r_0^{(1)}$, $r_0^{(2)}$ significantly influence the value of the acoustic power modulus. The quantity for the circular frequencies smaller than the circular frequency related to the first maximum and in the vicinity of the maxima associated with the symmetric vibrating
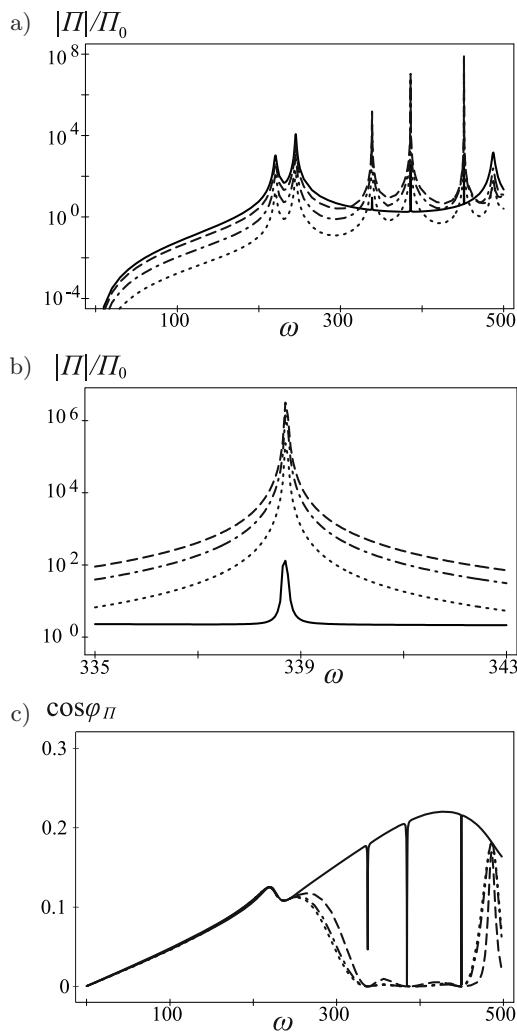
a) $|\Pi|/\Pi_0$

b) $|\Pi|/\Pi_0$

c) $\cos\varphi_\Pi$

Fig. 3. Normalized acoustic power: (a), (b) modulus, (c) phase cosine. Lines: solid – $\overline{r}_0^{(1)} = 0$, $\overline{r}_0^{(2)} = 0$, dashed – $\overline{r}_0^{(1)} = 0.5\,a_1$, $\overline{r}_0^{(2)} = 0.5\,a_2$, dashed-dotted – $\overline{r}_0^{(1)} = 0.75\,a_1$, $\overline{r}_0^{(2)} = 0.75\,a_2$, dotted – $\overline{r}_0^{(1)} = 0.9\,a_1$, $\overline{r}_0^{(2)} = 0.9\,a_2$. The values of the other parameters of the vibroacoustic system have been assumed based on Table 1.

modes reaches the largest values for the symmetric excitations. This fact results from a significant excitation of the symmetric modes for those frequencies. The maxima of the acoustic power modulus associated with the asymmetric vibrating modes also appear in the case of both symmetric excitations. They cover a narrow frequency range and their values are low (Fig. 3b). These maxima result from the sound radiation by a neighboring source and from the existence of acoustic waves reflected from the transverse baffles. The asymmetric modes are excited only by vibrating medium particles. This fact can explain the low values of the acoustic power modulus observed for the maxima associated with asymmetric modes, as well as a narrow circular frequency range covered by these maxima (Fig. 3a, 3b). The analysis of Fig. 3a allows to conclude that the acoustic power modulus decreases

with an increase in the parameters $r_0^{(1)}$, $r_0^{(2)}$ for the asymmetric excitations and within the low frequency range. Moreover, the parameters do not influence the locations of the maxima of the acoustic power modulus in the circular frequency scale. The acoustic power phase essentially depends on the parameters $r_0^{(1)}$, $r_0^{(2)}$ only for the circular frequencies larger than the circular frequency related to the first maximum of the acoustic power modulus (Fig. 3c). The greatest differences are observed between the acoustic power phase radiated for the symmetric and asymmetric excitations. They result from significant excitations of the asymmetric modes for the high frequencies.

The influence of the parameter on the acoustic power can be investigated based on an analysis of the relative changes defined as

$$
E_{|\Pi|} = \frac{\left|\,|\Pi| - \left|\Pi^{(\mathrm{ref})}\right|\,\right|}{\left|\Pi^{(\mathrm{ref})}\right|},
$$

$$
E_{\varphi_\Pi} = \frac{\left|\varphi_\Pi - \varphi_\Pi^{(\mathrm{ref})}\right|}{\left|\varphi_\Pi^{(\mathrm{ref})}\right|},
\tag{39}
$$

where $\left|\Pi^{(\mathrm{ref})}\right|$ and $\varphi_\Pi^{(\mathrm{ref})}$ are the modulus and phase of the reference acoustic power, respectively. It has been assumed that the reference acoustic power is radiated by the vibroacoustic system described by the values of parameters presented in Table 1.

The quantities given by Eqs. (39) have been presented in Fig. 4 as functions of the circular frequency. Some sample values of the parameters $\varphi_0^{(1)}$, $\varphi_0^{(2)}$ have been analyzed. It can be concluded that the changes in the parameters $\varphi_0^{(1)}$, $\varphi_0^{(2)}$ cause the relative changes in the acoustic power modulus $E_{|\Pi|}$ smaller than 0.1 for almost all the analyzed frequencies (Fig. 4a). The only exceptions are the narrow circular frequency ranges containing the circular frequencies related to the maxima of the acoustic power modulus. Within these frequency ranges, the quantity $E_{|\Pi|}$ reaches very large values exceeding even unity. The angles $\varphi_0^{(1)}$, $\varphi_0^{(2)}$ weakly influence the acoustic power phase. The relative changes E_{φ_Π} are much smaller than 0.01 for all the analyzed cases (Fig. 4b).

The influence of the difference between excitations' phases $\Delta\theta$ on the acoustic power has been analyzed. The relative changes in the modulus $E_{|\Pi|}$ and phase E_{φ_Π} of the acoustic power have been shown in Fig. 5 as functions of the circular frequency. The different values of the parameter $\Delta\theta$ have been discussed. Figure 5 shows that the difference between excitations' phases significantly influences the value of the acoustic power modulus (Fig. 5a). The relative changes $E_{|\Pi|}$ are greatest within the circular frequency range containing the first two maxima of the acoustic power modulus. They can reach even 50%. The influence of the parameter $\Delta\theta$ on the acoustic power modulus is weakest for the
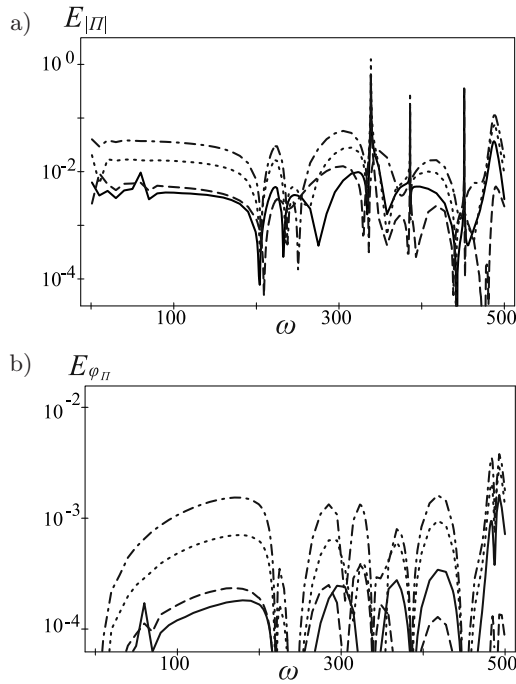
a) $E_{|\Pi|}$

b) E_{φ_Π}

Fig. 4. Relative changes in: a) the acoustic power modulus, b) the acoustic power phase, obtained according to Eqs. (39). Lines: solid – $\overline{\varphi}_0^{(1)} = \overline{\varphi}_0^{(2)} = 0$, dashed – $\overline{\varphi}_0^{(1)} = \overline{\varphi}_0^{(2)} = \pi/2$, dashed-dotted – $\overline{\varphi}_0^{(1)} = \overline{\varphi}_0^{(2)} = 5\pi/4$, dotted – $\overline{\varphi}_0^{(1)} = \overline{\varphi}_0^{(2)} = 7\pi/4$. The values of the other parameters of the vibroacoustic system have been assumed based on Table 1.

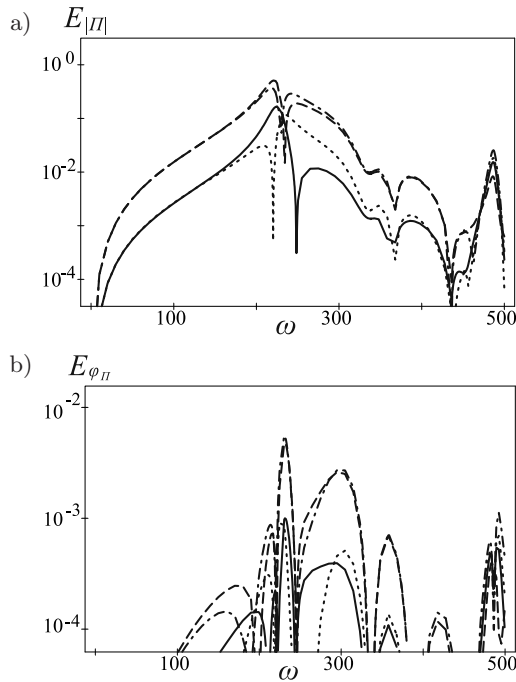a) $E_{|\Pi|}$

b) E_{φ_Π}

Fig. 5. Relative changes in: a) the acoustic power modulus, b) the acoustic power phase, obtained according to Eqs. (39). Lines: solid – $\Delta\theta = \pi/4$, dashed – $\Delta\theta = 3\pi/4$, dashed-dotted – $\Delta\theta = 5\pi/4$, dotted – $\Delta\theta = 7\pi/4$. The values of the other parameters of the vibroacoustic system have been assumed based on Table 1.

circular frequencies $\omega < 80$ rad/s, where $E_{|\Pi|} < 0.01$. The difference between excitations' phases does not significantly influence the acoustic power phase. Figure 5b shows that the quantity E_{φ_Π} is much smaller than 0.01 for all the analyzed cases. Moreover, the relative changes E_{φ_Π} are much smaller than 0.001 for the circular frequencies $\omega < 200$ rad/s. This means that the influence of the parameter $\Delta\theta$ on the acoustic power phase can be neglected for low circular frequencies.

The acoustic attenuation influences significantly the value of the modulus and phase of the acoustic power. Moreover, this factor changes the locations of maxima of the acoustic power modulus in the circular frequency scale. It has been assumed that $T_1 = C_M^{(1)^2} \sigma_1$, $T_2 = C_M^{(2)^2} \sigma_2$, where $C_M^{(1)} = 170$ m/s, $C_M^{(2)} = 100$ m/s. This assumption means that the membranes' eigenfrequencies are fixed. Their values are shown in Table 2. Changing the surface densities of the membranes influences only the values of the parameters ε_1, ε_2 describing the acoustic attenuation. It allows to determine the changes in locations of the maxima of the acoustic power modulus caused only by the acoustic attenuation. The normalized modulus and phase cosine of the acoustic power have been presented in Fig. 6 as functions of the circular frequency.
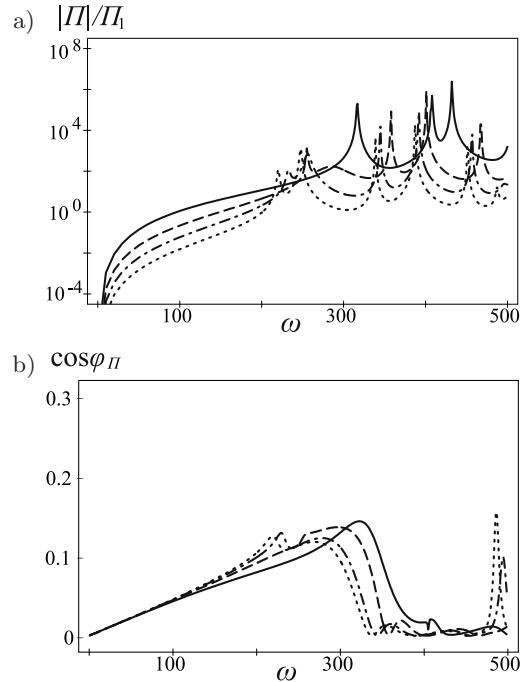
a) $|\Pi|/\Pi_1$

b) $\cos\varphi_\Pi$

Fig. 6. Normalized acoustic power: a) the modulus, b) the phase cosine. Lines: solid – $\sigma_1 = \sigma_2 = 0.2\,kg/m^2$, dashed – $\sigma_1 = \sigma_2 = 0.5$ kg/m^2, dashed-dotted – $\sigma_1 = \sigma_2 = 1$ kg/m^2, dotted – $\sigma_1 = \sigma_2 = 2$ kg/m^2. It has been assumed that $T_1 = C_M^{(1)^2} \sigma_1$, $T_2 = C_M^{(2)^2} \sigma_2$ where $C_M^{(1)} = 170$ m/s, $C_M^{(2)} = 100$ m/s. The values of the other parameters of the vibroacoustic system have been assumed based on Table 1.

The different surface densities of membranes $\sigma_1 = \sigma_2$ have been analyzed. The normalized value of the acoustic power modulus has been assumed as $\Pi_1 = c\rho_0 F_1^2 / \left(\omega_{0,1}^{(1)^2} \sigma_0^2 S_1 \right) \cong 142.3$ mW, where $\sigma_0 = 1$ kg/m². It enables a comparison of the values of the acoustic power modulus obtained for a different surface density σ_1. Based on Fig. 6a, it can be concluded that the maxima of the acoustic power modulus shift towards the high frequency when the acoustic attenuation grows. The parameters ε_1, ε_2 significantly influence the acoustic power phase only for the circular frequencies $\omega > 100$ rad/s (Fig. 6b).

7. Conclusions

The performed numerical analysis of the considered problem allows to conclude that the distance between the excited source point and its central point in the case of both membranes influences significantly the value of the acoustic power modulus. The acoustic power phase depends noticeably on this distance only for high frequencies. The radiated acoustic wave directly by the sound sources and the waves reflected from the transverse baffles cause excitation of the asymmetric vibrating modes of both membranes. It is particularly evident when both external excitations are symmetric. Angular locations of excited points influence weakly the acoustic power. The presented numerical analysis also shows that the difference between excitations phases has a significant influence on the value of the acoustic power modulus. The influence is greatest within the circular frequency range containing two lowest maxima of the acoustic power modulus. It has been concluded that the maxima of the acoustic power modulus shift towards the high frequency when the acoustic attenuation increases.

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