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Trombone Transfer Functions: Comparison Between Frequency-Swept Sine Wave and Human Performer Input

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Source/filter models have frequently been used to model sound production of the vocal apparatus and musical instruments. Beginning in 1968, in an effort to measure the transfer function (i.e., transmission response or filter characteristic) of a trombone while being played by expert musicians, sound pressure signals from the mouthpiece and the trombone bell output were recorded in an anechoic room and then subjected to harmonic spectrum analysis. Output/input ratios of the signals' harmonic amplitudes plotted vs. harmonic frequency then became points on the trombone's transfer function. The first such recordings were made on analog 1/4 inch stereo magnetic tape. In 2000 digital recordings of trombone mouthpiece and anechoic output signals were made that provide a more accurate measurement of the trombone filter characteristic. Results show that the filter is a high-pass type with a cutoff frequency around 1000 Hz. Whereas the characteristic below cutoff is quite stable, above cutoff it is extremely variable, depending on level. In addition, measurements made using a swept-sine-wave system in 1972 verified the high-pass behavior, but they also showed a series of resonances whose minima correspond to the harmonic frequencies which occur under performance conditions. For frequencies below cutoff the two types of measurements corresponded well, but above cutoff there was a considerable difference. The general effect is that output harmonics above cutoff are greater than would be expected from linear filter theory, and this effect becomes stronger as input pressure increases. In the 1990s and early 2000s this nonlinear effect was verified by theory and measurements which showed that nonlinear propagation takes place in the trombone, causing a wave steepening effect at high amplitudes, thus increasing the relative strengths of the upper harmonics.

Keywords: brass acoustics, trombone, transfer function, nonlinear propagation, input impedance.

1. Introduction

While publications giving measurements of input impedance vs. frequency for wind instruments have been quite numerous (e.g., Backus, 1974, 1976; Caussé *et al.*, 1984), there have been relatively few that have shown output/mouthpiece pressure transfer functions (also known as transmission responses). Early exceptions were by Benade (1976) and Elliott *et al.* (1982). The latter paper presented measurements of both input impedance and transfer functions of frequency for a trumpet and a trombone. They concluded that, despite the exceedingly high pressure levels that can occur in the trombone mouthpiece (greater than 165 dB SPL), "the magnitudes of the (nonlinear) effects are small compared to the overall, linear behavior of the instrument under normal playing conditions". This had already been discussed by Backus and Hundley (1971), who had also concluded that brass systems are linear and that harmonics are entirely generated in the mouthpiece due to a nonlinear variation of the slit resistance of the vibrating lips. However, the measurements I made with real players were indicating otherwise (Beauchamp, 1969, 1980, 1988, 1996).

In the 1990s a few papers began to appear confirming that harmonic pressure amplitudes at the output of a trombone are not strictly caused by linear filtering of a mouthpiece input signal. Hirschberg *et al.* (1996) showed that waveforms in the trombone become progressively more *steepened* as they move down the trombone's cylindrical tubing, and this effect becomes more pronounced as the acoustic pressure in the mouthpiece increases. Msallem *et al.* (1997, 2000) constructed a model for the trombone which included viscothermal losses and effects of nonlinear propagation and could be used for sound synthesis. Thompson and Strong (2001) showed that, given a measured mouthpiece pressure spectrum, a distributed finite-element method of simulation based on a trombone bore profile could be used to predict the trombone's output spectrum at both low and high amplitudes. 152 cylinders were used to approximate the trombone bore. Using the acoustical properties of a generalized cylinder and boundary conditions at the mouthpiece and bell, the output of each cylinder was computed for both the linear and nonlinear cases and accumulated to calculate the entire transfer response. While the difference between linear and nonlinear predictions was modest for a soft tone, it was dramatic for a loud tone of the same pitch. Moreover, the error between measured and predicted output spectra was much smaller for the nonlinear case over a range of pitches and intensities.

2. Early measurements

2.1. Transfer function measurement under performance conditions

In general, the transfer function $T(f)$ of a wind instrument, treated as a linear two-port system, is a frequency-domain measurement of the ratio between the instrument's acoustic pressure output $P_{\text{out}}(f)$ and its mouthpiece pressure input $P_{\text{in}}(f)$, given by

$$T(f) = P_{\text{out}}(f)/P_{\text{in}}(f). \tag{1}$$

In decibels this would be written as

$$\begin{aligned} T_{\text{dB}}(f) &= 20\log_{10}(P_{\text{out}}(f)/P_{\text{in}}(f)) \\ &= 20\log_{10}(P_{\text{out}}(f)) - 20\log_{10}(P_{\text{in}}(f)). \end{aligned} \tag{2}$$

In 1968 as part of a project to determine a source/filter model for a trombone (Beauchamp, 1969), I attempted to measure the pressure transfer function under performance conditions using four different student trombonists playing a Bach Model 36 Stradivarius trombone with a 6½-AL mouthpiece in an anechoic chamber at the University of Illinois at Urbana-Champaign (UIUC). Compared to other methods, measuring the transfer function in this way was actually very convenient because it only required two microphones, one in the mouthpiece and one at the output, and no other special equipment. The trombone was mounted on a fixed stand in the chamber. The mouthpiece pressure was monitored using a B & K 4136 ¼″ (0.635 cm) condenser microphone mounted flush with the inside of the mouthpiece backbore. The output pressure was measured by a B & K 1613 sound level meter with a 1″ (2.54 cm) omnidirectional capsule 2 m on axis from the trombone bell, making it easy to calibrate the output sound pressure level. Mouthpiece and output pressure signals for tones performed at a series of closed-position pitches (B^b_2, F_3, B^b_3, D_4, and F_4) and dynamics (*pp*, *mf*, and *ff*) were recorded on separate tracks of a stereo analog tape. Then, the transfer functions were estimated by taking ratios between the amplitudes of the corresponding harmonics of the output and input, as measured by a computer program. Thus, for each harmonic h and fundamental frequency f_1, the equation

$$T(hf_1) = P_{\text{out}}(hf_1)/P_{\text{in}}(hf_1) \tag{3}$$

gave a discrete point on the continuous transfer function $T(f)$ at frequency hf_1. Assuming a linear system and constant f_1, $T(f)$ should have been independent of dynamic level. However, measurements demonstrated a dependence of the transfer function on dynamic, i.e., the intensity of the tones. Figure 1 shows curves similar to those published in (Beauchamp, 1980) for B^b_2 tones performed at *pp*, *mf*, and *ff*. Further details on the measurement method are given in (Beauchamp, 1988).

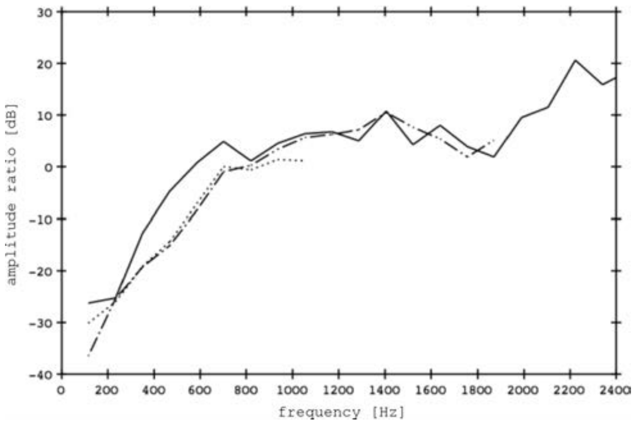

Fig. 1. Trombone transfer functions made under performance conditions for B^b_2 ($f_1 \approx 117$ Hz) tones played at dynamics *pp* (dotted curve), *mf* (dash-dot curve), and *ff* (solid curve). Curves are discontinued when input harmonic falls below a noise floor.

2.2. Swept sine measurements

In 1972 I made swept-sine measurements of $T(f)$ on a Holten tenor trombone in the UIUC anechoic chamber with an output microphone positioned 2 m from the bell, using a setup similar to that used in 1968, except that a University Sound 75 w driver supplied the input pressure. The transfer-function results in terms of decibels vs. linear frequency are shown in Fig. 2. The basic decibel level-vs.-frequency curve is a high-pass type with approximately uniform-spaced maxima and minima superimposed on it. The degree of difference between the maxima and minima decreases with frequency until roughly $f_{\text{cut}} = 1000$ Hz, the approximate bell cutoff frequency. For frequencies above f_{cut}

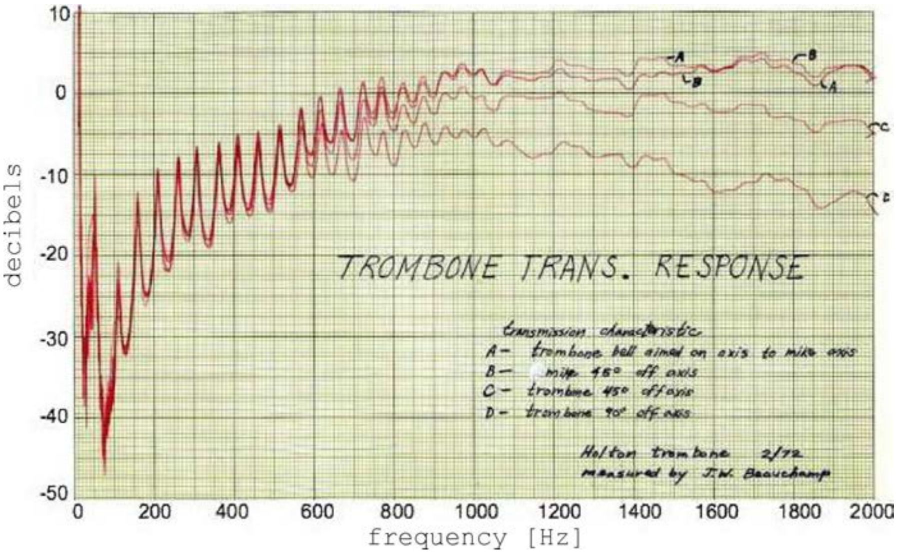

Fig. 2. Tenor trombone pressure transfer function (closed position). Four different cases for trombone output pressure: A – on-axis; B – on-axis with output microphone rotated 45°; C – positioned 45° off-axis; D – positioned 90° off-axis. Output microphone positioned 2 m from bell.

the response is close to flat when the microphone is on axis but decreases gradually when the microphone is off axis by 45° or 90°. The minima frequencies correspond closely to the playing frequencies of the trombone. The behavior for frequencies below cutoff can be explained as follows: At low frequencies the open end appears to be approximately closed and thus there are strong standing waves, reinforcing large fluctuations in pressure and velocity at the mouthpiece end, which also acts as a closed end. In this case, both the input impedance and transfer function vary widely with respect to frequency, and only a small amount of energy is radiated. However, this situation changes gradually as frequency rises. At the bell the reflection coefficient gets smaller and smaller, allowing more energy to be radiated, causing the transfer function to rise. This accounts for the high pass characteristic for $f < f_{\text{cut}}$.

In 1973 I visited Professor Arthur Benade at Case Western Reserve University in Cleveland, Ohio, and together we performed a simultaneous measurement of the transfer and the input impedance functions for a Conn 80A B^b cornet, using a swept-sine/chart recorder method. In this case, the microphone was positioned at the bell opening, and the entire assembly was enclosed in a box with absorbing material lining it. The graphs shown in Fig. 3 clearly demonstrate that the local minima of the transfer function curve correspond to the local maxima of the input impedance curve (given by $Z_{\text{in}}(f) = P_{\text{in}}(f)/U_{\text{in}}(f)$, where U_{in} is the mouthpiece particle velocity). It is well known that the performance fundamental frequencies of a wind instrument correspond to the local maxima of the input impedance magnitude, and therefore these frequencies also correspond to the local minima of the transfer function.

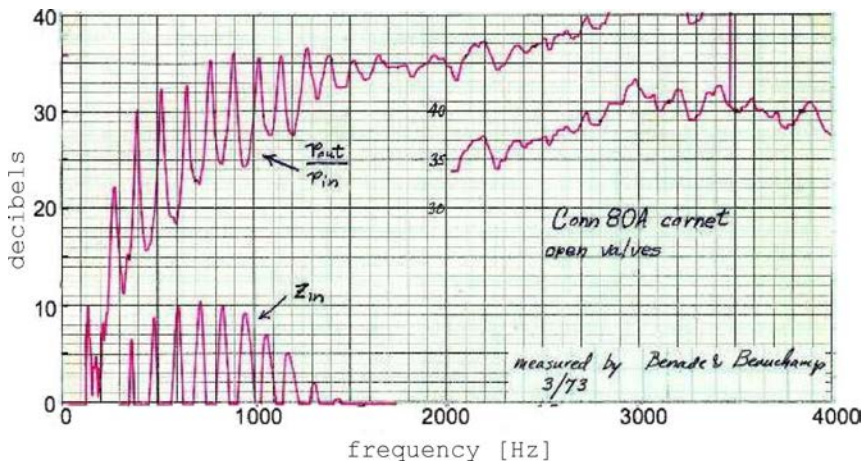

Fig. 3. Simultaneous measurement of the transfer function (upper curve) and the input impedance magnitude (lower curve) for a B^b cornet (open valves). Output microphone positioned at the bell opening.

2.3. Theoretical relationship between T(f) and $Z_{in}(f)$

That the peaks and valleys of the transfer function and the input impedance are interlaced can be seen by assuming a lossless system and equating input power and output power. For the input in the frequency domain we have

$$W_{\text{in}}(f) = \text{Re}\left\{P_{\text{in}}(f)U_{\text{in}}(f)\right\} = \text{Re}\left\{P_{\text{in}}^2(f)/Z_{\text{in}}(f)\right\} = P_{\text{in}}^2(f)\text{Re}\left\{1/Z_{\text{in}}(f)\right\}, \tag{4}$$

where $U_{\text{in}}(f)$ is the input particle velocity in the frequency domain. Note that without loss of generality, under the assumption that $P_{\text{in}}(f)$ is real, we can move the real part inside the brackets in the last term to affect only the $1/Z_{\text{in}}(f)$ term.

The output power is the average output intensity I_{ave} on a sphere of radius r at which this intensity is measured. Thus, the total output power is

$$W_{\text{out}}(f) = 4\pi r^2 I_{\text{ave}}(f) = 4\pi r^2 \frac{P_{\text{out}}^2(f)}{D(f)Z_o}, \tag{5}$$

where $D(f)$ is a frequency-dependent directivity index, Z_o is the characteristic impedance of air (415 rayls), and P_{out} is the on-axis acoustic pressure output. Equating the input and output powers and solving for $T(f) = P_{\text{out}}/P_{\text{in}}$ leads to

$$T(f) = \left|\frac{P_{\text{out}}(f)}{P_{\text{in}}(f)}\right| = \sqrt{\frac{Z_o D(f)\text{Re}\{1/Z_{\text{in}}(f)\}}{4\pi r^2}} \doteq \frac{5.75}{r}\sqrt{D(f)\text{Re}\{1/Z_{\text{in}}(f)\}}, \tag{6}$$

which demonstrates an important aspect of Fig. 3, namely that local maxima and minima of the transfer function and the input impedance function are interchanged. This is most important for frequencies below cutoff, i.e., $f < f_{\text{cut}}$, where $f_{\text{cut}} \approx 200/d$ and d is the bell diameter (Fletcher, Rossing, 1991). For a trombone, the bell diameter is approximately 0.2 m, resulting in a cutoff frequency of about 1000 Hz. We don't have to be concerned about $D(f)$ for $f < f_{\text{cut}}$ because it is approximately unity (Kinsler *et al.*, 1982). Above cutoff, however, assuming a plane circular piston model, D increases proportional to f^2 (actually, $D \cong ((\pi/c)df)^2 = (.0916df)^2$). Meanwhile, if $\text{Re}\{1/Z_{\text{in}}(f)\}$ converges to a relatively constant $1/Z_o$ for $f \gg f_{\text{cut}}$, from Eq. (6) we would expect

$$T(f) \doteq 0.0026\frac{d}{r}f, \qquad f \gg f_{\text{cut}}. \tag{7}$$

However, Eqs. (6) and (7) ignore frequency-dependent thermoviscous losses which occur at the interior boundary walls of the trombone. Losses can be accounted for by defining power efficiency (Elliott *et al.*, 1982) as the ratio of output power to input power:

$$E(f) = \frac{W_{\text{out}}(f)}{W_{\text{in}}(f)} = \frac{4\pi r^2}{D(f)Z_o}\frac{T^2(f)}{\text{Re}\{1/Z_{\text{in}}(f)\}}, \tag{8}$$

leading to

$$T(f) = \sqrt{\frac{E(f)Z_o D(f)\text{Re}\{1/Z_{\text{in}}(f)\}}{4\pi r^2}}. \tag{9}$$

Note that Eq. (9) is the same as Eq. (6) except for the multiplication by $\sqrt{E(f)}$. By measurement, Elliott *et al.* showed $E(f)$ to be a high-pass function which reaches an approximately constant value for $f > 700$ Hz. This, unfortunately, does not help resolve the discrepancy between the on-axis swept-sine measurement (Fig. 2) and the prediction of Eq. (7) for frequencies above cutoff.

3. Recent performance-condition transfer function measurements

Comparisons of the early swept-sine and the performance-condition transfer function results based on the 1968 and 1972 measurements were presented at two talks (Beauchamp, 1988, 1996). However, there was always some doubt about the accuracy of the performance-condition data because analog tape has a limited signal-to-noise ratio (approx. 55 dB) as well as significant distortion, so that accurate calculation of the FFT ratio between output and input when the upper harmonics of the input are weak (especially for the *pp* case) could have been compromised.

Therefore, in 2000 I made new direct-to-digital stereo recordings of the mouthpiece and output pressure of a trombone played by Jay Bulen (professor of trombone at Truman State University, Missouri) in the University of Iowa anechoic chamber (the UIUC chamber was unfortunately decommissioned in the early 1980s). This allowed much more accurate calculations of performance-condition transfer functions. Figure 4 shows a block diagram of the measurement system.

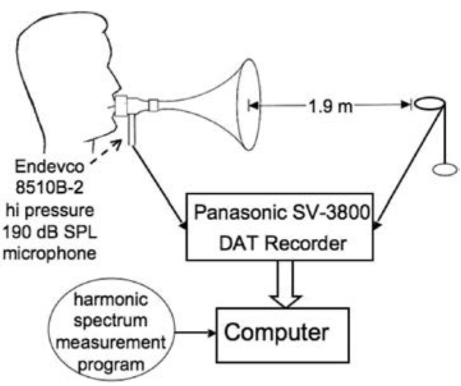

Fig. 4. System for measurement of trombone transfer functions under performance conditions. A stereo file is stored on the computer with the mouthpiece pressure signal $p_{\text{in}}(t)$ as the left channel and output signal $p_{\text{out}}(t)$ as the right channel.

The Endevco 8510B-2 piezoelectric microphone was accurate to 190 dB SPL and was much sturdier than the B & K condenser microphone used in 1968, so it was possible to position the mouthpiece microphone directly in the mouthpiece cup, flush with the inside surface. The output microphone, an omni-directional Neumann KM83, was positioned on axis at 1.9 m from the bell. The microphone outputs were recorded on digital audio tape (DAT) and subsequently transferred as stereo files to a computer for spectral analysis.

3.1. Calculation of T(f) under performance conditions

The trombone mouthpiece (input) and on-axis far-field (output) signals were then copied to separate monaural files, and a "phase vocoder" program (Beauchamp, 2007) was used to perform harmonic analysis on the signals. Using a different program, the amplitudes of $p_{\mathrm{in}}(f_h)$ and $p_{\mathrm{out}}(f_h)$, where $f_h = hf_1$ is the harmonic frequency and f_1 = fundamental frequency, were averaged over the durations of the sounds before computing the transfer function ratios $T_{\mathrm{dB}}(f_h) = 20\log_{10}(p_{\mathrm{out}}(f_h)/p_{\mathrm{in}}(f_h))$.

Figures 5–7 show graphs of $p_{\mathrm{in}}(f_h)$, $p_{\mathrm{out}}(f_h)$ (in decibels) vs. frequency, where $f_1 = 58$ Hz for the case pitch B_1^b, for dynamics *pp*, *mf*, and *ff*, respectively.

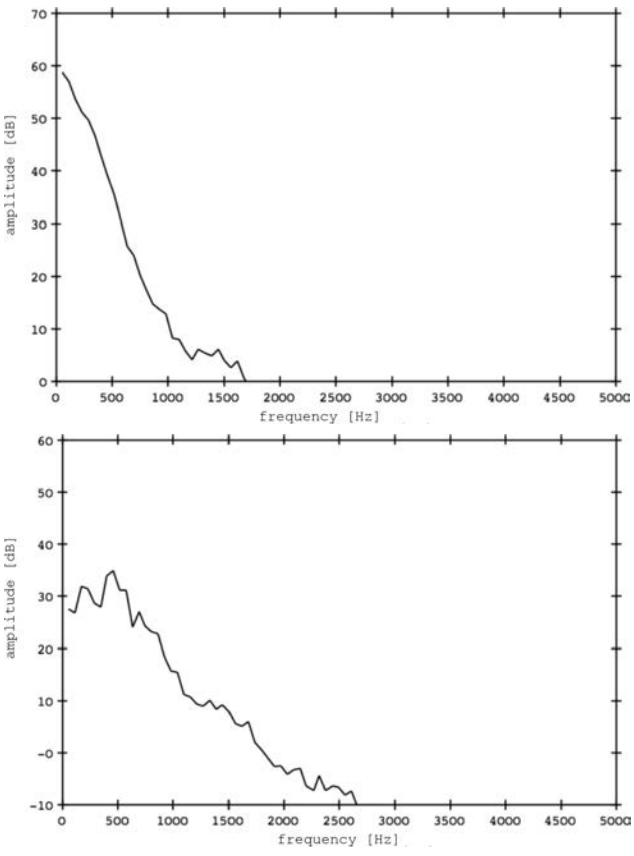

Fig. 5. Trombone mouthpiece spectrum (top) and output spectrum (bottom) for pitch B_1^b and dynamic *pp*.

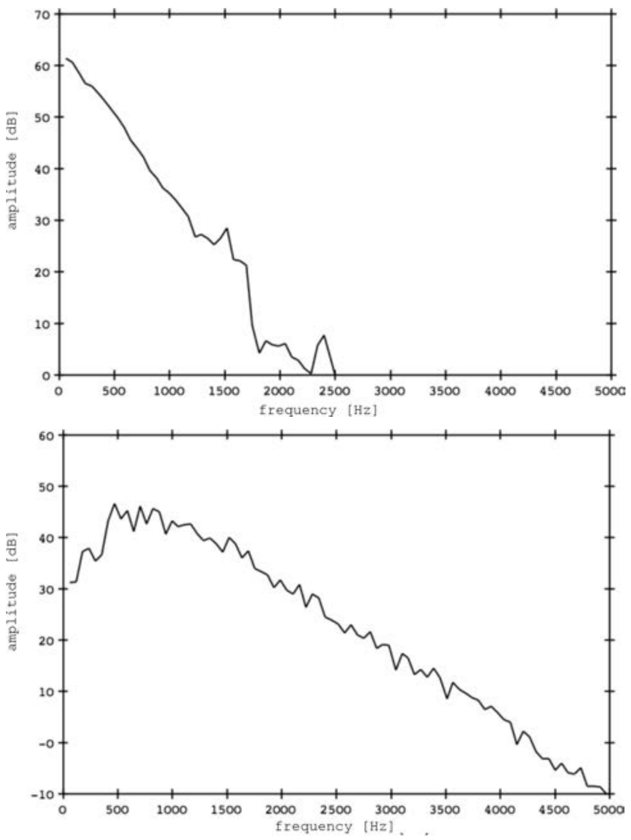

Fig. 6. Trombone mouthpiece spectrum (top) and output spectrum (bottom) for pitch B_1^b and dynamic *mf*.

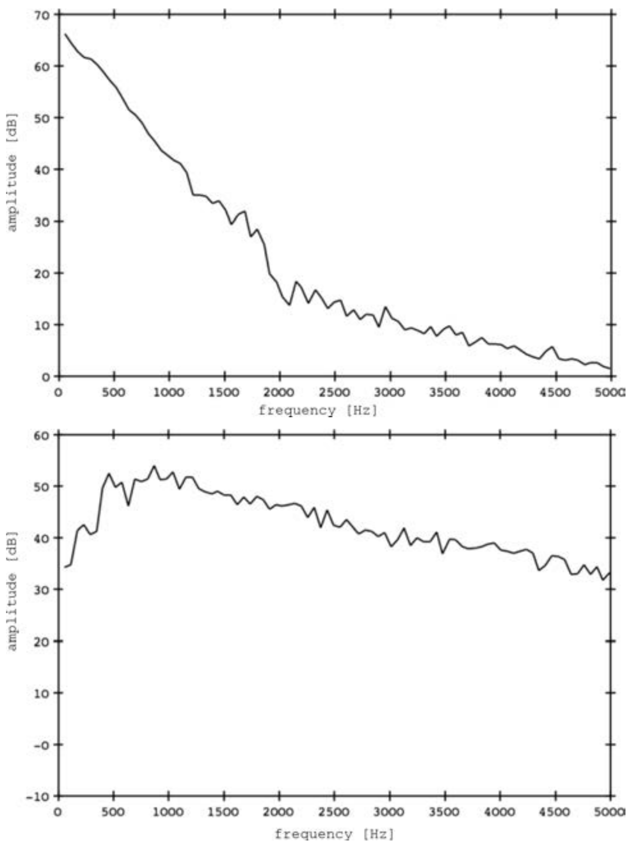

Fig. 7. Trombone mouthpiece spectrum (top) and output spectrum (bottom) for pitch B_1^b and dynamic *ff*.

Figures 8–10 show a comparison of computed transfer functions for the *pp*, *mf*, and *ff* cases for pitches B^b_1, B^b_2, F_3, B^b_3, D_4, and F_4 (corresponding approximately to $f_1 = 58.3$, 116.5, 174.6, 233.1, 293.7, and 349.2 Hz). The maximum frequency for each graph is limited by excluding points where the mouthpiece spectra fall below 0 dB, which was judged to be the level of the noise floor. (Note that although the $T(f)$ curves are shown continuous, this does not imply anything about the response characteristics between the harmonics, which are only revealed by the swept-sine measurements.) Two things are obvious from the graphs: First, the transfer functions are nearly identical for $f < 1000$ Hz. The average standard deviation of the curves over this range is about 0.6 dB for the seven natural closed-position tones between B^b_1 and B^b_4. Second, for $f > 1000$ Hz the curves are quite different. In general, $T(f)$ becomes greater as the dynamic level increases. Figures 9 and 10 show that at $f = 5000$ Hz the separation between the *mf* and *ff* cases is greater than 20 dB. At 2500 Hz the *mf* transfer functions are more than 15 dB greater than the *pp* versions.

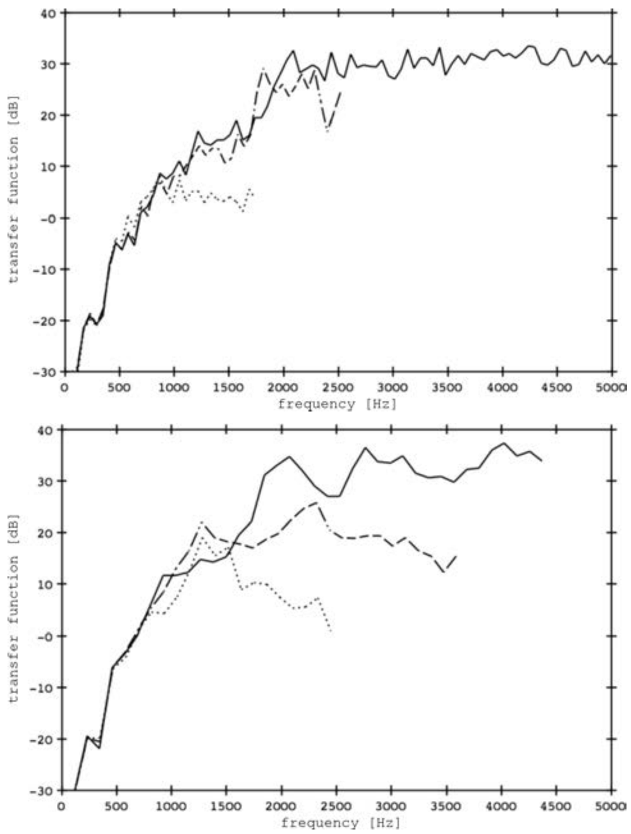

Fig. 8. Trombone transfer functions compared: *pp* (dotted lower), *mf* (dash-dot middle), *ff* (solid upper) for B^b_1 (top), B^b_2 (bottom).

The invariances of the transfer functions with respect to dynamic for $f < 1000$ Hz seem to indicate that the wave steepening effect is small for Fourier components with frequencies below cutoff. It is be-

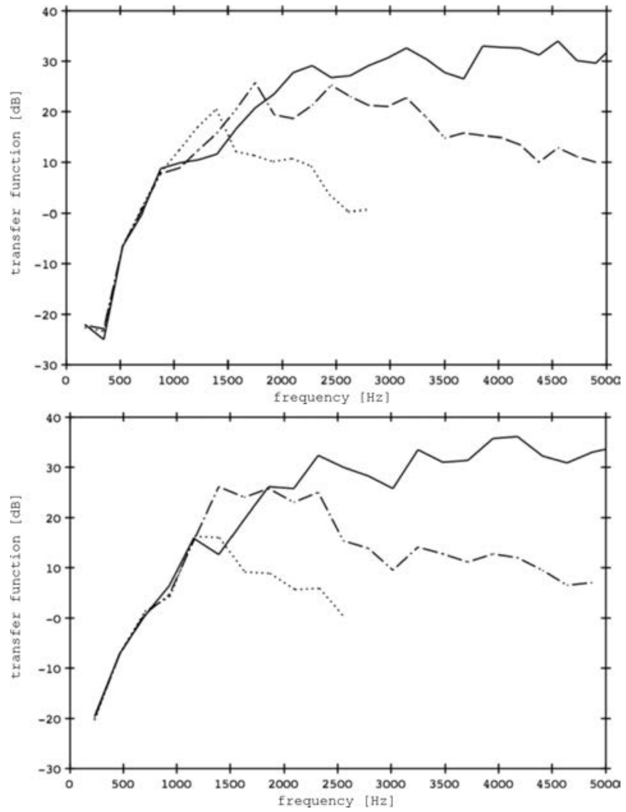

Fig. 9. Trombone transfer functions compared: *pp* (dotted lower), *mf* (dash-dot middle), *ff* (solid upper) for F_3 (top), B^b_3 (bottom).

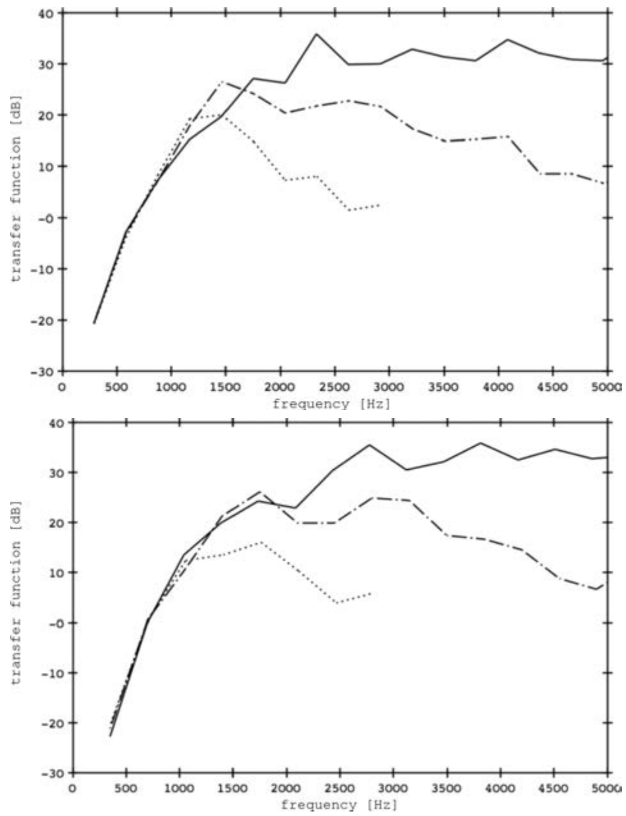

Fig. 10. Trombone transfer functions compared: *pp* (dotted lower), *mf* (dash-dot middle), *ff* (solid upper) for D_4 (top), F_4 (bottom).

Table 1. For each harmonic of a B_1^b (58 Hz) *mf* tone, decibel amplitudes (on a relative scale) for input pressure, output pressure, and output-minus-input (in dB) corresponding to Figs. 6 and 8 (top), as well as values sampled at the harmonic frequencies from Fig. 2 (upper curve) with 3 added, are given. The average magnitude error is 2.4 dB.

Harmonic	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
Frequency [Hz]	58	116	174	232	290	348	406	464	522	580	638	696	754	812	870	928
p_{in} perf. cond. [dB]	61	60	58	57	56	54	53	51	49	48	45	44	42	39	38	35
p_{out} perf. cond. [dB]	31	31	37	38	35	36	43	46	43	45	41	46	42	45	45	40
$T(f)$ perf. conf. [dB]	−30	−29	−21	−19	−21	−18	−10	−5	−6	−3	−4	2	0	6	7	5
$T(f)+$ 3 swept sine [dB]	–	−25	−22	−18	−14	−14	−7	−4	−5	−4	−3	−3	0	4	4	5
Error	–	−4	+1	−1	−7	−3	2	5	−1	1	−1	5	0	2	3	0

lieved that most wave steepening occurs within the cylindrical portions of trombones, which are typically 2 m long (Hirschberg *et al.*, 1996; Smyth, Scott, 2011). Peak-to-peak mouthpiece waveform amplitudes (Δp_m) can be as much as 2×10^4 Pa for fortissimo tones (Hirschberg *et al.*, 1996; Thompson, Strong, 2001). It can be shown (Cooper, Abel, 2010) that the difference between peak and trough time dilations caused by finite-amplitude plane waves in a cylindrical tube is given by

$$\Delta t = \frac{2L}{c_0}\left\{\frac{\beta\Delta p_m/(2P_0)}{1-\left(\beta\Delta p_m/(2P_0)\right)^2}\right\}, \qquad (10)$$

where in our case $L = 2$ m, $c_0 = 343$ m/s (speed of sound), $\beta = 1.2$ (coefficient of nonlinearity), $\Delta p_m = 2 \times 10^4$ Pa, and $P_0 = 10^5$ (atmospheric pressure). This gives $\Delta t = .00142$ s. While this time difference is significantly greater than the period corresponding to 1000 Hz, it is considerably smaller than the fundamental periods of trombone tones. Moreover, mouthpiece waveforms are heavily dominated by the lower harmonics, which are most important for time dilation. These observations constitute a clue for why the transfer functions behave in a linear fashion for $f < 1000$ Hz, but they lack the rigor needed for a proof of the reason for this effect.

3.2. Agreement between swept-sine and performance-condition measurements for $f < 1000$ Hz

The question arises: If we take transfer function values calculated from performance-condition measurements for the trombone and compare them to values taken from the swept-sine data, how well do they agree? Since the *pp*, *mf*, and *ff* performance-condition transfer functions agree for $f < 1000$ Hz, it seems that the propagation system behaves in a linear fashion for that frequency region and that therefore the performance-condition and swept-sine-wave results should coincide. Table 1 shows a data alignment between the B_1^b *mf* tone's performance-condition measurements of Figs. 6 and 8 (top) and the swept-sine measurement of Fig. 2 for harmonics below 1000 Hz. The results are surprisingly close considering that the trombones were different – a Holton TR602 tenor trombone with a 6½ AL mouthpiece was used for the swept-sine measurement, whereas a Bach 42 B0 tenor trombone with a Stork 5S mouthpiece was used for the most recent performance-condition measurement.

4. Conclusions

Swept-sine measurements of trombone (and cornet) pressure transfer functions show high-pass characteristics with superimposed resonance minima corresponding to harmonic performance frequencies. While transfer function measurements under performance conditions for harmonics below the trombone cutoff frequency (approx. 1000 Hz) show very little dependence on performance dynamic and closely follow a swept-sine measurement sampled at harmonic frequencies, they deviate strongly above cutoff. For example, performance-condition transfer functions for harmonics above cutoff are typically 20 dB stronger for tones played *ff* than for tones played *mf*. Assuming a lossless linear system, a theoretical derivation based on power conservation indicates that the transfer function should be proportional to the square root of the real part of the inverted input impedance function. Below cutoff, this relationship has been qualitatively verified by swept-sine measurement. However, the detailed relationship depends on the exact nature of the losses and the radiation directivity index, which have not yet been determined.

Acknowledgment

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