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Recognizing the Sequences of Code Manipulated Short LFM Signals

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Noise-like binary sequences combined with signals with linear frequency modulation might be successfully used to increase the reliability of the recognition of both probe and communication signals in the presence of natural and artificial interference. To identify such formed sequences the usage of the two-step matched filtering was suggested and the probabilistic model of the recognition of noise-like code sequences transferred by LFM signals was developed.

Keywords: sonar, LFM signal, matched filtering.

1. Introduction

In this study the signals recognition is understood in terms of distinguishing them from the noise or detecting them in the presence of other signals. The recognition of a signal (e.g. the echo of a probe signal) incoming at the input of the receiver is based on finding the correlation between this signal and the model of the probe signal.

Noise-like binary sequences are widely used in telecommunication, measuring, echo location and navigation systems. The collection of these sequences includes: Barker's, Willard's Neuman–Hoffman's and Alexis codes, the sequences of maximum length, golden codes, Kasami sequences, etc. (Davies, 1971; Glover, Grant, 1998; Howard, 2002; Luke, 2001; Proakis, 2001). This study is devoted to the recognition of the pseudo-noise binary sequences formed on the basis of the signals with linear frequency modulation (LFM signals, chirp signals). In digital compression, the detection of broadband signals and the identification of binary codes one can successfully use matched filtering realized in the time domain. Zeros and ones in a noise-like code sequence might be represented by *up-chirp* signals (the instantaneous frequency changes from low to high) and *down-chirp* signals (the instantaneous frequency changes from high to low). To identify such formed sequences the two-step matched filtering has to be used. The first filtering is related to the recognition of every single chirp signal forming a sequence, which results in obtaining a binary sequence, which is subjected to the second matched filtering.

In order to implement digital matched filtering in the time domain a chirp signal is presented as time series $\{x_n\}$ with the sampling frequency $f_s = 1/T_s \geq 2f_2$, so the number of samples equals N, where $N = \mathrm{ENT}(Tf_s)$, ENT – the integer part of a number. Each sample value of a chirp signal is determined from the formula:

$$x_r = x(rT_s) = A\cos\left[2\pi\left(\frac{\Delta f}{2N}r + f_1\right)rT_s + \varphi_0\right], \quad (1)$$

where $a = \Delta f/\tau_i$, $\Delta f = f_2 - f_1$ is the deviation of the frequency, f_1 is the initial frequency, f_2 is the final frequency, τ_i is the duration of the chirp signal, φ_0 is the initial phase, $r = \overline{0, N-1}$.

The coefficients of the impulse response of a matched filter *IR* without a smoothing window is defined as the mirror reflexion of the input signal $\{x_n\}$, $n = \overline{1, N}$. In this case the coefficients of the impulse characteristics are determined from the formula:

$$h_n = x_{N-n}, \quad (2)$$

where $n = \overline{1, N}$.

In order to decrease Gibb's oscillations, which occur during the matched filtering of chirp signals, the smoothing windows $\{w_n\}$ are used. In the time domain the application of windows on the impulse responses *IR* of the filter is done by the multiplication of the weight factors of a *IR* by corresponding weight factors of the smoothing window $\{h_n w_n\}$. In that case the algorithm of matched filtering with the use of a window is:

$$y_n = \sum_{m=0}^{N-1} x_{n-m} h_m w_m, \tag{3}$$

where y_n – n-th convolution result, $\{w_n\}$ – the collection of smoothing window coefficients. For the implementation of a specialized processor it is advisable to present formula (3) in a matrix form.

$$y_n = \mathbf{X} \cdot \mathbf{H}_w, \tag{4}$$

where

$$\mathbf{X} = [x_n \dots x_{n-N+1}],$$

$$\mathbf{H}_w = \begin{bmatrix} h_0 w_0 \\ \vdots \\ \vdots \\ h_{N-1} w_{N-1} \end{bmatrix}.$$

The research results presented in the studies (Leszczyński, 2010; Pogribny, Leszczyński, 2008) demonstrate that the initial phase and frequency, as well as the sampling rate of both short LFM signals and matched filter impulse response have an important influence upon the compression of these signals within the time domain. In matched filtering in the time domain convolutions form the main lobe as well as side lobes. The main lobe is formed by the result of central convolution, whereas side lobes are formed with the remaining ones. The ratio of *PSR* (Peak-to-Side lobe Ratio) of the main lobe value to the maximum value of a side lobe, expressed in dB, is the measure of the degree of signal compression and significantly affects its recognition:

$$PSR = 20 \log \frac{y_c}{|y_m|}, \tag{5}$$

where y_c – the central convolution value, $|y_m|$ – the maximum absolute value of the convolution chosen among all convolutions without those forming the main lobe.

The convolution results form the set on the basis of the formula (3) and some of them might have negative values. It can also be noticed that the convolutions adopting negative values become the biggest after setting their absolute values and are usually located near the central convolution, becoming a part of and extending the main lobe, which worsens time resolution and *PSR*. Performing nonlinear operations on the convolutions results (replacing negative convolution values with zeros) increases the *PSR* coefficient and decreases the width of the main lobe, which significantly affects the recognition of the chirp signal. The parallel matched filters systems presented in the studies (Leszczyński, 2010; Pogribny, Leszczyński, 2011) allow one to increase the resolution of recognition to the borderline – only one sample in the main lobe with the location of its position within the actual sampling period. Concurrently, such parallel systems allow one to obtain a result after N periods (where N is the number of all signal samples), which is $log_2 N$ faster than in systems using "fast convolutions".

2. Forming code sequences

Forming code sequences of *up-chirp* and *down-chirp* signals brings to the situation in which an *up-chirp* signal is sent to the filter detecting *down-chirp* signals, or inversely. It causes the occurrence of the increased noise level of an output signal at the matched filters output in comparison with the recognition of the proper signal, when the noise level is lower as compared to the main lobe. It results from a high correlation between an *up-chirp* and a *down-chirp* signal. When a *down-chirp* signal enters the input of a filter responding to *up-chirp* signals at this filter output there is the increase of the level of side lobes in the absence of the main lobe. The matched filters systems presented in the studies (Leszczyński, 2010) determine the value of the *PSR* coefficient identifying two consecutive maxima within the time interval determined by the number $N$of time steps (clocks), where N is the number of the coefficients of matched filter impulse characteristics. Such an approach allows one to increase the reliability of identification of a chirp signal by using simultaneously two criteria – reaching by an output signal the appropriate threshold δ_y and minimal *PSR* value. To avoid the misinterpretation of the increased level of a noise signal at the output of a filter responding to *up-chirp* signals when a *down-chirp* signal is sent at its input an interval between signals, at least equal to the duration of the signal, has to be introduced. This imposes appropriate limitations on the presented method of coding zeros and ones of code sequences by *up-chirp* and *down-chirp* signals. The results of the simulation of matched filtering of *up-chirp* and *down-chirp* signal pairs as well as *up-chirp* signal pairs are presented in Figs. 1 and 2. As shown in Fig. 2 chirp signals of one type coming directly one after another at the input of a filter designed for their detection slightly increase the level of side lobes in comparison with the case when the distance between them equals the pulse duration.

The presentation of ones and zeros of code sequences using appropriate chirp signals should take into account the presence of identical symbols of a code sequence coming directly one after another. This paper proposes two ways of forming code sequences. The first way uses *up-chirp* and *down-chirp* signals, whereas the other one uses only chirp signals of one type (*up-chirp* or *down-chirp*).

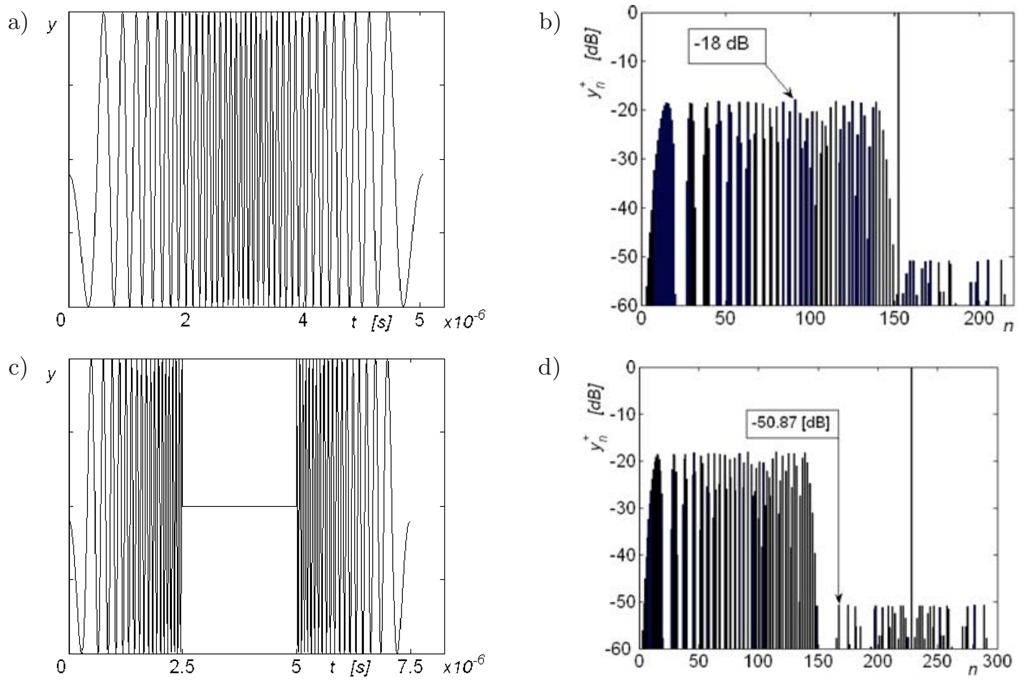

Fig. 1. The result of the matched filtering of a pair of *up-chirp* (0–15 MHz) and *down-chirp* (15 MHz – 0) signals coming at the input of a filter detecting *down-chirp*, the duration of both signals is identical and equals $T = 2.5$ μs. In the filtration process a rectangular window was used as well as non-linear operations. The signal initial phase and sampling frequency are optimal: a) the *up-chirp* signal and the *down-chirp* signal coming directly after it, b) the signal at the output of a matched filter detecting *down-chirp* signals at the input, which signal was provided (a), c) an *up-chirp* signal with an interval equal to the impulse duration and then a *down-chirp* signal, d) a signal at the output of a matched filter detecting a *down-chirp* signal at the input, which signal was provided (c).

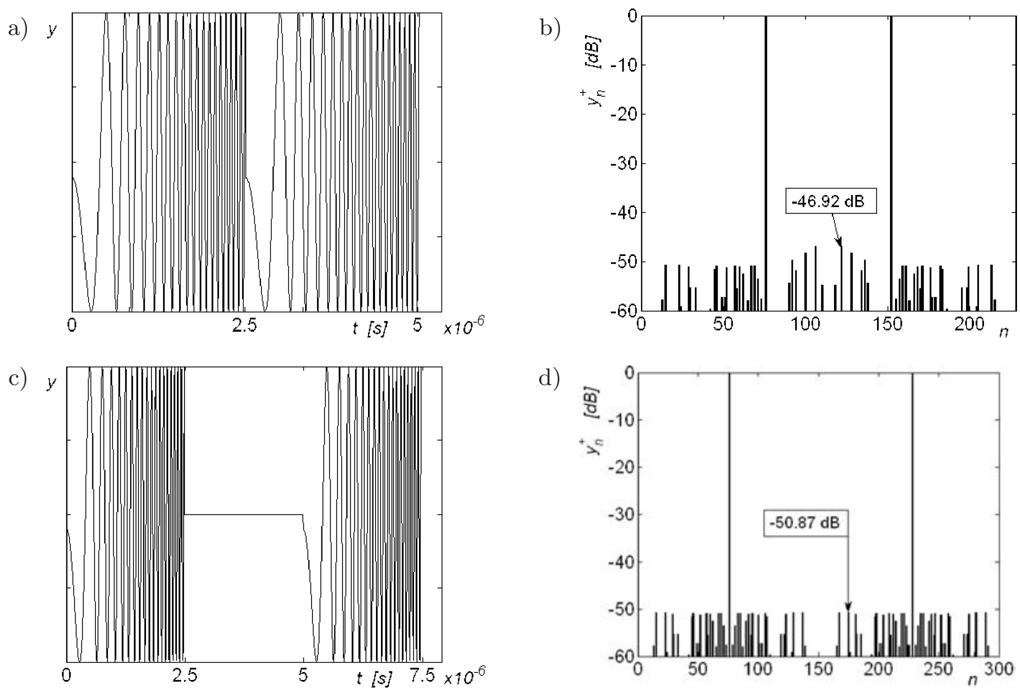

Fig. 2. The result of the matched filtering of a pair of *up-chirp* signals (0–15 MHz) sent at the input of a filter detecting *up-chirp* signals, the signal duration is $T = 2.5$ μs. In the filtration process a rectangular window was used as well as non-linear operations. The signal initial phase and sampling frequency are optimal: a) two *up-chirp* signals coming directly one after another, b) the signal at the output of a matched filter detecting *up-chirp* signals at the input, which signal was provided (a), c) two *up-chirp* signals with an interval equal to the impulse duration, d) a signal at the output of a matched filter detecting an *up-chirp* signal at the input, which signal was provided (c).

Let us consider the example of coding ones with *up-chirp* signals and zeros of a noise-like sequence 11010 with *down-chirp* signals.

Then the probing signal has the form of:

upup_down_up_down

where **up** is an up-chirp signal, **down** is a down-chirp signal, _ is an interval between the signals.

When using chirp signals of one type (e.g. *up-chirp*) the coding process is as follows:

- one is represented by two consecutive *up-chirp* signals (without an interval between the signals),
- zero is represented by a single *up-chirp* signal.

The probing signal formed on the basis of the same noise-like sequence, where only an *up-chirp* signal is used to code ones and zeros might be presented as follows:

upup_upup_up_upup_up

3. Identifying binary noise-like sequences

Using matched filtering to identify binary codes the replacement of 0 with −1 must be made. As a result of such a transformation the binary sequence $a_0, a_1, a_2, \ldots, a_i, \ldots, a_{n-1}$ converts to the sequence of $b_0, b_1, b_2, \ldots, b_i, \ldots, b_{n-1}$, where $a_i \in \{0, 1\}$ and $b_i \in \{-1, 1\}$ for $i = 1, 2, \ldots, n-1$. Such transformation can be performed using the formula:

$$b_i = 2a_i - 1 \qquad \text{for} \qquad i = 0, 1, \ldots, n-1. \tag{6}$$

The matched filtering algorithm for the input sequence $b_0, b_1, b_2, b_3, \ldots, b_i, \ldots, b_n$ can be written as:

$$y_j = \sum_{m=0}^{n-1} b_{j-m} h_m, \tag{7}$$

where $h_j = b_{n-j}$, $j = 1, 2, \ldots, n$.

The coefficients of the impulse response for this filter are defined by the terms of a searched noise-like code written in the reverse order. In the matched filtering of the input sequences $b_0, b_1, b_2, b_3, \ldots, b_i, \ldots, b_n$ non-linear operations are also used, which increases the probability of their recognition in the presence of incorrect symbols in a code sequence.

Figure 3 presents the structure of a system using two-step matched filtering to recognize code sequences transmitted by *up-chirp* and *down-chirp* signals (Fig. 3a) and transmitted by the signals of one type *up-chirp* or *down-chirp* (Fig. 3b).

The structure presented in Fig. 3a consists of two channels of strictly equalled delays depended on the transition process taking place in these channels. The upper channel with a $[f_1, f_2]$ band responds to a *down-chirp* signal, whereas the lower one with a $[f_2, f_1]$ band responds to a *up-chirp* signal. The compressed output

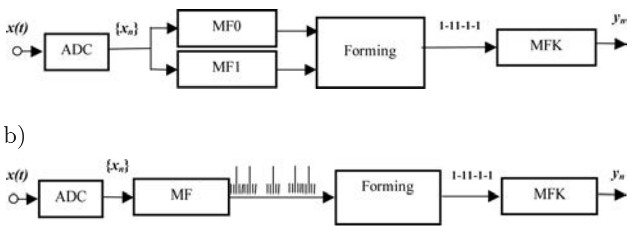

Fig. 3. The structures of the system identifying code sequences transmitted by chirp signals using two-step matched filtering: a) the structure using two matched filters MF0 and MF1, where one detects an *up-chirp* signal, whereas the other one detects a *down-chirp* signal, b) the structure using one matched filter detecting *up-chirp* signals.

signals of the MF0 and MF1 filters are used for creating a binary pulse train in a forming system. The zero-one sequence is transposed in a sequence consisting of +1, −1 elements, which goes to the input of an MFK matched filter responding to a specific code sequence.

The second method is illustrated in Fig. 3b. This structure consists of only one channel, and ones and zeros are represented by one type of a chirp signal (e.g. *up-chirp*). The forming system, on the basis of a signal level at the filter output and asked intervals, which should occur between the high levels, creates a zero-one sequence transposed in a sequence $\{+1, -1\}$, which goes to the input of an MFK matched filter responding to a specific code sequence. The systems identifying code sequences transmitted by chirp signals using two-step matched filtering were realized on the basis of the evaluation module X5-400M produced by Innovative Integration Company.

4. The influence of noises on the code sequences recognition

To examine the influence of noises on the recognition of code sequences transmitted by chirp signals the author suggested using a probabilistic model presented below.

The probability of the correct recognition of a noise-like code, in which zeros and ones are represented by chirp signals might be defined as follows.

Let us denote by:

- A – an event consisting in a correct recognition of a code sequence,
- B_k – an event consisting in the occurrence of k errors within a received code sequence,
- C_i – an event consisting in an incorrect recognition of a chirp signal representing the ith-position in a code sequence,
- $P(A/B_k)$ – the conditional probability of the correct recognition of a code sequence with the occurrence of k errors in it,

- $P(C_i) = p_i$ – the probability of an incorrect recognition of a chirp signal representing the ith-position in a code sequence,
- $P(B_k)$ – the probability of the occurrence of k errors within a received code sequence.

The collection $\{P(B_k)\colon k = \overline{0,n}\}$ forms a complete set of events, so:

$$\sum_{k=0}^{n} P(B_k) = 1. \tag{8}$$

Applying the formula for total probability and assuming that the distribution of the probability of events $\{C_i\,(i{=}\overline{1,n})\}$ is Bernoulli's distribution and conditional probabilities $P(A/B_k)$ are calculated using a computer simulation method, then the probability of the correct recognition of a code sequence might be calculated from the formula:

$$\begin{aligned} P(A) &= (1-p)^n + \sum_{k=1}^{n} \frac{m_k}{\binom{n}{k}} \binom{n}{k} p^k (1-p)^{n-k} \\ &= (1-p)^n + \sum_{k=1}^{n} m_k p^k (1-p)^{n-k}, \end{aligned} \tag{9}$$

where n is the length of a code sequence, m_k is the number of cases of the correct recognition of the code distorted on k positions (calculated using a computer simulation method, which involves conducting the matched filtering of the tested sequence for all possible distortions on k positions, and checking if the central convolution has the highest value among all possible convolutions), p is the probability of the incorrect recognition of a chirp signal.

If the probabilities of the incorrect recognition of the signals representing one (p_1) or zero (p_0) in a code sequence are different ($p_0 \neq p_1$), with the remaining invariable assumptions, then the probability $P(A)$ has to be calculated from the formula:

$$\begin{aligned} P(A) = \sum_{k_1=0}^{n_1} \sum_{k_0=0}^{n_0} & \frac{m_{k_1+k_0}}{\binom{n_0+n_1}{k_1+k_0}} \binom{n_1}{k_1} p_1^{k_1} \\ & \cdot (1-p_1)^{n_1-k_1} \binom{n_0}{k_0} p_0^{k_0} (1-p_0)^{n_0-k_0}, \end{aligned} \tag{10}$$

where n is the number of ones in a code sequence, n_0 is the number of zeros in a code sequence, $n_0 + n_1 = n$ is the length of a code sequence, p_1 is the probability of the incorrect recognition of one, p_0 is the probability of the incorrect recognition of zero.

The probabilities p, p_0, p_1 of the incorrect recognition of chirp signals representing ones and zeros in a code sequence might be calculated using Monte-Carlo method for the given level of interference. Knowing the probabilities of the incorrect recognition of chirp signals one can, using the developed approach, evaluate the probability of the correct recognition of a probing signal formed of the sequence of chirp signals created according to a chosen code sequence. Figure 4 shows the graphs of the probability changes $P(A)$ of the correct interpretation of sample code sequences according to the probability p of the incorrect recognition of a chirp signal.

a)

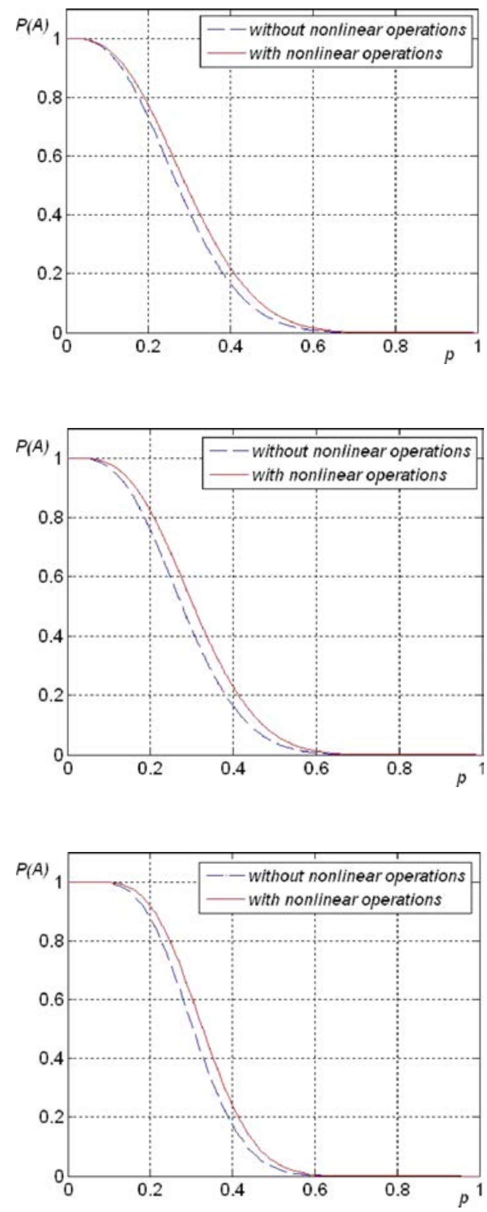

Fig. 4. The dependence of the probability of the recognition of a code sequence for a code word on the probability p of not recognizing a chirp signal: a) a sequence 1111100110101 type AB of length $n = 13$, b) 001111100110101 type N of length $n = 15$, c) a sequence 000001110011101010110110 type N of length $n = 24$.

The calculation results presented in Fig. 4 were obtained assuming that the probabilities of the distortion of zeros and ones in a code sequence are the same ($p_0 = p_1 = p$). The obtained results show that the use of non-linear operations together with matched filtering of code sequences increases the probability $P(A/B_k)$ of their recognition with the occurrence of incorrect symbols in a code sequence. For every code sequence there is a maximum value of the probability $p_{\max}$ of the incorrect recognition of a chirp signal, for which a code sequence is still accurately recognized. The value $p_{\max}$ depends on the length and type of a code sequence. Using the proposed model of the recognition of code sequences transmitted by chirp signals one can evaluate the noise level SNR_{in} disturbing the reception of single chirp signals, which does not influence significantly the accuracy of the recognition of these sequences.

5. Conclusion

The forming methods and the systems of the recognition of code sequences transmitted by the signals with the linear frequency modulation presented in this work can be effectively used in relation to probe and communication signals. Moreover, these systems work in the real time. The developed probabilistic model of the recognition of noise-like code sequences transmitted by chirp signals allows one to choose the optimal code sequence providing its accurate recognition at a given probability of the incorrect recognition of a single chirp signal.

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