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Estimation of Acoustic Impedance for Surfaces Delimiting the Volume of an Enclosed Space

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Several methods can be applied for analyses of the acoustic field in enclosed rooms namely: wave propagation, geometrical or statistical analysis. The paper presents problems related to application of the boundary elements method to modelling of acoustic field parameters. Experimental and numerical studies have been combined for evaluation of acoustic impedance of the material used for the walls of a model room. The experimental studies have been carried out by implementing a multichannel measuring system inside the constructed model of an industrial room. The measuring system allowed simultaneous measurements of the source parameters – the loudspeaker membrane vibration speed, the acoustic pressure values in reception points located inside the model space as well as phase shifts between signals registered in various reception points. The numerical modelling making use of the acoustic pressure values measured inside the analyzed space allowed determination of requested parameters of the surface at the space boundary.

Keywords: acoustic impedance, acoustic measurement, numerical methods, sound field.

1. Introduction

Modelling of the distribution of acoustic field parameters in enclosed rooms is often accompanied by a presence of considerable errors. Origins of those errors most often can be traced down to phenomena taking place at the boundary of the analyzed volume (Kosała, 2008). Two types of approach are encountered in description of those phenomena. A practical or engineering approach is based on a description of the boundary defining its absorption coefficient. The material's compressibility or the surface structure details are simply neglected. Such an approach is widely applied because it offers a relatively easy procedure for determination of the absorption coefficient values, both reverberation and physical ones. The problem that has to be coped with is the necessity to measure the values for all possible versions of the boundary elements fastening, as it can have a considerable effect on the values of the absorption coefficients. In the second approach that is applied in the acoustic field modelling by a wave propagation analysis the phenomena taking place on the boundary are described by the values of the complex acoustic impedance defining the dependence between the acoustic pressure and the acoustic particle velocity at the boundary of the analyzed volume (Weyna, 2007). Such a description introduces more serious practical problems. The task of determination of acoustic impedance values for real model structures becomes much more difficult (Alba, 2011). There are methods offering the solution of this problem but, similarly to the case of absorption coefficient determination, modification of the boundary elements fastening may considerably change acoustic impedance values of the surface. Because of the above, the authors have decided to combine the experimental methods (Piechowicz, 2007a; 2007b) with numerical modelling of the acoustic field (Björk, 1987; Kincaid, 2005), which may lead to a successful evaluation of acoustic impedance for the surfaces delimiting the analyzed volume with the enclosed acoustic field.

The analyzed problem is formulated as a task to determine such values of acoustical impedance for surfaces delimiting the analyzed space that during the numerical modelling process reproduce as closely as possible the measured results. The surface acoustical impedance values are treated as optimal when an objective function built of differences between the experimental and model values reaches the minimum.

In order to solve such a problem it is necessary to execute the following partial tasks:

- build a model connecting the impedance values at the boundary of an analyzed volume and the acoustic pressure values in selected reception points,
- define a function (norm) for determination of the interval between experimental and simulation results,
- implement the model and the appropriate objective function.

2. Problems of acoustic field modelling in an enclosed space

Acoustic field in a selected volume of space can be modelled in transitional states (the wave equation) or in a steady state (the Helmholtz equation) (Kuttruff, 1991). The process of solving the differential equations used for modelling of the acoustic field can be practically realized in many ways, among which the most frequently used are the ones listed below:

- analytical method – for simple room geometries and non-complicated boundary conditions;
- finite element method – fast, universal method which fails for large volumes and higher frequencies;
- boundary element methods – reduces the model rank by one, reducing 3D problems to 2D – which is, however, very demanding with respect to computational resources;
- non-grid methods.

In our work, the boundary element method has been used for modelling the acoustic field in an enclosed room (Ciskowski, 1991; Gołaś, 1995; Kirkup, 1998). If the acoustic pressure is labelled as p, the acoustic wave equation, with additional assumption that the solution is harmonic in time, takes the following form (the Helmholtz equation):

$$\nabla^2 p + k^2 p = 0, \tag{1}$$

where k is the wave number, it defines the relationship between the frequency ω and the velocity of propagation c of the sound wave: $k = \omega/c$.

The boundary conditions imposed on the boundary surface for this equation can be written as follows:

- Dirichlet condition: $p|_\Gamma = p_i$,
- Neumann condition: $\left.\dfrac{\partial p}{\partial n}\right|_\Gamma = i\omega\rho_0 v_n$,
- impedance condition, called also mixed or Robin's condition: $p|_\Gamma = Zv_n = Zi\omega\rho_0 \dfrac{\partial p}{\partial n}$,

where p is the acoustic pressure, v_n is the normal component of the particle velocity, ω is the angular frequency, $\dfrac{\partial p}{\partial n}$ is the partial derivative with respect to the normal, ρ_0 is the density of the air ≈ 1.21 kg/m^3, and Z is the acoustic impedance of a material.

After employing the Green's identity the following integral boundary equation can be written in the space delimited by the surface S:

$$c_p p = \int_S \left(g\frac{\partial p}{\partial n} - p\frac{\partial g}{\partial n} \right) \mathrm{d}S, \tag{2}$$

where $g = \dfrac{1}{4\pi r} e^{ikr}$ provides the fundamental solution, and c_p is a coefficient dependent on the point's location.

After carrying out the boundary discretization and assuming appropriate shape functions one obtains the following equation for every sub-surface (Ciskowski, 1991; Kirkup, 1998):

$$c_p p_i - \sum_j \int_{S_j} p\frac{\partial g}{\partial n}\,\mathrm{d}S = -\sum_j \int_{S_j} \left(g\frac{\partial p}{\partial n} \right) \mathrm{d}S. \tag{3}$$

Each of the sub-surfaces is a boundary element. The behaviour of the variables in each element is defined by a suitable shape function. The shape function can be constant, linear, or parabolic. Constant shape functions were used by the authors, which means that in the whole sub-surface there is a constant acoustic pressure and velocity. This treatment makes determination of integral values in the above equation much easier (Ciskowski, 1991). The above equation describes the acoustic pressure in i-th node caused by the pressures and velocities in all other nodes. After executing the following substitutions:

$$h_{ij} = \int_{S_j} \frac{\partial g}{\partial n}\,\mathrm{d}S \quad \text{and} \quad g_{ij} = \int_{S_j} g\,\mathrm{d}S, \tag{4}$$

one can write:

$$c_p p_i - \sum_j h_{ij} p_j = -\sum_j g_{ij} \left.\frac{\partial p}{\partial n}\right|_j. \tag{5}$$

Taking into the account that $\dfrac{\partial p}{\partial n} = i\omega\rho_0 v$, where v is the acoustic velocity, one can execute a transformation of the above formula. After performing a multiplication of the coefficients g_{ij} by $i\omega\rho_0$, the set of equations can be expressed in a matrix form as:

$$\mathbf{Hp} = \mathbf{Gv}, \tag{6}$$

where $\mathbf{p}$, $\mathbf{v}$ are column vectors containing the node values for acoustic pressures and velocities respectively, while $\mathbf{H}$ and $\mathbf{G}$ are square matrices of coefficients called influence matrices.

A necessary condition for solving the problem is the knowledge of the acoustic pressure or acoustic velocity value in each of the boundary nodes. Then, by the power of the equation mentioned above, one can determine the unknown values of the velocity or pressure at the boundary.

The acoustic impedance of the boundary Z is taken into the account as a mixed (Robin's) boundary condition in the nodes in the following way:

$$Z = \frac{p}{v} \tag{7}$$

or

$$v = \frac{1}{Z}p$$

which in the matrix form can be written as:

$$\mathbf{v} = \mathbf{Ep}, \tag{8}$$

where the $\mathbf{E}$ is a diagonal matrix that contains the known values of the inverse of acoustic impedance, or admittance, in the node points. After taking into the account the dependence, the written above Eq. (6) takes the following form:

$$\mathbf{Hp} = \mathbf{GEp}. \tag{9}$$

In the boundary element method the calculations are executed in two stages:

- determination of the acoustic pressure and velocity values in all boundary nodes,
- determination of the pressure values in the analyzed space in selected points.

Therefore, it is difficult to use directly Eq. (9) for determination of the acoustic impedance of the delimiting surfaces. Equation (9) is just a starting equation for determining of the acoustic impedance of the surfaces delimiting the room (Fig. 1).

The same acoustic impedance determination was carried out using the numerical model. To test the room, a model was prepared in a 1:3 scale made of 6 Oriented Strand Boards. In one of the walls a speaker driven sinusoidal signal was placed. A laser vibrometer was used to make non-contact velocity and phase measurements of the speaker's membrane. The linear array of 24 microphones was moved down to the length of the cuboid in steps of 5 cm on the central plane. Each of the microphones recorded the sound pressure and phase shifts of the signal. Figure 2 shows the distribution of the ratio of the sound pressure and vibration velocity at the frequency of 100 Hz in the model room. The measurement results collected in the real model of the room (see Fig. 2) have been used as starting values for the tuning process of the numeric model. The model tuning has been carried out taking into the account the values of the acoustic impedance on the bounding walls.

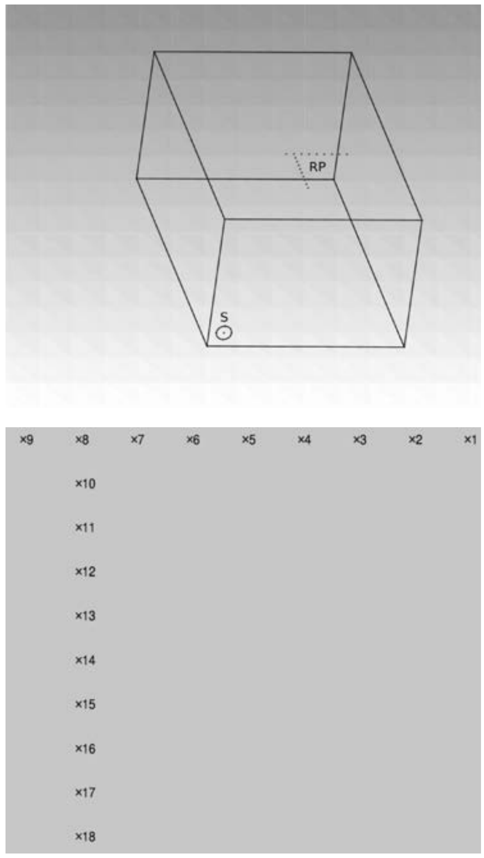


Fig. 1. Model of the analyzed room and distribution of the reception points used for identification of the boundary conditions: S is the sound source, RP are reception points (numbered in the figure on the bottom).

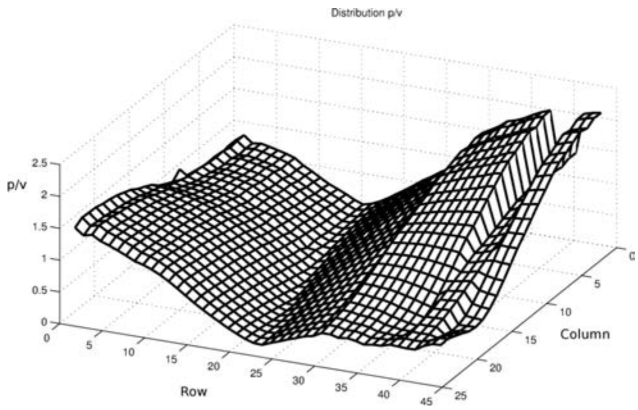


Fig. 2. Distribution of the ratio of the sound pressure and vibration velocity at the frequency $f = 100$ Hz.

3. Determination of the acoustic impedance of the model surface

Determination of the acoustic impedance values for the bounding surfaces was based on a measurement of acoustic pressure values in selected reception points located inside the model room and a simultaneous measurement of the signal phase shifts with respect to the stimulation signal (Piechowicz, 2011).

The scheme of the experiment was as follows:

1. Selection of the reception points used in the model tuning;
2. Estimation of effective values of the acoustic pressure, phase shifts between the pressure and velocity of the stimulating membrane, as well as the effective value of the membrane's velocity;
3. Assumption of the impedance values at the volume's boundary;
4. Estimation of the acoustic pressure values at selected points by numerical calculations;
5. Specification of the objective function and calculation of its minimum.

In order to evaluate the quality of the chosen numeric model in relation to the measured values it is necessary to assume the form of the respective function (see Fig. 3). At the present stage of the study the error measure has been assumed as a sum of squares of differences. If one assumes that p_i is the acoustic pressure value at the i-th reception point, then one can write it down in the form:

$$F(Z_1, Z_2, ..., Z_n) = \sum_{i=1}^{m} (\widehat{p}_i - p_i)^2, \qquad (10)$$

where $\widehat{p}_i$ is the value of the acoustic pressure determined by the numerical calculations. The objective function defined above is a good tool for evaluation of the model quality. However, it should be remembered that the obtained function minimum might not ensure a complete identity of the experimental and simulation results (Fig. 4).

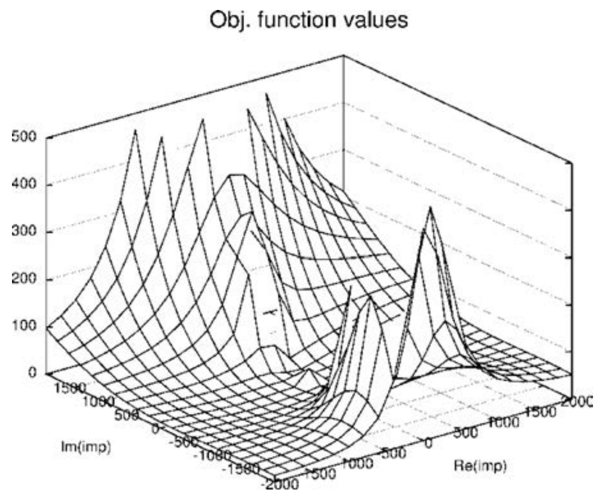


Fig. 3. Shape of the objective function.

The shape of the objective function shown in Fig. 3 was obtained by iterative calculations for successive values of the acoustic impedance for the boundary of the analyzed space. The solution was searched for the minimum objective function values (Fig. 4). Determination of the acoustic impedance values at the boundary of the analyzed volume has been carried out by an

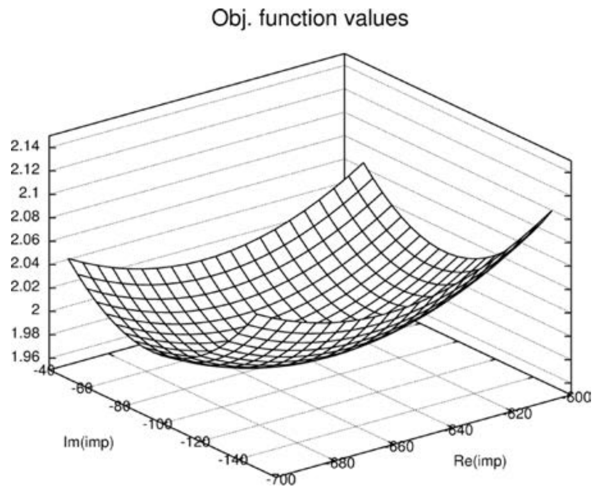


Fig. 4. Shape of the objective function in the vicinity of the global minimum.

appropriate optimization process. Values of the acoustic pressure $p_i = p(x_i, y_i, z_i)$ generated in the analyzed volume by a source located at its boundary have been determined. Momentary values of the acoustic pressure in the reception points have been registered together with momentary values of the membrane vibration velocity for the stimulating loudspeaker. The results of those measurements have been used to determine the pressure amplitudes and the signal phase shifts with respect to the stimulating signal (Table 1).

Table 1. Measured values of the sound pressure, phase shifts, and membrane vibration velocity for stimulating the loudspeaker for 18 measurements points.

Point	pressure [Pa]	phase [°]	velocity [m/s]
1	0.0142	30.5	00280
2	0.0147	31.5	0.0280
3	0.0148	31.0	0.0280
4	0.0159	28.8	0.0280
5	0.0160	26.5	0.0280
6	0.0166	22.4	0.0280
7	0.0175	22.6	0.0280
8	0.0181	22.3	0.0280
9	0.0179	23.0	0.0280
10	0.0018	165.3	0.0117
11	0.0173	23.5	0.0303
12	0.0135	25.9	0.0276
13	0.0109	25.5	0.0259
14	0.0102	28.3	0.0300
15	0.0075	31.7	0.0296
16	0.0012	32.2	0.0072
17	0.0011	67.6	0.0107
18	0.0010	135.0	0.0113

A numerical model consistent with the layout of the measurement system has been constructed. Acoustic impedance values Z_i have been set on the surfaces delimiting the analyzed volume, with each value constant within the respective wall surface. Then values of the acoustic pressures $\widehat{p}_i$ have been determined in 18 reception points (see Fig. 1) attributed to respective measurement points in the real system.

The collected data allowed determination of an objective function in the form given by Eq. (10). In the next step, the Nelder-Mead optimization method has been applied, as implemented in the statistical software package "R".

4. Numerical model of the room

The presented problem of determination of acoustic impedance values for the surfaces delimiting the analyzed room has been solved by applying numerical modelling. Such an approach turned out to be necessary because analytical determination of the requested impedance value from Eq. (5) was not possible. The numerical model comprised a spatial mapping of the interior geometry for the analyzed volume. The principal difference between the model and the study of the real object (Fig. 5) was the assumption of stiff delimiting surfaces.

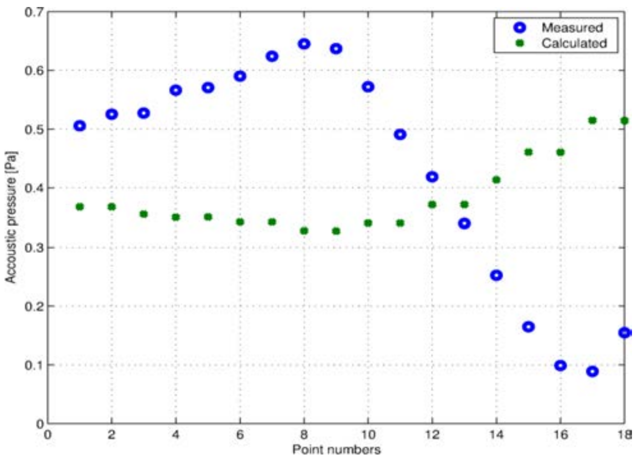


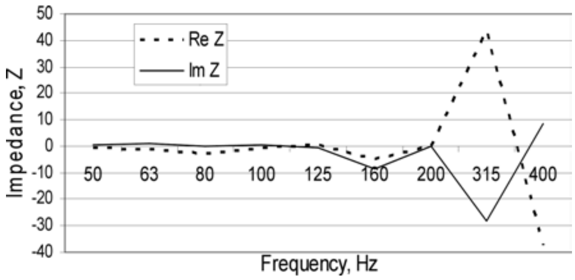
Fig. 5. Differences between the experimental and numerically determined values for individual reception points.

The elaborated model has been used for determination of acoustic impedance values of the walls. The stimulation in the form of the acoustic velocity at the volume's boundary has been applied on a surface equivalent to the membrane area of the stimulating loudspeaker. For all the remaining surfaces of the walls one value of the acoustic impedance has been assumed, and it was later subject to optimization. For the calculations 18 reception points have been selected, located along two perpendicular straight lines (see Fig. 1).

As a result of the calculations, the acoustic impedance value Z on the OSB surface of the delimiting walls has been determined and presented in Table 2.

Table 2. Specific acoustic impedance Z of Oriented Strand Board (22 mm thick panel) as a function of the frequency.

frequency [Hz]	Impedance Z	
	Re Z	Im Z
50	0.75	0.30
63	−1.00	0.87
80	−2.53	−0.02
100	−0.77	0.68
125	0.49	−0.46
160	−5.03	−8.81
200	−0.01	−0.25
315	43.58	−28.15
400	−37.33	8.40
500	−178.15	21.99
630	−5.37	−6.64



5. Summary

The study presents a possibility of appointing a normal wall acoustic impedance using an inverse BEM approach from a set of sound pressures at different points measured in an interior steady-state sound field. This is important because distribution of the sound pressure in the interior shows the effect of wall coverings on the sound field. For the calculation a model with stiff boundary surfaces has been used, while in the real system the surfaces have been realized as boards fastened along their edges. It seems that this fact is the source of errors in determination of the acoustic impedance values of the delimiting surfaces. The error is introduced by a presence of an additional component of the wall impedance, resulting from its elasticity. Another consequence of such a fastening of the bounding boards is an inhomogeneous distribution of the acoustic impedance value within the whole bounding surface.

The calculations have shown that scaling of the objective function values may be very helpful for improving a convergence of the optimization procedure – as a slow convergence seems to be a problem for some starting points. The analyses have shown that the objective function exhibits only one local minimum in the analyzed variable range covering $\langle -10\,000, 10\,000\rangle \times \langle -10\,000i, 10\,000i\rangle$. It seems that it is also a global minimum. This allows determination of acoustic impedance values for the indicated frequencies. Subsequent studies were conducted for various configurations of wall materials. Refined computational algorithms for the model space will allow a transfer of the method to determine the acoustic impedance of actual walls of small factory rooms.

Further research directions that should be indicated include determination of the influence of reception points distribution on a determined value of the acoustic impedance of walls, estimation of the error bound for a determined impedance value resulting from the assumption of stiff boundary surfaces delimiting an analyzed volume.

It would also seem plausible to test other methods of the acoustic field analysis to find a solution of the discussed problem, for example, some algorithms from the non-grid methods family.

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References

1. Alba J., del Rey R., Ramis J., Arenas J.P. (2011), *An inverse method to obtain porosity, fibre diameter and density of fibrous sound absorbing materials*, Archives of Acoustics, **36**, 3, 561–574.
2. Björk Å., Dahlquist G. (1974), *Numerical methods*, Prentice-Hall.
3. Ciskowski R.D., Brebbia C.A. (1991), *Boundary element methods in acoustics*, Elsevier, London-New York.
4. Gołaś A. (1995), *Computer methods in interior and environmental acoustics* [in Polish], AGH, Cracow.
5. Kincaid D., Cheney W. (2002), *Numerical analysis*, Mathematics of Scientific Computing, Third Edition, The Wadsworth Group, Brooks/Cole.
6. Kirkup S. (1998), *The boundary element method in acoustics (Integral Equation Methods in Engineering)*, Integrated Sound Software.
7. Kosała K. (2008), *Global index of the acoustic quality of sacral buildings at incomplete information*, Archives of Acoustics, **33**, 2, 165–183.
8. Kuttruff H. (1991), *Room Acoustics*, Elsevier Applied Science, London – New York.
9. Piechowicz J. (2007a), *Determination of acoustic fields in industrial room*, Archives of Acoustics, **32**, 2, 313–319.
10. Piechowicz J. (2007b), *Acoustic field in the mechanical workshop*, Archives of Acoustics, **32**, 4, 221–226.
11. Piechowicz J. (2011), *Estimating surface acoustic impedance with the inverse method*, International Journal of Occupational Safety and Ergonomics, **17**, 3, 271–276.
12. Weyna S. (2007), *Some comments about the existing theory of sound with comparison to the experimental research of vector effects in real-life acoustic near fields*, Archives of Acoustics, **32**, 4, 859–870.