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A Single Number Index to Assess Selected Acoustic Parameters in Churches with Redundant Information

Krzysztof KOSAŁA

AGH University of Science and Technology
Department of Mechanics and Vibroacoustics
Al. Mickiewicza 30, 30-059 Kraków, Poland
e-mail: kosala@agh.edu.pl

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The paper presents an innovative approach for the index assessment of the acoustic properties of churches. A new formula for an approximate single number index to assess selected acoustic parameters of church interiors, such as reverberation time (RT), speech intelligibility index (RASTI) and music clarity index (C_{80}), is presented in the paper. The formula is created by means of the Singular Value Decomposition (SVD) method. An innovative approach for calculating the weights of partial indices is shown by solving the problem of redundant information, i.e., the system of overdetermined linear equations, using a computed pseudoinverse matrix. The new procedures for calculating the values of three partial indices and the single number index to assess selected acoustic parameters are presented. The proposed method was verified by measurements in several selected churches.

Keywords: acoustic quality, index method, sacral objects, Singular Value Decomposition (SVD), redundant information.

1. Introduction

Many studies on church acoustics have been carried out in different geographical areas (Carvalho, 2000; Meyer, 2003; Crillo, Martellotta, 2005; Galindo *et al.*, 2005; Martellotta, 2006; Soeta *et al.*, 2010; Vodopija *et al.*, 2010).

Most of the studies have been based on acoustic parameters defined by ISO 3382 (1997). Measured acoustic parameters were compared with preferred ones as well as subjective listeners' evaluation tests (Carvalho, 1997; Vodopija *et al.*, 2010). A set of guidelines for acoustical measurements in churches was defined by Martelotta *et al.* (2009).

An innovative approach for the assessment of the acoustic properties of churches is shown in the paper.

It is necessary to elaborate a uniform method for assessing the acoustic quality of a church interior as there is no one typical tool used to assess the acoustics of existing and currently-built church interiors – in the design stage (Engel *et al.*, 2007). The long-term scientific research performed in order to elaborate such a method was based on the designed partial indices of assessment (Engel, Kosała, 2004; 2007). The partial indices calculated from some relations were components of the proposed approximate global assessment of the acoustic quality of churches W_{AQS}.

The Index Method, proposed by Engel and Kosała (2007), has been applied by Carvalho and Silva (2010) to assess acoustic quality of the 9000-seat Holy Trinity Church in Fatima in Portugal. Using that version of the method (2007), the church achieved a rating of very good, which was in accordance with the previous author's research (Carvalho, Silva, 2010).

The subsequent index method research involved the Singular Value Decomposition (SVD) method (Kosała, 2007; 2008b). By means of the SVD method, the relations between the partial indices were analysed and a quantitative analysis of the weights used for the global assessment, based on the weighted average, was performed (Kosała, 2008a; 2009).

The research described in the paper features a proposal of a new formula for the single number index, based on three selected partial indices as functions of the acoustic parameters, i.e., reverberation time RT, music clarity index C_{80} and $RASTI$ index. The partial indices are mutually correlated. In the paper it is presented that through the use of the SVD method it is possible to obtain a number, i.e., a singular value that can be used for the single number assessment. As compared with previous research, a different approach for calculating the weights of partial indices is shown that features a pseudoinverse matrix to solve the problem of redundant information (solution of the overdetermined system of equations).

The paper presents a general description of the concept of the approximate single number assessment by means of the SVD method and presents an application of the proposed algorithm used to assess the acoustic quality of real church interiors.

2. Single number index to assess selected acoustic parameters W_{SAP}

Single number indices are useful because church designers are not as experienced in the field of acoustic problems as acousticians, and therefore an index is important for a designer. The general and approximate measure of acoustic quality is a single number index to assess selected acoustics parameters W_{SAP}, whereas a more accurate assessment of the acoustic quality of an interior is performed by means of partial indices.

The structure of the single number index W_{SAP} determining the assessment of the acoustic quality of selected acoustic parameters is based on the calculation model in which the relations between the partial indices W_i of the index W_{SAP} are analysed. The main component of the calculation model is the observation matrix **A** containing data interconnected by the mutual relations (Fig. 1).

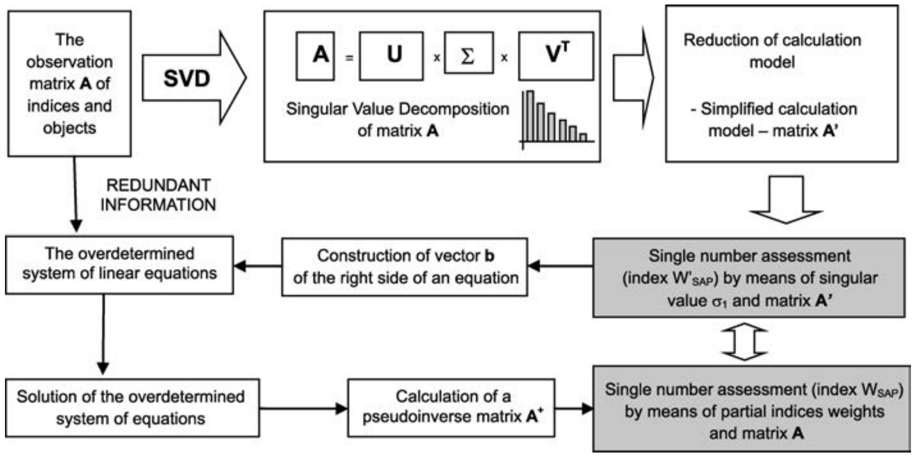

Fig. 1. The determination of the assessment of acoustic quality by the single number index.

The SVD (Singular Value Decomposition) method was used as the basic analytical tool for testing the content of the observation matrix. The SVD method is the calculation method commonly used in numerical linear algebra (Lanczos, 1961; Kiełbasiński, Schwetlick, 1992). This method has applications in many fields of science such as diagnostics (Cempel, 1980) and vibroacoustics (Engel Z., Engel J., 2007).

In this case, the SVD method was used for factor analysis, i.e., obtaining an explanation of the information concerning independent indices through consecutive singular components. Identification of the most informational component makes it possible to reduce the calculation model and obtain the largest singular value from the decomposition to be applied in the single number index formula. Then the simplified calculation model obtained through the matrix $\mathbf{A}'$ will be used.

However, the single number assessment can also be expressed by another relationship containing weights assigned to every partial index, i.e., components of the matrix **A**. Therefore, the system of numerous linear equations with a few unknowns (weights) shall be solved. Such a system is an overdetermined system of equations where the number of equations is greater than the number of unknowns. The problem of redundant information also has to be managed. In order to solve the problem a pseudoinverse matrix and the vector **b** of the right side of the equation are calculated.

The basis for creating the observation matrix is data obtained from the measurements. The measured values of the acoustic parameters are calculated into partial indices of assessment for the interior's acoustic quality within the range

$\langle 0,\ldots,1\rangle$. The columns of the matrix $\mathbf{A}$ of the size $m\times n$ are partial indices and the rows are particular objects in which the measurements were performed. The observation matrix contains mutually correlated partial indices.

The acoustic quality index of an object, W_{SAP}, can be any value within the range 0 to 1. If a tested object has a W_{SAP} equal to 1, it features very good acoustic properties corresponding to the preferred values. If W_{SAP} is equal to 0, an interior has poor acoustic properties and is disqualified according to the acoustic criteria. Therefore, the values of the acoustic parameters measured in a real object are considerably different than the preferred values.

2.1. Index W'_{SAP} with a singular value

By means of the SVD decomposition of the matrix $\mathbf{A}$, three other matrices containing right and left singular vectors in the matrix $\mathbf{U}$ and $\mathbf{V}$ and singular values in the matrix Σ are obtained (Engel *et al.*, 2008; Kosała, 2008a). The analysis of singular values makes it possible to identify the percentage of information concerning independent indices explained through the consecutive components. The first component is usually the most informational and, if it is large in relation to other components, the calculation model can be simplified through applying the approximation of the first rank:

$$\mathbf{A}' = \sigma_1 \mathbf{u}_1 \mathbf{v}_1, \tag{1}$$

where $\mathbf{A}'$ is the matrix obtained as the result of the approximation of the first rank (k=1), $\mathbf{u}_1$ is the first column of the matrix $\mathbf{U}$, $\mathbf{v}_1$ is the first row of the matrix $\mathbf{V}^{\mathrm{T}}$, σ_1 is the first singular value of the matrix $\boldsymbol{\Sigma}$.

The matrix $\mathbf{A}'$, which approximates the matrix $\mathbf{A}$, contains components as the partial indices mutually correlated in a perfect way (the coefficient of the linear correlation between the indices is 1).

The matrix $\mathbf{A}'$ is obtained using the orthonormal vector u_1 of the values identical for all partial indices, v_1 of the values identical for all objects, and the singular value σ_1 identical for all objects and indices.

The Eq. (1) can be expressed in the following way:

$$\mathbf{A}'\sigma^{-1} = \mathbf{u}_1 \mathbf{v}_1. \tag{2}$$

Therefore, the rows of the matrix $\mathbf{A}'$ contain the partial indices divided by a "common" component identical for all objects and indices, i.e., the singular value σ_1. Through discarding the common element of the perfectly correlated indices in such a way it is proposed to calculate their sum determining the approximate single number assessment of the acoustic quality of the object W'_{SAP_i} which can be expressed as follows:

$$W'_{SAP_i} = \sigma_1^{-1} \sum_{j=1}^{n} W'_j \tag{3}$$

or

$$W'_{SAP_i} = \mathbf{u}_{i1} \sum_{j=1}^{n} \mathbf{v}_{1j}, \tag{4}$$

where W'_{SAP_i} is the single number index for assessment of the selected acoustic parameters of an i-th object, i is the i-th row corresponding to an i-th object of the matrix $\mathbf{A}'$, j is the j-th column corresponding to a j-th partial index of the matrix $\mathbf{A}'$, $\mathbf{v}_{1j}$ is the right singular vector of the matrix $\mathbf{A}'$ (the first row of the matrix $\mathbf{V}^{\mathrm{T}}$), and $\mathbf{u}_{i1}$ is the left singular vector of the matrix $\mathbf{A}'$ (the first column of the matrix $\mathbf{U}$).

The equivalent single number index relationships (3) and (4) result from the properties of the SVD method. As for the single number assessment using the index W'_{SAP_i}, the values of indices from the matrix $\mathbf{A}'$ or the singular value σ_1 are applied. The matrix $\mathbf{A}$ with a certain Frobenius error approximates the observation matrix $\mathbf{A}$. As for the local assessment, it is also possible to use the input indices which are not fully correlated, and then each index would have to feature a certain weight assigned to each index of the matrix $\mathbf{A}$.

2.2. Index W_{SAP} with weights

Determination of the weights of partial indices is related to the problem of redundant information. Redundant information appears when there are more equations than unknowns. The unknowns are the weights x_j of partial indices W_j, whereas the number of linear equations is equal to the number i of the tested objects in the observation matrix $\mathbf{A}$.

The problem of redundant information can be overcome by solving the system of equations with the following form:

$$\mathbf{A}\mathbf{x} = \mathbf{b}, \tag{5}$$

where $\mathbf{A}$ is a matrix $m \times n$, $\mathbf{b} = [\mathbf{b}_1, \mathbf{b}_2, \ldots, \mathbf{b}_m]^{\mathrm{T}}$ is a vector of the right side of the equation, $\mathbf{x} = [\mathbf{x}_1, \mathbf{x}_2, \ldots, \mathbf{x}_n]^{\mathrm{T}}$ is the searched solution (weights of partial indices).

A method for solving such problems is to find a vector $\mathbf{x}^*$ of the Euclidean norm as small as possible, so that it minimises the Euclidean norm of a residual vector for a given matrix $\mathbf{A}$ and a vector $\mathbf{b}$.

Having the SVD decomposition determined,

$$\mathbf{A} = \mathbf{U}\mathbf{S}\mathbf{V}^{\mathrm{T}}, \tag{6}$$

it is easy to find the solution of the problem, i.e., the linear least squares problem:

$$\mathbf{x}^* = \mathbf{A}^{+}\mathbf{b}, \tag{7}$$

where $\mathbf{A}^{+} = \mathbf{V}\mathbf{S}^{+}\mathbf{U}^{\mathrm{T}}$ is a pseudoinverse matrix in relation to $\mathbf{A}$,

$$\mathbf{S}^{+} = \mathrm{diag}\left(\frac{1}{\sigma_1}, \ldots, \frac{1}{\sigma_k}, 0, \ldots, 0\right) \in R_{n \times m}. \tag{8}$$

The components of the vector **b** are the values of the obtained single number indices:

$$\mathbf{b} = [W'_{SAP_1}, \ldots, W'_{SAP_i}]. \tag{9}$$

The system of linear equations is formed from the following equation:

$$x_1 W_{1i} + x_2 W_{2i} + \ldots + x_{ji} W_{ji} = W_{SAP_i}, \tag{10}$$

where i is the i-th equation corresponding to an i-th tested object, j is the number of indices, and $x_1, \ldots, x_j$ are weights of indices.

If the solution for the system of equations is found, i.e., the weights are calculated, the single number acoustic assessment is performed by means of the equation (10):

$$W_{SAP_i} = \sum_{j=1}^{n} x_{ji} W_{ji}, \tag{11}$$

where j is the j-th weight corresponding to an j-th index and i is the i-th object.

3. Application of the single number index to assess the acoustic quality of church interiors

3.1. Assumptions of the index assessment method

The proposed index assessment for the acoustic quality of church interiors is an approximate method. The single number assessment of the acoustic quality of an interior is performed in an approximate way by means of the proposed global index W_G which is a function of several partial indices. The purpose of this method is not to perform a precise synthesis of all detailed information describing the acoustic field but of the information that is the most useful from the functional perspective of an object, such as a church. It is assumed that a global index W_G is a general measure through which the acoustic properties of different churches can be compared in a complex way. So far this has not been possible due to the lack of a uniform method of assessment for churches. More accurate information about the acoustic properties of a tested interior are also determined as single numbers by means of the partial indices.

A global index of the acoustic quality of a church, W_G, is a function of many partial indices:

$$W_G = f(W_1, W_2, W_3, \ldots, W_j), \tag{12}$$

where $W_1, \ldots, W_j$ are partial indices.

The created relationships for partial indices are used to assess the values of the acoustic parameters measured in real objects and averaged as the function of frequency and space in comparison with the preferred values. The assessment, including acoustic parameters in the domain of frequency and space, can be the subject of future method-related research if it is necessary to obtain more accurate results.

The basic assumption of the proposed index assessment of church interiors is the creation of all assessment indices (including a global index W_G) so that the assessment scale (of values of indices) is within the range 0 to 1. The value of 1 determines good acoustic parameters corresponding to the preferred values of the parameters, whereas the value of 0 determines poor acoustic parameters diverging considerably from the preferred ones.

Such an approach makes it possible to perform a quick mutual comparison of the different acoustic properties of a single object and the acoustic properties of different objects. The method is designed for religious buildings that do not feature any acoustically coupled spaces, such as the interiors of Roman Catholic churches of simple shape. The sound amplification system of a church is not included at this stage of development of the index method. The acoustic properties of an interior influenced by architecture, finishing materials, volume, floor shape, sound direction, and dispersion elements are tested.

The proposed index method is an objective method. As for future research, the results obtained by means of the method should be compared with subjective research. Then it would be possible to propose the scale of the acoustic assessment not only as numbers within the range 0 to 1, but also as assessment grades, such as bad, satisfactory, good, and very good.

It is proposed to group the partial indices $W_1, \ldots, W_j$ into uncorrelated and correlated ones. A single number index to assess selected acoustic parameters in the church is a function of new correlated partial indices which are described in the article.

3.2. Modified partial indices used for the single number assessment

The paper is focused on three partial indices that were analysed for the simplified calculation model characterised in (Kosała, 2009). The indices' computation procedures were modified in order to obtain more accurate results. In addition to simplifying the calculations, excluding the congregation during computation of the reverberation index W_R provides extra advantages, as additional errors related to calculation of the reverberation time corrected by the congregation's presence (as in (Engel, Kosała, 2007) or (Kosała, 2008b)) are not brought into a new calculation model (the observation matrix). The exclusion of the congregation in all partial indices reflects the measuring conditions in a real object in a more precise way. The presence of the congregation can be included in the simulation tests. The time of calculating the values of partial indices was shortened by making two other indices dependent only on single acoustic parameters and not on many parameters (as in (Kosała, 2009)). The music sound index is the function of a single index, i.e., the clarity C_{80}, and the speech intelligibility index is the function of the *RASTI* index. Such simplifications make it possible to apply the index method for the simulation tests of the geometric model of a church.

3.2.1. Reverberation index

It is recommended that the reverberation index featured in (Engel, Kosała, 2004) and later in (Kosała, 2008b) is simplified into the following form:

$$W_R = 1 - \frac{|RT - T_p|}{4.5}, \tag{13}$$

where W_R is a reverberation index, RT is a reverberation time, [s], measured in a real church without congregation, T_p is a preferred reverberation time for Roman Catholic churches, [s], according to the following relation (Engel *et al.*, 2007; Engel, Kosała, 2007):

$$T_p = 0.24 \ln(V_S) - 0.24 \quad [\mathrm{s}], \tag{14}$$

where V_S is the church interior volume, [m^3].

The reverberation index W_R has any value within the range 0 to 1 on the assumption that $RT \le T_P + 4.5$. If $RT = T_P$ then $W_R = 1$ and the tested church has good acoustic properties corresponding to the preferred value. If $RT = T_P + 4.5$ then $W_R = 0$, i.e., the tested object has bad acoustic properties.

If $RT > T_P + 4.5$ the formula (13) provides negative values and $W_R = 0$ shall be assumed. Thereby, the reverberation time of a tested space exceeds the reverberation time preferred for a given volume by more than 4.5 seconds.

Figure 2 presents values of the modified reverberation index W_R calculated according to the formula (13) for the eleven tested Roman Catholic churches in comparison with the previous index described in (Kosała, 2008b) including presence of the congregation.

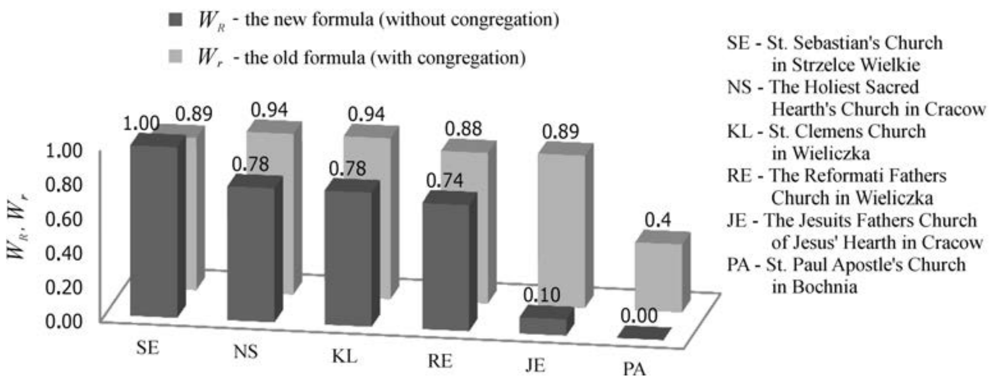

Fig. 2. The reverberation indices used for acoustic assessment of interiors of eleven Roman Catholic churches with and without the congregation.

Figure 2 proves that the formula (13) provides good results as there is a broader result range from 1 for objects of good acoustic parameters ($W_R = 1$) to 0 for objects of poor acoustic parameters ($W_R = 0$) in comparison with the previous formula including presence of the congregation.

3.2.2. Music sound index

Assessment of the quality of the music sound in a church by means of the music sound index is presented in (ENGEL *et al.*, 2007) and (Engel, Kosała, 2007). By simplifying the computation procedure it is proposed to make the index W_M dependent only on a single acoustic parameter, i.e., the music clarity index C_{80}. Thereby, the music sound index of a church is the function of C_{80}:

$$W_M = f(C_{80}). \tag{15}$$

The nomogram elaborated in (ENGEL *et al.*, 2007) and (Engel, Kosała, 2007) is used to calculate the value of the index W_M. In order to do it in a more precise way the following relations can be used:

$$W_M = \begin{cases} 0 & \text{for} \quad -15 < C_{80} > 15 \text{ dB}, \\ 0.04C_{80} + 0.6 & \text{for} \quad -15 \le C_{80} < -10 \text{ dB}, \\ 0.06C_{80} + 0.8 & \text{for} \quad -10 \le C_{80} < -5 \text{ dB}, \\ 0.1C_{80} + 1 & \text{for} \quad -5 \le C_{80} < 0 \text{ dB}, \\ 1 & \text{for} \quad 0 \le C_{80} < 5 \text{ dB}, \\ -0.12C_{80} + 1.625 & \text{for} \quad 5 \le C_{80} < 10 \text{ dB}, \\ -0.083C_{80} + 1.25 & \text{for} \quad 10 \le C_{80} < 15 \text{ dB}. \end{cases} \tag{16}$$

The music sound index W_M can be any value within the range 0 to 1. The W_M values equal to 1 correspond to the most preferred conditions for listening to music in the tested space. The W_M values equal to 0 determine unfavourable acoustic conditions for listening to music.

3.2.3. Speech intelligibility index

The speech intelligibility of a church interior determined by means of the intelligibility index as the function of three auxiliary indices (ALCONS, C_{50}, RASTI) was presented in (ENGEL *et al.*, 2007) and (Engel, Kosała, 2007). Currently it is proposed to make the W_S index dependent on only a single acoustic parameter, i.e., the most characteristic RASTI.

According to the index method assumptions (Subsec. 3.1) assessment of the acoustic quality of the church concerns the interiors with sound amplification systems turned off. The speech intelligibility is assessed with the same conditions because this parameter, as well as the rest ones, is obtained from the impulse response of the room.

The values of the W_S index are within the range from 0 (bad speech intelligibility) to 1 (very good speech intelligibility) and are equal to the RASTI with the same range of values. However, assessment of the RASTI (or W_S index) without sound amplification system in interiors like churches as the average value obtained from all measurement points in practice does not exceed the value of 0.5. The range of RASTI from 0.45 to 0.6 means fair intelligibility of speech, which is satisfactory for interiors with sound amplification systems off. Therefore, the 0 to 1 range of W_S index is more suitable to assess church interiors with sound amplification system on. Such a research has not been done by the author, however, this will be the subject of further investigations.

3.3. Verification of the index assessment for selected churches

In order to calculate the partial indices W_R, W_M, and W_S described in subsection 3.2 the values of the following three acoustic parameters are required: interior reverberation time *RT*, music clarity index C_{80} and *RASTI*, as well as the architectural parameter, i.e., church interior volume V (Fig. 3). The values of the coefficients of linear correlation between the partial indices are high and within the range 0.92 to 0.98.

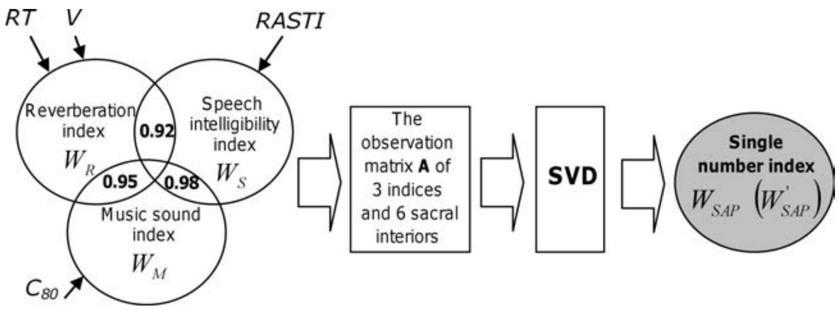

Fig. 3. Determination of the single number assessment of the quality of a church interior.

The calculated values of the indices for the tested churches are the components of the observation matrix **A** of indices and church interiors. As a result of the SVD method (Fig. 3), the observation matrix **A** is decomposed in relation to the singular values. By using the approximation k of the first rank it is possible to obtain the singular value σ_1 as a component of the single number assessment. The graphic interpretation of decomposition of the observation matrix **A** is presented in (Kosała, 2009).

After the new procedures for partial indices were applied the SVD decomposition of the observation matrix **A** was performed in Matlab software.

The SVD decomposition of the matrix **A** featuring columns containing the values of three indices (i.e., W_R, W_M, W_S) and rows as the interiors of six churches is shown as graphs in the Fig. 4. The content of the index observation matrix **A** with three mutually-correlated partial indices was shown in the Fig. 4a. The content of the matrices $\mathbf{U}$, $\mathbf{\Sigma}$, $\mathbf{V}^{\mathrm{T}}$ obtained from SVD was shown in the Figs. 4b, 4c, and 4d. All left singular vectors in the matrix $\mathbf{U}$, $\mathbf{u}_1$, $\mathbf{u}_2$, $\mathbf{u}_3$, are orthogonal to each other, as such they are entirely uncorrelated (Fig. 4b). The singular vectors of the matrix $\mathbf{V}^T$, $\mathbf{v}_1$, $\mathbf{v}_2$, $\mathbf{v}_3$, have the same properties (Fig. 4d). The first singular value σ_1 (Fig. 4c) is far bigger than σ_2 and σ_3.

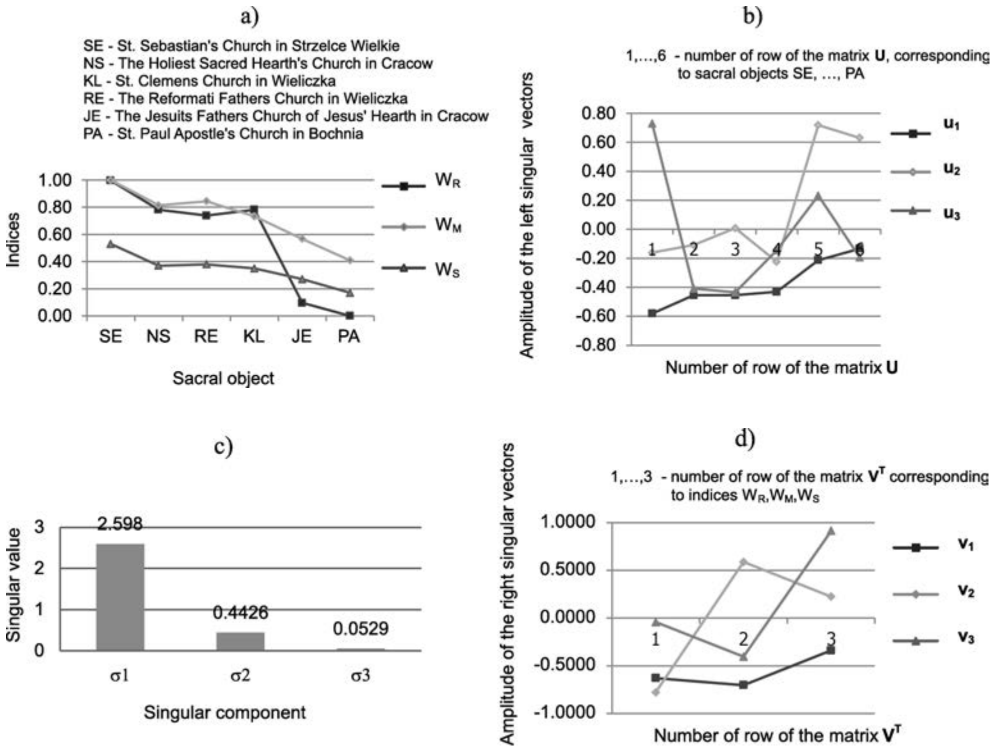

Fig. 4. Content of the index observation matrix $\mathbf{A}$ (a) and matrices obtained from SVD of the matrix $\mathbf{A}$; b) left singular vectors of the matrix $\mathbf{U}$; c) singular values of the matrix $\mathbf{\Sigma}$; d) right singular vectors of the matrix $\mathbf{V}^T$.

The analysis of the percentage shares of the explanation of information concerning the independent indices (Fig. 5a) obtained from the SVD decomposition

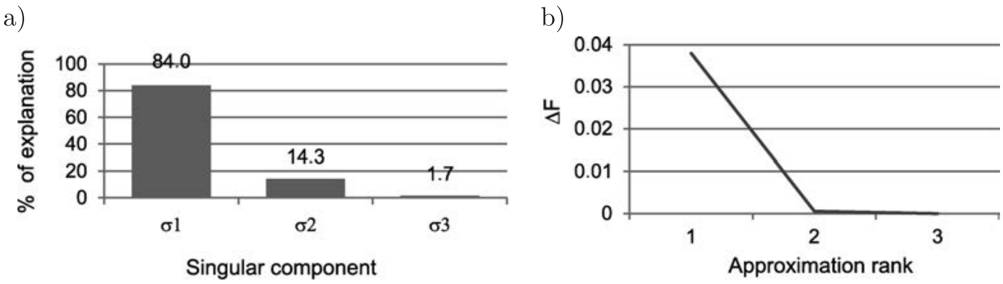

Fig. 5. Content of information concerning independent indices in the matrix $\mathbf{A}$ (a) and approximation error in the function of the k approximation rank (b).

proves that σ_1 is the most informational component as it contains 84% of the total information.

Therefore, the other two singular components can be rejected and approximation of the matrix $\mathbf{A}$ of the first rank can be applied for future analyses using the first component σ_1. Such an approximation containing the Frobenius error ΔF is shown in Fig. 5b.

The matrix $\mathbf{\Sigma}$ obtained from the SVD decomposition contains three singular values. Every singular value corresponds to an approximation of the matrix of the rank $k = 1, \ldots, 3$. Approximation of the first rank is defined by the formula (1). As for the analysed calculation model, the following is obtained:

$$\underset{\mathbf{A}'}{\begin{bmatrix} 0.95 & 1.06 & 0.51 \\ 0.74 & 0.83 & 0.40 \\ 0.74 & 0.83 & 0.40 \\ 0.70 & 0.79 & 0.38 \\ 0.35 & 0.39 & 0.19 \\ 0.22 & 0.24 & 0.12 \end{bmatrix}} = 2.598 \times \underset{\mathbf{u}_1}{\begin{bmatrix} -0.5805 \\ -0.4552 \\ -0.4550 \\ -0.4310 \\ -0.2132 \\ -0.1328 \end{bmatrix}} \times \underset{\mathbf{v}_1}{\begin{bmatrix} -0.6273 & -0.7013 & -0.3387 \end{bmatrix}}. \tag{17}$$

The obtained matrix $\mathbf{A}$ contains partial indices W'_R, W'_M, W'_S that are perfectly mutually correlated (Fig. 6). The results that are shown in the Fig. 6 are slightly different in comparison to mutually-correlated indices (Fig. 4a).

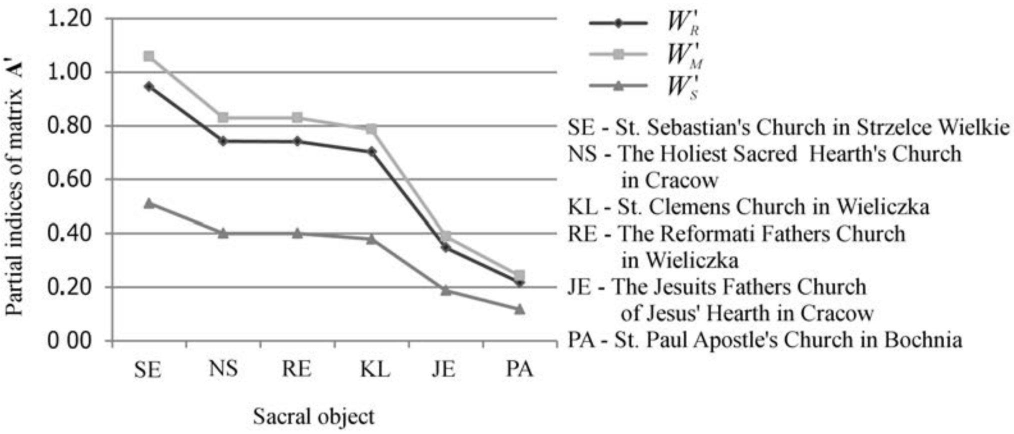

Fig. 6. Partial indices of matrix $\mathbf{A}'$.

According to the relation (3), the values of the index W'_{SAP} for every church as the components of the vector $\mathbf{b}$ of the right side of the Eq. (7) are calculated. The weights for every partial index are calculated in Matlab software by means of the computation procedure described in the Subsec. 2.2. The obtained solution vector has the following form:

$$\mathbf{x}^* = [0.3880 \;\; 0.4381 \;\; 0.3314]. \tag{18}$$

After the weights are applied to the formula (11) the single number index is defined by the following relation:

$$W_{SAP_i} = \sum_{j=1}^{3} x_{ji} W_{ji} = 0.39 W_R + 0.44 W_M + 0.33 W_S. \tag{19}$$

Table 1 contains values of the W_R, W_M, and W_S partial indices calculated for six real interiors of Roman Catholic churches, as well as the computed values of the single number index for two cases: W'_{SAP} calculated according to the formula (3) featuring the singular value σ_1 and the approximate matrix $\mathbf{A}'$, and W_{SAP} calculated according to the formula (20) featuring input partial indices (from the matrix $\mathbf{A}$). The comparison proves that the W'_{SAP} and W_{SAP} values are very similar.

Table 1. Partial indices and single number indices for acoustic assessment of sacral interiors W_{SAP} and W'_{SAP}.

Church	Symbol	Volume [m^3]	RT	C_{80}	RASTI	W_R	W_M	W_S	σ_1	weights			W'_{SAP}	W_{SAP}
										x_1	x_2	x_3		
St. Sebastian's Church in Strzelce Wielkie	SE	1102	1.4	2.5	0.53	1	1	0.53					0.97	1.00
The Holiest Sacred Heart's Church in Cracow	NS	2750	2.6	−1.8	0.37	0.78	0.81	0.37					0.76	0.78
The Reformati Fathers Church in Wieliczka	RE	4450	3	−1.6	0.38	0.74	0.84	0.38	2.598	0.39	0.44	0.33	0.76	0.78
St. Clemens Church in Wieliczka	KL	6380	2.8	−2.7	0.35	0.78	0.73	0.35					0.72	0.74
The Jezuits Fathers in Cracow	JE	9120	6	−4.3	0.27	0.1	0.57	0.27					0.36	0.38
St. Paul Apostle's Church in Bochnia	PA	13740	8.1	−6.5	0.17	0	0.41	0.17					0.22	0.24

Figure 7 presents the single number assessment of the acoustic quality of church interiors by means of the index W'_{SAP} and partial indices.

According to Fig. 7, despite the fact that the input indices are mutually correlated after applying the formula (3) obtained through the SVD decomposition or the equivalent formula (20) the assessment by means of the index W_{SAP} provides good results as a broad result range – for a church with good acoustics W_{SAP} is equal to 1, whereas for a church with bad acoustics W_{SAP} is close to 0, i.e., 0.2.

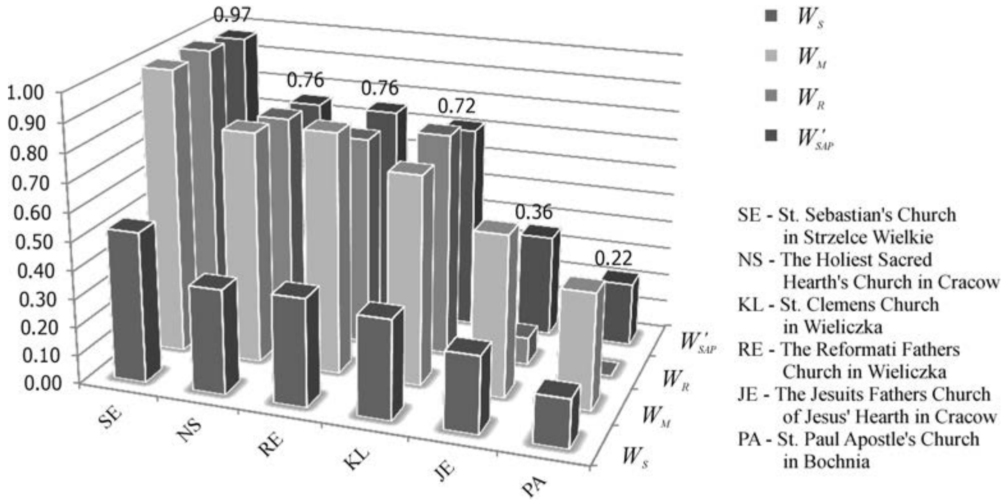

Fig. 7. Single number assessment of the acoustic quality of church interiors by means of the index W_{SAP} in comparison to the partial indices (1 – very good acoustic properties, 0 – bad acoustic properties).

4. Conclusions

The paper presents an innovative approach for the index assessment of the acoustic properties of churches. The method-related tests involved three correlated partial indices as the components of the single number approximate assessment of the acoustic properties of a church. It was possible to perform the single number assessment through application of the singular value decomposition of the object and index observation matrix. The singular value as a component of the single number assessment was obtained from the SVD decomposition. The computed pseudoinverse matrix used for the inverse methods made it possible to set the formula for the single number acoustic quality by means of the weights of partial indices. The proposed method was verified by experiments in several selected churches. Despite the fact that the partial indices are correlated, it has been proved that it is possible to perform the single number assessment using the single number index with a broad number range by means of the SVD method.

An application of the proposed assessment method for simulation test featuring geometrical models of churches will be presented in the next publication.

It is planned to expand the observation matrix through applying more tested churches in the matrix and creating other partial indices. Then new uncorrelated indices with the single number index of selected acoustic parameters will be the components of the new global assessment of the acoustic quality of churches.

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