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Nonlinear Distortions in Electroacoustic Devices

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The task of electroacoustic devices is a transmission of audio signals. The transmitted signal should be distorted as little as possible. Nonlinear distortions are the distortions depending on signal level. The types of nonlinear distortions as well as their measures are presented in the paper. The weakest device in an electroacoustic chain is a loudspeaker. It causes the greatest degradation of the signal. It is usually the most nonlinear part of the electroacoustic system. The nonlinearities in loudspeakers are described in details. Other types of nonlinear distortions as transient intermodulation in power amplifiers and distortions caused by the A/C sampling are also presented.

Keywords: nonlinear distortion, electroacoustics.

1. Introduction

The nonlinear distortions are important cause of degradation of the quality of transmitted audio signals. They can appear in various places of an electroacoustic chain. The paper presents some aspects of evaluation of nonlinear distortions, they causes, description and measurement. The paper is a review of recent works realized by many scientific teams around the world, among others by the Author and his team. First publications of the Author concerned the nonlinear distortion in loudspeaker for low frequencies (Dobrucki, Szmal, 1986; Dobrucki, 1988; 1992; 1994; 1995). In these works the behavior of loudspeaker under harmonic excitation has been developed. Nonlinearity produces not only harmonics, but also constant component, which is a good detector of a cause of nonlinearity. For some frequency ranges and high-amplitude excitation the dynamic instability appear and subharmonics and deterministic chaos is observed. The research in years 2000–2010 concerned mainly modeling of nonlinearities in loudspeakers (together with Pruchnicki) (Dobrucki, Pruchnicki, 1999; 2001; 2003;

Pruchnicki, Dobrucki, 1999; Pruchnicki, 2002), and measurements (together with Siczek) (Siczek, 2008; Dobrucki, Siczek, 2008; 2009; Siczek, Dobrucki, 2010). Author also realizes research about nonlinearities in loudspeaker's motor and voice-coil inductance (together with V. Lemarquand *et al.*) (Merit *et al.*, 2009; Dobrucki *et al.*, 2010). The works of other researcher and teams are considered in this review, among others W. Klippel (Klippel, 1990; 1992; 1996a; 1996b; 2006), Lemarquand (Ravaud *et al.*, 2010; Remy, Lemarquand, 2009) and Voishvillo *et al.* (Czerwinski *et al.*, 2001; 1999; Voishvillo, 2006; Voishvillo *et al.*, 2004). The paper consists of introduction, 4 main sections and conclusions. The problems connected with description of nonlinearities in audio system as well as with their measurements are presented in Sec. 2. The nonlinearities in loudspeakers are described in Sec. 3. The main causes of nonlinearity, i.e. nonhomogeneity of magnetic field in the air gap and nonlinearity of suspension stiffness has been treated marginally, because they are known very well. An extensive discussion of these issues is in (Dobrucki, 2006). Much attention has been done the nonlinearity of voice-coil inductance and produced intermodulation distortion. The influence of different causes of nonlinearity has been discussed. In Sec. 4 the transient intermodulation distortion has been described. The implementation of new original method of measurement of this distortion is also presented. In Chapter 5 the nonlinear distortion produced by quantization of analog signal during the A/C conversion process is discussed. This distortion is higher for lower values of signal. Dither (a low amplitude noise) is a tool for reduction of this type distortion.

2. Types of nonlinear distortions and their measures

Nonlinear distortions are the distortions depending on signal level. Two input signals transmitted with the nonlinear channel which have the same shape in the time domain but different amplitudes give at the output signals of different shapes. It means that new spectral components, absent in input signal, appear in output signal. Then, the description of nonlinear systems is a complex problem. We can recognize two types of nonlinearities: static, called also memoryless, and dynamic – the systems with memory. The static nonlinearity is a simple way for description of nonlinear systems, and it is often used although inaccurate. The static nonlinearity it is a nonlinear function which transform input signal into output. The dependence between input $x(t)$ and output $y(t)$, where t – time is given in following form:

$$y(t) = F[x(t)]. \tag{1}$$

Nonlinear function $F(\dots)$ usually is presented in the form of Taylor series (Straszewicz, 1976):

$$F(x) = a_0 + a_1 x + a_2 x^2 + a_3 x^3 + \dots . \tag{2}$$

This way of presentation is particularly convenient, if the input signal is a sinusoid of magnitude A, and frequency f:

$$x(t) = A \sin 2\pi ft. \tag{3}$$

Substitution (3) into (1) and (2) yields:

$$\begin{aligned}
y(t) = a_0 + a_1 \cdot (A \sin 2\pi ft) + a_2 \cdot (A \sin 2\pi ft)^2 + a_3 \cdot (A \sin 2\pi ft)^3 + \ldots \\
= \left(a_0 + \frac{1}{2}A^2 a_2 + \ldots\right) + \left(a_1 + \frac{3}{4}A^2 a_3 + \ldots\right) A \sin 2\pi ft \\
+ \left(-\frac{A^2}{2} a_2 + \ldots\right) \sin(2 \cdot 2\pi ft) + \left(-\frac{A^3}{4} a_3 + \ldots\right) \sin(3 \cdot 2\pi ft) + \ldots .
\end{aligned} \tag{4}$$

The constant component as well as higher harmonics appear at the output. All spectral components, including fundamental (first harmonic), depend on the input signal's magnitude. If the input signal is a two- or multi-tone, at the output appear, apart of harmonics of all input components, the combination tones with frequencies equal to sums or differences of harmonics. There are so called intermodulation distortions. The description of nonlinearity with Eq. (2) is possible when the function $F(x)$ is a continuous function with all derivatives. In general case the continuity is not necessary, e.g. in electronic and electroacoustic systems the following functions appear:

- Peak cutter:

$$\begin{aligned}
y(t) &= x(t) && \text{for} \quad |x(t)| < X, \\
y(t) &= X \cdot \operatorname{sign}(x(t)) && \text{for} \quad |x(t)| \geq X.
\end{aligned} \tag{5}$$

- Noise gate:

$$\begin{aligned}
y(t) &= x(t) - x0 \cdot \operatorname{sign}(x(t)) && \text{for} \quad |x(t)| > x0, \\
y(t) &= 0 && \text{for} \quad |x(t)| \leq x0.
\end{aligned} \tag{6}$$

The peak cutter processes the sinusoidal signal in the following way (Fig. 1).

Distortion depends on the magnitude of signal A and on the threshold of cutting X. If $A \leq X$, the signal at the output remains undistorted.

The function of noise gate is presented in Fig. 2.

The output signal is distorted for all magnitudes of input signal, however the distortion is higher when the magnitude is lower. If the magnitude A is lower than the threshold of gating $x0$, the output signal is equal to zero. This type of distortion causes a big degradation of the signal. Voishvillo (2006) presented an example, when perception of distortion caused by peak cutter equal to 22% is less than the distortion caused by the noise gate equal to 2%.

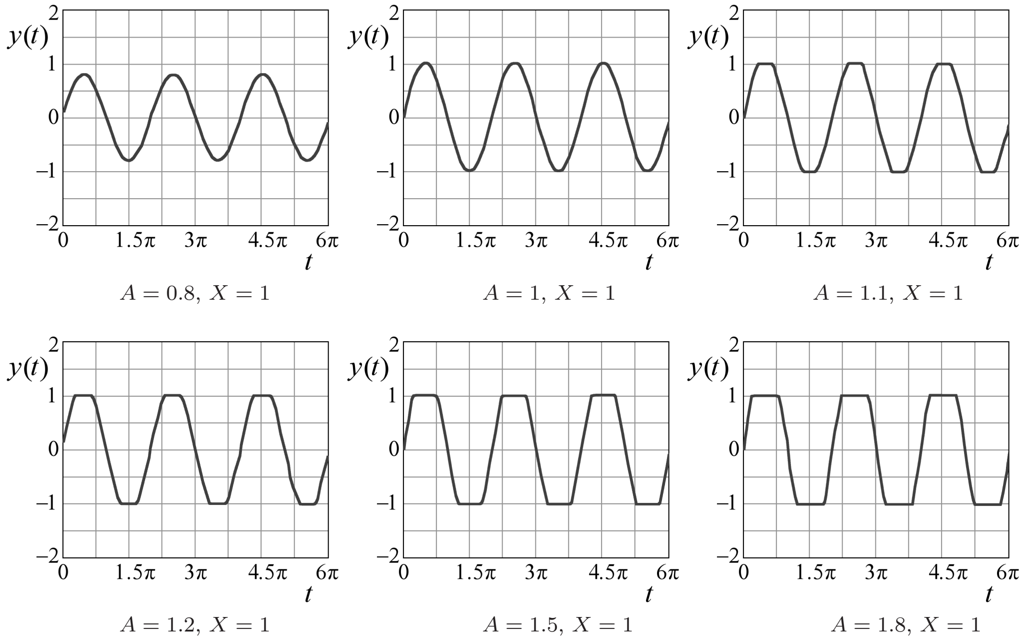

Fig. 1. Distortion of sinusoidal signal caused by peak cutter.

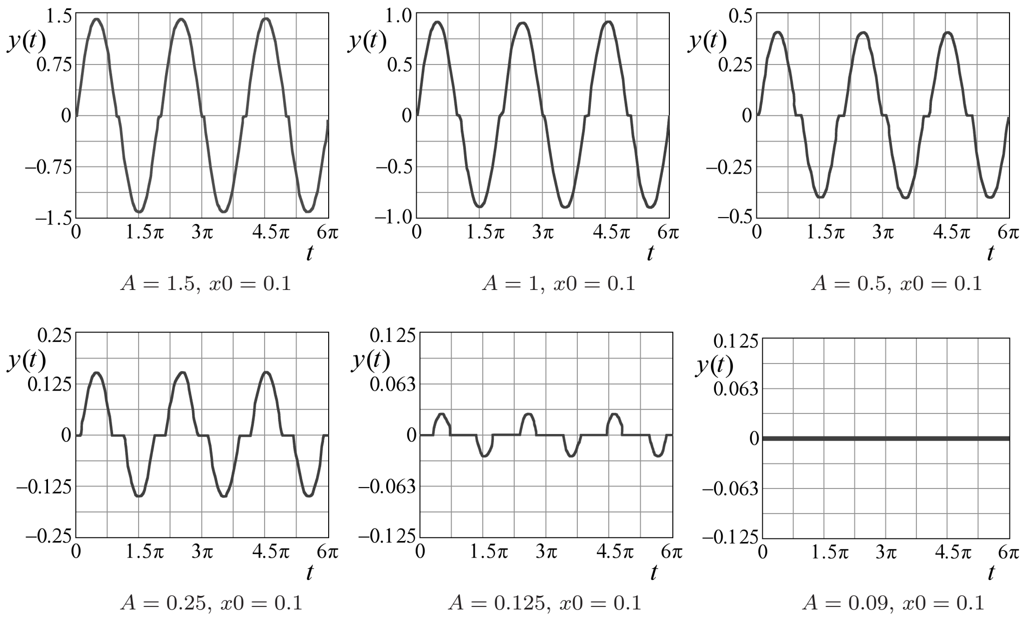

Fig. 2. Distortion of sinusoidal signal caused by noise gate.

More complex systems are systems with memory. They can be described with ordinary or partial nonlinear differential equations. These equation are derived from physical considerations, and they are valid only for particular transmitting system, e.g. loudspeaker or power amplifier. The system approach for nonlinear objects is given with the Volterra or Wiener series (Schetzen, 1989). The Volterra series is a generalization of description of transmission system using an impulse response. This Volterra series is given by the following formula:

$$y(t) = h_0 + \int_{-\infty}^{\infty} h_1(\tau)x(t-\tau)\,\mathrm{d}\tau + \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} h_2(\tau_1,\tau_2)x(t-\tau_1)x(t-\tau_2)\,\mathrm{d}\tau_1\,\mathrm{d}\tau_2$$
$$+ \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} h_3(\tau_1,\tau_2,\tau_3)x(t-\tau_1)x(t-\tau_2)x(t-\tau_3) + \ldots . \qquad (7)$$

It should be noted that the Volterra series is also generalization of the description of static nonlinearity using Eq. (2). If the input signal is sinusoidal, the substitution of sinusoidal excitation (3) into (7) yields:

$$\begin{aligned} y(t) = {} & B_0(h_0, h_2, \ldots, A) \\ & + B_1(h_1, h_3, \ldots, A) \cdot A \sin[2\pi f t - \varphi_1(h_1, h_3, \ldots, A)] \\ & + B_2(h_2, \ldots, A) \cdot A^2 \sin[2 \cdot 2\pi f t - \varphi_2(h_2, \ldots, A)] \\ & + B_3(h_3, \ldots, A) \cdot A^3 \sin[3 \cdot 2\pi f t - \varphi_3(h_3, \ldots, A)] + \ldots . \end{aligned} \qquad (8)$$

A constant component and harmonics appear in the output signal. The harmonics have magnitudes and phase shifts depending on the shape of kernels of the given nonlinearity order and higher orders of the same parity. The magnitude coefficients B_0, B_1, B_2, $B_3, \ldots$ and phases φ_1, φ_2, $\varphi_3, \ldots$ are obtained by the integration over τ_1, τ_2 etc. of products of kernels and sinusoidal and cosinusoidal harmonics of appropriate order. Similarly, if the excitation is a combination of sinusoids, the Volterra series gives as the output signal the harmonics and intermodulation components with different amplitudes and phase shifts. Each term of Volterra series (7) produces distortion of the order determined with number of arguments of the kernel and lower orders of the same parity. Then, although the interpretation of Volterra series is simple, the measurement of the kernels is difficult. This disadvantage can be omitted using orthogonal form of the Volterra series, called Wiener series. It can be presented in the form:

$$y(t) = \sum_{n=0}^{\infty} G_n[k_n, x(t)], \qquad (9)$$

where $G_n[k_n, x(t)]$ is a Wiener operator of the n-th order:

$$G_n(k_n, x(t)) = \int_{-\infty}^{\infty} \dots \int_{-\infty}^{\infty} k_n(\tau_1, \dots, \tau_n) x(t-\tau_1) \cdot \ldots \cdot x(t-\tau_n) \, \mathrm{d}\tau_1 \dots \mathrm{d}\tau_n + \sum_{m=1}^{\mathrm{int}(n/2)} \left[\int_{-\infty}^{\infty} \dots \int_{-\infty}^{\infty} k_{n-2m,n}(\tau_1, \dots, \tau_{n-2m}) x(t-\tau_1) \cdot \ldots \cdot x(t-\tau_{n-2m}) \, \mathrm{d}\tau_1 \dots \mathrm{d}\tau_{n-2m} \right], \tag{10}$$

$k_n(\tau_1, \dots, \tau_n)$ are the Wiener kernels of the n-th order, $k_{n-2m}(\tau_1, \dots, \tau_{n-2m})$ are the Wiener kernels of the order $n-2m$ produced by the kernel of the n-th order. Term of order n in Wiener series produces only distortion of this order.

The Wiener term of order 0 is a constant component occurring in output signal. The Wiener term of order 1 is the linear impulse response of the system. It is a function of one argument – the delay τ_1, which represents the influence of past values of the input signal on the current value of the output signal. The Wiener term of the order 2 is a function of two delays τ_1 and τ_2. It can be drawn as the color map on the plane. The Wiener term of the order 3 is a function of three delays τ_1, τ_2 and τ_3.

It can be presented in the form of color maps for fixed one delay. The examples of Wiener kernels of three orders measured for a loudspeaker are presented in Figs. 3, 4, and 5.

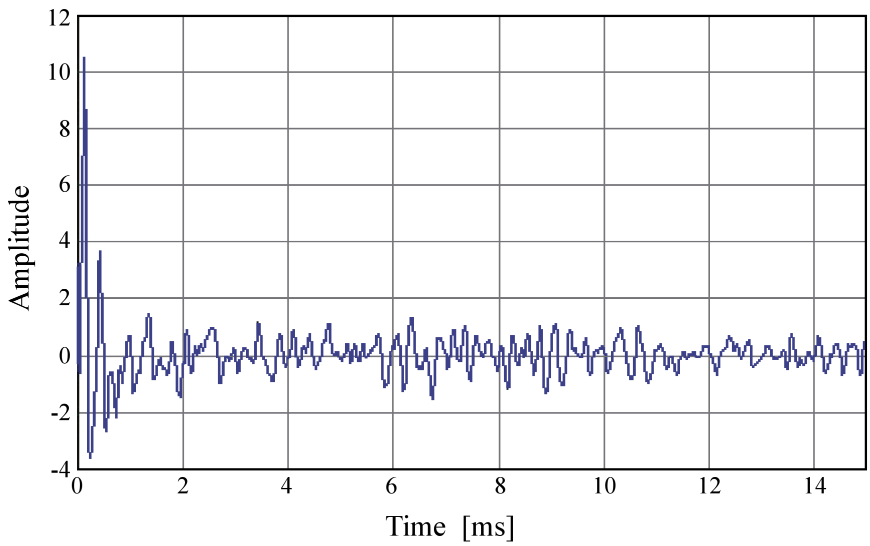

Fig. 3. Wiener kernel of the order 1 of an electrodynamic loudspeaker (Dobrucki, Pruchnicki, 2001).

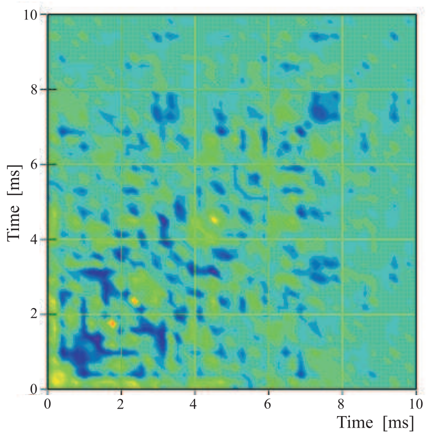

Fig. 4. Wiener kernel of the order 2 of an electrodynamic loudspeaker (Dobrucki, Pruchnicki, 2001).

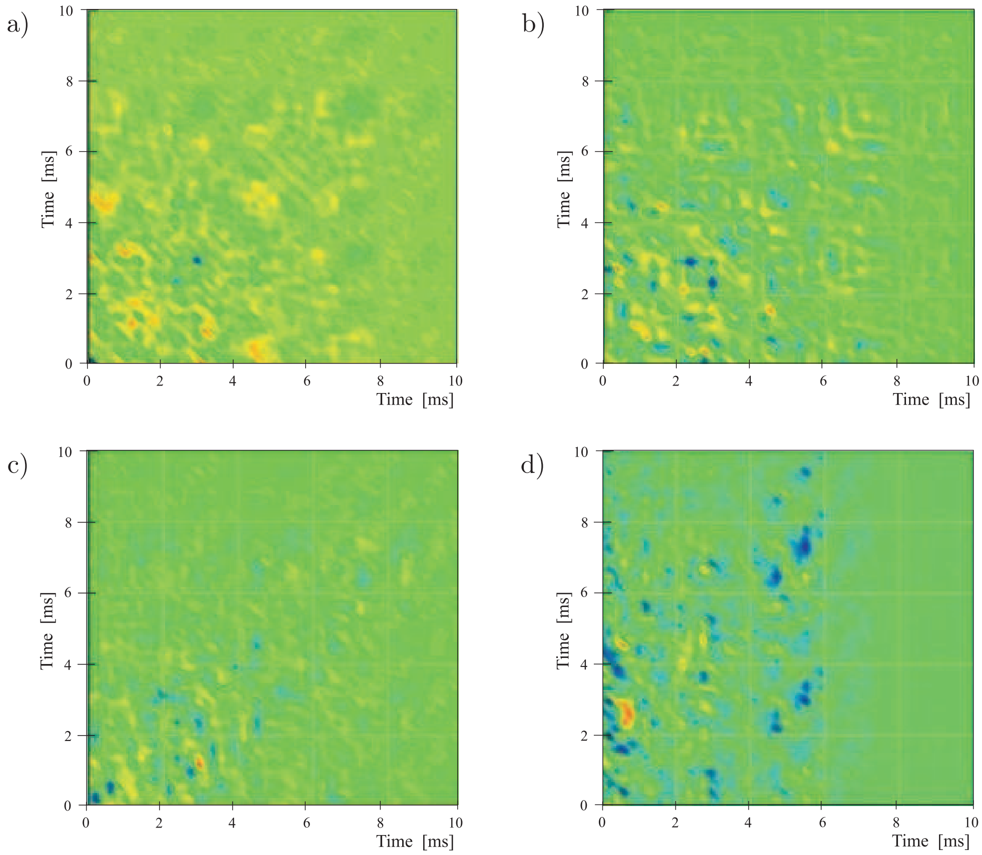

Fig. 5. Wiener kernel of the order 3 of an electrodynamic loudspeaker (Dobrucki, Pruchnicki, 2001), a) $\tau_3 = 0$, b) $\tau_3 = 7$ ms, c) $\tau_3 = \tau_2$, d) $\tau_3 = \tau_2 + 7$ ms.

The Wiener and Volterra series can be used for prediction of the response of nonlinear system on any excitation. Unfortunately, the measurement of Wiener and Volterra kernels is a time consuming process and visual representation is difficult for interpretation. Then, usually the nonlinearity of the system is evaluated using its response for specific signals. The measures of nonlinear distortions are based on this proceeding. The simplest procedure is based on the excitation of a nonlinear system with sinusoidal signal and observation of harmonic distortions. Then, the measures of nonlinear distortions are coefficients of harmonic distortions. It can be defined the coefficient of total harmonic distortion THD, or coefficients of successive harmonics: second, third etc. Total harmonic distortion is defined by the following equation:

$$\mathrm{THD} = \frac{\sqrt{\sum\limits_{i=2}^{N} y_i^2}}{\sqrt{\sum\limits_{i=1}^{N} y_i^2}}, \tag{11}$$

where y_i are magnitudes or root mean squares of successive harmonics in the output signal. Fundamental component has index i equal to 1, the distortion products have indexes $i > 1$. The coefficients of particular harmonics are defined as:

$$h_j = \frac{y_j}{\sqrt{\sum\limits_{i=1}^{N} y_i^2}}. \tag{12}$$

Most often $j = 2$ and 3. Second and third harmonics are usually the greatest and causes of harmonics of even and odd orders are different. Then, the values of coefficients of second and third harmonics can facilitate the diagnosis of the causes of nonlinearity. The coefficients of harmonics are usually drawn as functions of exciting frequency. Unfortunately, the correlation between THD and subjective evaluation of nonlinear distortion is poor. It has been proved, that human ear is more sensitive for higher harmonics. Then, sometimes the weighted THD are used (Shorter, 1950; Straszewicz, 1976; Voishvillo, 2006):

$$\mathrm{THD}_{w1} = \frac{\sqrt{\sum\limits_{i=2}^{N} \left(\frac{i}{2}\right) y_i^2}}{\sqrt{\sum\limits_{i=1}^{N} \left(\frac{i}{2}\right) y_i^2}}, \tag{13}$$

$$\mathrm{THD}_{w2} = \frac{\sqrt{\sum\limits_{i=2}^{N} \left(\frac{i}{2}\right)^2 y_i^2}}{\sqrt{\sum\limits_{i=1}^{N} \left(\frac{i}{2}\right)^2 y_i^2}}. \tag{14}$$

The correlation between THD_{w2} and subjective evaluation is the best but it is still unsatisfactory. The signal transmitted in electroacoustic channel has usually the complex spectral structure. This is the reason that measures based on sinusoidal excitation are poorly correlated with subjective evaluation of nonlinear distortion. The better results are obtained using two-tone as the exciting signal (Fig. 6). The intermodulation products between tones of high and low frequencies give impression of high degradation of sound quality.

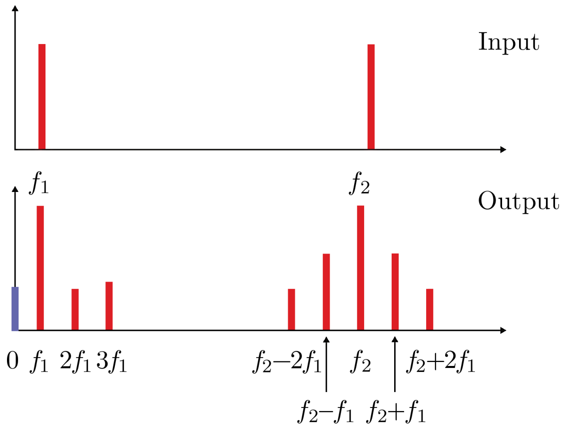

Fig. 6. Spectra of excitation (top) and response (bottom) for nonlinear system.

According to the International Standard (IEC Publ. 60268-5, 2007) the frequency of high component of input signal should be at least 8 times higher than the frequency of low component. The coefficient of intermodulation distortion is defined as:

$$\mathrm{ID} = \sqrt{\frac{y^2_{(f_2-f_1)} + y^2_{(f_2+f_1)} + y^2_{(f_2-2f_1)} + y^2_{(f_2+2f_1)} + \ldots}{y^2_{f_2} + y^2_{(f_2-f_1)} + y^2_{(f_2+f_1)} + y^2_{(f_2-2f_1)} + y^2_{(f_2+2f_1)} + \ldots}}\,. \tag{15}$$

The more general results are obtained for a non-harmonic multitone as an exciting signal. The method based on multitone was proposed by Voishvillo *et al.* (Czerwinski *et al.*, 2001). Figure 7 shows the spectrum of output signal when a 10-tone with frequencies logarithmically increasing in the range between 50 and 510 Hz (Czapiewski, 2009). The components of this multitone at the input had equal amplitudes, the differences of amplitudes at the output result from the frequency response of the system (a dynamic loudspeaker). The response of the nonlinear system contains a large number of intermodulation products.

The continuous line at the Fig. 7 presents the product of multitone nonlinear distortion averaged in 1/3-octave frequency bands:

$$h_{\mathrm{MTND}}\left(f_i\right) = \sqrt{\frac{1}{K}\sum_{k=1}^{K} y_k^2}, \tag{16}$$

where K is number of product components in i-th 1/3-octave band with center frequency f_i. The dashed line represents total energy in successive 1/3-frequency bands:

$$h_{\mathrm{MTNDE}}(f_i) = \sqrt{\sum_{k=1}^{K} y_k^2}. \tag{17}$$

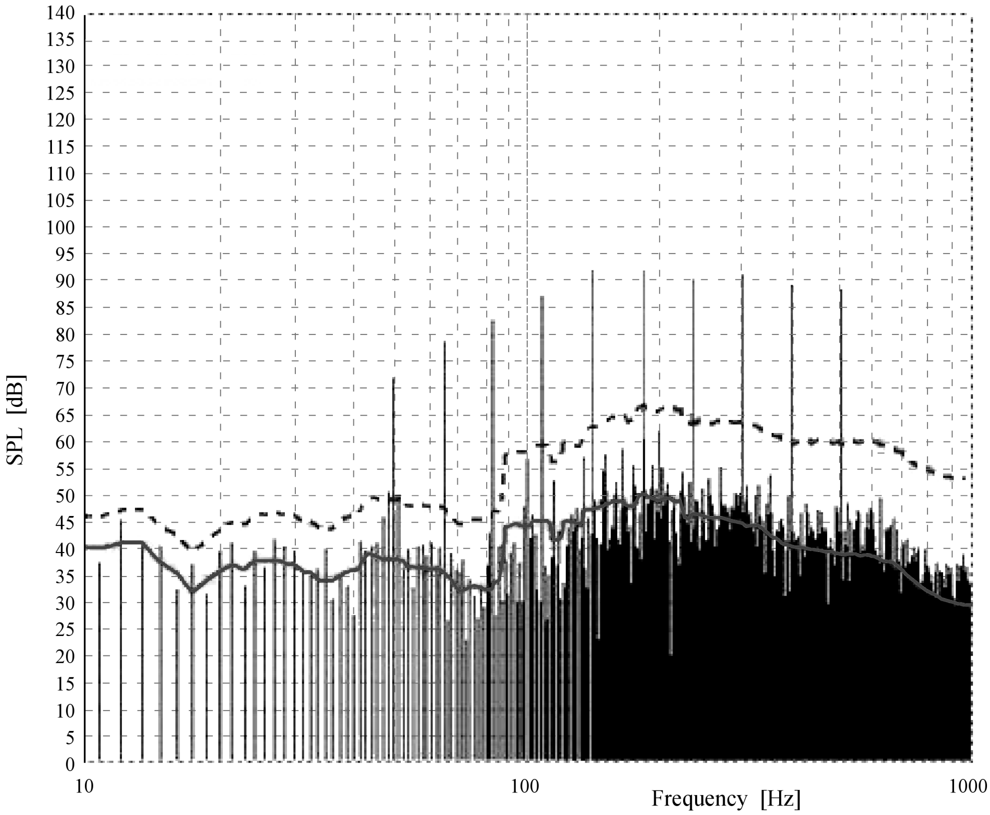

Fig. 7. Product of multitone distortion.

Of course, h_{MTNDE} is always greater than h_{MTND}. For calculation of both measures, the components with frequencies of excitation were not taken into account.

The stationary signal which has the spectral structure most similar to the program usually transmitted in an electroacoustic channel is a broadband noise. Unfortunately, the spectrum of product of nonlinearity appears in the same frequency range as the program and it is covered by the program. Wolf (1953) developed the method of measurement of nonlinear distortion using broadband program as the excitation signal. The principle of Wolf's method is presented in Fig. 8. (Siczek, 2008).

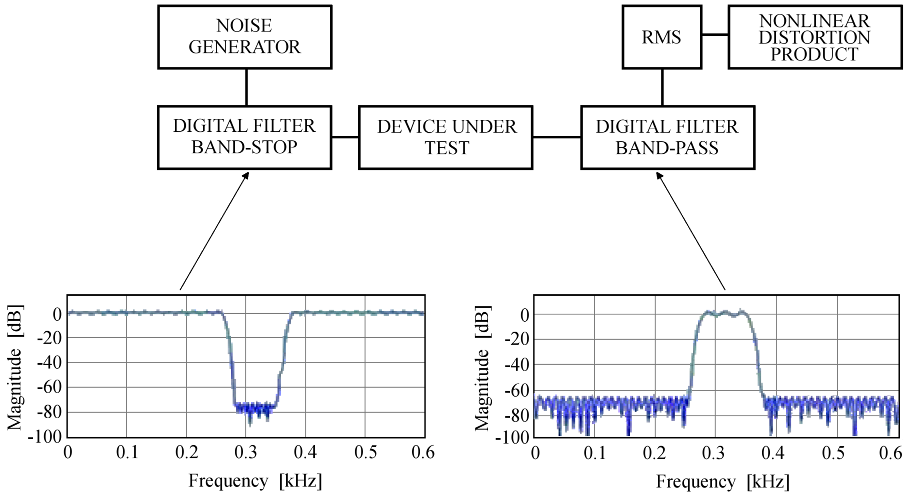

Fig. 8. The setup for measurement of nonlinear distortion with a broadband noise.

The narrow frequency band is cut out from the broadband noise with a band-stop filter. This signal is given to the input of measured device. The narrow frequency band of the same center frequency as band-stop filter is cut out from response of the measured system with a band-pass filter. If the system under test is linear it does not produce any component in this frequency band. Then, the signal measured in this frequency band is a product of nonlinearity. Changing the frequency band it can be obtained dependence of the product of nonlinearity on frequency. Unfortunately, the band-stop filter at the input and band-pass one at the output are not ideal, and they have a finite slope in rejected frequency band. It causes the occurring of a residual signal which disturb the result of measurement. The level of this residual signal should be significantly less than the measured product of nonlinearity. In order to fulfill this condition the slopes of both band-stop and band-pass filters should be very high and the bandwidth of the band-pass should be narrower than the bandwidth rejected with the band-stop filter. These requirements can be fulfilled using digital filters in the measuring arrangement. Figure 9 presents the signals of noise from generator, noise with removed 5/9-octave frequency band which is the excitation and product of nonlinearity obtained after band-pass filter with 1/3-octave bandwidth. The center frequency of both filters was 315 Hz.

It should be noted, that the levels of signals in Fig. 9a and b are almost the same. The difference of both levels can be neglected even for bands in high frequency range. The signal in Fig. 9c has the level very low in comparison with levels of signals from Fig. 9a and b. However, this level is higher than level of residual noise and it is the product of nonlinearity. The problems with high requirements for applied filters caused, that the relatively old method was not used for a long time, and it has been forgotten.

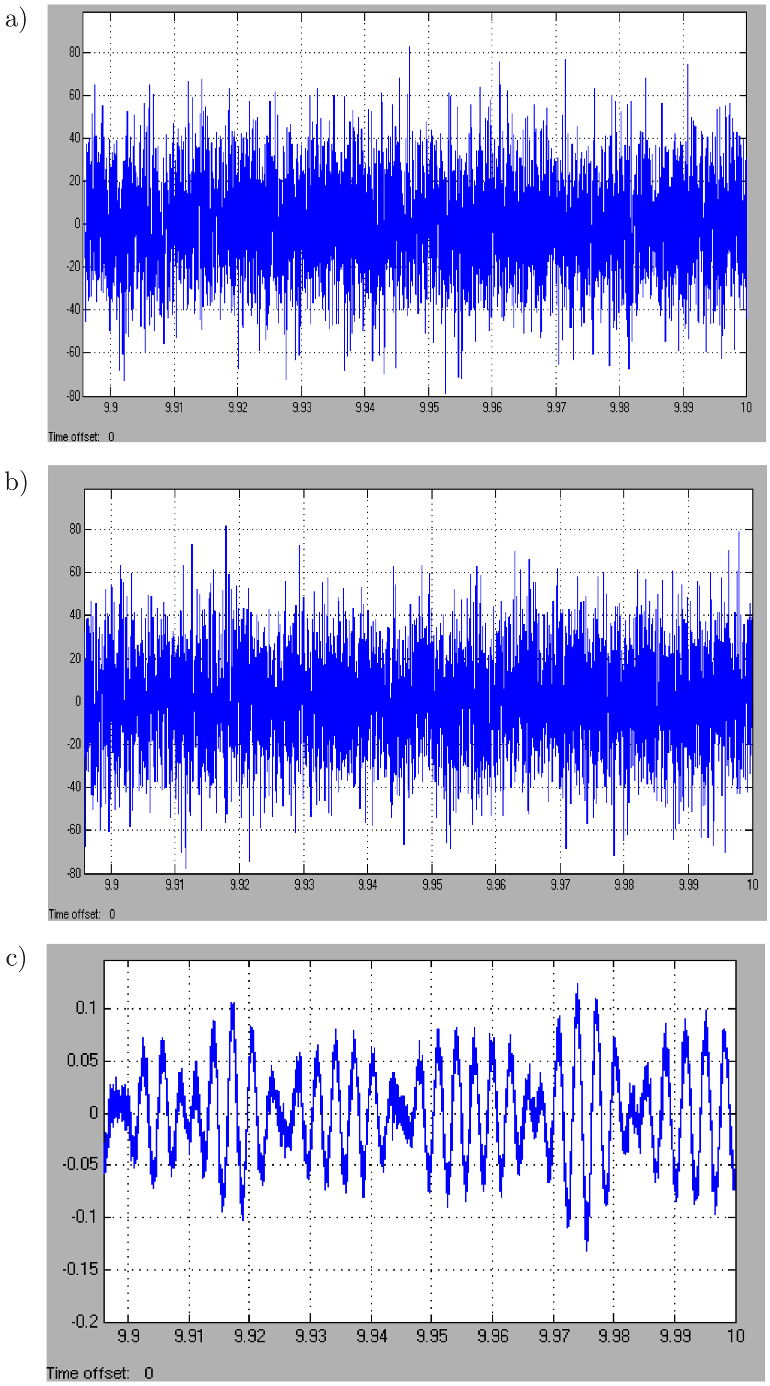

Fig. 9. Signals in various points of setup for measurement of nonlinear distortion using Wolf's method, a) signal from noise generator, b) signal after band-stop filter, c) signal after band-pass filter. X-axis: time in seconds, Y-axis: amplitude in arbitrary units.

3. Nonlinear distortion in loudspeakers

Loudspeaker is a weakest link in an electroacoustic channel. The cause of this fact is among others nonlinear distortion. There are many causes of the nonlinear distortions in loudspeakers. There are: electric – nonlinearity of voice-coil inductance (dependence of inductance on electric current and displacement of voice-coil), magnetic – nonhomogeneity of the magnetic flux density in the air gap, mechanic – dependence of a stiffness of surround on displacement, and acoustic – nonlinearity of propagation of the acoustic wave (KLIPPEL, 1996a; 1996b; ŻÓŁTOGÓRSKI, 1999a; 1999b; CZERWINSKI *et al.*, 1999) and Doppler effect (ŻÓŁTOGÓRSKI, 1993). Most effects depend on a displacement of moving system. This displacement is the larger for low frequencies. Then, the nonlinear distortions are the highest in low frequency range. For low frequencies loudspeaker can be considered as the system of lumped parameters – described with a system of ordinary differential equations. These equations can be derived from an equivalent electrical circuit. The system of nonlinear differential equations describing the moving system of the loudspeaker has the form (DOBRUCKI *et al.*, 2010):

$$\begin{aligned}
&\left[L_E(x,i)+\frac{\mathrm{d}L_E(x,i)}{\mathrm{d}i}\cdot i\right]\frac{\mathrm{d}i(t)}{\mathrm{d}t} \\
&\qquad = U(t)-R_E\cdot i(t)-Bl(x)\cdot v(t)-\frac{\mathrm{d}L_E(x,i)}{\mathrm{d}x}\cdot i(t)\cdot v(t), \\
&\frac{\mathrm{d}x(t)}{\mathrm{d}t}=v(t), \\
&\frac{\mathrm{d}v(t)}{\mathrm{d}t}=-\frac{r}{mms}\cdot v(t)-\frac{k(x)}{mms}x(t)+\frac{Bl(x)}{mms}\cdot i(t) \\
&\qquad\qquad +\frac{1}{2mms}\cdot\frac{\mathrm{d}L_E(x,i)}{\mathrm{d}x}\cdot i^2(t),
\end{aligned}\tag{18}$$

where t – time, x – displacement, v – velocity, i – electric current, L_E – voice-coil inductance, R_E – voice-coil resistance, Bl – force factor (product of a flux density in the air gap and length of the voice-coil's wire), mms – mass of the voice coil, r – mechanical resistance of the surround, k – mechanical stiffness of the voice-coil, U – exciting voltage. The acoustical pressure is the quantity which is measured and used for determination of the signal quality. The dependence of the acoustical pressure on the velocity of the moving system of the loudspeaker is given by the following formula (DOBRUCKI, 2006):

$$p(t)=\rho\frac{S}{2\pi d}\cdot\frac{\mathrm{d}v(t)}{\mathrm{d}t},\tag{19}$$

where p – acoustic pressure, S – effective surface of the loudspeaker's diaphragm, ρ – density of air, d – distance of the observation point (usually $d = 1$ m).

The nonlinearity of the voice-coil is usually neglected in low frequency range because of low value of the electrical impedance of the voice-coil. However, the intermodulation distortion caused by the voice coil can be cause of significant degradation of the signal quality. The calculation has been done for the following dependence of the voice-coil dependence on electric current:

$$L_E(i) = L_{E0}(1 + ai + bi^2). \tag{20}$$

Following parameters were used for modeling: $L_{E0} = 1.7$ mH, $a = –1.68$ mH/A, $b = 7.58$ mH/A^2, $R_E = 3.3\ \Omega$, $mms = 0.009$ kg, $r = 1.1$ N·s/m, $k = 7.692$ N/m, $Bl = 5.5$ T·m. The system was excited with two-tone of frequencies 50 Hz and 1250 and voltages 10 V and 2.5 V, respectively. The spectrum of the acoustic pressure is presented in Fig. 10 (Dobrucki *et al.*, 2010).

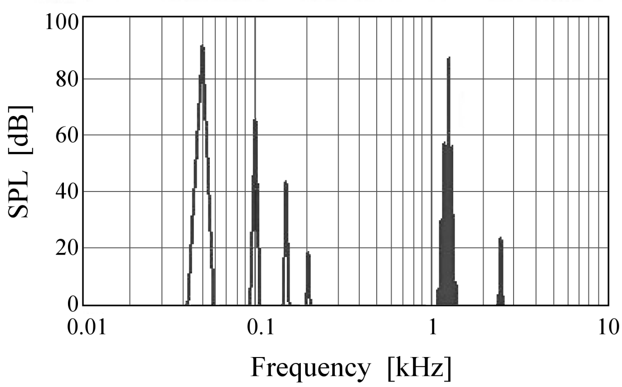

Fig. 10. Intermodulation distortion produced by the system with nonlinear inductance, depending on electric current. Excitation: $f_1 = 50$ Hz, $U_1 = 10$ V, $f_2 = 1250$ Hz, $U_2 = 2.5$ V.

In order to test the influence of different causes of nonlinearity depending on displacement, the modeling has been done for the following nonlinear inductance, stiffness and force factor:

$$L_E(x) = L_{E0}\left(1 - 200x + 20000x^2\right), \tag{21}$$

$$k(x) = k_0\left(1 + 200x + 20000x^2\right), \tag{22}$$

$$Bl(x) = Bl_0\left(1 + 200x - 20000x^2\right). \tag{23}$$

The values L_{E0}, k_0 and Bl_0 are the same as in previous example. The nonlinear parameters have the same absolute values, the different signs result from typical dependences of considered parameters on the displacement. The results of modeling are presented in Figs. 11, 12 and 13.

The highest distortion is produced by nonlinear force factor. The lowest harmonics of lower frequency is produced by nonlinear inductance. However, the nonlinear inductance is the cause of the highest second harmonic of higher fre-

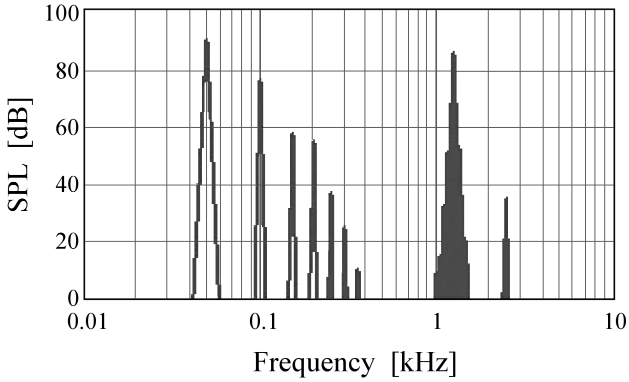

Fig. 11. Harmonic and intermodulation distortion produced by the system with nonlinear inductance, depending on displacement according to Eq. (21). Excitation the same as in Fig. 10.

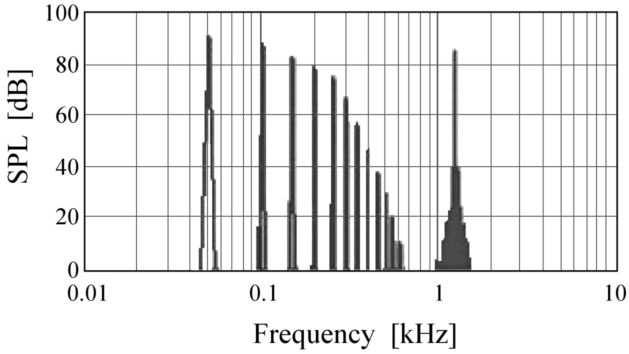

Fig. 12. Harmonic and intermodulation distortion produced by the system with nonlinear stiffness, depending on displacement according to Eq. (22). Excitation the same as in Fig. 10.

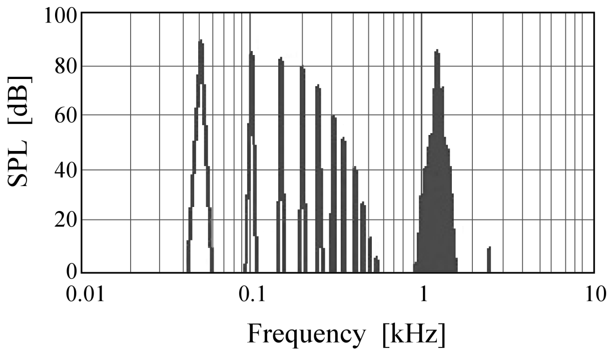

Fig. 13. Harmonic and intermodulation distortion produced by the system with nonlinear *Bl*, depending on displacement according to Eq. (23). Excitation the same as in Fig. 10.

quency. The lowest intermodulation is produced by nonlinear suspension stiffness. Then, the nonlinearity of voice-coil inductance should be taken into account in consideration of nonlinear distortions in loudspeakers.

4. Transient intermodulation distortion in audio power amplifiers

Other type of nonlinear distortion are produced by power amplifiers. In audio amplifiers the negative feedback is usually used. It improves linearity of amplifiers and reduces noise. A part of output signal is fed in antiphase to the input. The operation of the feedback circuit is not immediate, a delay occurs between the output and the input. If the input signal changes rapidly (i.e. during transmission of transients) this delay causes that both signals: input and feedback are in the same phase. The amplification increases rapidly and causes momentary overload of the amplifiers (Otala, 1972). This type of distortion is called transient intermodulation distortion (TIM). In transistor amplifiers the feedback is usually deeper than in vacuum tube ones, the TIM is higher in transistor amplifiers. It causes higher degradation of the signal quality. It is a cause that audiophiles prefer vacuum tube amplifiers. The measurement of TIM is not simple. Neither the sinusoidal signal nor a multitone do produce transient intermodulation distortion. The International Standard (IEC Publ. 60268-3, 2000) defines a method for measurement of nonlinear distortion using the rectangular signal of frequency 3.15 kHz and sinusoidal signal of frequency 15 kHz. The peak-to-peak values ratio of rectangular and sinusoidal signals is equal to 4:1. The springs of rectangular signal are the transients. Their interaction with sinusoid produces TIM. The product of TIM are the sum and difference frequencies of sinusoid and successive harmonics of rectangle: 3.15 kHz, 6.3 kHz, 9.45 kHz, 12.6 kHz, 15.75 kHz etc. The product of TIM have frequencies: 0.75 kHz (a difference between 5th harmonic of rectangle and sinusoid), 2.4 kHz (a difference between sinusoid and 4th harmonic of rectangle), 3.9 kHz, 5.55 kHz etc. Two first frequencies are lower than the fundamental component and they can be extracted using low-pass filter of cutoff frequency 2.9 kHz. The intermodulation products can be also observed with frequency analyzer. The result of measurement of a vacuum-tube power amplifier with the pentode GU50 is presented in Fig. 14 (Dobrucki *et al.*, 2009; Maleczek, 2009).

The increase of TIM product with output power can be observed.

The interesting method of measurement of TIM distortion has been developed by Antognetti *et al.* (1981). The method is called an "inverted sawtooth" – IS. The idea of this method is presented in Fig. 15.

The fast changing slope is the transient. It can be either decreasing or increasing slope. If the transient is decreasing (Fig. 15a), TIM causes that the minimum (negative peak) slightly increases and positive constant component appear. The constant component is drawn in Fig. 15 with a continuous thick line. The neutral value (lack of constant component) is drawn with thin line. If the transient is increasing (Fig. 15b), TIM causes that the maximum (positive peak) slightly decreases and negative constant component appear. If the slopes are periodically switched, the constant component changes from positive to negative. If the saw-

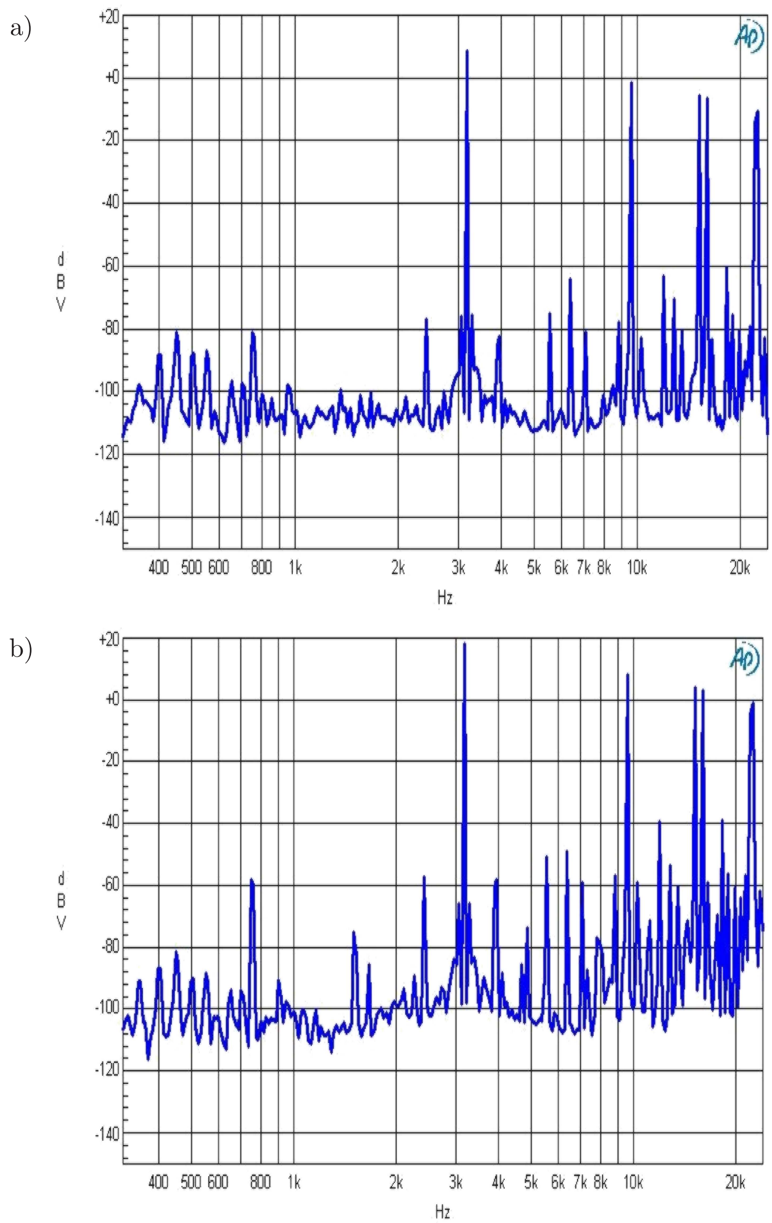

Fig. 14. TIM distortion in amplifier with pentode GU50 a) output power 1 W, b) output power 10 W.

tooth signal is removed with low-pass filter, the rectangular signal appears at the output.

The measuring setup has been developed by Grzesiak (2010). The sawtooth frequency is 20 kHz, and slopes are inverted each 256 period. The rectangu-

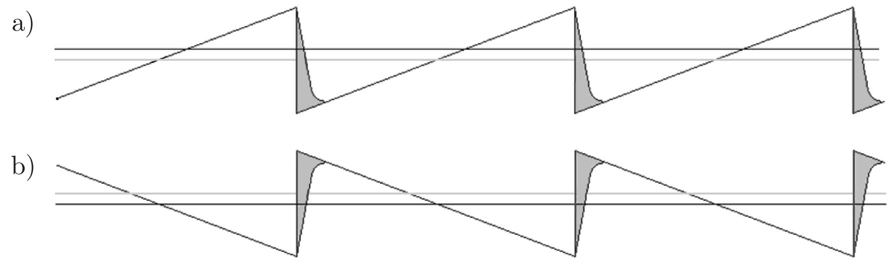

Fig. 15. Sawtooth signals used in the IS method, a) increasing slope slow, decreasing – fast, b) decreasing slope slow, increasing – fast.

lar signal of frequency ca. 78 Hz is filtered with a low-pass filter of cutoff frequency 250 Hz. The measured oscillogram of this signal is presented in Fig. 16 (Dobrucki, Grzesiak, 2010).

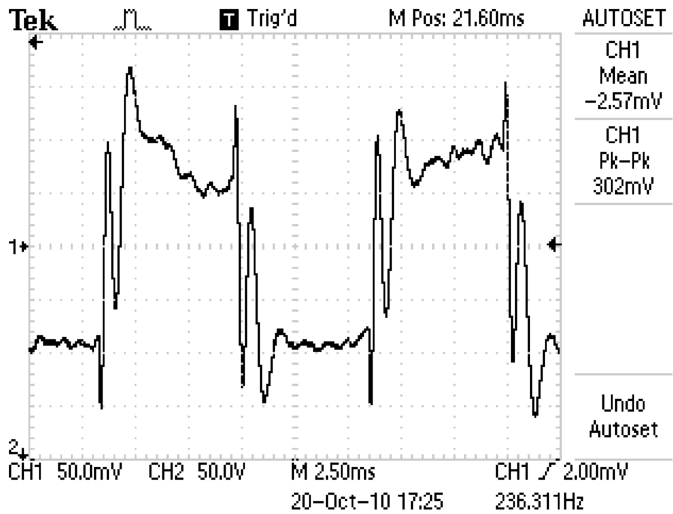

Fig. 16. Oscillogram of output signal in IS method (X-axis: time, Y-axis: voltage).

A measure of TIM is the ratio of peak-to-peak values of rectangular signal after filtering to sawtooth signal before filtering:

$$\mathrm{TIM} = \frac{U_{Rp-p}}{U_{Sp-p}} \cdot 100\%, \tag{24}$$

where U_{Rp-p} is rectangular peak-to-peak voltage, U_{Sp-p} is sawtooth peak-to-peak voltage.

The experiment shows, that IS method is more sensitive that the standard method. The TIM values of 0.1% can be detected.

5. Distortions in digital signals

The process of analog to digital converting consists of three main stages. In the first stage, called sampling the actual values of analog (continuous) signal are taken at regular intervals. The time interval between two successive samples is an inverse of sampling rate. In the next step, called quantization the sample values are assigned to integers. Last step of A/C process is coding – the quantized samples are transferred into a sequence of zeros and ones. During the reverse process digital to analog, the analog signal is reconstructed from its digital representation. The most sensitive step of D/A conversion is the quantization. The difference appears between an actual value of sample and its quantized representation. This difference changes in time and it is a quantization noise. The conversion is more accurate when number of levels of quantization increases. For example for 8-bit converter, the number of levels is equal to $2^8 = 256$ and 256 different values can be recognized. For a 16-bit converter, this number is equal to $2^{16} = 65536$, i.e. this number of different values can be coded. The signal-to-noise of quantization ratio can be determined from a simple formula

$$\mathrm{SNR} \approx 6 \cdot n \text{ dB}, \tag{25}$$

where n is number of levels of quantization.

A problem appears when n is a low number or when the signal is very low and only a few least significant bits are used in quantization. With low signal level, the quantization error is not random, it's character changes as it becomes correlated to the input signal, and potentially audible distortion results. The shape of signal after reconstruction differs significantly from original analog signal. The "stairs" are visible in the form of the signal (Figs. 17a and 18a). Spectrum of the analog sinusoid it is a single component. The spectrum of reconstructed signal contains many components, i.e. nonlinear distortion occurs (Figs. 17b and 18b). In the extreme case when only one least significant bit is applied in quantization, the form of such signal becomes rectangular instead of sinusoidal.

The nonlinear distortion caused by quantization process can be significantly reduced using dither (Vanderkooy, Lipshitz, 1987). Dither is a noise of very low amplitude, 1/2 least significant bit, added to the original signal. Dither influences the quantized signal modifying its values of $+1$ or -1 level (Fig. 19).

Optimal probability density function for a dither is triangle (Vanderkooy, Lipshitz, 1992). It can be observed, that nonlinear isolated components of signal with dither disappeared, but the noise floor increased. A noise degrades the quality of signal less than the nonlinear distortion. The noise spectrum is often transferred into high frequencies, beyond of the hearing range of human ear, using noise shaping. The specific types of nonlinear distortions called idle tones (typically in a form of limit cycles oscillations) often occur in the sigma-delta A/C converters. They will yield spurious spectral lines on the output, and potentially audible distortion. Dithering technique is employed to spread out the energy of

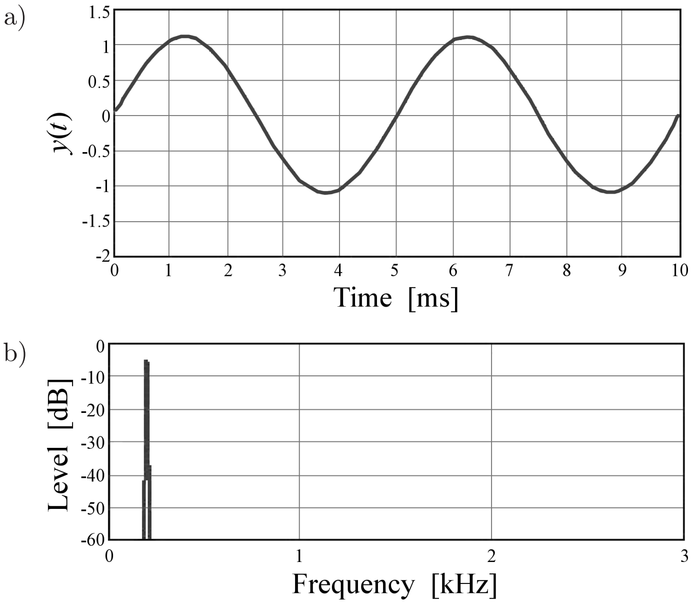

Fig. 17. Analog sinusoidal signal of frequency 200 Hz, a) form of the signal, b) spectrum.

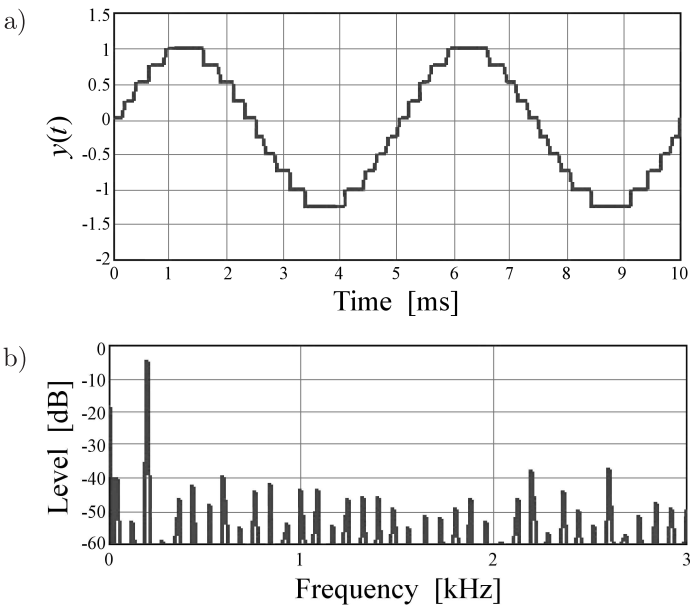

Fig. 18. Sinusoidal signal reconstructed from digital one quantized with 10 levels, a) form of the signal, b) spectrum.

the idle tone to out of the range of the signal band, but this increases the overall level of baseband noise.

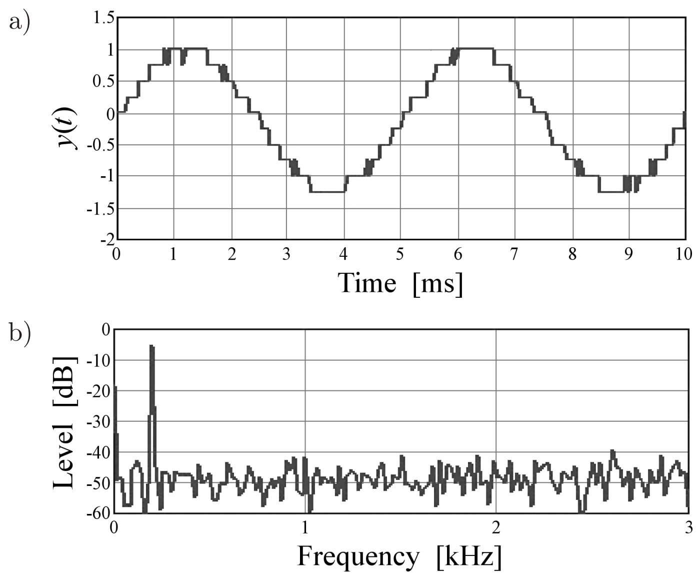

Fig. 19. Reconstructed sinusoidal signal with dither, a) form of the signal, b) spectrum.

6. Conclusions

The different types and causes of nonlinearities have been discussed in the paper. The nonlinear distortion depends on the amplitude of exciting signal. Usually it increases when signal level increases, but it is not a general rule. Two examples of distortions which increases when the signal level decreases has been given. There are noise gate and A/C quantizer. Problem of measuring of nonlinear distortion in order to an objective measure give the results correlated with subjective evaluation is still waiting for the solution. The highest distortion is produced by loudspeaker. Analysing nonlinearity causes in loudspeaker, the nonlinear voice-coil inductance (depending on current and on displacement) should be taken into account. This type of nonlinearity produces particularly high intermodulation product. The signal quality can be also degraded with transient intermodulation distortion. The nonlinear distortion in digital system can be effectively reduced using a dither. However, the noise floor increases when dither is used.

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