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Wave Generation by a Finite Baffle Array in Application to Beam-Forming Analysis

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Directional excitation of sound in an aperiodic finite baffle system is analyzed using a method developed earlier in electrostatics. The solution to the corresponding boundary value problem is obtained in the spatial-frequency domain. The acoustic pressure and normal particle velocity distribution in acoustic media can be easily computed by the inverse Fourier transform from their spatial spectra on the baffle plane. The presented method can be used for linear acoustic phased arrays modeling with finite element size and inter-element interactions taken into account. Some illustrative numerical examples presenting the far-field radiation pattern and wave-beam steering are given.

Keywords: phased array, beamforming, mixed boundary-value problem, BIS-expansion.

1. Introduction

The role of phased array transducers in ultrasound diagnostics and nondestructive testing can be hardly overestimated (see (Tasinkevych, Danicki, 2010) and references therein). The full-wave analysis of the periodic baffle system for acoustical beamforming applications was presented in the previous work (Tasinkevych, Danicki, 2010). A similar boundary-value problem is considered for the case of a finite system in the current paper. The wave excitation case is examined specifically. However, the scattering or sound detection can also be addressed by this method. The structure consists of a finite number of acoustically hard baffles (strips) separated by acoustically soft domains. A similar system modeling the phased array transducer was analyzed for example in (Kuhnicke, 2007). In this paper we deal with a mixed boundary-value problem: the normal acoustic vibration vanishes on baffles and between them the acoustic pressure is

given constant values, which models the wave-beam generation. A wave field excitation by a uniform harmonic pressure distribution is not novel and was earlier dealt with for instance in (Selfridge, *et al.*, 1980) where a model of a narrow strip transducer is presented. An efficient method developed earlier in electrostatics (Danicki, Tasinkevych, 2006; Tasinkevych, Danicki, 2005a) for a finite aperiodic planar system of conducting strips is found suitable for the solution of the above-mentioned problem. Next, the finite baffle system of interest is approximated by a periodic one with a certain large period, comprised by the multiple replica of the analysed structure. This enables one to use the BIS-expansion method as in (Tasinkevych, Danicki, 2010) to find the solution of the considered problem. The paper is organized as follows. In the next section the boundary value problem for strips is formulated. In Sec. 3 the method of solution is discussed and in Sec. 4 some numerical results are presented.

2. Formulation of the boundary-value problem

Let us consider a finite system of N acoustically hard baffles distributed along the x-axis on the boundary plane $z = 0$ of the acoustic medium spanning for $z > 0$, as shown in Fig. 1. Their edges are defined by x-coordinates (a_i, b_i), $i = 1, \ldots, N$. The baffles are assumed to be infinitely long along the y-axis. Without loss of generality we consider here the baffles having the same width $2d$ and being equally spaced along the x-axis with the pitch P, which is usually the case in practical linear arrays. The slots between baffles are denoted by $w = P - 2d$. The boundary-value problem for the case of acoustic wave-field generation by a finite system shown in Fig. 1a is formulated here similar to the way as it was done in the earlier paper (Tasinkevych, Danicki, 2010) for an infinite periodic baffle array. Likewise, the time harmonic wave-field is assumed $e^{j(\omega t - \xi x - \eta z)}$, where t is the time, ω is the temporal frequency, and ξ, η are the spatial frequencies corresponding to x, z, respectively. The wave-field on the baf-

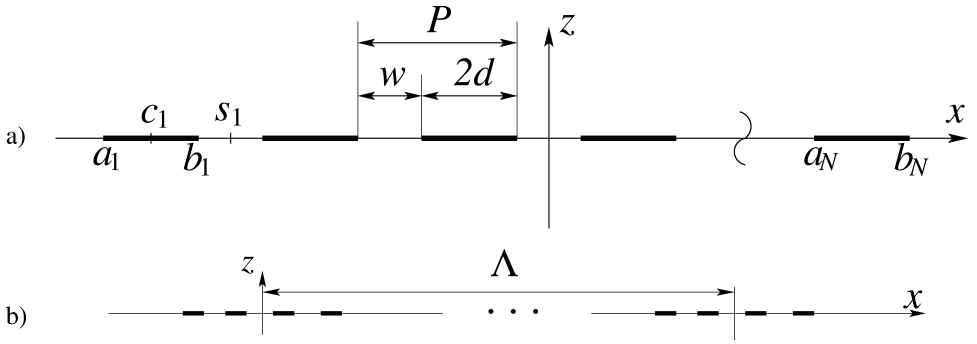

Fig. 1. a) A system of N rigid baffles (strips) on the boundary of acoustic media spanning for $z > 0$; b) multi-periodic structure with certain large period Λ comprised by the replicas of the finite baffle system (a).

fle plane is of the main importance in the applied analysis. Following the same considerations as in (TASINKEVYCH, DANICKI, 2010), we write the relationship between the normal component of the particle velocity $v \equiv v_z = -\varphi_{,z}$ and the x-derivative of the pressure distribution $q \equiv p_{,x}$, $p = \rho_a \varphi_{,t}$ (φ is the scalar acoustic potential satisfying the wave equation) on the baffle plane $z = 0$ as follows:

$$v = g(\xi)q, \qquad g(\xi) = \frac{j}{\omega\rho_a}\frac{\eta}{\xi}, \qquad \eta = \sqrt{k^2 - \xi^2} = -j\sqrt{\xi^2 - k^2}, \tag{1}$$

where $k = \omega/c$ is the wave-number, c is the sound velocity in the acoustic media, ρ_a is the mass density of the media, $S_\xi = -1$ for negative ξ and $S_\xi = 1$ otherwise.

For the case of acoustic wave-filed generation considered here the normal component of the particle velocity v vanishes on baffles. A harmonic pressure of amplitude p_l (constant over the entire slot) excites the wave-field in the medium $z > 0$; lP describes the position of the given l-th slot centre along the x-axis. Thus, the boundary conditions are:

$$\begin{aligned} q = 0,\ x \in (b_l, a_{l+1}),\ l = 0, \ldots, N \quad &\text{– between baffles,} \\ v = 0,\ x \notin (a_l, b_l),\ l = 1, \ldots, N \quad &\text{– on baffles,} \\ p(s_l) = p_l,\ l = 1, \ldots, N-1 \quad &\text{– at the } l\text{-th slot centre,} \end{aligned} \tag{2}$$

where $s_l = (a_{l+1} + b_l)/2$ is the coordinate of the slot centre between l-th and $(l+1)$-th baffles, b_0 and a_{N+1} correspond to $\pm\infty$ respectively, and p_l are given constant values in corresponding slots between baffles due to the condition $q = 0$ there. The solutions that we seek here are the functions $p(\xi)$ and $v(\xi)$, i.e. the spatial-frequency representations of the $p(x)$ and $v(x)$ on the boundary plane $z = 0$. The field in the medium, $z > 0$, can be easily evaluated (see Eq. (5) in (TASINKEVYCH, DANICKI, 2010)) if $p(\xi)$ and $v(\xi)$ are known.

3. Method of solution

To find the solution fulfilling conditions given by Eq. (2) the method developed earlier in electrostatics for the finite system of conducting strips (DANICKI, TASINKEVYCH, 2006; TASINKEVYCH, DANICKI, 2005a,b) can be successfully adopted. The set of template functions introduced in (TASINKEVYCH, DANICKI, 2005b) as partial solutions to the corresponding electrostatic problem is referred below:

$$\begin{aligned} \Phi^{(N)}(x) &= j^{N-1} \prod_{m=1}^{N} \frac{1}{\sqrt{d_m^2 - (x - c_m)^2}}, \\ \Phi^{(N,i)} &\sim x^i \Phi^{(N)}, \qquad i = 0, \ldots, N-1, \end{aligned} \tag{3}$$

where d_m and c_m are the half-width and centre coordinates of the i-th baffle. The function $\Phi^{(N)}$ is the basis template function and the rest of $\Phi^{(N,i)}$ can be derived

from $\Phi^{(N)}$, as in Eq. (3). The above functions have known spectral representations in the form of multiple convolutions of Bessel functions of the first kind $J_0(\xi d_m)$ and $J_1(\xi d_m)$. For the basis template function $\Phi^{(N)}$ the spatial-frequency counterpart is:

$$\Phi^{(N)}(\xi) = \Phi_1(\xi) * \Phi_2(\xi) * \cdots * \Phi_N(\xi), \tag{4}$$

where

$$\Phi_m(\xi) = \mathcal{F}\left\{ \frac{1}{\sqrt{d_m^2 - (x - c_m)^2}} \right\} = \left\{ \begin{array}{ll} J_0(\xi d_m)\, e^{jrc_m}, & \xi \geq 0 \\ 0, & \xi < 0 \end{array} \right\} \tag{5}$$

and $\mathcal{F}$ denotes the Fourier transform. Note the semi-finite support of the above functions, which feature is of great importance for further numerical analysis. The function $\Phi^{(N)}(x)$ has the property that its real and imaginary parts vanish in subsequent domains of the x-axis, as required by the boundary conditions given by Eq. (2). We introduce the template functions for the acoustic wave-field generation problem as follows:

$$\begin{aligned} Q^{(N)}(\xi) &= \left\{ \begin{array}{ll} \Phi^{(N)}(\xi), & \xi \geq 0 \\ \Phi^{*(N)}(-\xi), & \xi < 0 \end{array} \right\}, \\ V^{(N)}(\xi) &= S_\xi Q^{(N)}(\xi) = \left\{ \begin{array}{ll} \Phi^{(N)}(\xi), & \xi \geq 0 \\ -\Phi^{*(N)}(-\xi), & \xi < 0 \end{array} \right\}. \end{aligned} \tag{6}$$

As shown in (Danicki, Tasinkevych, 2006), the functions defined in Eq. (6) have their spatial counterparts vanishing on the x-axis in accordance with Eq. (2), namely, $Q^{(N)}(x)$ vanishes between baffles (as $q(x)$) and $V^{(N)}(x)$ vanishes on baffles (as $v(x)$). These functions, evaluated at discrete values of the spectral variable $\xi_n = n\Delta\xi$, are the discrete series in the numerical analysis and actually they represent, on the basis of the theory of FFT (Press *et al.*, 1992), the periodic functions in spatial domain with a certain large period $\Lambda = 2\pi/K,\ K = \Delta\xi$ (see Fig. 1b):

$$\begin{aligned} Q^{(N)}(x) &= \sum_n Q_n^{(N)} e^{-j\xi_n x}, \qquad Q_n^{(N)} = Q^{(N)}(\xi_n), \\ V^{(N)}(x) &= \sum_n V_n^{(N)} e^{-j\xi_n x}, \qquad V_n^{(N)} = V^{(N)}(\xi_n). \end{aligned} \tag{7}$$

The functions from Eq. (7) will help us satisfy the boundary conditions given by Eq. (2). Namely, following the same considerations as in (Danicki *et al.*, 1995; Danicki, 2004), we multiply the functions from Eq. (7) by $\exp(-jmKx)$ and take linear combinations of the resulting terms. After a simple rearrangement of summations we obtain the following representation of the wave-fields $(q, v)(x)$ by the inverse Fourier transform, written in a discrete form for the assumed large

period Λ (formally, $\Lambda \to \infty$, but in the applied approximation Λ is large but finite; see Sec. 4 for more details):

$$q(x) = \sum_{n=-\infty}^{\infty} q_n e^{-j\xi_n x}, \qquad v(x) = \sum_{n=-\infty}^{\infty} v_n e^{-j\xi_n x}, \tag{8}$$

where

$$q_n = \sum_m \alpha_m Q^{(N)}_{n-m}, \qquad v_n = \sum_m \beta_m V^{(N)}_{n-m} = \sum_m \beta_m S_{n-m} Q^{(N)}_{n-m}. \tag{9}$$

The expansions in Eq. (9) are the convolutions in the spectral domain, written in a discrete form, which correspond in the spatial domain to the products of the template functions in Eq. (6) with certain unknown functions $(\alpha, \beta)(x)$ represented by their Fourier transforms (in a discrete form):

$$\alpha(x) = \sum_{n=-\infty}^{\infty} \alpha_n e^{-j\xi_n x}, \qquad \beta(x) = \sum_{n=-\infty}^{\infty} \beta_n e^{-j\xi_n x}. \tag{10}$$

The corresponding spectral samples $(\alpha, \beta)_n$ occur in Eq. (9) as unknown expansion coefficients that have to be determined. The functions in Eqs. (8), (9), being the solutions to the considered boundary-value problem for the finite system of N baffles, satisfy the boundary conditions given by Eq. (2) due to the properties of the template functions from Eqs. (6), (7). Now we only need to check whether the applied solutions from Eqs. (8), (9) satisfy the wave equation in the media, which equation is represented on the baffle plane $z = 0$ by the harmonic admittance $g(\xi)$, defined by Eq. (1). Only the part of the wave-field $(q, v)(x)$ that satisfies the radiation condition at $z \to \infty$ is involved in the solution, yielding the following relation for the n-th spectral line having wave-number ξ_n:

$$v_n = g(\xi_n) q_n. \tag{11}$$

Following the same considerations as for the case of an infinite periodic baffle system considered in (Tasinkevych, Danicki, 2010), the following system of linear equations can be deduced:

$$g_\infty \sum_m \alpha_m \left[S_{n-m} - j(\eta_n/\xi_n) \right] Q^{(N)}_{n-m} = 0, \qquad m, n \in [-N_1, N_1]. \tag{12}$$

To obey the last condition in Eq. (2) stating that the pressure takes given constant values in the slots between baffles, we use a similar technique as described in (Danicki, 2006; Tasinkevych, Danicki, 2005b). Having N baffles there are $N_s = N - 1$ slots and constraints that have to be satisfied. For this purpose the number of coefficients α_m in Eq. (12) is enlarged to $2N_1 + 1 + N_s$ and the above N_s constraints are added:

$$p(x = s_i) = \int q(x)\, \mathrm{d}x \Big|_{x=s_i}, \qquad i = 1, \ldots, N_s, \tag{13}$$

where s_i is the i-th slot centre (see Fig. 1a). Here we benefit from the known spatial-frequency representation of the template solution $Q^{(N)}(\xi)$ from Eq. (6) and, following the same procedure as described in (Tasinkevych, Danicki, 2005b), we can numerically evaluate the pressures in Eq. (13) as follows:

$$p(x=s_i) = j\sum_m \alpha_m \mathcal{F}^{-1}\left\{\frac{Q^{(N)}(\xi_{n-m})}{\xi_n}\right\}\Bigg|_{x=s_i}, \qquad i=1,\ldots,N_s. \tag{14}$$

Summarizing, the system of linear equations for unknown α_m for

$$m \in [-N_1 - M_l, N_1 + M_u],$$

where $M_u = M_l = N_s/2$ for even N_s and $M_l = (N_s-1)/2$ and $M_u = (N_s+1)/2$ for odd N_s, is:

$$[A_{nm}][\alpha_m] = [b_n], \qquad n \in [-N_1, N_1 + N_s]. \tag{15}$$

The elements of the matrix A_{nm} are given by Eq. (12) and $b_n = 0$ for $n \in [-N_1, N_1]$ and

$$A_{nm} = \mathcal{F}^{-1}\left\{\frac{Q^{(N)}(\xi_{n-m})}{\xi_n}\right\}\Bigg|_{x=s_i}, \tag{16}$$

$$b_n = p_i, \quad n \in [N_1+1, N_1+N_s], \quad i \in [1, N_s].$$

Thus, solving the system of equations (15) for unknown α_m,

$$m \in [-N_1 - M_l, N_1 + M_u],$$

the solution to the considered boundary-value problem can be obtained from Eq. (8) using Eqs. (9) (note that $\beta_m = \alpha_m/(\omega\rho_a)$).

4. Numerical examples

In this section some numerical examples of the sound beamforming by the finite baffle systems are given. We consider here the far-field radiation pattern of the acoustic pressure field. It should be noted, that the method of the analysis discussed here yields the spatial spectrum of the acoustic pressure on the baffle plane directly. Taking the advantage of this, the radiation pattern can be evaluated as the inverse Fourier transform of $p(\xi)$ which is related to $q(\xi) = -j\xi p$ (note, $p(\xi \to 0) = 0$) in a similar way as in (Tasinkevych, Danicki, 2010):

$$p_R(\theta) = p\left(k\sin\theta\right)\cos\theta\,\frac{k}{K}\sqrt{\frac{j2\pi}{kR}}\,e^{-jRk}. \tag{17}$$

The angular dependence in the far-field region can also be written in terms of $q(\xi)$ as follows:

$$p_R(\theta) \sim q\left(k\sin\theta\right)\cot\theta. \tag{18}$$

In the numerical example presented in Fig. 2a, a far-field radiation pattern is shown for $N = 8$ baffles and the given pressures $p_l = \exp(jlPk \sin\beta), l = 1, \ldots, 7$, where β is a steering angle. The cases of $\beta = 0°$ and $\beta = 15°$ are considered. In Fig. 2b corresponding distributions of the pressure field on the baffle plane are shown. In Fig. 3 a pressure field distribution in the media $z > 0$ is illustrated for the considered 8 element baffle array and different steering angles.

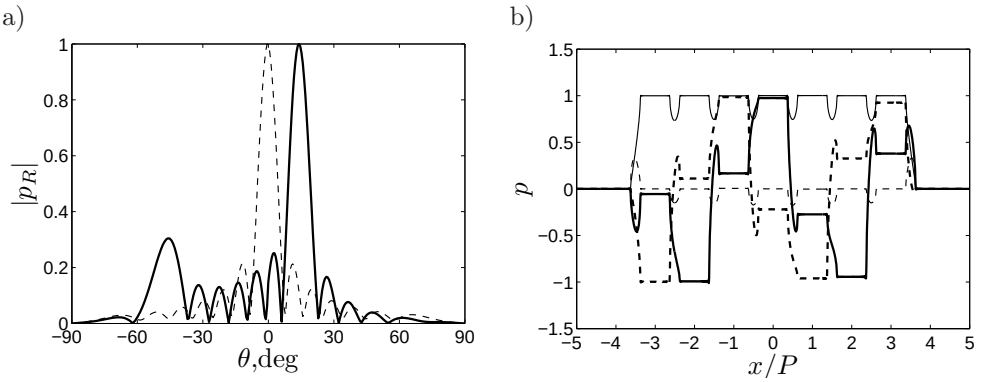

Fig. 2. a) Far-field radiation pattern for an 8 element baffle array; steering angle $\beta = 15°$ (a solid line) and $\beta = 0°$ (a dashed line); $P = \lambda$, $w = 0.75\lambda$; b) spatial distribution of $p(x)$ on the baffle plane; steering angle $\beta = 15°$ (thick lines) and $\beta = 0°$ (a thin line); solid lines – Re(p), dashed lines – Im(p).

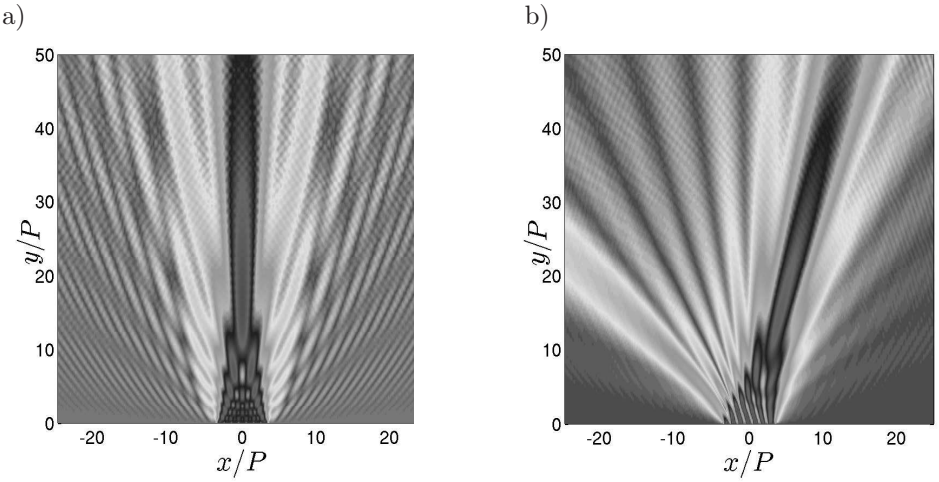

Fig. 3. Pressure distribution generated in an 8 element baffle array in the media $z > 0$: a) steering angle 0°, b) steering angle 30°.

In the numerical example presented in Fig. 4a a far-field radiation pattern for the case of one active slot in 5 element baffle array ($p_l = \delta_{l0}$, $l = -2, \ldots, 2$) is shown and a corresponding distribution of the pressure field on the baffle

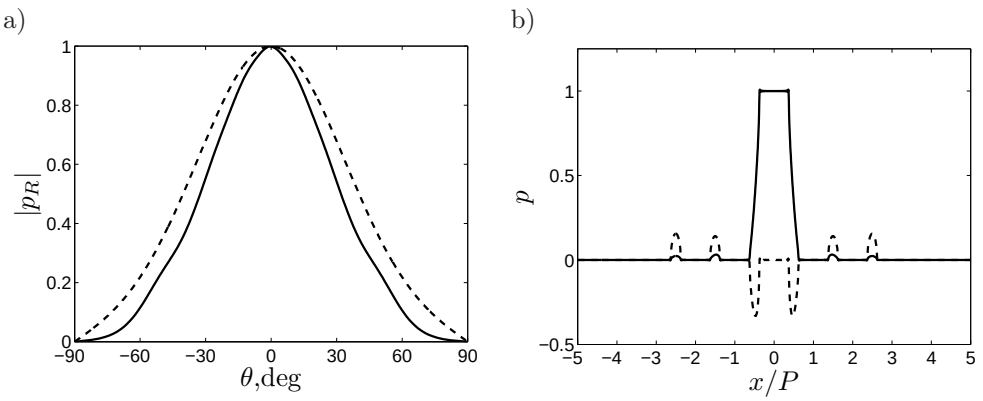

Fig. 4. a) Far-field radiation pattern of one active slot in a 5 element baffle array (a solid line) and of a narrow strip acoustic transducer excited by a time harmonic uniform pressure distribution (a dashed line); $P = \lambda$, $w = 0.75\lambda$; (b) spatial distribution of $p(x)$ on the baffle plane; a solid line – $\mathrm{Re}(p)$, a dashed line – $\mathrm{Im}(p)$.

plane is illustrated in Fig. 4b. For comparison, a far-field radiation pattern of a narrow strip acoustic transducer excited by a time harmonic uniform pressure distribution is marked by a dashed line in Fig. 4a. According to (Selfridge *et al.*, 1980), the analytical expression for the later case is:

$$f(\theta) = \frac{\sin(\pi w/\lambda \sin\theta)}{\pi w/\lambda \sin\theta)} \cos\theta. \tag{19}$$

The comparison of the far-field radiation patterns calculated for the case of a single active slot in a 5 element baffle array and a narrow strip transducer indicates influence of neighboring baffles on the radiated wave-field. In Fig. 5 the pressure field distribution in the media $z > 0$ is shown for the considered 5 element baffle

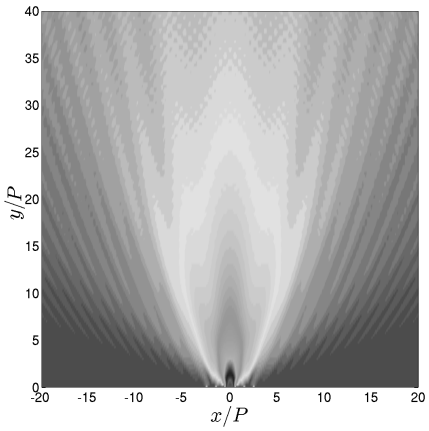

Fig. 5. Pressure distribution generated in a 5 element array with one active slot in the media $z > 0$.

array with one active slot computed by the presented method. In the numerical examples shown in this section the period $\Lambda \approx 100P$ is applied, which is sufficient for considering the finite baffle array as isolated within an approximating periodic structure (see Fig. 1). This can be easily observed from the examples, shown in Figs. 2b, 4b where pressure distribution on the boundary plane vanishes rapidly at a distance of several P away from the baffle system.

5. Conclusions

In this paper a mixed boundary-value problem for a finite array of rigid baffles in acoustic medium was solved for the case of wave-field generation. The method developed earlier in electrostatics (Danicki, Tasinkevych, 2006; Tasinkevych, Danicki, 2005a) was adopted here. The finite baffle system was approximated by some multi-periodic structure with a certain large period and an approach similar to the one developed in our previous work (Tasinkevych, Danicki, 2010) was used to find a solution to the considered problem by means of the BIS-expansion method (Blotekja *et al.*, 1973). The presented numerical examples show that the method yields all interesting characteristics of a linear transducer array for beamforming analysis. Direct evaluation of the spatial spectrum of acoustic pressure distribution on the baffle plane is favourable since it is used for far-field radiation pattern evaluation. The developed method delivers a model of a linear phased array which accounts for a finite element size and inter-element interactions. Also, transducers with different elements' width and spacing can be modeled by this approach. Such a modification may help reducing spurious effects connected with abrupt ends of a transducer system, which is considered as quite difficult for analysing. Besides, the problem of wave detection can be also addressed by this approach, which requires solving a corresponding boundary-value problem formulated for the case of plane acoustic wave scattering.

Acknowledgments

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