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Statistics of Envelope of High-Frequency Ultrasonic Backscatter from Trabecular Bone: Simulation Study

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The paper considers the application of statistical properties of backscattered ultrasonic signal for assessment of the trabecular bone status.

Computer simulations were conducted to investigate the properties of the ultrasound pulse-echo signal, as it is received on the transducer surface after scattering in trabecular bone. The micro-architecture of trabecular bone was modeled by a random distribution of long and thin cylindrical scatterers of randomly varying diameters and mechanical properties, oriented perpendicular to the ultrasound beam axis. The received echo signal was calculated as a superposition of echoes from all the scatterers present in the scattering volume.

The simulated signal envelope was used for statistical processing to compute various parameters like the mean amplitude, the amplitude MSR defined as the ratio of the mean to the standard deviation and the amplitude histogram.

Results indicated that while for the well-defined trabeculae properties within the simulated bone structure the signal envelope values are Rayleigh distributed the significant departures from Rayleigh statistics may be expected as the thickness of trabeculae become random. The influence of the variation of mechanical properties of the bone tissue building the trabeculae on the bone backscattered signal parameters was not observed.

Keywords: trabecular bone, Rayleigh distribution, scattering, simulations.

1. Introduction

Ultrasonic examinations of soft tissues, based on the analysis of scattered ultrasonic signal, demonstrated potential research opportunities to characterize and to differentiate the tissues (Bamber *et al.*, 1981; Insana *et al.*, 1990; Saha, Wehrli, 2004; Klimonda *et al.*, 2009). Similarly, signals that have been scattered in trabecular bone contain information about the properties of the bone

structure. Therefore, the scattering-based ultrasonic methods potentially enable assessment of microscopic structure of bone. In addition, very important zones of human skeleton (due to osteoporosis hazard) are examinable by the backscattering techniques whilst those areas are inaccessible for the transmission measurements (Laugier *et al.*, 2008).

A lot of investigations have been focused on measurements and calculations of the frequency-dependent backscattering coefficient of trabecular bone (Chaffai *et al.*, 2000; Laugier *et al.*, 2000; Padilla *et al.*, 2003; Wear, 1999; Wear, Garra, 1998). It has been demonstrated that ultrasound backscatter measurements have ability to discriminate patients with osteoporosis from controls (Laugier *et al.*, 1997). Theoretical studies of ultrasonic scattering by trabecular bone were performed by Wear (1999). The model of a bone, proposed by Wear, consisted of a random space-distribution of long cylinders with a diameter much smaller than the wavelength, aligned perpendicularly to the acoustic beam axis.

In our previous study (Litniewski *et al.*, 2009) we have developed the simulation technique that enables determination of the ultrasound signal received at the pulse-echo transducer surface after interrogation of cancellous bone. The simulation can be applied for different scattering models of a trabecular structure. In this study Wear's bone model was adopted with modifications that allowed for random variations of diameters and mechanical properties of cylindrical elastic scatterers describing the trabeculae. We have focused on statistical properties of backscattered signal trying to find dependencies between characteristic features of the bone structure and histograms of the signal envelope values.

This paper tries to answer the questions how much alterations in the cancellous bone structure may affect the effective number of trabeculae within the resolution cell and whether such alterations may cause significant and measurable changes in statistics of the envelopes of signals received on the transducer during ultrasonic pulse-echo examination of a bone.

2. Statistics of the scattered signal envelope

It can be assumed that the wave backscattered in trabecular bone consists of a number of individual scatterings on the single trabeculae and the received signal is a result of the summation of these partial components. It can be shown basing on the central limit theorem (Goodman, 1985) that for a large number of randomly distributed scatterers within the resolution cell the value of instantaneous amplitude of RF signal generated on the transducer surface is Gauss distributed with the mean value equalled to zero while the signal envelope values (A) are Rayleigh distributed with the probability density function ($p(A)$, see Fig. 1) expressed by the formula:

$$p(A) = \frac{A}{\Psi} \exp\left(-\frac{A^2}{2\Psi}\right). \tag{1}$$

The parameter Ψ depends on the number and the scattering cross-section of elements in the volume of measurements. The most probable estimator of the Ψ is the following

$$\Psi \approx \frac{1}{2n} \sum_{i=0}^{n} A_j^2, \tag{2}$$

where A_j stands for the amplitude values (subsequent points of the signal envelope) and n denotes total number of such points.

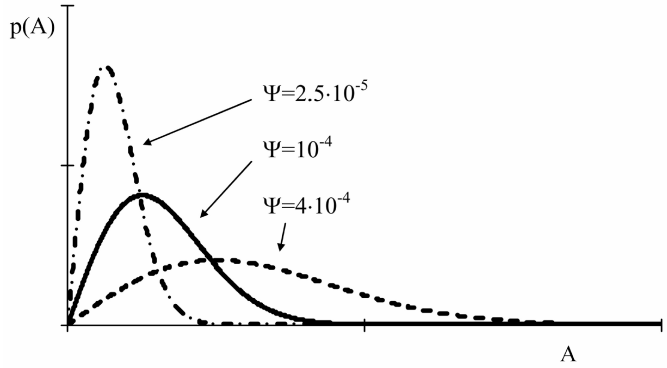

Fig. 1. Rayleigh probability density function for various values of the Ψ parameter.

It happens that the scattering cross-sections of individual scatterers are random. Then, the actual number of scattering centres located within the resolution cell should be substituted with the effective number. When alterations of scatterers cross-sections become more stochastic (their variance increases) the effective number of scattering centres decreases and in some cases it may reach value substantially lower than the real number of elements. In such case the assumptions of the central limit theorem are not fulfilled and the deviations in distribution of the signal amplitudes from the Rayleigh model are visible (Shankar, 2000; Wagner *et al.*, 1987).

The envelopes of backscattered signals were used to calculate the mean-to-standard deviation ratios (MSR) coefficient defined as a ratio of the mean ($\langle A \rangle$) to the standard deviation (σ) and to calculate amplitude histograms. Theoretical value for the MSR $= \langle A \rangle / \sigma$ for Rayleigh distribution equals to

$$(\pi/(4\pi))^{1/2} \approx 1.913.$$

Any deviation of MSR from this value can be considered as deviations from the Rayleigh statistics. The histograms were also compared with Rayleigh probability density function calculated for the Ψ estimator (2) derived from the signal envelope values.

3. Numerical generation of the backscattered signal envelopes

3.1. Scattering model of the cancellous bone

The trabecular bone was modelled as a collection of randomly distributed in water long, elastic cylinders with small diameter compared with wavelength aligned perpendicularly to the ultrasound beam axis. The signal that is received on the transducer surface after scattering in trabecular bone structure was simulated in the following way: Every cylinder (trabecula) was considered as a secondary source of an ultrasonic wave. Each cylinder was associated with a complex backscattering coefficient. The value of this coefficient was defined on the basis of the theory of scattering of ultrasound from elastic cylinder (Flax *et al.*, 1981) and depended on the cylinder diameter d (see Fig. 2), mechanical properties of the cylinder and immersion liquid and frequency of the probing wave. The distance between the scatterers and transducer was assumed to be large comparing to the diameters of scatterers. A spectrum of every elementary pulse scattered from a cylinder was obtained as a product of the emitted pulse spectrum and the complex backscattering coefficient of the scatterer. The receiving transducer action was simulated by superposing all the elementary scattered pulses taking into account the phase differences that result from various locations of individual trabeculae. The model assumes that the effects of multiple scattering are negligible.

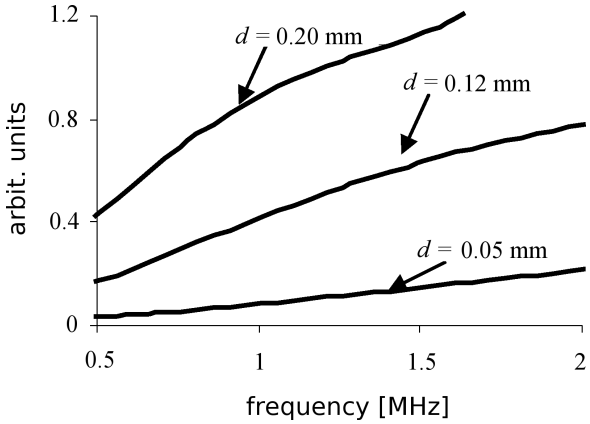

Fig. 2. Frequency dependent backscattering coefficient for a cylinder calculated with varying diameter $d = 0.2$, 0.12 and 0.05 mm and constant velocity $c = 4 \times 10^3$ m/s and density $\rho = 1900$ kg/m^3.

Variations of the structural properties of cancellous bone were modelled by changing spatial density of cylinders and changing mean value and variance of cylinders diameters and their mechanical properties such as wave velocity and density. In this paper (if not specified) velocity indicates the longitudinal wave velocity and density stands for the material density.

The simulation procedures were carried out with a transmitted pulse identical to the short 1 MHz pulse produced by the ultrasonic transducer used in ultrasonic densitometry. The spectrum of the simulated signal was limited in accordance with the frequency transfer function of the transducer.

3.2. Data for simulations

Reported measurements for ranges of morphometric parameters and values for material properties of cancellous bone are given in Table 1. In the bone model, coordinates that define location of trabeculae were random (uniformly distributed). The Gauss distribution was assumed for velocity and density values distribution in simulations. Thickness distributions of trabeculae are right-skewed as reported in experimental studies (Dagan *et al.*, 2004; Saha, Wehrli, 2004). The published trabeculae thickness distributions well fit Gamma distribution. Therefore trabeculae thickness values in the bone model were Gamma distributed.

Table 1. Trabecular bone properties assumed in simulations of the backscattered signal, in bracket reference number.

Porosity (2, 6, 11)	80–97%
Mean length of trabecula (6)	4 mm
Thickness of trabecula (2, 6, 7, 17, 20)	0.05–0.23 mm
Longitudinal wave velocity in trabecula (8, 15)	3200–3900 m/s
Density of trabecula tissue (8, 15)	1200–1950 kg/m^3
Trabeculae thickness, ratio of mean value to standard deviation (7, 11)	1.7–12

3.3. Procedures of simulations

Every simulation consisted of one RF-line, of 512 samples corresponding to a bone structure depth of 35 mm and sampled at the rate of 10 MHz (Fig. 3). Attenuation in trabecular bone tissue was neglected in this study. Statistical properties of the backscattered signal were calculated in every case on the basis of 32 independent spatial arrangements of trabeculae (32 RF-lines = 16384 amplitude values) whereas geometrical and mechanical properties of trabeculae were unchanged. For the calculation of MSR coefficient all 16384 amplitude values were used.

The study investigated the impact of three parameters that define structural properties of trabecular bone. These included:

1. Spatial density of trabeculae;
2. Significant change in one of physical parameters of trabecula, such as density, velocity of longitudinal waves or diameter and
3. Variation of physical parameters defined by standard deviations from their mean value.

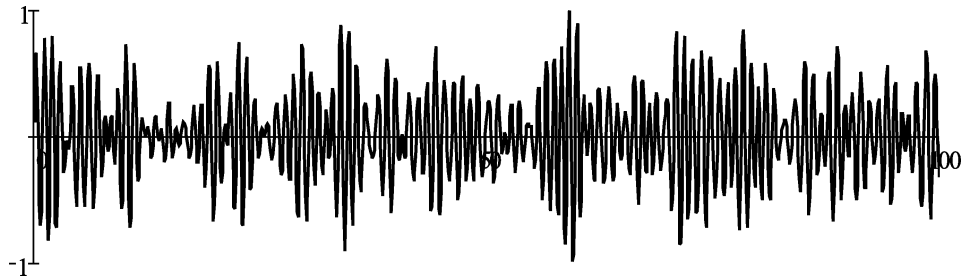

Fig. 3. An example of 1MHz RF-echo simulated for the trabecular bone model consist of identical cylinders ($d = 0.12$ mm, $\rho = 1900$ kg/m^3, $v = 3800$ m/s).

3.4. Simulation results

3.4.1. The impact of cancellous bone porosity

The sensitivity of simulated signal envelopes to bone porosity was investigated assuming a constant length and diameter of trabeculae, equal to 4 mm and 0.12 mm respectively. These values are often assumed as a means for the length and diameter of a healthy trabecula (HÄUSLER *et al.*, 1999). The geometrical parameters define the volume of trabecula (cylinder) what together with the assumed value of bone porosity enables to calculate the spatial density of trabeculae. For the healthy trabecular bone the porosity is ranging from 80% to 90%. For the osteoporotic bone the porosity increases up to 97% (see Table 1). In simulations the osteoporotic bone was considered therefore the bone porosity higher than 90% was assumed. The mechanical properties of the bone tissue were also fixed ($\rho = 1900$ kg/m^3, $v = 3800$ m/s). Porosity of bones ranging from 90% to 99.5% corresponds to a change of the number of scatterers in the volume of measurements (30 mm^3) from 70 to about 3. Finally, for the simulated signals, histograms of signal amplitudes and the values of the MSR coefficient were calculated.

It was found that within the range of trabecular bone porosity from 90% to 98% the MSR parameter is nearly constant and equals to 1.91 (Fig. 4). This

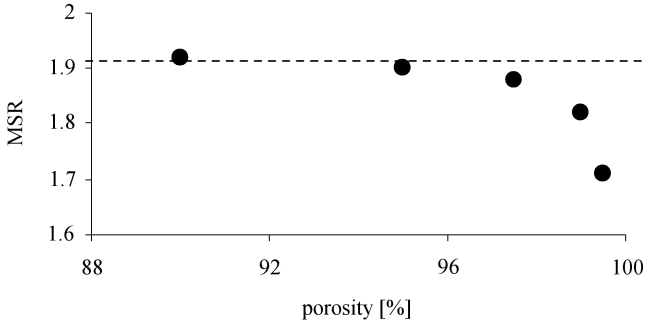

Fig. 4. The dependence between the MSR factor and structural porosity ($d = 0.12$ mm). The dashed line corresponds to the MSR value = 1.91.

means that number of scattering centres in the volume of measurements is large and the amplitude of the backscattered signal is Rayleigh distributed. Only when the porosity was really high (99%, $N = 7$) the assumptions of the central limit theorem were not fulfilled and rapid decrease of the MSR value was observed. In clinical practise no case was observed when trabecular structure represented only 1% of cancellous bone volume.

3.4.2. The impact of material properties and thickness of trabeculae

The second part of simulations was carried out to determine how mechanical properties of bone tissue constituting the trabeculae and trabeculae thicknesses affect statistics of backscattered signals. One of the consequences of the assumed model of trabecular bone and backscattered signal generation is that an equal change in the material properties of all trabeculae affects only the amplitude of simulated signals. The statistical properties of the backscattered signals should remain unchanged.

The scattered signals were simulated for two average densities of trabecular tissues ($\langle\rho\rangle = 1900$ kg/m^3 and $\langle\rho\rangle = 1200$ kg/m^3), two average values of velocity of longitudinal wave ($\langle v\rangle = 3200$ m/s and $\langle v\rangle = 3800$ m/s) as well as for two average thicknesses of trabeculae ($\langle d\rangle = 0.12$ mm and $\langle d\rangle = 0.05$ mm). For every simulation only one parameter was altered. Other parameters were assumed as constant and equal to the values for non-pathological tissues ($\rho = 1900$ kg/m^3, $v = 3800$ m/s, $d = 0.12$ mm).

As expected the obtained results show that properties of bone tissue that constitute trabeculae have practically no influence on statistical properties of the backscattered signal (Fig. 5). Similarly, alterations of average thickness (diameter) of trabeculae induce no changes in the nature of scattering (calculated MSR values were ranging from 1.89 to 1.90 for all simulations).

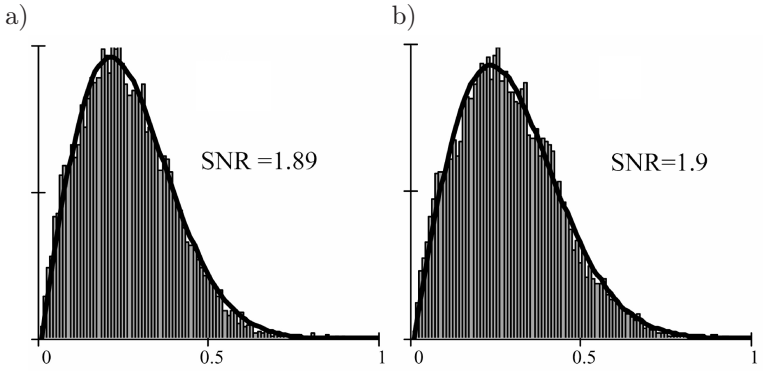

Fig. 5. The histograms of normalized amplitudes of backscattered signals simulated for two structures, substantially different in velocity value of trabeculae material: a) – $v = 3800$ m/s, b) – $v = 3200$ m/s. The continuous line corresponds to the Rayleigh distribution. Porosity – 97.5%.

3.4.3. The impact of statistical spread of trabeculae parameters

The third part of simulations was aimed at investigation of how variation of the parameters defining trabeculae affects the backscattering of ultrasonic waves and consequently how the statistical properties of amplitudes of that signal are modified.

First, the simulations were performed with a constant thickness and length of trabeculae equal to 0.05 mm and 4 mm respectively. The variation of random, Gauss distributed values of mechanical properties of trabeculae were simulated by modifying the standard deviation values for density (from $\sigma = 100$ kg/m^3 to 400 kg/m^3) and velocity (from $\sigma = 100$ m/s to 500 m/s) while keeping the mean values constant and equal to 1500 kg/m^3 and 3500 m/s respectively. Simulations were carried out for porosities of 97.5% and 99%.

Calculated values of the MSR coefficient clearly indicate that spread of velocity and density values of the trabeculae material (even for unrealistically high porosity of 99%) have nearly no impact on statistical properties of the backscattered signal (MSR value ranged from 1.89 to 1.91).

In the next step of simulations the material properties of the bone tissue were kept constant (1900 kg/m^3, 3800 m/s respectively for density and velocity) while the impact of randomness of Gamma distributed trabeculae thickness values on the scattered ultrasonic signal was investigated. Thickness of trabeculae in cancellous bone is affected by a number of factors such as the anatomical size of bones, their age, character and scale of transferred loads and metabolic illnesses of the osseous system. These illnesses have also a substantial impact on the variation of trabeculae thickness. Images from an acoustic microscope (Fig. 6) as well as optical images of samples of osteoporotic cancellous bones show both trabeculae with thickness that corresponds to nonpathological bones as well as atrophic trabeculae with much less thickness.

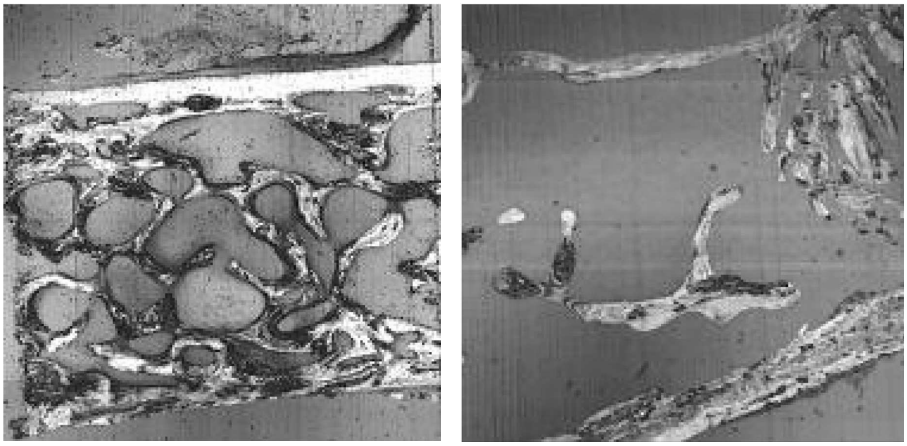

Fig. 6. Scanning Acoustic Microscope images of healthy (left) and osteoporotic (right) trabecular bone samples. Frequency 100 MHz.

Simulations were carried out for three values of bone porosity, namely 99%, 97.5% and 95%. To restrict effect of structural parameters of bones to merely variation of their thickness, the simulation used the assumption of the constant average thickness ($\langle d \rangle = 0.05$ mm). The value of standard deviation of the thickness varied in the range of $\sigma = 0.004$ mm – 0.05 mm.

Substantial impact of trabeculae thickness variation on the statistical properties of amplitudes for the backscattered signal is clearly visible for all the considered porosities. At the very beginning decreasing of the MSR value is insignificant (for low values of standard deviation of thickness $\sigma = 0.004$ mm – 0.01 mm). Low spread for thickness values of trabeculae is characteristic for non-pathological bones. The degradation of trabeculae (caused by metabolic illnesses) enhances variation of trabeculae thickness. As variation of trabeculae thickness increases the value of the MSR factor rapidly drops (Fig. 7). The effect concerns variation of the standard deviation of thickness within the range of $\sigma = 0.02$ mm – 0.05 mm.

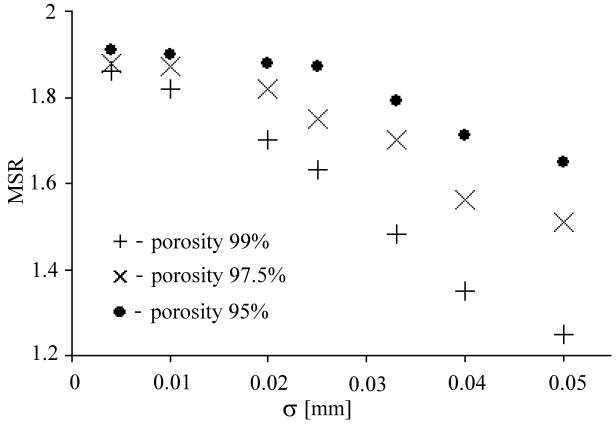

Fig. 7. The MSR factor calculated using the envelopes of signals that are backscattered in the trabecular structure. Thickness of trabeculae is Gamma distributed with the mean equal 0.05 and variable value of standard deviation (σ).

Alterations of trabeculae thickness result in variations of the backscattering coefficient. Thus, the spread of trabeculae thickness values is equivalent to the spread of the backscattering coefficient. The more random variations of the backscattering coefficient are the more significant decrease of the effective number of scatterers in the volume of measurements occurs that leads to deviations in distribution of signal envelopes from the Rayleigh distribution.

4. Conclusions

The performed simulations have revealed that the variable trabeculae thickness in structure of cancellous bone is the main reason for decrease of the MSR value calculated for amplitudes of backscattered signals. Mechanical properties

of the tissue constituting the trabeculae and the spread of these parameters have no impact on the statistics of the backscattered signal amplitude. The backscattered coefficient of trabecula significantly depends on trabecula thickness. Thus, the spread of thickness values represent a primary reason for significant variation of backscattered coefficient. Eventually, the volume of measurements comprises a number of constituents that variably affect the final result of backscattered echoes addition, which leads to formation of instantaneous amplitude values. The number of scatterers essentially contributing to the final signal (trabecula with large thickness, defining the effective number of scattering centres) is lower than total number of the scatterers within the resolution cell. Backscattered signals that are produced under such conditions present substantially increased variance of their amplitudes, which leads to the decrease of the MSR factor and departures of the amplitude histograms from the Rayleigh model.

The model of trabecular bone assumes that the cylindrical trabeculae are arranged perpendicularly to the beam axis. That simplifies calculations very much. Anatomically in the mesh of trabeculae only the mean orientation of thick trabeculae can be identify and in the experimental evaluation of the heel bone the acoustic beam is perpendicular to this mean orientation. The amplitude of the scatterings on trabeculae located within the resolution cell varies additionally with the trabeculae orientation. That influence the effective number of scatterers and can change the value of measured MSR. But the results of presented simulations are still valid because the variation of the diameters of scatterers lowers the MSR regardless of the initial value of this coefficient.

The method proposed in this paper exploits the information about structural properties of bone. The MSR factor calculated using backscattered signal envelopes is insensitive to bone density, expressed as mean trabeculae thickness or density of the bone tissue. The usefulness of the presented approach must be verified experimentally. If confirmed, the MSR value could be used to differentiate between the bone structures with well-defined trabeculae thickness and the structures with diffuse values of trabeculae dimensions.

Acknowledgments

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