Annales Societatis Geologorum Poloniae (2002), vol. 72: 271–282.

AN APPLICATION OF MARKOV CHAIN ANALYSIS TO STUDIES ON LITHOFACIES SEQUENCES IN THE ALLUVIAL FANS FROM THE "BEŁCHATÓW" LIGNITE DEPOSIT (POLAND)

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Mastej, W., 2002. An application of Markov chain analysis to studies on lithofacies sequences in the alluvial fans from the "Bełchatów" lignite deposit (Poland). *Annales Societatis Geologorum Poloniae*, 72: 271–282.

Abstract: The Markov chain analysis was used to study on lithofacies sequences in alluvial fans enclosed in the main seam of the "Bełchatów" lignite deposit. Quantitative approach supported the qualitative results of sedimentological analysis – the origin of muds, clays and coaly clays as well as the presence of a barrier (peat bog) between the fans and the lake with carbonate deposition. It was found that ephemeral lakes located on the fan surface were favourable sites for development of peat bogs. Similarly, the same lakes evolving into the peat bogs were favourable environments for expansion of sand lobes of the fans. It was discovered that cyclicity in sediment succession was disturbed by erosion. The new model of lacustrine limestone deposition was proposed for the northwestern part of lake which borders the fans from the northwest.

Key words: Markov chain analysis, vertical lithofacies succession, "Bełchatów" lignite deposit, alluvial fans, lacustrine limestones.

Manuscript received 31 January 2002 , accepted 15 November 2002

INTRODUCTION

In the central part of the "Bełchatów" lignite deposit, close to the southern margin of the Kleszczów Graben, Słomka et al. (2000) reported on the splitting and pinching-out of the main lignite seam (PG) within the thick sand succession or on the facial transition of splitted lignite seam into the coaly clays. Detailed sedimentological analysis proved the presence of two alluvial fans: upper and lower, and allowed the determination of deposition model (Słomka *et al.*, 2000). However, this conventional analysis did not include the statistical approach to the vertical sequence of lithofacies. Such an approach based upon the Markov chain analysis has already gained an approval in sedimentology. The Markov chain analysis enables a relatively easy recognition of general regularities in vertical succession of facies, which supports the conclusions obtained from "conventional" approach and helps to decipher the genetic controls of depositional processes. Due to a large number of data required for successful analysis, the method is usually applied for studies of flysch, coal-bearing and alluvial sequences. The author attempted to apply the Markov chain analysis to lithofacies succession in alluvial fans discovered in the main seam of the "Bełchatów" lignite deposit. The Markov chain analysis has already been used to study depositional conditions of lacustrine limestones bordering the Bełchatów alluvial fans from the southwest (Wagner *et al.*, 2000). Hence, the present research covered only a small fragment of fans area, i.e. their interfingering zone with the lacustrine limestones.

GEOLOGICAL SETTING

Geology of the "Bełchatów" lignite deposit is quite well- known. Detailed information can be found in the following papers: Felisiak & Szewczyk (1994), Gotowała (1994), Hałuszczak (1994), Kasiński & Piwocki (1994), Matl (2000) and Wagner (2000).

The deposit occuppies the Kleszczów tectonic graben filled with the Miocene coal-bearing formation. The four lignite seams were distinguished: main seam (marked PG or D) and A, B and C seams (Fig. 1). The B+C+D seams merge in the central, currently exploited part of the deposit forming so-called "lignite complex". The identification of seams is based upon paratonsteins (Wagner, 2000). The bottom and the top of the main seam (which is the principal mining target) are defined by the Ts-10 and Ts-4 paratonstein layers, respectively. In the central part of the deposit, close to the southern margin of the Kleszczów Graben, the main seam is facially replaced by sandy sediments of alluvial fans (Słomka *et al.*, 2000). The fans are bordered from the south-

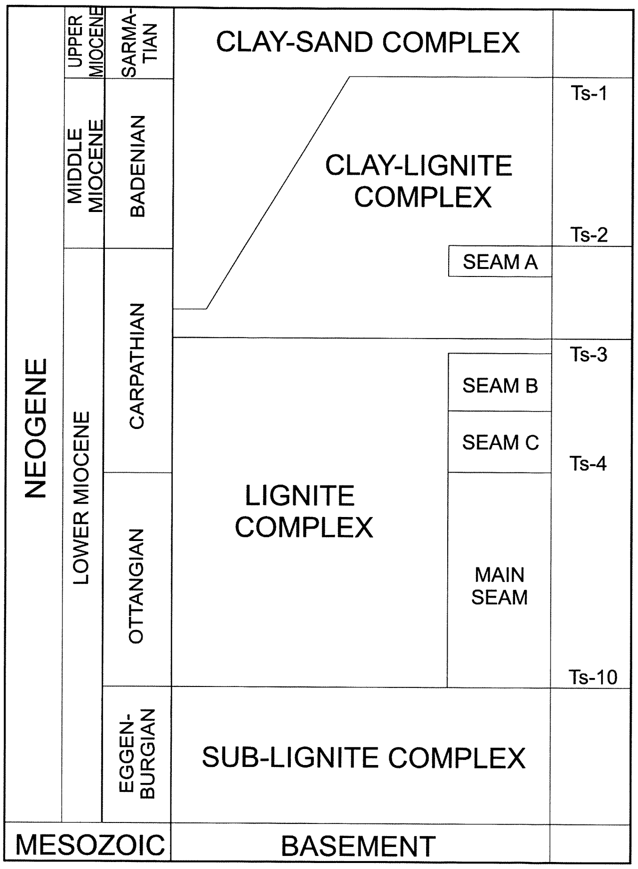

Fig. 1. Lithostratigraphy of Tertiary lignite formation in the Bełchatów deposit (after Czarnecki *et al.*, 1992 and Matl, 2000)

east with the lacustrine limestone deposit. The limestones partly interfinger with the upper fan sediments. Main mass of lacustrine limestones is located within the main lignite seam, partly replacing the coal. Only in a few drill cores the single limestone layers were observed also in sub-coal and clay-coaly complexes but their occurrence have never been noticed outside the main lignite seam. Depositional environment of lacustrine limestones was described Ciuk & Piwocki (1967), Szwed-Lorenc & Rascher (1982), Tomaszewski & Cygan (1986), Wagner *et al.* (2000).

MATERIAL

The study area covered about 1.5 km^2 of alluvial fans (Fig. 2), including the southeastern, interfingering zone with the lacustrine limestones. As the fans occur exclusively within the main lignite seam (Fig. 3) only the seam sequence was studied. Totally, 5557 layers (lithofacies occurrences) were analysed of cummulative length 18.5 km. The database comprised descriptions of drill cores from 286 boreholes provided by the Geological Department of the "Bełchatów" Mine. The boreholes were drilled in a regular grid over the whole area of the fans (Fig. 2). Detailed description of facial development of alluvial fans can be found in Słomka *et al.* (2000). The analysis included only those lithofacies, which were sufficiently common in the drill cores to provide credible statistical results. This criterion was met by the 6 lithofacies marked as: P – sands, M – muds, I – clays, IW – coaly clays, W – lignites and J – lacustrine limestones. The P and M lithofacies are alluvial fan or meandring river sediments whereas I, IW and rare M lithofacies represent the local lakes located on the fan surface. The W lithofacies sedimented in the swamp environment whereas the J lithofacies was deposited in a larger lake (about 0.7 km^2) (Wagner *et al.*, 2000).

METHODS

The application of Markov chain analysis to studies of depositional processes has a long history and numerous references. Among the important theoretical publications the handbook by Schwarzacher (1975) must be first mentioned as it provides a systematic explanation of principles of the Markov chains theory applied to sedimentology. Xu & MacCarthy (1998) reviewed current methods of the analysis. The earliest, Gingerich-Read's (Gingerich, 1969; Read, 1969; further abbreviated as GR) and Selley's (Selley, 1970) methods did not solve correctly the problem of randomness tests as they ignored the lack of lithofacies transitions into themselves (the presence of zero values at the diagonal of the matrix of the number of lithofacies transitions). Therefore, some other authors – Turk (1979), Powers & Easterling (1982), Carr (1982), Harper (1984), Le Roux (1994) – proposed the improved versions. Their publications contain also the results of Markov chain analysis of vertical lithofacies succession. Among numerous applications the papers of Doveton (1971), Radomski & Gradziński (1978, 1979, 1981), Krawczyk (1980) and Słomka (1986, 1995) should be recommended to the Reader.

The credible results of statistical analysis of the Markov process can be obtained only if the individual elements of the so-called "matrix of the number of (facial) transitions" are represented by sufficienly large values (at least 5 by presumption, although single cases with lower number of transitions are acceptable), which means that the defined lithofacies should repeat sufficiently frequently in a given succession. It is difficult to achieve if transitions between some lithofacies are rare despite the length of analysed succession (i.e. large number of elements in a succession). Usually, the presumed condition can be achieved by decreasing the ratio of defined lithofacies to the length of succession. In practice, it is done by combining the relatively short successions into a single, long, artificial sequence. Transitions between the border elements of neighbouring successions are prohibited. An alternative solution is the limitation of the number of defined lithofacies, which, however, may give rise to an erroneous generalizations leading to trivial results. In the following paper both methods had to be used. After numerous tests it was found that best results are obtained if all successions are combined into a single sequence with 6 defined lithofacies. The calculation procedure is presented below.

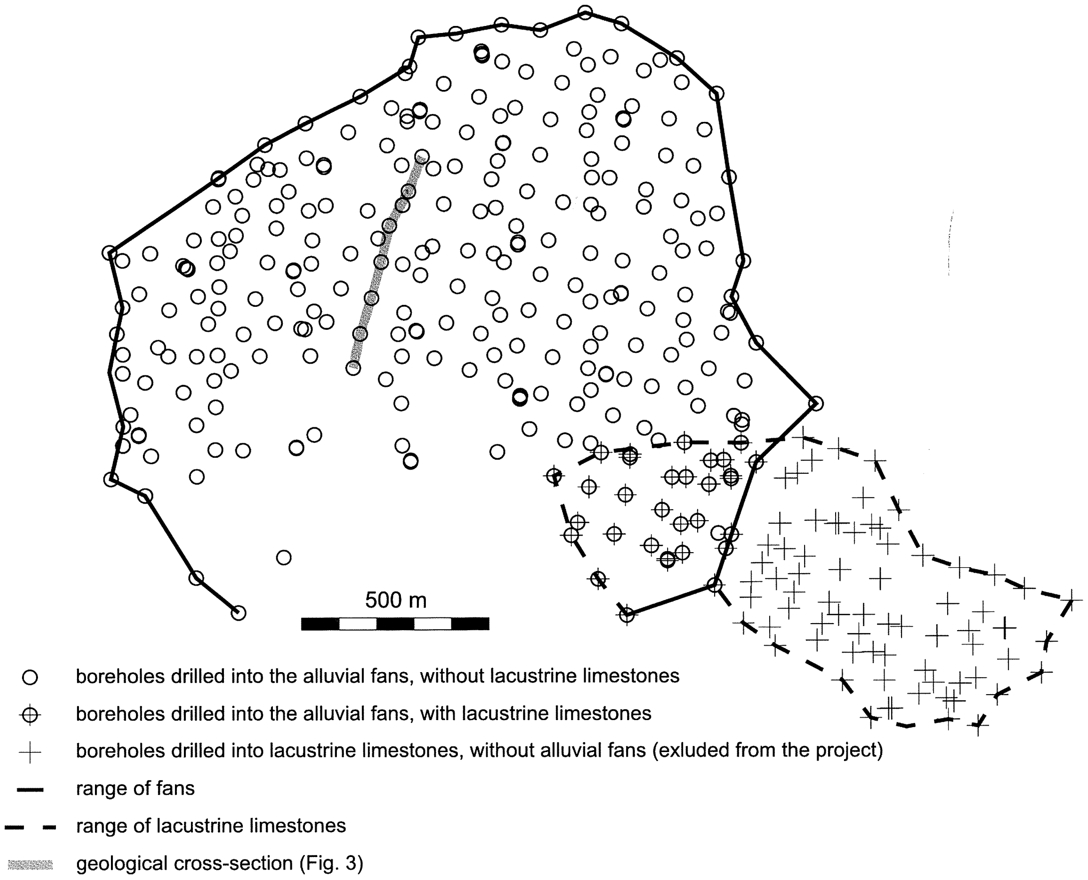

Fig. 2. Distribution of alluvial fans and lacustrine limestones in southwestern part of the Bełchatów lignite deposit (modified after Słomka *et al.*, 2000)

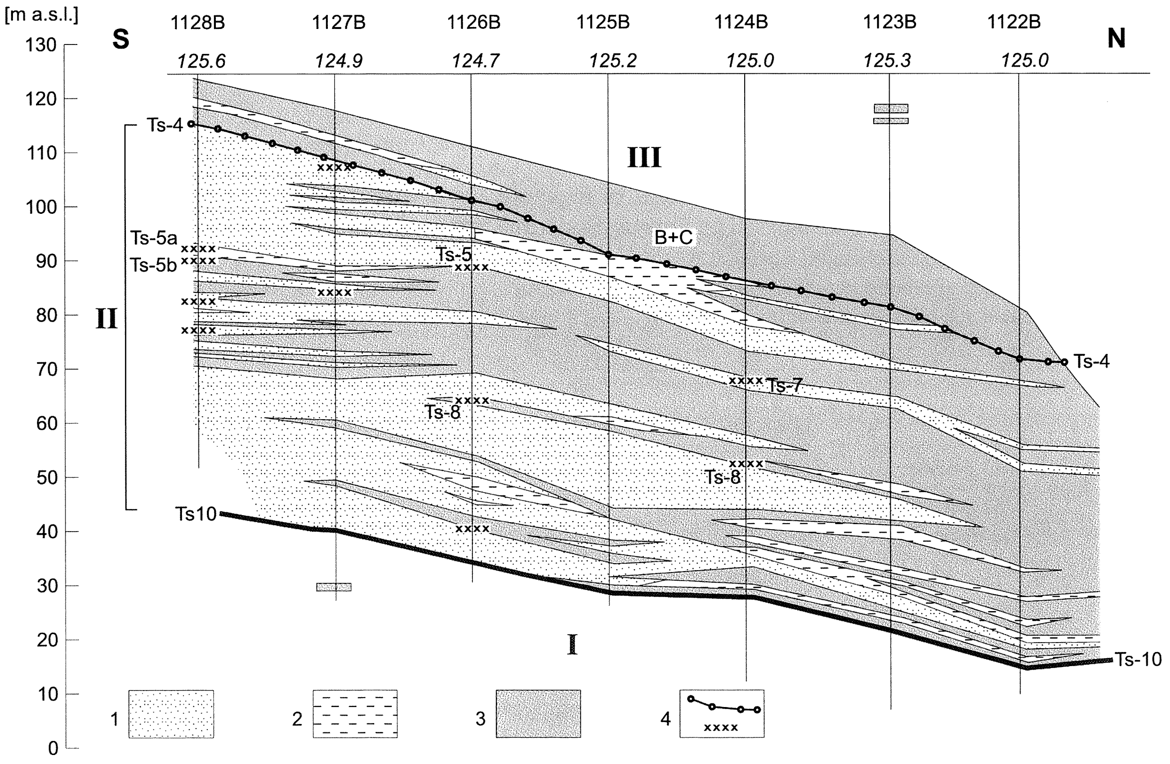

Fig. 3. Geological cross-section through the zone of alluvial fans (after Słomka, 2000) in the main lignite seam (PG). I – sub-lignite complex, II – lignite complex, III – clay-lignite complex; *1* – sand, *2* – clay, *3* – lignite, *4* – paratonstein markers (Ts-10 – bottom of the PG, Ts-4 – top of the PG), B+C – symbols of younger lignite seams

STUDIES ON THE NUMBER OF TRANSITIONS

At this stage of calculation the following matrices were considered: matrix of the number of observed transitions [*Q*], matrix of the number of expected transtions in a random sequence [*E*], matrix of differences between the number of observed transitions and the number of expected transitions in a random sequence [*D*] = [*Q*] – [*E*]. The hypotheses were also tested for consistency with random sequence and for randomness of specific transitions between lithofacies (significance test of specific elements of the matrix [*D*]). As mentioned above, the older methods of sequence randomness testing, particularly the most popular GR, should be eliminated. Among the latest improvements of this method the most credible is the Powers & Easterling's proposal (Powers & Easterling, 1982, see also discussion in Xu & MacCarthy, 1998) further abbreviated as PE. The calculation method of χ^2 statistics used for the randomness test of a sequence is identical in the GR and the PE methods:

$$\chi^2 = \sum_{i=1}^{m} \sum_{j=1}^{m} \frac{D_{ij}^2}{Eij}$$

In the PE method the number of degrees of freedom is defined as $(m\text{-}1)^2\text{–}m$ where: m – number of lithofacies types, D_{ij} – element of matrix [*D*], E_{ij} – element of matrix [*E*]. In the GR method the number of degrees of freedom is defined as $m^2\text{–}2m$.

The new Powers & Easterling's idea (1982) is an original calculation of [*E*] matrix elements, different in comparison with the GR method. The importance of their modification is not limited to the randomness test of a sequence but has some much deeper implications because [*E*] matrix influences [*D*] matrix. It causes different results of significance test of [*D*] matrix elements and gives a diagram of lithofacial transitions significantly different from random transitions. In the following paper the PE method was applied (see Table 1) whereas the GR method was used only for testing differences in the results (see Table 2).

In the PE method the element of [*E*] matrix located at the crossing of *i*-row and *j*-column, i.e., the expected value (in the case of sequence randomness test) of the number of transitions from state "*i*" to state "*j*" is calculated as:

$$E_{ij} = a_i\, b_j, \text{ for } i \neq j$$

$$E_{ij} = 0, \text{ for } i = j.$$

The parameters a_i and b_j $(i, j = 1, \ldots, m$, where m –[*E*] matrix order) are calculated by iteration.

For the first iteration:

$$a_i^{(1)} = n_{i+} \,/\, (m-1) \quad \text{for } i = 1, \ldots, m;$$

$$b_j^{(1)} = n_{j+} \,/ \sum_{i \neq j} a_i^{(1)} \quad \text{for } j = 1, \ldots, m;$$

where: n_{i+} and n_{j+} are sums of frequencies of *i* and *j* columns of the [*Q*] matrix, respectively.

For k iteration:

$$a_i^{(k)} = n_{j+} \,/ \sum_{i \neq j} b_j^{(k-1)} \quad \text{for } i = 1, \ldots, m;$$

$$b_j^{(k)} = n_{j+} \,/ \sum_{i \neq j} a_i^{(k)} \quad \text{for } j = 1, \ldots, m.$$

The procedure terminates if two conditions are satisfied (for k iteration):

$$\left| a_i^{(k)} - a_i^{(k-1)} \right| < 0.01 \quad \text{for } i = 1, \ldots, m;$$

$$\left| b_j^{(k)} - b_j^{(k-1)} \right| < 0.01 \quad \text{for } j = 1, \ldots, m.$$

Table 1

Matrix of transitions [*Q*] and results of randomness tests of sequence and of differences between numbers of observed and expected, random transitions. The Powers-Easterling method (Powers & Easterling, 1982)

Results of randomness test of sequence: $\chi^2_{obl.} = 429.67$; at $(m\text{-}1)^2 - m = 19$ degrees of freedom (where $m = 6$ is dimension of matrix [*Q*]), $\chi^2_{\alpha=0.05} = 30.14$. Randomness hypothesis has been rejected

	I	IW	M	P	J	W
I	–	**34 (-0.00)**	**74 (-0.00)**	**343 (+0.05)**	**15 (-0.01)**	**655 (+0.00)**
IW	**43 (-0.00)**	–	50 (+0.71)	149 (+0.40)	**31 (+0.00)**	**313 (+0.00)**
M	**82 (-0.02)**	40 (-0.28)	–	**164 (+0.00)**	**2 (-0.01)**	**184 (-0.13)**
P	**359 (+0.00)**	**165 (+0.03)**	**165 (+0.00)**	–	**10 (-0.00)**	**463 (-0.00)**
J	**15 (-0.01)**	**27 (+0.00)**	**1 (-0.00)**	**14 (-0.00)**	–	**82 (+0.00)**
W	**648 (+0.00)**	**324 (+0.00)**	**190 (-0.28)**	**523 (-0.00)**	**87 (+0.00)**	–

I – clays, IW – coaly clays, M – muds, P – sands, W – lignite, J – lacustrine limestones.
First cell in table – elements of a matrix of number of interfacial transitions.
Second character (in brackets) is a probability p_{ij} (explained in the text). If $p_{ij}<\alpha$, where α is significance level, the randomness hypothesis of differences between the number of observed and expected transitions was rejected which means significant excess or deficit in transitions.
"+" means excess, "-" means deficit of transitions. Significant excess and deficit transitions at significance level 0.05 or less typed in bold.
Grey background – excess transitions at significance level 0.05 or less.

Table 2

Matrix of the number of transitions [*Q*] and randomness test of sequence and of differences between numbers of observed and expected, random transitions. The Gingerich-Read method (Gingerich, 1969; Read, 1969)

Results of randomness test of sequence: $\chi^2_{obl.} = 678.32$; at $(m\text{-}1)^2 - m = 25$ degrees of freedom (where $m = 6$ is dimension of matrix [*Q*]), $\chi^2_{\alpha=0.05} = 37.65$. Randomness hypothesis has been rejected

	I	IW	M	P	J	W
I	–	**34 (-0.00)**	**74 (-0.00)**	343 (+0.26)	**15 (-0.00)**	**655 (+0.00)**
IW	**43 (-0.00)**	–	**50 (-0.09)**	149 (+0.55)	31 (+0.76)	**313 (+0.00)**
M	**82 (-0.02)**	**40 (-0.01)**	–	**164 (+0.00)**	**2 (-0.00)**	184 (+0.33)
P	**359 (+0.00)**	165 (+0.56)	**165 (+0.00)**	–	**10 (-0.00)**	**463 (-0.05)**
J	**15 (-0.00)**	27 (+0.87)	**1 (-0.00)**	**14 (-0.00)**	–	**82 (+0.00)**
W	**648 (+0.01)**	**324 (+0.10)**	**190 (-0.00)**	**523 (-0.00)**	**87 (+0.00)**	–

Excess and deficit numbers of transitions at significance level 0.10 and less typed in bold. Grey background – excess transitions at significance level 0.10 and less.
Other explanations as in Tab. 1.

Testing of randomness hypotheses for lithofacial transitions (i.e. significance tests of differences between numbers of observed and expected transitions) allows us to disclose the significant excess and deficit of the numbers of observed transitions related to the random ones. Randomness of differences is determined with the "*z*" test. The "*z*" statistics was calculated from the Powers-Easterling test (Powers & Easterling, 1982). For transition of "*i*" into "*j*" lithofacies: $z_{ij}=[(Q_{ij} - E_{ij})/(E_{ij})]^{1/2}$, where Q_{ij} – element of [*Q*] matrix, E_{ij} – element of [E] matrix of expected random transitions. In order to estimate an error which occurs if the true hypothesis on randomness of the transition is rejected, the probability was calculated $p_{ij}=2*[1\text{-}F(z_{ij})]$ where $F(z_{ij})$ is a value of normal distribution function for z_{ij} statistics (multiplication by 2 due to bilateral test). If $p_{ij}<\alpha$, where α is a presumed significance level, the hypothesis is rejected (the z_{ij} value is significant at α significance level).

The significant excess of transitions is conventionally displayed in a diagram, which facilitates the interpretation. Despite the formal confirmation of the significance of such transitions, the transitions of limited size may also appear. If the frequency distribution of such transitions is highly irregular the analyzed dataset may contain transitions which constitute a fraction of per cent of overall transitions. Such results must be treated with great caution. From the other side, the so-called "modal" (i.e. most frequent) transitions can be distinguished. Such transitions enable the determination of modal sequences (Duff & Walton, 1962; Schwarzacher, 1975) which dominate the overall population (in the studied case – the whole lithosome of alluvial fans). According to Słomka and Słomka (2001), the modal sequences occur most commonly in successions, despite their origin, and represent the final result of depositional processes disturbed to various extent by erosion and interactions between various depositional mechanisms.

Concluding so far, it must be emphasized that determination of statistically significant excessive transitions in relation to random ones is the most important part of the whole sequence analysis. Such transitions are usually genetically interrelated, e.g. due to their origin from the same formative processes. The analysis of modal sequences is also valuable. In such sequences the random transitions may occassionally appear (even in those showing significant deficits), which is an effect of the fact that lithofacies participating in these transitions are so common in the sequence that transitions become modal. The most probable genetic explanation of this effect is the long-lasting coexistence of various depositional environments in the adjacent areas with random overlapping. Statistically significant deficits of transitions can also be subjected to interpretation although it is a difficult process, attempted by only a limited number of sedimentologists. It seems that statistically significant deficits of facial transitions may reflect the genetic links.

STUDIES ON FREQUENCY (PROBABILITY) OF TRANSITIONS

At this stage of calculation the following matrices were calculated: matrix of the frequency of transitions [*P*], matrix of the expected transitions in random sequence [E_P] (based upon the expected sizes calculated with both the PE and GR methods) and matrix of differences between observed and expected transitions in a random sequence [D_P] = [*P*] – [E_P]. Similarly to the first stage, the signifcance of [D_P] matrix elements was tested (Słomka & Słomka, 2001).

The elements P_{ij} of [*P*] matrix are calculated from the formula: $P_{ij} = Q_{ij}/n_{i+}$, where Q_{ij} are corresponding elements of [*Q*] matrix and n_{i+} is a sum of elements in *i*-row of [*Q*] matrix. Hence, the sum of elements of *i*-row in [*P*] matrix (i.e. the sum of transition frequencies of a selected lithofacies into all other lithofacies) equals 1, which suggests the link between the frequencies. Statistically significant excess (disclosed by "*z*" test) between frequencies observed [*P*]

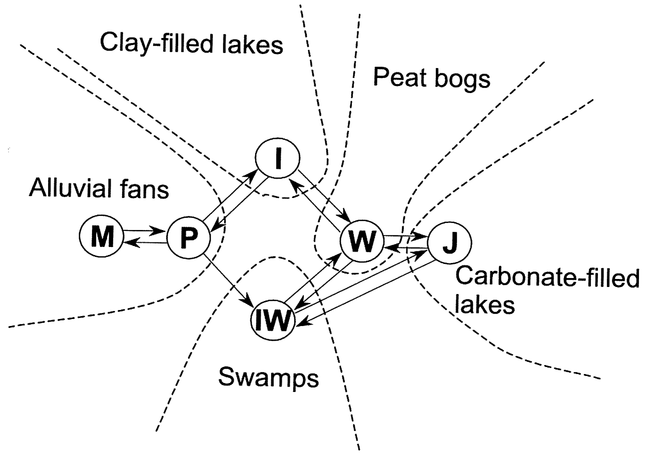

Fig. 4. Diagram of interfacial transitions with significant excess ($\alpha \le 0.05$) of the number of observed transitions related to the random transitions. The Powers-Easterling method (Powers & Easterling, 1982). I – clays, IW – coaly clays, M – muds, P – sands, W – lignite, J – lacustrine limestones

and expected in a random sequence $[E_P]$, i.e. positive and significant elements of $[D_P]$ matrix enable the reconstruction of so-called "model sequences" (Duff & Walton, 1962; Hobday *et al.*, 1975; Radomski & Gradziński, 1978). According to some authors (e.g., Słomka & Słomka, 2001), such sequences reflect the effects of depositional processes and exclude the disturbances originating from other factors. The remarkable difference between modal and model sequences implies the importance of effects of other processes.

The study included calculation of so-called stable probability vector (Schwarzacher, 1975), which coordinates are mean values of columns of $[E_P]$ matrix. Then, the matrix of transitions frequency $[P]$ was multiply raised to the power until the stable probability vector is achieved, i.e. until the mean values of columns of this matrix differ from the stable probability vector by less than 0.005). The rate at which the $[P]$ matrix reaches the stable probability vector, expressed by expotent L is a measure of the system entropy. The lower are L values, the higher is entropy, i.e. the higher is the "chaos" in lithofacies sequence of alluvial fans, which point out that the Markov chain quickly "forgets" its preceding states.

ADDITIONAL STUDIES

In the main study stage described above analysis included so-called Markov's I-order processes, which refer to a one-step transition from one lithofacies to another. The following chapter presents the results of higher-order Markov chains analysis ($k>1$). First, additional calculations were made of so-called first passage probability after exactly k steps, i.e. probability that, starting from lithofacies A, the Markov chain will pass into lithofacies B after exactly k steps (Schwarzacher, 1975). If A=B the so-called recurrence probability appears. For $k=1$ first passage probability is recorded in $[P]$ matrix. Calculations were run for $k=2$ to $k=10$, which resulted in matrices of first passage from $[P_2]$ to $[P_{10}]$. Moreover, the plots were drawn of probability values depending on the steps, separately for specific transitions of lithofacies into a selected, single one. The appearance of high values of first passage probability (particularly, the first return) after $k>1$ steps proves the cyclicity of sediments. The k value is a measure of cycle periods.

Additionally, $[S]$ matrix was calculated of variability coefficients of distance between lithofacies in a sequence. Generally, the low values of coefficients indicate the presence of cyclicity in studied sequence. Low values along the principal diagonal of the matrix suggest that relevant lithofacies appears regularly in the sequence (see e.g. Krawczyk, 1980).

DISCUSSION

Due to the size of a sample representative for the succession of lithofacies in fan sequences, the obtained results are clear and relatively easy for interpretation. It is obvious from Table 1 where high values of χ^2 statistics significantly exceed critical value and from the fact that elements of [D] matrix are almost always significant, i.e. they describe either significant excess or signifcant deficite of lithofacial transitions (with three exceptions – random transitions IW→M, IW→P and M→IW marked in Table 1).

Graphic presentation of most important results is the diagram of interfacial transitions (Fig. 4). Arrows mark significant ($\alpha \le 0.05$) excess of transitions over the random ones. Apart from the three exceptions mentioned above, all the possible transitions, which were not marked in the diagram belong to the group with significant deficit in relation to random transitions.

Lithofacies typical of the alluvial fans environment (M and P) are strongly mutually linked by the presence of significant excess of transitions. Muds (M) are replaced only by sands (P). This supports the common opinion on the origin of these lithofacies (Słomka *et al.*, 2000), i.e. that muds are formed from sand granular flows in the final phase of deposition or during high-water periods when overflows take place of finer fractions over the levees of braided streams flowing atop the fans. It must be emphasized that clastic sediments (M and P) might have been partly deposited as channel sediments of a river meandring at the bottom of the Kleszczów Graben, northeast of the alluvial fans (Wagner *et al.*, 2000). However, such deposition was absent from the fans themselves, which is proved by the lack of bituminous coals typical of abandoned channels closely connected with meandring rivers (Słomka *et al.*, 2000). Also a suggestion made by Słomka *et al.* (2000) is meaningful that muds can be to only limited extent deposited in marginal lakes. It is supported by the lack of significant number of excessive lithofacial transitions from muds to clay or coaly clays, which are typical oxbow deposits. An alternative proposal is the existence of a number of oxbows in which sedimentation included muds but not clays, which is less probable.

Słomka *et al.* (2000) proposed that clays and coaly clays (I and IW) were laid down in ephemeral lakes developed over sand fans. It is supported by the results of quanti-

Table 3

Statistics of the modal sequences. The Powers-Easterling method (Powers & Easterling, 1982)

Modal sequences	Number of sequences in succession	Number of lithofacies identified in sequences	Ratio [%]: number of lithofacies in sequences/general number of litofacies	Thickness of lithofacies identified in sequences [m]	Ratio [%]: Thickness of lithofacies identified in sequences [m]/ general thickness of lithofacies [m]
P → M	165	330	5.94	1098.30	5.90
M → P	164	328	5.90	1029.00	5.53
P → I	359	718	12.92	1931.90	10.38
I → P	343	686	12.34	1872.70	10.06
P → IW	165	330	5.94	1012.70	5.44
W → IW	324	648	11.66	2473.80	13.29
IW → W	313	626	11.27	2358.40	12.67
W → I	648	1296	23.32	3653.40	19.62
I → W	655	1310	23.57	3048.20	16.37
*IW → P	149	298	5.36	841.10	4.52
*P → W	463	926	16.66	3414.30	18.34
*W → P	523	1046	18.82	4508.10	24.21
*M → W	184	368	6.62	1027.40	5.52
*W → M	190	380	6.84	1084.40	5.82
P → I → W	168	504	9.07	1208.40	6.49
W → I → P	148	444	7.99	1264.40	6.79
*W → P → I	126	378	6.80	1175.90	6.32
*I → P → W	96	288	5.18	723.30	3.89
*P → W → I	151	453	8.15	1211.70	6.51
*I → W → P	185	555	9.99	1510.90	8.12

* cycles including random transitions ($\alpha = 0.05$) or significant deficit of transitions

tative studies as these lithofacies are underlain by sands (significant excess of transitions) and may return back to sands (P) or coals (W). Such feature documents typical succession of depositional environment of alluvial fans with local lakes and peat bogs. Sediments of lakes resting upon the fan surfaces, if not covered with sands, were covered by peat bogs. Significant deficit in I→IW transitions suggests the action of an unknown genetic factor, which hampered the eutrophication of clay-filled lakes into swamps. Hence, deposition of coaly clays is not linked to the eutrophication of ponds with clayey deposition. It seems that the principal source of organic matter in coaly clays could be the older peat bogs subjected to river erosion. Słomka *et al.* (2000) accepted the possibility of such erosion. Considering also the significant deficit of reversed IW→I transitions, it is suggested that both the IW and I lithofacies were formed generally in separated lakes. Some of these environments were supplied with organic matter (swamps), some were not. It does not preclude the possible formation of swamps at the sites of former lakes with clayey deposition and *vice versa* – clayey sedimentation in lakes developed at the sites of former swamps. However, such events must have been accompanied by the change in alimentation area.

Transitions P→W and W→P reveal significant deficit in comparison with the random transitions. From the other side, such transitions appear in bi- and tri-elemental modal sequences (Table 3) which include also I lithofacies together with P and W ones. This can be explained by considerable percentages of P (22.19%), I (21.07%) and W (34.21%) lithofacies in the sequences (Table 5). All bi- and tri-elemental combinations of these lithofacies form modal sequences. If these transitions showed random character – it might have been explained in terms of localization of an area in which the active lobe of the fan covered the peat bog (W→P transition). Such localization depends on many random factors thus, it is of random character. Similarly, the active lobes could randomly transform into the passive ones, which, in turn, could be more probably occupied by peat bogs (P→W transition). However, appearance of significant deficits in bi-elemental modal sequences P→W and W→P suggests the role of a genetic link – sandy lobes do not “readily” cover peat bogs and peat bogs do not “readily” cover the lobes, even the passive ones. The relationships change when the third, intermediate lithofacies appears – clays (modal sequences P→I→W and W→I→P with significant excesses in all transitions). Despite the fact that

Table 4

Matrix of passage probabilities in II-order Markov process (after two itinerations). The Powers-Easterling method (Powers & Easterling, 1982)

	I	IW	M	P	J	W
I	**0.29**	0.15	0.10	0.19	0.03	0.18
IW	0.27	**0.14**	0.09	0.20	0.03	0.22
M	0.24	0.12	**0.10**	0.18	0.03	0.28
P	0.16	0.09	0.07	**0.27**	0.03	0.32
J	0.24	0.12	0.09	0.24	**0.04**	0.22
W	0.12	0.07	0.07	0.19	0.02	**0.49**

First passage probability typed in bold. Lithofacies explanations as in Tab. 1

Table 5

Statistics of lithofacies appearance

	Number of lithofacies in sequence	Relative number of lithofacies in sequence [%]	Total thickness of lithofacies in sequence [m]	Relative total thickness of lithofacies in sequence [%]
M	487	8.76	1006.9	5.41
P	1233	22.19	4936.4	26.51
I	1171	21.07	1543	8.29
IW	619	11.14	1229.2	6.60
W	1901	34.21	9237.9	49.62
J	146	2.63	664	3.57
Σ	5557	100	18617.4	100

coaly clays rarely appear in the succession and do not form modal cycles, this lithofacies may also play the role similar to clays as P→IW and IW→W transitions show significant excess. Hence, the important conclusion can be made that lakes, which occupy local depressions in fans underlain by clay layer isolating from the groundwaters, were favourables sites for the formation and growth of peat bogs. The marginal lakes were favourable sites where filling of peat bogs with sands has commenced. Presumably, the reason was their depressional character resulting from higher compaction of fitogenic and clay sediments loaded with sand bodies in comparison with "pure" sands. Higher compaction rate of phytogenic sediments was suggested by Słomka *et al.* (2000).

In the eastern part of the alluvial fans area the lacustrine limestones were encountered (J lithofacies). These sediments cover larger area and continue eastward (Wagner *et al.*, 2000). Limestone layers appear many times in this succession which suggests that lake in which sediments were deposited was subjected to rejuvenation due to subsidence. This subsidence was caused not only by compaction of peat but also by tectonic factor (Wagner *et al.*, 2000). In the western part of study area this process had to be genetically related solely to the peat-bogs (W) and marshy lakes (IW) environments with the characteristic, cyclic succession of J and W lithofacies. Such cyclicity has not been found by Wagner *et al.* (2000) who applied the Markov chain analysis to sedimentation of lacustrine limestones in whole area along the southern margin of the Kleszczów Graben. These authors proved random succession of coals and lacustrine limestones, and interpreted this feature in terms of long-lasting, lateral neighbourhood of lacustrine (carbonate sedimentation) and peat-bog environments "with scarce and areally limited interference episodes". Logically, the link between carbonate deposition and peat-bogs proposed in the present paper has been forced by the presence of sand fans and, may be, is valid only for the zone of facial interfingering between the lacustrine limestones and the sands. Presumably, the process responsible for such an interfingering was the higher and more variable subsidence rate of carbonate deposition adjacent to alluvial fans caused by higher (and more variable in time and space) compaction of peats loaded with sinking sand bodies. It is supported by the fact that the study area is a downthrown block of a high-angle, synsedimentary fault where thickness of limestones reaches about 100 meters whereas further eastward, in the upthrown block, this thickness is 50 meters, in average (Wagner *et al.*, 2000). It seems that the factor directly responsible for cyclicity in the western part of lacustrine sediments is an unstable and commonly breaking balance between subsidence and deposition rate (mainly peat bogs). If subsidence dominated the lake expanded or another lake with carbonate deposition formed on the peat bog (W→J transition). At higher subsidence and resulting elevated water table the concentration of $Ca(HCO_3)_2$ might have been reduced and carbonate precipitation migh have been interrupted. At the balance state or at the dominance of peat-bog deposition the lake might have been eutrophicated from the shoreline (J→W transition) and boundary between J and W lithofacies might have oscillated repeatedly as demonstrated by their succession in the studied sequences. Finally, the development of peat bogs along the lake margins could interrupt carbonate precipitation due to decreasing pH of water. At the decreasing subsidence rate the peat bog could cover the entire lake. The eastern part of carbonate deposition area was dominated by sedimentation model described by Wagner *et al.* (2000). Here, compaction had to be much lower and less variable in time and space.

The important observation is that transitions between lacustrine limestones (J) and other lithofacies (except for coals (W) and coaly clays (IW)) reveal significant deficit in comparison with the random transitions. The presence of natural barrier – the low and, periodically, high peat bog (Wagner *et al.*, 2000) between the lake in the east (where lacustrine limestones precipitated) and alluvial fans zone in the west (with ephemeral lakes on the surface) was demonstrated also by quantitative results. Despite that fact that this barrier was occassionally broken by sandy lobes, its existence caused the lake with limestone deposition being supplied by streams other than those supplying the sand lobes. The streams were cutting through the Oxfordian limestones at the southern margin of the Kleszczów Graben and sup-

Table 6

Matrix [*S*] of variability coefficients for distance between sequence elements. The Powers-Easterling method (Powers & Easterling, 1982).

	I	IW	M	P	J	W
I	–	2.15	1.10	0.64	0.98	7.00
IW	1.71	–	1.31	0.96	0.94	7.23
M	1.48	1.05	–	2.85	0.90	1.51
P	0.75	1.06	1.13	–	1.04	2.90
J	1.28	1.03	1.24	1.01	–	6.89
W	0.80	1.09	1.18	1.95	1.01	–

Explanations as in Tab 1

plied calcium carbonate to the lake (Słomka *et al.*, 2000) whereas the alimentation zone of alluvial fans was located further westward, behind the zone of strike-slip faults (Folwark Fault), at the surface of Albian sandy sediments. The relationship between J and W lithofacies (significant excess of facial transitions) and their cyclic interfingering is more difficult for explanation because (as mentioned above) coaly clays are genetically linked to the fans (main P lithofacies). However, more detailed analysis of transition matrix (Tab. 1) indicates that less common IW lithofacies under- or overlies the [J] lithofacies two times more often than I lithofacies typical of ephemeral lakes covering the fans. Similar relationship is valid for common P lithofacies typical of the fan environment. Two explanations are possible:

– the relationship between J and IW lithofacies is not associated with the links between lacustrine environment with limestone precipitation and the fan environment. Not all coaly clays must have been laid down in swampy lakes covering the fan surface but some might have sedimented in the same basin dominated by carbonate precipitation. The diagram of interfacial transitions (Fig. 4) reveals a theroretically possible existence of cycles with significant excess of J→IW→W→J or J→W→IW→J transitions. This feature is not reflected in modal sequences because J and IW lithofacies are rather rare in the succession. Therefore, some observed transitions between lacustrine and peat-bog environments were separated by deposition of coaly clays,

– the marshy lakes on the surface of fans (IW lithofacies), which occasionally expanded over the peat-bog barrier and entered the carbonate-dominated lakes or shoreline peat-bogs might have been the favourable sites where peat bogs or carbonate lakes developed again. On the contrary, the non-favourable sites were lakes with clay deposition and sandy lobes of the fans) Such concept could be supported by theoretically possible P→IW→W→J or P→IW→J→W transitions (Fig. 4, the absence of modal sequences caused by the same reason).

The obtained results allow us to suggest that in the western part of study area (where lacustrine limestones are lacking) coaly clays were deposited in lakes developed at fan surface. Generally, these clays include redeposited organic matter whereas in the eastern part (where limestones occur) the validity of any of the two above presented concepts cannot be proven. Apparently, more investigations are necessary of facies succession in the area of lacustrine limestone deposition, in a basin adjacent to the fans but outside the interfingering zone of limestones and sands. It would verify if significant excess of IW→J transitions does not disappear after the elimination of the influence of thick sand bodies in successions. Although such a study would not exclude definitely any of the above-proposed models, it can allow to select a model dominating in the eastern part of study area.

With an only one exception (IW→P transition), each facial transition has its corresponding recurrence transition (significant excess of transitions). It proves the cyclic succession of sedimentary environments. Cyclicity is also confirmed by somewhat increased values of recurrence probability at *k*=2 (cycle period) listed in Table 4: P→P (0,27), I→I (0,29) and W→W (0,49). For *k*>2, these values quickly decrease. The alluvial fan, local, ephemeral lake and peat bog environments appear cyclically at the characteristic, short periods as the return to P, I or W lithofacies is separated by only a single depositional episode. The interfacial transition diagram (Fig. 4) demonstrates such first-return patches to P, I or W lithofacies, which include only (*k*=1) transitions of significant excess, hence, are most credible. The P→P transition may proceed as P→I→P or P→M→P transition sequences. Consequently, I→I transition may occur as I→P→I or I→W→I sequences and the W→W one – as W→I→W, W→IW→W or W→J→W sequences. At *k*=2 higher probability values appear of first transition to lithofacies P (J→P – 0.24), I (IW→I, J→I, M→I – from 0.24 to 0.27) and W (P→W, M→W – from 0.28 to 0.32) (Fig. 4). Except of J→P and M→W transitions (at *k*=2), the interfacial diagram (Fig. 4) demonstrates most credible pathes of first transitions to P, I and W lithofacies (similarly to first return pathways). Specifically, at k=2 the IW→I transition corresponds to IW→W→I sequence, J→I transition corresponds to J→W→I sequence, M→I transition corresponds to M→P→I sequence and P→W transition corresponds to P→I→W (modal sequence) or P→IW→W sequences. For (k=2) M→M, I→I, IW→IW and J→J transitions, no regularities were encountered due to low percentage of M, IW and J lithofacies in successions. Similarly to the first transition probability, the probability values of first return at *k*>2 quickly decrease.

The genetic controls of these processes have been already commented on. Analysis of first passage and recurrence probabilities provides additional information on cyclicity. Generally, this cyclicity reveals short periods of cycles as probability values decreases quickly for *k*>2. It is also reflected in predominance of bi-elemental modal sequences (Tab. 3), low number of tri-modal sequences and complete absence of multi-element sequences. One of the reasons is certainly a non-uniform distribution of lithofacies (low number of M, IW and J lithofacies in comparison with P, I and W ones (see Tab. 5). However, it is also possible that older layers might have been removed by erosion before deposition of P, I or W lithofacies.

Such an idea is supported by the fact that in the matrix of differences between observed frequencies and expected frequencies in random sequence [D_P] statistically significant elements are absent. Hence, the selection of model sequences is impossible and, consequently, the reconstruction of undisturbed depositional processes cannot be made. Presumably, erosion strongly influenced deposition episodes in the area of fan sedimentation. Such disturbances are reflected also by high entropy of the system as [P] matrix quickly reaches the stable probability vector ($L = 10$). Another support of this concept comes from relatively high values of variability indexes of distance between all lithofacies in the sequence (Tab. 6), which directly points to irregularity in the appearance of lithofacies in the sequence and, indirectly (in consistency with previous results), demonstrates that cyclicity is weak and strongly disturbed.

SUMMARY

The application of Markov chain analysis to lithofacies succession in alluvial fans from the upper seam of the Bełchatów lignite deposit allowed to: (i) support quantitatively several qualitative results presented by Słomka *et al.* (2000) and Wagner *et al.* (2000), (ii) precise quantitative results of Wagner *et al.* (2000) and (iii) disclose and interpret the new regularities in lithofacies succession. Moreover, it was proved that the applied Powers-Easterling method (Powers & Easterling, 1982) provides results significantly different from the Gingerich-Read method (Gingerich, 1969; Read, 1969; Tab. 2).

Specifically, the concept was confirmed (Słomka *et al.* 2000) that muds are generally the alluvial fans not the lakes sediments and that the link exists between the local lakes with clay deposition (and a part of swampy lakes), and the alluvial fans environments. The lakes were supplied with finest fractions by streams flowing onto the fan surfaces. The existence of a barrier (peat bog) between alluvial fans with ephemeral lakes developed on their surface and larger lake with lacustrine limestone deposition located eastward (Wagner *et al.* 2000) was also evidenced.

The studies revealed the new features. It was found that coaly clays sedimented partly in local lakes on the fan surface and partly in a larger lake located east from the fans. Generally, the former environment was supplied by organic matter derived from eroded peat bogs (as suggested by Słomka *et al.*, 2000) and unrelated to eutrophication of the lakes.

It was found that favourable sites for development of peat bogs on alluvial fans were ephemeral lakes. The important factors were the presence of groundwater and isolating clay layer. Similarly, the lakes transforming into peat bogs were favourable sites for expansion of sand lobes due to their depressional character resulting from higher compaction rate of phytogenic and clay sediments additionally loaded with sand bodies.

Both Słomka *et al.* (2000) and Wagner *et al.* (2000) proved the cyclicity of lithofacies succession. The present study allowed to precise the character of cyclicity. The discovered cycles are very short and consist of only two lithofacies. Such feature originates from strong obliteration caused by disturbing processes, most probably the erosion.

The present study did not confirm random succession of lignite and lacustrine limestones in the western part of carbonate sedimentation area, as suggested by Wagner *et al.* (2000). Such a succession was interpreted by these authors as a result of long-lasting, lateral neighbourhood of lake and peat-bog environments with "scarce and areally limited interfingering episodes of both environments". The present results demonstrate that such model cannot be applied to the whole area of carbonate deposition along the southern margin of the Kleszczów Graben. Instead, it is applicable only outside the zone of interfingering carbonates and sands, or, maybe, only in the eastern, upthrown block of high-angle, synsedimentary fault where subsidence was low and presumably constant in time and space. In the western part (interfingering zone or even the whole downthrown block) strong links exist between both environments. Certainly, it is a result of higher and more variable subsidence caused by high rate of peat compaction under the load of sand bodies. In this area a cyclic alternation of sedimentary environments was found, occassionally with transitional coaly clays. It seems that the main factor responsible for cyclicity is an unstable and regularly broken balance between subsidence and deposition rates (mostly peats). As the subsidence rate is connected with the position of water table in the lake, the prevailing subsidence caused the expansion of lake with carbonate sedimentation whereas the balance or the dominance of deposition limited the range of the lake. At high subsidence the concentration of $Ca(HCO_3)_2$ might have decreased and carbonate deposition might have been interrupted. All these processes caused multiple oscillation of boundary between lacustrine limestones and lignites well visible in the studied successions. Another factor influencing the relation between lacustrine and peat-bog environments could be the interruption of carbonate precipitation after acidification of lake waters with humic acids derived from peat bogs.

Acknowledgements

The research project was financed by the University of Mining and Metallurgy grant No. 10.10.140.576. Sincere thanks are due to Prof. T. Słomka and Prof. M. Wagner for their kind help and critical reading of the manuscript, and to the staff members of the Geological Dept. of the Bełchatów Mine who provided data for the project.

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Streszczenie

ZASTOSOWANIE ANALIZY ŁAŃCUCHÓW MARKOWA DO BADAŃ SEKWENCJI LITOFACJI W STOŻKACH NAPŁYWOWYCH W ZŁOŻU WĘGLA BRUNATNEGO "BEŁCHATÓW"

Wojciech Mastej

Zastosowanie analizy łańcuchów Markowa do zbadania następstwa litofacji w stożkach napływowych w pokładzie głównym złoża bełchatowskiego pozwoliło potwierdzić w sposób ilościowy wiele wyników badań jakościowych Słomki *et al.* (2000) i Wagnera *et al.* (2000), uściślić wyniki ilościowych badań Wagnera *et al.* (2000) oraz wyłowić nowe prawidłowości w następstwie litofacji i dokonać ich interpretacji. Ponadto stwierdzono, iż użyta tu wiarygodniejsza metoda Powersa-Easterlinga (Powers & Easterling, 1982) daje znacząco różne wskazania niż metoda Gingericha-Reada (Gingerich, 1969; Read, 1969; tab. 2).

W szczególności, potwierdzono tezy Słomki *et al.* (2000), że mułki są generalnie osadem stożków napływowych, a nie osadem powstającym w zastoiskach oraz, że istnieje związek lokalnych zastoisk ilastych i części zastoisk bagnistych ze środowiskiem stożków napływowych. Zastoiska te były zasilane najdrobniejszym materiałem dzięki ciekom wodnymi, płynącym po stożku. Odkryta przez Wagnera *et al.* (2000) bariera (torfowisko) między obszarem stożków napływowych z efemerycznymi zastoiskami na ich powierzchniach a większym zastoiskiem, położonym dalej na wschód, w którym powstawały wapienie jeziorne, również została potwierdzona w niniejszych badaniach ilościowych.

Ujawniono także nowe zjawiska i dokonano ich interpretacji. Stwierdzono, że część iłów węglistych powstawała w lokalnych zastoiskach na powierzchni stożków a część – w większym zastoisku, usytuowanym na E od obszaru stożków. Depozycja tych pierwszych nie wiązała się generalnie z eutrofizacją zastoisk z sedymentacją ilastą, ale raczej z tym, że do tych zastoisk była dostarczana materia organiczna pochodząca z erozji torfowisk (na możliwość taką wskazywał Słomka *et al.*, 2000).

Stwierdzono, że uprzywilejowanymi miejscami do powstawania i rozrostu torfowisk na powierzchni stożków były rejony efemerycznych zastoisk. Znacząca była tam zapewne obecność wody gruntowej i izolującej warstwy ilastej. Podobnie, te same rejony zarośniętych torfowiskiem zastoisk były uprzywilejowanymi miejscami dla ekspansji piaszczystych lobów stożków. Powodem mogło być niższe ich położenie, spowodowane wyższą kompakcją osadów fitogenicznych i ilastych, dodatkowo obciążonych grzęznącymi ciałami piaszczystymi.

Zarówno Słomka *et al.* (2000) jak i Wagner *et al.* (2000) stwierdzają istnienie cykliczności w następstwie litofacji. Niniejsze badania pozwoliły jednak dokładniej określić jej charakter. Ujawnione cykle są bardzo krótkie – składają się tylko z dwóch litofacji. Jest to w głównej mierze efekt mocnego maskowania cykliczności przez procesy zaburzające depozycję – najprawdopodobniej erozję.

W odniesieniu do zachodniej części obszaru sedymentacji węglanowej, nie zostało potwierdzone sugerowane przez Wagnera *et al.* (2000) losowe następstwo węgli i wapieni jeziornych, co interpretował on jako długotrwałe lateralne sąsiedztwo środowiska jeziornego (z sedymentacją węglanową) i torfowisk "z nielicznymi i powierzchniowo ograniczonymi epizodami wkraczania jednego środowiska w drugie". Uzyskane wyniki wskazują, że takiego modelu sedymentacji nie można stosować dla całego obszaru sedymentacji węglanowej przy południowej krawędzi rowu Kleszczowa, ale tylko poza strefą zazębiania się osadów węglanowych i piasków, a być może nawet tylko w rejonie wschodniego, wiszącego skrzydła stromego uskoku synsedymentacyjnego, gdzie była słaba i najprawdopodobniej stała w czasie i przestrzeni subsydencja. Natomiast w zachodniej części (strefa wspomnianych zazębień albo nawet całe skrzydło zrzucone) zachodzi silny związek między tymi dwoma środowiskami. Spowodowane jest to zapewne większą i bardziej zmienną subsydencją, spowodowaną większą kompakcją torfów, obciążonych grzęznącymi w nich ciałami piaszczystymi. Ujawniono tam cykliczną alternację tych środowisk, czasem z udziałem pośredniczącej litofacji iłów węglistych. Wydaje się, że głównym czynnikiem, odpowiedzialnym za to jest chwiejna i regularnie naruszana równowaga między subsydencją a tempem przyrastania osadów, głównie torfów. Ponieważ z tempem subsydencji związany był poziom wód w jeziorze, przy przewadze subsydencji obszar istniejącego zastoiska z sedymentacją węglanową powiększał się a w sytuacji odwrotnej lub przy równowadze – ulegał zmniejszeniu. Przy znacznej subsydencji mogło dochodzić do zmniejszenia stężenia $Ca(HCO_3)_2$ i przerwania sedymentacji węglanowej. Wszystkie te procesy powodowały widoczne w profilach, wielokrotne oscylacje granic między wapieniami jeziornymi a węglami. Innym czynnikiem, wpływającym na wspomniany związek między środowiskiem jeziornym a torfowiskami mogło być przerywanie sedymentacji węglanowej przez zakwaszanie wody kwasami humusowymi pochodzącymi z torfowisk.