# Stable Gait Synthesis and Analysis of a 12-degree of Freedom Biped Robot in Sagittal and Frontal Planes

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**Abstract:**

*Legged machines have not been offered biologically realistic movement patterns and behaviours due to the limitations in kinematic, dynamics and control technique. When the degrees of freedom (DOF) increases, the robot becomes complex and it affects the postural stability. A loss of postural stability of biped may have potentially serious consequences and this demands thorough analysis for the better prediction and elimination of the possibility of fall. This work presents the modelling and simulation of twelve degrees of freedom (DOF) biped robot, walking along a pre-defined trajectory after considering the stability in sagittal and frontal planes based upon zero moment point (ZMP) criterion. Kinematic modelling and dynamic modelling of the robot are done using Denavit-Hartenberg (DH) parameters and Newton-Euler algorithm respectively. This paper also proposes Levenberg-Marquardt method for finding inverse kinematic solutions and determines the size of the foot based on ZMP for the stable motion of biped. Biped robot locomotion is simulated, kinematic and dynamic parameters are plotted using MATLAB. Cycloidal gait trajectory is experimentally validated for a particular step length of the biped.*

**Keywords**: *Denavit-Hartenberg parameters, sagittal and frontal plane, zero moment point (ZMP), Levenberg-Marquardt algorithm, Cycloidal gait trajectory.*

## Nomenclature

$\theta_i$ $i^{th}$ Joint angle
$a_i$ Legth of $i^{th}$ link
$d_i$ offset distance of $i^{th}$ link
Twist angle about $x_i$ axis
$m_{iR}$ ; $m_{iL}$ Masses of $i^{th}$ right and left leg
$q_i$ $i^{th}$ joint parameter
$B, L$ Breadth and length of the foot
Foot pose function
Hessian matrix
Jacobian
$\lambda$ Damping factor
Inertial/mass Matrix
Coriolis and centrifugal matrix
Gravity matrix
$D$ Coefficient matrix of torque $\tau$
$R$ Orientation matrix
Moments at the $i^{th}$ joint
$X_{zmp}$; $Y_{zmp}$; $Z_{zmp}$ X, Y and Z coordinates of ZMP
Force at the $i^{th}$ joint
$F_g$; $F_r$ ; $F_i$ Total inertial and gravity force; Reaction force; inertial force
$M_g$; $M_r$ ; $M_i$ Total inertial moment; Reaction moment; Inertial moment.
$g$ Acceleration due to gravity
Rolling angle of circle
$S_L$ Step length
$v_i$ Instantaneous velocity of $i^{th}$ link
$I_i$ Inertial matrix of $i^{th}$ link,
$\omega_i$ Angular velocity of $i^{th}$ link
$\ddot{x}$, $\ddot{y}$ , $\ddot{z}$ x,y and z accelerations

## 1. Introduction

Robots of current generation have been used in various fields isolated from the human society. They suffer major shortcomings because of their limited abilities for the manipulation and interaction with humans. Humanoid/biped robots are better suited for working in human environment and have a better degree of mobility, especially in environment with obstacles. The main motive behind the development of bipeds is its adaptability to human environment, so that there is no need to make special working environment for bipeds.

Early studies on bipeds were mostly on its locomotion and not on its real industrial applications. Now it has reached the level of designing customized bipeds for specific applications. Still, there are issues yet to be addressed, among them the most basic being stable dynamic locomotion and gait synthesis. Bipeds can perform both Static and dynamic walking. In static walking, the complete system stays balanced by always keeping the centre of mass (COM) of the system vertically over the support polygon formed by the feet during locomotion [1]. In dynamic balance or walking the vertical projection of COM does not stay within the support polygon during the motion, i.e. during motion the COM may leave the support polygon for certain periods of time. Therefore some complicated and coordinated movement of other body parts only can balance the biped. This makes the dynamic walking more difficult from a design point of view.

Location of COM and ZMP are the two important issues in biped locomotion. The concept of ZMP was put forward by M. Vukobratovic *et al.* [2] which revolutionized and accelerated the studies in dynamic walking of bipeds. ZMP is termed as the point on the ground about which the robot's resultant moments at the ground is zero. This is used as a stability criterion for dynamic walking in this work. If the ZMP is inside the support area, the walking is considered dynamically stable, because the foot can control the robots posture. The ZMP criterion cannot be applied to biped robots that do not continuously keep at least one foot on the ground or to those which do not have active ankle joints.

The motion of a humanoid comprises of time-functions of angular positions and velocities of the joint angles of the robot. The straight forward approach is to generate the joint time trajectories by solving inverse kinematics, to maintain the physical stability of the humanoid. It becomes increasingly difficult to compute the inverse kinematics as the DOF of the biped increases. However, such an approach is suitable for off-line generation of joint trajectories. Generation of low-energy gait is an open and nontrivial issue over a considerable period [3] during the motion of robot.

This paper mainly concentrates on inverse kinematics and dynamics by using Levenberg-Marquardt algorithm (LMA) and Newton-Euler algorithm (NEA) respectively to analyze the stability of biped locomotion during dynamic walking. It also proposes a methodology to find the foot size for the smooth motion of joints. In almost all previous works related to humanoid walking analysis and synthesis, stability in sagittal and frontal planes are analyzed separately with the kinematic modelling based on the geometrical approach. DH parameters are used in the present work for the kinematic modelling and ZMP concept is used for the stability analysis in sagittal and frontal planes.To the best knowledge of the authors, no work based on LMA and DH modelling for analysis and synthesis has been reported in the area of humanoid or biped robots with the minimization of foot size.

This paper is structured as follows. In section 2, kinematic, LM algorithm and dynamic modelling are described. The fundamental theory of the centre of mass (COM) and ZMP, in single and double support phases are given in section 3. Section 4 deals with the simulation of the gait trajectory for stepping motion with stability in both sagittal and frontal planes. Results and discussion are shown in section 5. Section 6 presents the concluding remarks with outlook.

## 2. Modelling of biped robot

Fig. 1 shows a 12 DOF biped robot modelled in SOLID WORKS which is having six DOF per leg, two at the ankle, one knee and three at the hip. The ankle joint of both legs have yaw and pitch motions, the knee is having only pitch motion and hip joints of both the legs have roll, pitch and yaw motions. The proposed model consists of seven links in order to approximate the locomotion characteristics similar to those of the lower extremities of the human body.

The complete walking cycle consists of three single support phases (SSP) in which only one leg is on the ground while the other swings forward and four double support phases (DSP) in which both legs are on the ground. The stance leg in contact with the ground carries the whole weight of the robot. During the transition from single support phase to double support phase, swing leg decelerates to zero velocity. As a result of this, huge impact forces are developed at the contact phase for a short period of time.

At the end of the DSP the swing leg accelerates which creates jerk in the joints and links of the robot. In DSP, the robot will be stable when the projection of COM stays within the support polygon. As a result of transition between the single support and double support phases, the problem of instability of the humanoid arises. The contact phase in walking is almost 20% of the total gait period [4]. As it is difficult to find out the reaction forces accurately, it is assumed that the impact of swing leg is perfectly inelastic while ensuring that no slippage occurs. Another important assumption made is that during the SSP, stance foot remains in flat contact with the ground. In SSP, the robot will be stable when the ZMP stays within the support foot polygon.

*Table 1. Biped mass and dimensions*

| Mass | $m_{0R}$ | $m_{1R}$ | $m_{2R}$ | $m_6$ | $m_{2L}$ | $m_{1L}$ | $m_{0L}$ |
|---|---|---|---|---|---|---|---|
| Kg | 0.025 | 0.2 | 0.25 | 0.30 | 0.25 | 0.2 | 0.025 |
| Link length | ***d*** | ***d*** | ***d*** | ***d*** | ***d*** | ***d*** | ***d*** |
| m | 0.042 | 0.098 | 0.090 | 0.060 | 0.090 | 0.098 | 0.042 |

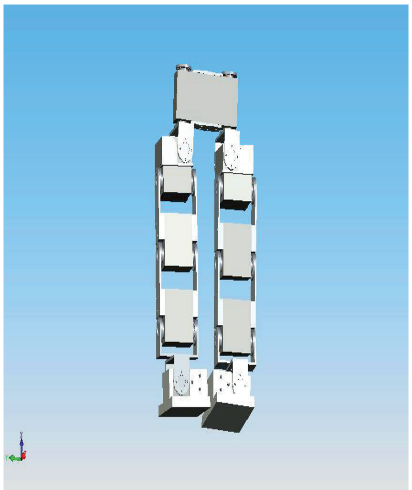

*Fig.1. Twelve DOF biped robot model*

### 2.1. Kinematic modelling

Kinematic diagram of the 12 DOF biped is given in Fig. 2. Biped robots can be described kinematically by using joint-link DH parameters namely joint angle ($\theta_i$), link length ($a_i$), offset distance ($d_i$) and link twist ($\alpha_i$). Table 1 shows link dimensions of the biped robot. Four DH parameters corresponds to each link of biped are given in table II based on the frame assignment as shown in Fig. 3. Forward kinematics determines the pose of robot end effector as a function of its joint and link parameters where as the inverse kinematics gives the values of the joint variable corresponding to end effector or foot pose.

#### 2.1.1. Inverse kinematics

A suitable step length is assumed for the biped walking analysis from the idea that the step length for minimum energy consumption is about 60% of hip height [5]. Cartesian space trajectory is planned to get the trajectory of the swing foot which follows a cycloidal trajectory profile during the motion of robot [6]. As this profile is made by superposition of linear and sinusoidal function,

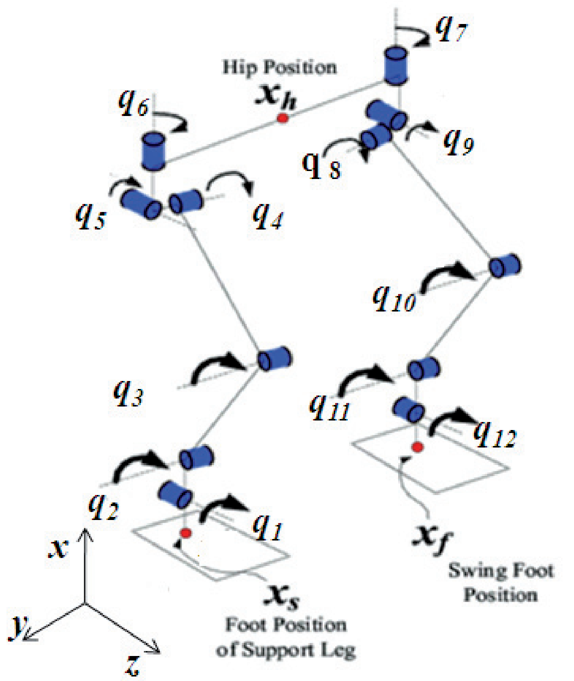


*Fig. 2. Kinematic diagram of biped robot*

it has a property of slow start, fast moving, and slow stop. This reduces the jerk during the start and end of walking. This characteristics can reduce the over burden at instantaneous high speed motion of the actuator. Points on the swing gait trajectory are taken as poses of the foot for getting the joint variables of each leg of the robot. Final pose matrix of the biped robot model ${}^{0}T_{12}$ is equal to the pose of the foot on the swing gait trajectory.

### 2.2. Levenberg-Marquardt algorithm (LMA)

The Levenberg-Marquardt method is a search method which gives the advantages of both Gauss-Newton direction and the steepest descent direction methods because it uses a search direction that is a cross between the Gauss-Newton direction and the steepest descent direction.

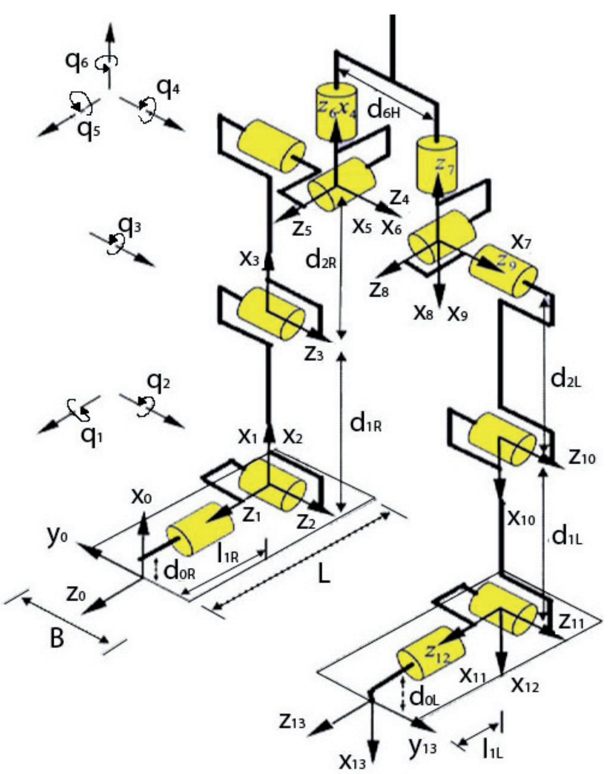


*Fig. 3. D-H modelling-Frame assignment for 12-dof biped*

*Table 2. D-h parameter table*

| Link(i) | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|
| $\theta$ | $\theta$ | $\theta$ | $\theta$ | $\theta$ | $\theta\ -\pi\ 2$ | $\theta$ |
| $d$ | 0 | 0 | 0 | 0 | 0 | 0 |
| $a$ | 0 | $d$ | $d$ | 0 | 0 | $d$ |
| $\alpha_i$ | $-\pi/2$ | 0 | 0 | $\pi/2$ | $-\pi/2$ | 0 |

| Link(i) | 7 | 8 | 9 | 10 | 11 | 12 |
|---|---|---|---|---|---|---|
| $\theta$ | $\theta$ | $\theta\ -\pi\ 2$ | $\theta$ | $\theta$ | $\theta$ | $\theta$ |
| $d$ | 0 | 0 | 0 | 0 | 0 | 0 |
| $a$ | 0 | 0 | $d$ | $d$ | 0 | $d$ |
| $\alpha_i$ | $\pi/2$ | $\pi/2$ | 0 | 0 | $-\pi/2$ | 0 |

LMA is used for finding inverse kinematic solution in this work. Solving inverse kinematics involves solution of twelve nonlinear equations with trigonometric functions. Six independent equations, three for orientation and three for position are to be solved. Since it is a bit laborious to do the inverse kinematics of the 12 DOF robot manually for the whole interpolated points, inverse kinematics is carried out in MATLAB and optimized results satisfying the boundary conditions are obtained. Levenberg-Marquardt iterative method is used for this purpose. It is a modification of Newton-Euler algorithm and gradient descent method. It is also called damped Gauss-Newton method, as it uses a damping factor to decide the accuracy level of solutions when the search approaches the minima. For starting, an initial guess is to be provided. An advantage of this method is that, the search direction is independent of the initial solution set given and it gives the actual minima even if the initial assumptions are far from the global minima. LM algorithm for the present context is explained below.

$$\text{Min } F(\theta)=[F_1(\theta)\ldots F_{12}(\theta)];\ \text{Sub to}$$
$$\pi/4\leq\theta\leq\pi/4,\ \text{Where,}$$
$$\theta=[\theta1\quad\theta2\ \ldots\theta12]^T.$$

The coefficient of the quadratic term of local Taylor series expansion of a function is, $Y=f(\theta+\delta\theta)\approx f(\theta)+J(\theta)\delta\theta+\delta\theta^T H(\theta)\delta\theta$.

The convergence criteria is

$$f(\theta+\delta\theta)\approx f(\theta);$$

Therefore, $J(\theta)\delta\theta+\delta\theta^T H(\theta)\delta\theta=0$

$$\delta\theta=-H(\theta)^{-1}J(\theta).$$

Modified Hessian is $H(\theta,\lambda)=2J^T J+\lambda I$

1. Set damping factor $\lambda=0.001$
2. Solve $\delta\theta=-H(\theta,\lambda)^{-1}g$
3. If $(\theta_n+\delta\theta)>f(\theta_n)$, increase $\lambda$ (x 10 say) and go to 2.
4. Otherwise, decrease $\lambda$(x 0.1 say), let $\theta_{n+1}=\theta_n+\delta\theta$, and go to 2.

When $\theta_n$ the algorithm has converged set $\lambda=0$ and compute the final solution

Where $\theta_n$ is the initial vector assumed and Hessian and Jacobian matrices, $H(\theta)\&J(\theta)$ are given in equation

1. a & 1.b respectively.

$$H(\theta) \approx F(\ddot{\theta}) = \begin{bmatrix} \frac{\partial^2}{\partial\theta_1\partial\theta_1}F(\theta) & \cdots & \frac{\partial^2}{\partial\theta_1\partial\theta_j}F(\theta) \\ \vdots & \ddots & \vdots \\ \frac{\partial^2}{\partial\theta_i\partial\theta_1}F(\theta) & \cdots & \frac{\partial^2}{\partial\theta_i\partial\theta_j}F(\theta) \end{bmatrix} \quad (1.a)$$

$$J(\theta) \approx F(\ddot{\theta}) = \begin{bmatrix} \frac{\partial}{\partial\theta_1}F(\theta) \\ \vdots \\ \frac{\partial}{\partial\theta_i}F(\theta) \end{bmatrix} \quad (1.b)$$

### 2.3. Dynamic modelling

The main challenges of gait planning are learning which includes the selection of specific initial conditions, constraints and their associated gait parameters [4]. In this section, different methodologies are adopted for dynamically constrained locomotion of biped robot in sagittal and frontal planes. From a control point of view, the inverse dynamics problem is of solving the joint torques from the joint angles along with their first and second order derivatives. In this work, we have used the Newton-Euler recursive algorithm for dynamic analysis. Since stance foot is assumed to be in flat contact, resultant ground reaction force/moment and torques can be computed using Newton-Euler algorithm [10].

The process consists of 2 iterations, (i) forward iteration to compute link velocities and accelerations and (ii) backward iteration to get the torque variation at joints. Initially velocity and acceleration of base frame is taken to be zero. While the stance leg is in motion no external forces are acting on it, except gravity loading. It is also assumed that the centroid of the link and the centre of mass of link coincide. A general dynamic model for biped walking related to the joint coordinates vector and joint torque vector without considering the friction and other disturbances is given below.

$$\mathbf{M}(\theta)\ddot{\theta}+\mathbf{C}(\theta,\dot{\theta})\dot{\theta}+\mathbf{G}(\theta)=\mathbf{D}\tau \quad (2)$$

Where, is the 12 x 12 inertia matrix, is the 12 x 12 coriolis and centrifugal matrix and is the 12 x 1gravity vector, is the 12 x 12 coefficient matrix of joint torques. Joint torques at different joints are determined through backward iteration by using the set of equation 3 and 4.

$$\bar{\tau} = i\eta_i^{T\,i}R_{i\text{-}1}\hat{z}_0 \quad (3)$$

$$^{i}\eta_i = R_{i+1}{}^{i+1}\eta_{i+1} + {}^{i}R_0^{\,i\text{-}1}D_i x\, {}^{i}R_{i+1}{}^{i+1}f_{i+1} + ({}^{i}R_0^{\,i\text{-}1}D_i + {}^{i}R_0{}^{i}\,\bar{r}_i)\, x\, {}^{iF}{}_i + {}^{i}N_i \quad (4)$$

Where, $i = 12,11\ldots,1$; $i\eta_i$ is the moment exerted on link $i$ by link $i$–1 and $^{i}R_{i\text{-}1}$ is the orientation matrix, $iF_i$ is the total external force acting at the centre of mass of the link, $iN_i$ is the total external moment acting on link at its centre of mass, is $if_i$ the force exerted on link $i$ by link $^{i}R_0^{i-1}D_i$ is the coordinates of the $i^{th}$ joint when referred to frame $i$ and $^{i}\bar{r}$ is the centre of mass of link referred to frame $i$.

## 3. Zero moment point and gait trajectory

Static walking stability condition is sufficient to ensure locomotion for very slow motion of biped robot. Some of the drawbacks of this technique of motion planning are the discrete nature of the motion of the robot and the time required for taking a single step being unusually long. It is not always necessary for the centre of mass (COM) of the robot to lie vertically above the base polygon. Another method of analyzing stability is based on the Zero moment Point criterion [6]. Zero Moment Point is defined as the point about which the moment of all the active forces acting on the robot turn out to be zero. In static gait planning problems, biped robot is stable if the projection of the centre of mass (COM) falls within the convex hull of the foot support polygon. In dynamic locomotion, link acceleration, inertial forces, and ground reaction force are also to be considered and the ZMP should be within the convex hull of the foot polygon for satisfying the stability criterion. The dynamic locomotion is highly nonlinear and difficult to analyze in real environment. The condition in the static and dynamic stability of the biped during the single support phase is the location of the ZMP must be inside the convex hull of the supporting foot. In double support phase ZMP or projection of COM should lie within the convex hull of support polygon formed by left and right foot. According to D'Alembert's principle, if all forces are balanced, the motion of the biped is physically realizable. By D'Alembert's principle the total forces and moments acting on the biped must be zero. This is given by:

$$F_r + F_e = 0 : M_r + M_e = \mathbf{0} \quad (5)$$

Where is the ground reaction force, $F_i$ is the total inertial and gravity force acting on biped, $M_r$ is the reaction moment and $M_e$ is the inertial moment acting on biped. Let $F_i$ be the inertial force, $M_i$ be the inertial moment, and $m_i$ be the $i^{th}$ mass of the $^{th}$ segment $(i = 1...n)$ . We have:

$$F_e = \sum_{i=1}^{n}(-F_i - m_i g) = \sum_{i=1}^{n} -m_i(\dot{v}_i + g) \quad (6)$$

$$M_e = \sum_{i=1}^{n} M_i = \sum_{i=1}^{n} -\frac{d}{dt}(I_i\omega_i) \quad (7)$$

Where, $v_i$ is the instantaneous velocity, $I_i$ is the inertial matrix, $\omega_i$ is the instantaneous absolute angular velocity of ith link at its COM, terms are relative to the fixed reference coordinate, say O as in Fig. 4 [10].

The balancing problem of the biped system can be reduced at an assigned ground point ($x$'= 0, $y$', $z$') called the ZMP, where the resultant moment (M) at the ground plane is zero ($Mx$'= $My$'= 0).From the relation of the equivalent force moment, one obtains:

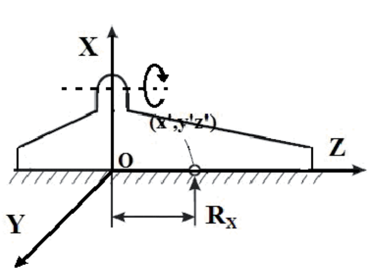


*Fig. 4. Reference co-ordinate system for foot base*

$$F_e = \sum_{i=1}^{n} m_i(\dot{v}_i + g) \quad (8)$$

$$y' F'_z - z' F'_y + M'_x = \sum_{i=1}^{n} -m_i\,(y_i \ddot{z}_i + z_i \ddot{y}_i) - \sum_{i=1}^{n} -M_{ix} \quad (9)$$

$$z' F'_x = \sum_{i=1}^{n} -m_i\,(z_i(-\ddot{x}_i - g) + x_i \ddot{z}_i)) - M_{iy} \quad (10)$$

$$y' F'_x = \sum_{i=1}^{n} -m_i\,(y_i(\ddot{x}_i + g) - x_i \ddot{y}_i))\ \sum_{i=1}^{n} M_{iz} \quad (11)$$

Where $F'$ and $M'$ are the resultant force and moment at the ZMP $(0, y', z')$ respectively, and $(x_i, y_i, z_i)$ is the vector from the origin O of the fixed reference coordinate O to the COM of link considered. $\ddot{x}, \ddot{y}, \ddot{z}$, are the corresponding components of accelerations in respective directions. From the above equations, one obtains:

$$X_{zmp} = 0 \quad (12)$$

$$Y_{zmp} = \sum \frac{\mathbf{m_i}\left(g + \ddot{x}\right) y_i - m_i\left(\ddot{y} x_i\right) - M_{iz}}{\mathbf{m_i(g + \ddot{x})}} \quad (13)$$

$$Z_{zmp} = \sum \frac{\mathbf{m_i}\left(g + \ddot{x}\right) z_i - m_i\left(\ddot{z} x_i\right) - M_{iy}}{\mathbf{m_i(g + \ddot{x})}} \quad (14)$$

The constraint on the dynamic motion of the biped during the single-support phase is the location of the ZMP $(0, Y_{zmp}, Z_{zmp})$ must be inside the convex hull of the supporting foot. In the single-support phase the stable convex hull is same as the area occupied by the supporting leg on the ground. Therefore, $Z_{min} < Z_{zmp} < Z_{max}$ and $Y_{min} < Y_{zmp} < Y_{max}$ where, we assume that the supporting foot is rectangular, parallel to the fixed reference coordinate O, and between points $(0, Y_{min}, Y_{max})$ and $(0, Z_{min}, Z_{max})$. Mathematical interpolation is one of the simplest methods for providing suitable gait trajectory in accordance to the given boundary conditions. Cartesian space trajectory planning is carried out to get the trajectory of the swing foot. Generally human's ankle joint motion trajectory is a cycloidal profile in normal walking (Kurematsu, Kitamura & Kondo, 1988) Cycloidal profile reduces effects of sudden acceleration at the beginning and deceleration at the end during the gait generation. Hence the cycloidal profile is used for the trajectories of the swing foot. As this profile is made by superposition of linear and sinusoidal function, it has a property of slow start, fast moving, and slow stop. This avoids the jerk that can happen during the start and end of walking. This characteristics can reduce the over burden at instantaneous high speed motion of the actuator. Equation of a cycloid in parametric form for selecting break points on the trajectory is given in equations 15 and 16. Gait trajectory pattern is shown in Fig. 5.

$$x_i = S_L\,(\varphi_i - \sin\varphi_i)/2\pi \quad (15)$$

$$z_i = S_L\,(1 - \cos\varphi_i)/2\pi \quad (16)$$

Where $i = 0, 1 \ldots N$, the number of poses of foot on the trajectory and $S_L$ is the step length.

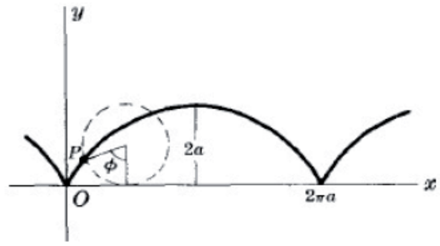


*Fig. 5. Cycloid curve*

## 4. Simulation

Numerical simulation of 12 DOF biped walking is done using MATLAB™.3-D kinematic pattern of the biped for a single step is shown in Fig. 6. Variations of kinematic and dynamic parameters and ZMP are plotted. Initially both legs of biped are in stance position then the left leg is stepping a length of 10 cm in 1 s. Simulations are carried on a biped robot having hip height of 25 cm and mass of 1.7 kg. Kinematic and dynamic modellings help to synthesis and analyze the biped robot at different scaled dimensions based on stability.

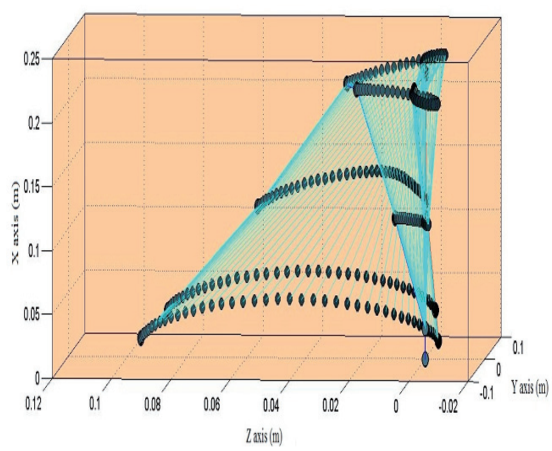


*Fig. 6. 3-D Biped walking pattern*

## 5. Results and discussion

Stable gait generation of a 12 DOF biped robot is demonstrated in this paper. Variations in parameters like joint angles, link velocities and link acceleration are plotted during the stable motion of biped. Torque and ZMP variations are also analyzed here. The variation of joint angles at ankle, knee and hip for right and left leg are varying smoothly and continuously for a single step as shown in Fig. 7 and this assures a smooth transition of the robots motion. Rolling angular variations at the ankle joint of the left leg and hip joint pitch angular variations ($4^{th}$ and $9^{th}$ joints)of both legs are high compared to other joint angle variations. Because these two joints plays vital role in the stability of biped motion in this analysis. Initially, when the left leg is about to lift, both hip and knee joints should have some angular variations for bringing the COM within the support foot polygon.

Link velocities and accelerations at the COM are given in Fig. 8 and Fig. 9, respectively. First link is fixed at the ground during the walking so the velocity and accelerations are zero. Velocity is maximum for the swing foot (link 12) and minimum for the lower part (link 2) of the stance leg. All other links the velocities are varying approximately in between 0 and 25 cm/s.

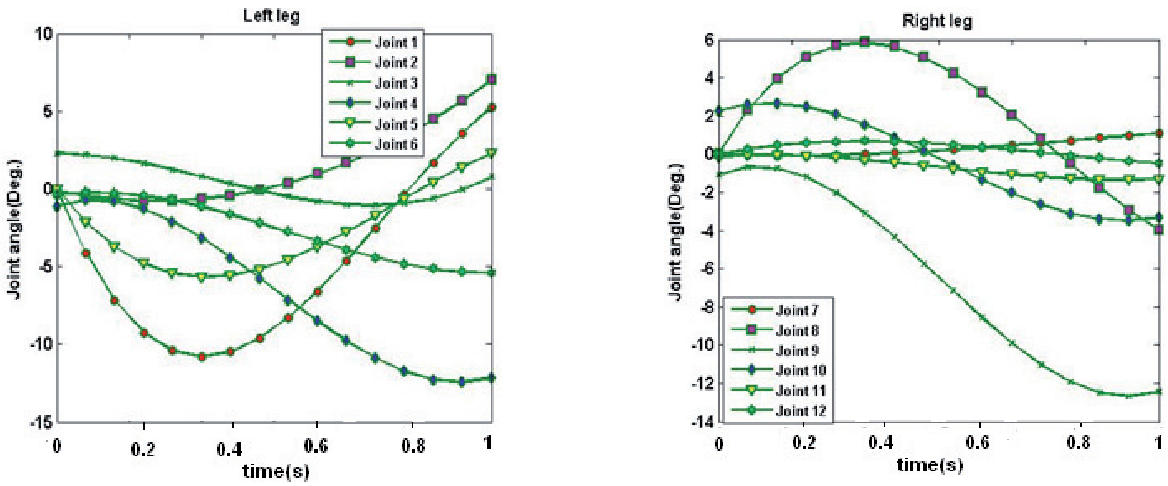


*Fig.7. Joint angle variations of left and right leg*

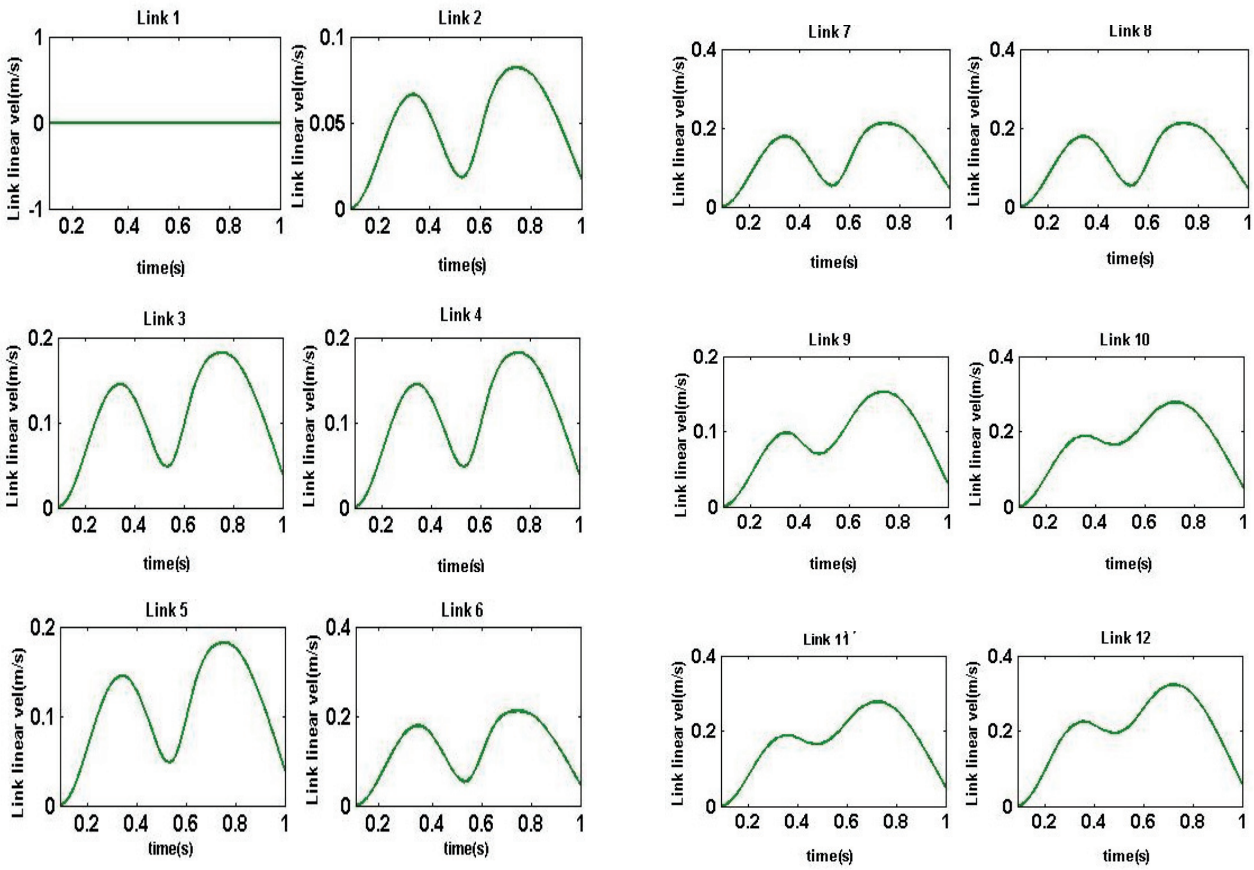


*Fig. 8. Link linear velocity*

Variations in accelerations are smooth but there are up and downs because the biped is moving in high speeds with step length of 10 cm. There are some values of accelerations at the beginning and end of the gait trajectory so that the jerk will be the minimum at these two locations. Up and downs of velocities result in irregular variations in the accelerations as shown in Fig. 9. This creates jerk at the joints and links of the robot at intermediate positions and biped can mostly be suited at slow speeds for small step lengths and moderate speed at higher step lengths. Jerk will be reduced for higher step lengths in moderate speeds but stability will be achieved with larger foot size. This will be clear from the ZMP variations plotted in Fig. 11.

It is clear from the velocity and acceleration diagrams that the velocity and acceleration variations are same for link 3, 4 and 5. Similarly velocity and acceleration variations are same for links 6, 7 and 8. This is because of the assumption that the joints $4^{th}$, $5^{th}$ and $6^{th}$ are at the same origin and also the joints $7^{th}$, $8^{th}$ and $9^{th}$ are at the same origin in modelling.

Fig.10 shows the continuous variations of torque for all joints. Starting torque for the first joint is high because this joint is only making the robot walk by swinging the whole system in the frontal plane. Geared motor can be used for getting high torque at joint 1. Torque is smallest for the ankle joint of the swing foot. By changing various kinematic and dynamic parameters it is possible to bring the ZMP within the limited size of Y-Z plane for attaining stable walking, and variations are plotted against the stepping time as shown in Fig. 11.

Variation is more in Z direction compared to  because the Z component of acceleration has an more effect on shifting of ZMP. The inertia components are small here due to the small size of biped. However those inertial

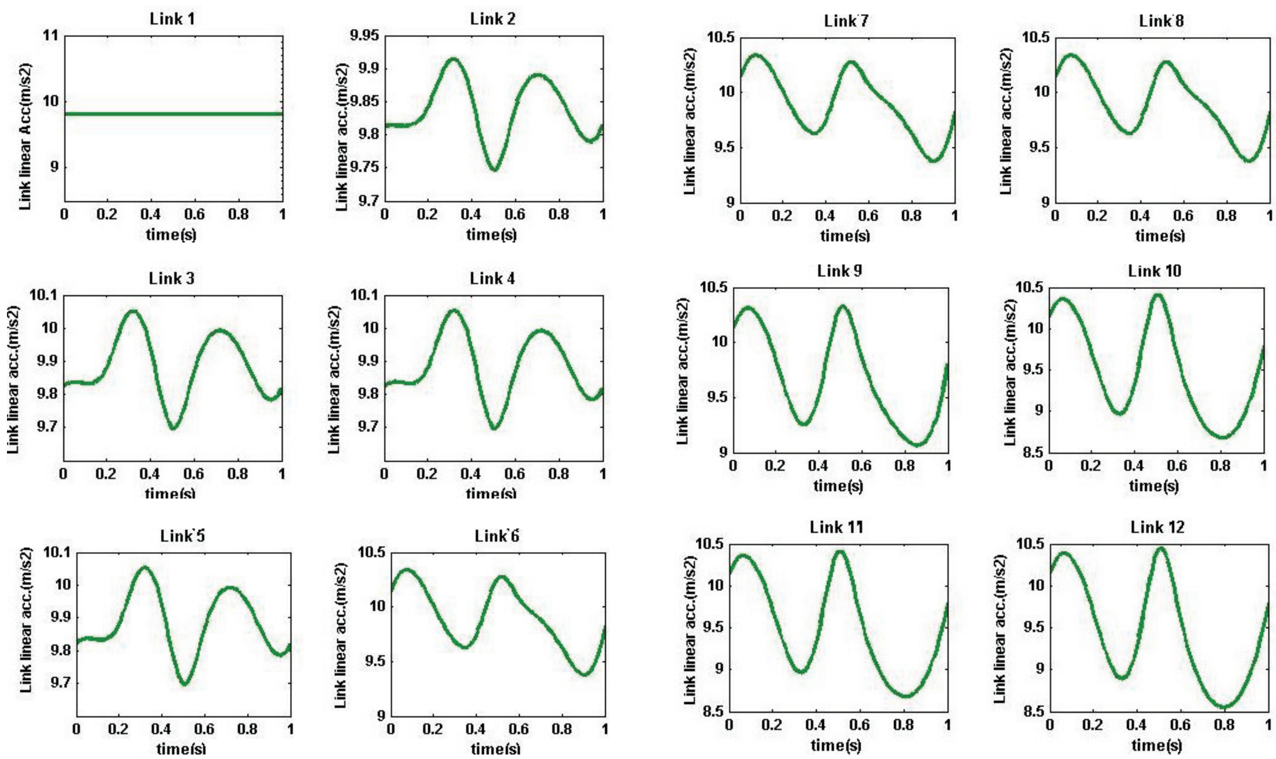


*Fig. 9. Link linear acceleration*

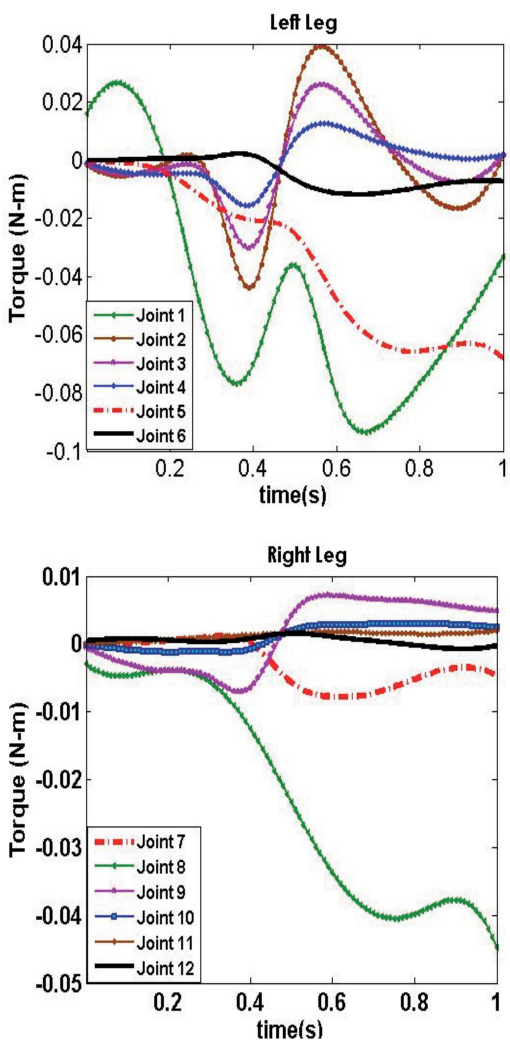


*Fig. 10. Joint torque*

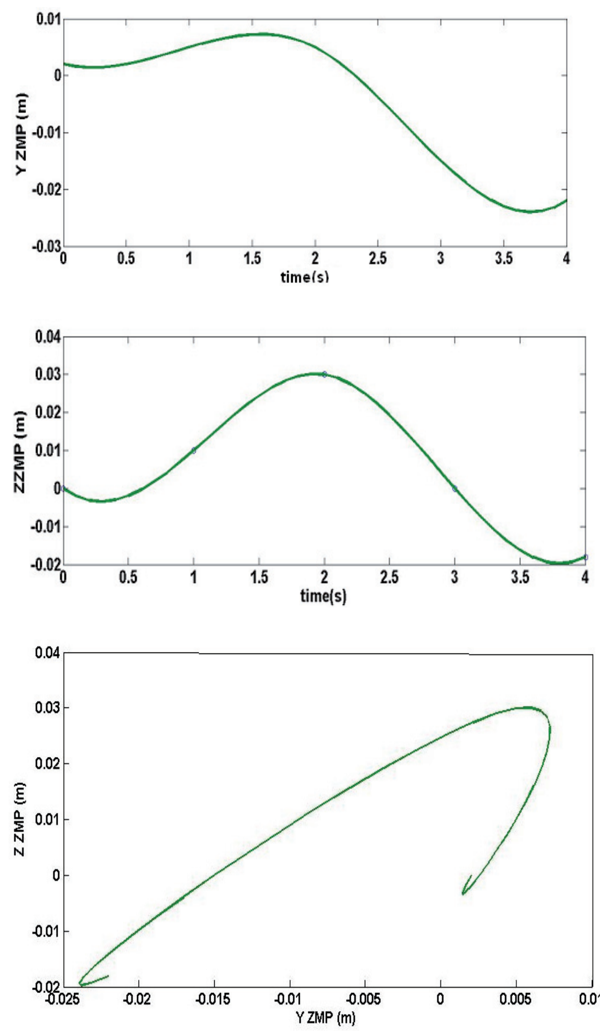


*Fig. 11. ZMP variation in first walking phase*

terms will not be negligible in case of fast bipedal activities like running and jumping, or when the link masses and dimensions are comparable to those of the actuators.

There will not be any difference if we neglect the inertial effects in slow motion. ZMP moves in Y-Z plane approximately in a parabolic path within the foot base. The maximum approximate range of $Y_{zmp}$ and $Z_{zmp}$ are -2.4 cm to 0.7 cm and -2 cm to 3 cm respectively. Fig.1 1 shows the movement of ZMP on the foot of the stance leg. This plot gives the feasible size of foot of stance leg for a particular step length. The resultant values of ZMP variations are represented graphically for step length of 0 to 20 cm in Fig. 12. This helps to decide the foot size for a range of step length based on kinematic and dynamic constraints. As per the Fig. 12, foot size of 10 cm x 10 cm is required for biped walking through a cycloidal gait for a step length of 20 cm.

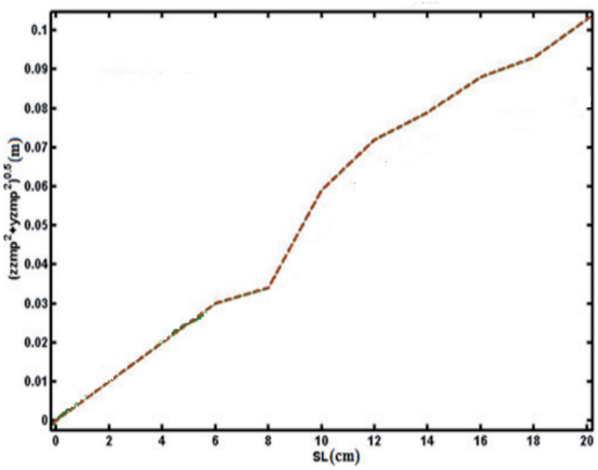


*Fig. 12. ZMP Vs Step length*

## 6. Experimentation

Walking gait generation is simulated and the results involving the relevant variables are analysed in the previous section. In this section, gait generated for a step length of 10 cm is experimentally validated in a 12 DOF biped. The experimental validation is done by matching simulated cycloidal trajectory with real time gait trajectory. Validation can also be done experimentally by evaluating and comparing ZMP variations along with the gait trajectory. Computer / Processor is interfaced with the biped robot through a mini maestro 12 channel servocontroller for controllling actuators for the required cycloidal trajectory.

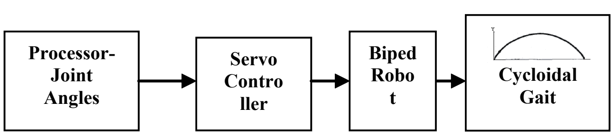


*Fig. 13. Data Flow diagram*

The block diagram shown in Fig. 13 depicts the details of data flow for testing and validation of bipedal gait. Joint angles corresponding to a single step swing foot trajectory is determined using MATLAB and the signals are sent to the biped for the required motion.

Fig 14 shows the snapshots of 12 DOF biped robot walking for a step length of 10cm. Instant motions are captured for the gait analysis during the foot step movement. Evaluated real time gait trajectory is compared with the cycloidal gait trajectory determined from the simulation result. Robot stable motion and Real time gait trajectory are shown in Fig. 15 and Fig. 16 ,respectively. Joint angles are fed to the biped for getting the cycloidal trajectory with a fixed step length. One of the experimental real time cycloidal trajectory is given in Fig. 16. Experiment is conducted five times for the same joint angles and steplength. Average step length obtained in the real time gait genaration is approximately 10.3 cm instead of 10 cm. A cycloid is constructed corresponding to the step length of 10 cm and its calibrated image is superimposed on the plot of 19 instataneous poses of swing foot. The dots in Fig. 15 are the instantaneous poses obtained during the experimentation.

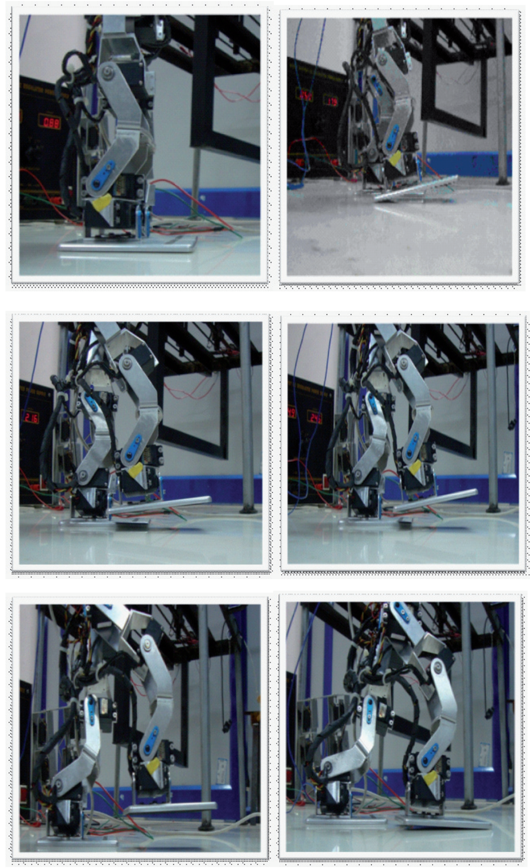

*Fig. 14. Snapshots of biped walking*

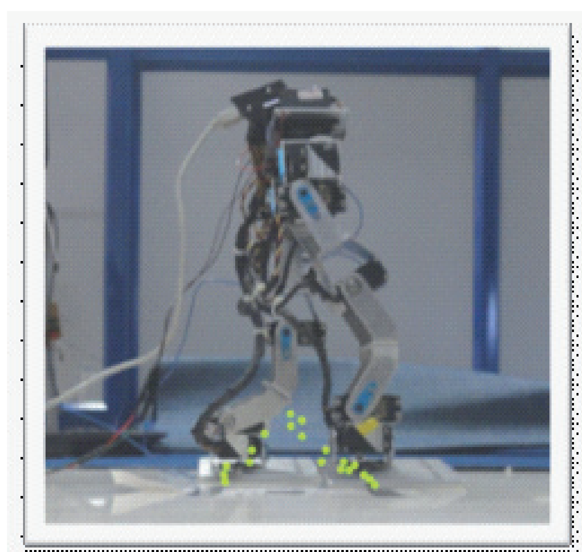

*Fig. 15. Points on the stable gait trajectory*

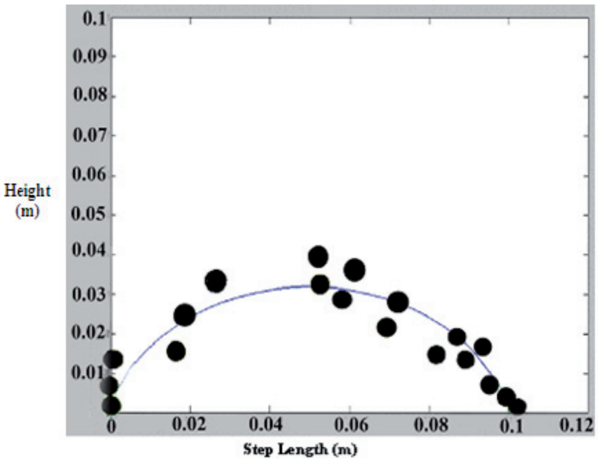


*Fig. 16 Real time gait trajectory*

Fig. 16 depicts the variations of points on the real time stable gait trajectory with the theoretical cycloidal gait. In this particular real time gait trajectory the step length obtained is approximately 10.1 cm instead of 10 cm. This Analysis shows the correctness of modelling and gait trajectory of the 12 DOF biped robot.

## 7. Conclusion and outlook

Stability analysis of a twelve dof biped robot in the sagittal and frontal plane for a cycloidal gait is presented in this paper. Generation of a gait for the stable walking based on zero moment point within a particular foot size is attempted. The inverse kinematic solutions are found by using Levenberg-Marquardt iterative method. Motion of the robot is constrained because of the limited number of DOF. The present synthesis and analysis gives idea of foot size for stable biped walking. Experimentation for determining the real time gait trajectory is attempted and trajectory is compared with theoretical trajectory. This experimental result authenticates the suitability of the model for the synthesis and analysis of biped robot. At present researchers are trying to minimize the foot size to avoid self collision and flexibility with higher level of stability during walking. This work gives a clear direction for generating an optimum gait trajectory based on ZMP with minimum foot size. It is expected that this will lead to optimization of the foot size by considering all the kinematic and dynamic constraints for achieving better stability. It may be considered that the synthesis and analysis procedure can be refined in many ways: some of them being, optimum smooth gait planning with minimum energy consumption using traditional and soft computing techniques by changing the walking parameters like step lengths, stepping time and height of the trajectory. Simulation can be extended to various structured and unstructured environments. Authors are attempting to verify experimental results by evaluating and comparing ZMP variations along with the gait trajectory in a complete walking cycle.

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