About Evaluation of Multivariate Measurements Results

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Abstract:

A brief review of few problems arising in the correct numerical expression and evaluation of results of indirect multi-parameter measurements is given. There is included a theoretical basis for determining the estimates of values, uncertainties and correlation coefficients of the indirectly obtained multi-measurand, which are processed from data of the simultaneously measured set of variables. The algebra of random vectors is used. A numerical example illustrates the linear transformation of two variables and the types of improperly evaluated results – that may occur with overrounding. There are given thresholds of the safe uniform rounding of mean vector and its scatter ellipsoid. There is proposed an upgrading of the GUM Example H.2 and of the uncertainty equation for nonlinear functions. It is also evidenced that correlation matrix of current 2010 data of fundamental physical constants recommended by CODATA has non-negligible negative eigenvalues. In the end of this work it is argued for the urgent needs of standardization of e-publication of the experimental data in two parts: e-printed traditional narrative part, and an attached computer readable file with all numerical input data and results, to allow "fast" numerical peer review of the proposed publication reporting new measurement results. This work is a result of an interdisciplinary cooperation of a metrologist and a nuclear physicist.

***Keywords**: uncertainty, indirect measurements, multi-measurand, correlated data*

1. Introduction

Simultaneous measurements of several statistically related quantities, i.e. correlated, are performed in science, education, technology, economy and many other disciplines. From the digitally processed on- or off-line data of m variables, directly measured on input, the n other variables (in physics called as observables) are determined indirectly on output, if their mutual relation is known. In addition to estimators of values and uncertainty the knowledge about correlation coefficients of output quantities also is of special importance for some or all of these variables to be jointly processed further.

Accuracy of evaluated output multi-measurand data depends on the statistical uncertainties of given parameters of input multi-measurand, as well as on the accuracy of their processing. Final rounding of indirectly obtained data of output multi-measurand must depend on a uncertainty of the input data [6]. The "safe rounding" of the digitally processed multi-measurand data should be done in such a way that they are not be damaged. If the accuracy of final uncertainties or number of repetitions of raw measurements are not given in publication of input data then it should not be assumed that the values of estimators of standard deviations and correlation coefficients of the initial variables are correctly found from measurement data and are as their values for whole populations.

In indirect multi-dimensional measurements there are two border types of relations of the uncertainty both components u_A and u_B [1].

First case: uncertainty $u_A << u_B$. In such situations it is enough to provide the necessary instrumental resolution and accuracy for measurement of input values and to determine cross-links to the output.

Second case: $u_A >> u_B$ when all environmental effects interacting on input measurements are carefully eliminated and the uncertainty of type B is small compared to the range of random scatter of observed variables. Here one should achieve maximum accuracy in measurements and then the information obtained in the experiment is not partially lost in the processing of the random input data and in the rounding of the obtained results. The number of observations should be as large as possible to minimize the statistical type uncertainty u_A.

2. Theoretical backgrounds in short

In multivariate indirect measurements the input multi-measurand can be expressed by random vector $\mathbf{X}=[X_1, X_2,... X_m]^T$ and output one – by vector $\mathbf{Y}=[Y_1, Y_2, ... Y_n]^T$. These random vectors $\mathbf{X}$ and $\mathbf{Y}$ of dimensions m and n, respectively, can be described by the multi-dimensional distributions. In general case the relation between them can be expressed by

$$\mathbf{Y} = \mathbf{F}(\mathbf{X})$$

If $\mathbf{F}$ is a linear operator, then

$$\mathbf{Y} = \mathbf{S} \cdot \mathbf{X}$$

Where: $\mathbf{S}$ is matrix of dimensions n x m and $n \le m$.

Two examples of multivariate indirect measurements are given in Fig 1.

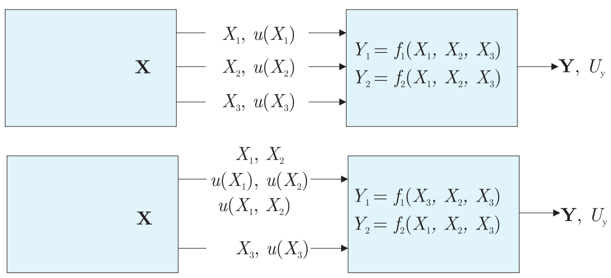

Fig. 1. Examples of indirect evaluation of measurement data of 2 jointed output variables $\mathbf{Y}=[Y_1, Y_2]^T$ *from measurements of 3 input variables* $\mathbf{X}=[X_1, X_2, X_3]^T$*: a) no correlated, b) correlated* X_1, X_2 *[10]*

The basic structure of the numerical estimation of the multi-measurand should contain averaged components of the random vector and a description of the multidimensional scatter region of it. The accuracy of both these data should be also known. Even if the relation $\mathbf{Y}=\mathbf{F}(\mathbf{X})$ is non-linear, in the most cases for small deviations of the random vectors **X** and **Y** their scatter regions can be defined by a model of joint n-dimensional normal probability distribution. Then, for a given probability density p_0 the distribution region for $p \geq p_0$ takes the form of a n-dimensional hyper-ellipsoid with its center at the end of the average vector. Relations between covariance matrices of hyper-ellipsoids of the output and input measurands (in a linear approximation of the observables in the vicinity of the end of mean vector $\overline{\mathbf{Y}}$) are described analytically by

$$\mathbf{c_Y} = \mathbf{S}\ \mathbf{c_X}\ \mathbf{S}^T$$

Where: $\mathbf{S} \equiv (\partial \mathbf{Y})/(\partial \mathbf{X})$ is the matrix of linear sensitivity coefficients.

The matrix **r** of the correlation coefficients is defined by the relation

$$\mathbf{r} = \boldsymbol{\sigma} \mathbf{c} \boldsymbol{\sigma}^T$$

It is called also shortly as the correlator.

A multidimensional distribution is normal if matrices **c** and **r** are positive definite, i.e. their eigenvalues λ_i, which are the roots of the characteristic equations

$$\det[\mathbf{c}-\lambda\mathbf{1}]=0, \text{ and } \det[\mathbf{r}-\lambda\mathbf{1}]=0,$$

should be positive [3].

This requirement was not included in Supplement 2 of GUM [1] in above form.

So, to express correctly the result of measuring or evaluating random vector quantity the **minimal data structure** should contain:

- Mean vector,
- Vector of standard deviations and their uncertainties (or number of measurements in each sample),
- Positive definite correlation matrix and uncertainties of their elements,
- Machine precision used to compute vector parameters and eigenvalues of correlation matrix.

With these data the user will have complete information to plan and control the safe usage of data in next computations.

In Fig 2 there is shown an example of a linear transformation of two dimension (2D) "Greek" vector $\mathbf{X}=[\eta, \zeta]^T$ to "Latin" vector $\mathbf{Y}=[x, y]^T$ [5].

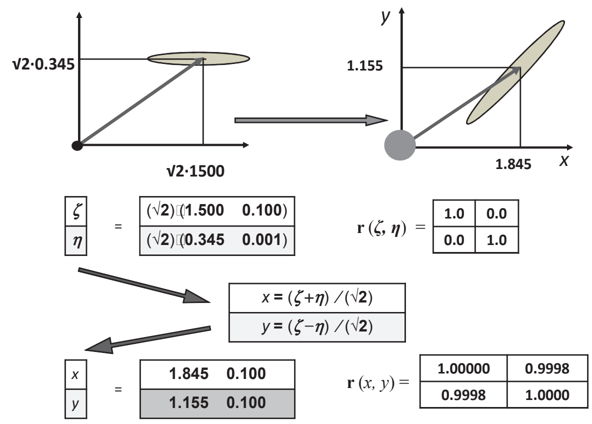

Fig 2. Linear transformation of 2D random vector [5]

Basic equations for the processing of 2D random vectors are given in Table 1 and typical distortions of output data by not proper – too high rounding are shown in Fig. 3 [5].

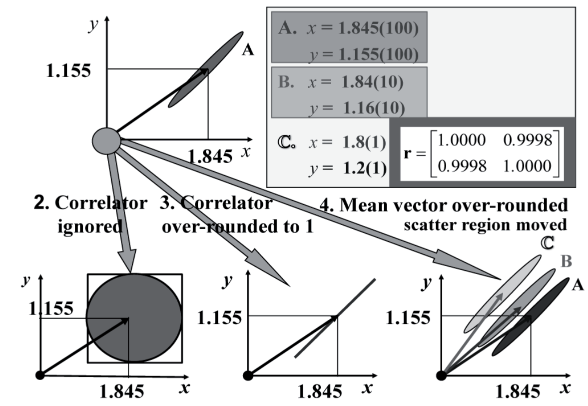

Fig 3. Cases of improper presentation of correlated 2D data

To express difference $\Delta\mathbf{Y}=\mathbf{Y}_i-\mathbf{Y}$ between the center of the ellipse of transformed original raw data **Y** and the end of rounded vector Y_i Mahalanobis distance χ is used, which is given by [12]

$$\chi^2 = \Delta\mathbf{Y}\frac{1}{\sigma_x\sigma_y}\mathbf{r}(x,y)^{-1}\Delta\mathbf{Y}^{\mathrm{T}}$$

Let us consider rounding of data **Y** given in Fig 2 [5]. Raw data **Y** rounded to 3 digits after decimal point:

Table 1. Basic formulas of the 2D random vector transformation of Fig 2

Transformation	Covariance matrix
$\begin{bmatrix}x\\y\end{bmatrix} = \mathbf{S}\begin{bmatrix}\varsigma\\\eta\end{bmatrix} = \begin{bmatrix}1 & 1\\1 & -1\end{bmatrix}\begin{bmatrix}\varsigma\\\eta\end{bmatrix} = \begin{bmatrix}\varsigma+\eta\\\varsigma-\eta\end{bmatrix}$	$\mathbf{c}_{xy} = \begin{bmatrix}1 & 1\\1 & -1\end{bmatrix}\begin{bmatrix}\sigma_\varsigma^2 & \sigma_\varsigma\sigma_\eta\rho_{\varsigma\eta}\\\sigma_\varsigma\sigma_\eta\rho_{\varsigma\eta} & \sigma_\eta^2\end{bmatrix}\begin{bmatrix}1 & 1\\1 & -1\end{bmatrix} = \begin{bmatrix}\sigma_x^2 & \sigma_\varsigma^2-\sigma_\eta^2\\\sigma_\varsigma^2-\sigma_\eta^2 & \sigma_y^2\end{bmatrix}$
Standard deviations $\sigma_x = \sqrt{\sigma_s^2+\sigma_\eta^2+2\sigma_s\sigma_\eta\rho_{s\eta}}$ $\sigma_y = \sqrt{\sigma_s^2+\sigma_\eta^2-2\sigma_s\sigma_\eta\rho_{s\eta}}$	Correlator $\mathbf{r}_{xy} = \begin{bmatrix}1 & \rho_{xy}\\\rho_{xy} & 1\end{bmatrix}$ Where: correlation coefficient $\rho_{xy} = \frac{\sigma_\varsigma^2-\sigma_\eta^2}{\sigma_x\sigma_y}$ **c** & **r** *positive define* $\det(\mathbf{A}-\lambda\mathbf{I})=0$ $\lambda_i \geq 0$

$\mathbf{Y}$=[1,845(100); 1,155(100)]
(ρ_{xy}=0,9998),

B. Rounding of **Y** to 2 digits after decimal point:

$\mathbf{Y_1}$= [1,84(10); 1,16(10)];

$$\Delta\mathbf{Y}_1 = \mathbf{Y_1} - \mathbf{Y} = [-0{,}005;\ 0{,}005];\ \chi_1^2 = 25 > 1$$

C. Rounding **of Y** to 1 digit after decimal point:

$\mathbf{Y_2}$= [1,8(1); 1,16(1)];

$$\mathbf{Y}_2 = \mathbf{Y_2} - \mathbf{Y} = [-0{,}045;\ 0{,}045];\ \chi_2^2 = 2500 > 1$$

For the ellipse border the Mahalanobis distance χ=1. Then the ends of both rounded vectors $\mathbf{Y_1}, \mathbf{Y_2}$ situated are outside of the ellipse of transformed original random input data **Y**. If the assumption proposed by V. Ezhela in [2–4] for safety processing of random vectors is to be satisfied, these ends has to be situated inside of this ellipse. Then in both cases the results are over-rounded. Such assumption can be valid only for the absolutely accurate input statistical data from the whole random populations of X_i or for very large going to infinity, number *N* of sample elements, and when instrumental errors are negligible. That condition is not fulfilled in many existing in measurement situations, where data samples of a small number of elements are only possible to obtain in a limited time of observation. Then all estimators of mean values, standard deviations and correlation coefficients obtained from the samples of limited number *N* of multivariate observations have their own uncertainties, which are quite high for small *N*. From above it follows that two different type requirements for precision of processing and rounding procedures of multivariate data can be used:

– very high for safety numerical processing of input random vector of statistical parameters treated as absolutely accurate (if they are given for whole population or for being safe if accuracy of them is unknown), and

– lower, dependent on given or possible to be estimated accuracy of statistical parameters of the input multimeasurand.

In the second case thresholds for limiting the rounding of output data parameters have to be established as dependent on the accuracy of statistical parameters of input samples.

3. Thresholds of the safe rounding of transformed multivariate data

Applying spectral theorems of matrix theory V. Ezhela in [4] proposed thresholds of the number of digits after decimal point for fully safe independent uniform rounding of the multivariate random vector data. All such safety thresholds of rounding are expressed in terms of decimal numbers [6], i.e. for:

standard deviations U_i of vector components V_i

$$A_i^U \geq A_i^{Uth} = \text{Upper Integr}\left[\frac{1}{2}\log_0\left(\frac{n}{4\lambda_{\min} T_C^2 \left(\frac{U_i}{\text{unit}_i}\right)^2}\right)\right]$$

$A_i^V \geq A_i^{Vth} = A_i^{Uth}$ – values of the components V_i of the average vector elements of the correlation matrix

$$A^C \geq A_{i\neq j}^{Cth} = \text{Upper Integr}\left[\log_0\left(\frac{n-1}{2\lambda_{\min}}\right)\right]$$

Where: $\lambda_{\min}$ – minimal eigenvalue of the correlation matrix, – "tolerance" factor at defined confidence level.

Above formulas look sophisticated, but are quite easy to be used – see examples in [4], [6] and [7]. For data of the Example H.2 of GUM [1] (the same as in chapter 9.4 of Supplement 2 [11]) after high precision processing of the input data correctness of the rounding of the multivariate output vector **Y**=[*R, X, Z*], when T_{CL}=1 (one standard deviation) are the following numbers of digits [4], [7]:

Upper Integer{$A^{th}[R]$} = 5
Upper Integer{$A^{th}[X]$} = 4
Upper Integer{$A^{th}[Z]$} = 5

Correlator of the output vector $[R, X, Z]^T$ has elements uniformly rounded to 9 digits and the smallest eigenvalue $\lambda_{\min}$=2,22711×10^{-8} and for this $\lambda_{\min}$ the minimum number of digits of the correlation coefficients is:

$A^C(\text{Cor}_{H3})$ = 8

As large number of significant figures are obtained from proposed by V. E. "thresholds of uniform rounding", such as for scalars by GUM, valid with assumptions:

- input data are treated as absolutely accurate,
- scatter region for $p \geq p_0$ is kept as *n*-ellipsoid,
- numerical processing is "safety rounded", and
- the output vector must be maintained inside of the scattered area of the transformed raw input data.

Then such of many digits thresholds are not needed in measurements. They are valid only for the save digital processing of random vector itself because of the assumption that mean values of components the input vector, their standard deviations and correlation coefficients are absolutely accurate known, which is not happen for any real experiments. Obtained experimentally measurement data are not absolutely accurate as number *N* of observations in samples are limited (uncertainty type Ais rising with decreasing of *N*) and unknown instrumental errors are not negligible (represented by uncertainty type B). Then in requirements for processing and rounding the uncertainties of estimators of mean value (or other the most probable values of measured vector components e.g. mid-range for uniform distribution and for trapeze distributions of the ratio of their bases from 1 to 0,65. The accuracy of standard deviations and of correlation coefficients have to be also taken in considerations. Then the rounding of real multivariate measurement data has to be done below thresholds given by V. Ezhela and dedicated for the save data

Table 2

Parameters	$\bar{x}$	σ_x	$\bar{y}$	σ_y	$2\sigma_x$	$2\sigma_y$
Raw results	0,3242	0,0664	0,1555	0,0256	0,1328	0,0512
A. rounding to 3 digits	0,324	0.067	0.156	0,026	0,133	0,051
B. rounding to 2 digits	0.32	0.07	0,16	0.03	0,14	0,05
C. rounding to 1digit	0.3	< 0.1	0,2	< 0.1	<0,2	<0,1

processing itself only. Approximately it should be enough if precision of processing is the one digit more than the accuracy of measured input data.

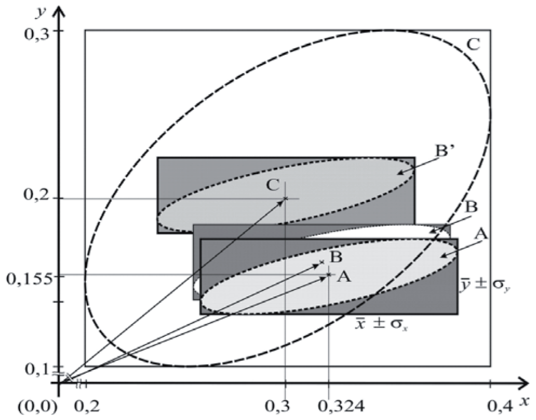

Fig. 4. Rounding with constant correlation coefficient ρ_{xy}

An graphical illustration of such rounding of data given in table 2 for constant correlation coefficient ρ_{xy} is shown in Fig. 4.

The largest ellipse C obtained after rounding standard deviations σ_x, σ_y to 1 digit do not fully cover the primary ellipse A. But it was checked also that for larger ellipses tangential to the rectangular of twice larger sides $\pm 2\sigma_x$, $\pm 2\sigma_y$ after their rounding to 2 or to 1 digit it is ok. For example for such ellipses A_2, B_2 (not given in Fig. 3) it is

$$2\sigma(\bar{x}_B) - (\bar{x} - \bar{x}_B) > 2\sigma(\bar{x}_A)$$

$$2\sigma(\bar{y}_B) - (\bar{y} - \bar{y}_B) > 2\sigma(\bar{y}_A).$$

Rounding of the correlation coefficients depend on their value and on accuracy. Then special care, not considered here, is needed,

The different rounding of multivariate data with changes of correlation coefficients values ρ_{ij}, also should be applied. Two following methods of such rounding below thresholds are preliminary tested:

• Method 1 (of Z. L. W) – author proposed to maintain a constant values of non diagonal elements of the positive covariance matrix, i.e.:

$$\bar{\rho}_{RX} = \frac{\sigma_R\ \sigma_X}{\sigma_R^+\ \sigma_X^+}\rho_{RX} \qquad \bar{\rho}_{RZ} = \frac{\sigma_R\ \sigma_Z}{\sigma_R^+\ \sigma_Z^+}\rho_{RZ} \qquad \bar{\rho}_{XZ} = \frac{\sigma_X\ \sigma_Y}{\sigma_X^+\ \sigma_Z^+}\rho_{XZ}$$

Where: signs in the upper index indicates the direction of change.

• Method 2 (of V. E.) – V. Ezhela proposed to use truncation, i.e. to omit further digits after no changed the last accepted digit [6].

Both methods are used for the output data of the Example H.2 GUM for rounding them to 3 and 2 digits after decimal point [6]. Results are given below in Table 3.

It was checked:

– positive definite of the rounded correlator;

– the relative distance by Mahalanobis $\chi 2$ [12] between the end of rounded vector and the center of raw original data after transformation.

Correlators of both methods are positive definite, but the full theoretical justification of the method 2 is not given jet. Method 2 gives a smaller values of Mahalanobis distances of ends of the rounded vectors from the ellipse center of transformed raw data, but its smallest eigenvalue is closer to zero than obtained in method 1.

Conclusion: since in multivariable measurements the rounding level of output vector **Y** depends not only on the precision of digital processing but mainly on the uncertainties of all statistical parameters of input vector **X**, then additional formulas of rounding thresholds then given by V. Ezhela [4], [6] are also urgently needed.

4. Upgrading the GUM proposals for multivariable measurements

GUM [1] and other official metrological documents about uncertainty are applied up to now only in measurements of the single quantity. But statements in the main text of GUM was formulated in such a manner that the reader gets the impression that a generalization to the multivariate case is straightforward. That was considered in details in Example H.2 of GUM which illustrates clauses 7.2.5 and 7.2.6.

Supplement 2 to GUM [11] – about extension of evaluation uncertainty of measurements to any number of quantities, has been published just now and time to implement its clauses in practice is not jet long enough.

About Example H.2 of GUM

In Table H.2 of GUM [1] there are given five (and in Supplement 2 – six) raw simultaneous measurements of input vector $\mathbf{X}=[U, J, \Phi]^T$ and vector $\mathbf{Y}=[R=U\cos\Phi/J, X=U\sin\Phi/J, Z=U/J]^T$ is evaluated. The results are presented there in Tables H.3 and H.4. Rounding of correlation coefficients is not properly done there, since the smallest eigenvalue of correlator matrix is negative and so the scatter region is not of the 3D-ellipsoide form. Also final output data of Example H.2 does not satisfy "physical law" of impedance of the two-terminal passive element which is: $X^2+Y^2=Z^2$ as $\sigma^2= -71{,}5$ [3], [6], [7].

Table 3

Method 1 (of Z.L.W)

Rounding to 3 digits

Mean value	Standard Deviation	Correlator		
127,732	0,160	1	–0,586	–0,483
219,847	0,661	–0,586	1	0,991
254,60	0,529	–0,483	0,991	1
Eigenvalue: [2,39983; 0,598631; 0,00154204]				
χ^2		8,39		

Rounding to 2 digits

Mean value	Standard Deviation	Correlator		
127,73	0,16	1	–0,59	–0,48
219,85	0,66	–0,59	1	0,99
254,26	0,53	–0,48	0,99	1
Eigenvalue: [2,39972; 0,598797; 0,00148649]				
χ^2		284,0		

Method 2 (of V. E.)

Rounding to 3 digits

Mean value	Standard Deviation	Correlator		
127,732	0,160	1	–0,588	–0,485
219,847	0,661	–0,588	1	0,992
254,260	0,529	–0,485	0,992	1
Eigenvalue: [2,40297; 0,596499; 0,000533094]				
χ^2		1, 108		

Rounding to 2 digits

Mean value	Standard Deviation	Correlator		
127,73	0,16	1	–0,58	–0,48
219,85	0,66	–0,58	1	0,99
254,26	0,53	–0,48	0,99	1
Eigenvalue: [2,26846; 0,657234; 0,0743036]				
χ^2		6,445		

For establishing requirements of safety digital processing purposes according, the input multivariate measured data of H.2 Example are firstly treated as absolutely accurate data. Then, for such theoretical case, according thresholds given by V. Ezhela [4] (see chapter 3) the required digit numbers after decimal point are as follows: for mean values and standard uncertainty of R and Z – 5 digits, of X - 4 digits and for correlation coefficients – 8 digits **!** [4, 7]. Therefore, high numbers of digits cannot be accepted for describing the measurement results as they are obtained under the assumptions that: input data are treated as fully accurate, numerical results of processing are safety and uniformly rounded to maintained vector end in the scattered area of the transformed original input data. They can be used only as a reference of processing of the absolutely accurate data of the random vector of similar component values as parameter estimators of samples of the input vector **X** in H.2 Example. All these samples have only N=5 **!** (or 6 in supl. 2) measurements each and the accuracy of SD of each variable and of correlator elements is very poor. Relative uncertainty of SD of such small samples is about 36% – see Table E.1 in GUM [1].

5. About Notice GUM on the nonlinear uncertainty propagation

In the multivariable case the linear propagation of errors from m-dimensional vector **X** to n-dimensional vector **Y** in some cases is misleading for $n>m$, i.e. for nonlinear functions.

5.1. Single measurand case

In the Notice to clause 5.1.2 of GUM [1] is determinate the uncertainty of highly non-linear single function $y=f(X)$ only. There is recommended that the linear propagation of variance is supplemented by higher-order components, i.e.

$$u_c^2(y) = \sum_{i=1}^{N} \left(\frac{\partial f}{\partial x_i} \right)^2 u^2(x_i) +$$

$$+ \sum_{i=1}^{N} \sum_{j=1}^{N} \left[\frac{1}{2} \left(\frac{\partial^2 f}{\partial x_i \partial x_j} \right)^2 + \frac{\partial f}{\partial x_i} \cdot \frac{\partial^3 f}{\partial x_i \partial x_j \partial x_j} \right] u^2(x_i)\, u^2(x_j)$$

V. Ezhela noticed in [4] that calculations of the variance $u^2(y)$ according to this formula may give a false negative value because of the component in parentheses with the third derivative. This is illustrated below by the example of the single nonlinear polynomial function

$$F(x) = 1 - x + 2x^2 + 3x^3 + 4x^4,$$

If measurand x is normally distributed around x= 0 with variance σ^2, then the variance $u^2(F)$ calculated according to the recommendation 5.1.2 is

$$u^2(F) = \sigma^2 F'(0)^2 + 1/2\sigma^4 F''(0)^2 + \sigma^4 F'(0)\, F'''(0) =$$
$$= \sigma^2 [1 + \sigma^2(16/2 - 18)] = \sigma^2 [1 - 10\, \sigma^2]$$

For $\sigma^2>0.1$ (i. e. $\sigma>0.316$) is obtained $u^2(F)<0$. So, the additional component of the uncertainty formula of nonlinear function given in Notice to clause 5.1.2 of GUM should be corrected by removing from the sum in brackets the second component with the third derivative.

5.2. Nonlinear processing of input vector

In case of the nonlinear processing of input vector the widely used as approximation differential "linear uncertainty propagation law" does not work properly in more accurate calculations for highly nonlinear functions. The nonlinear uncertainty propagation should be used with the obligatory positivity constraints

$$\boldsymbol{C}_i,\ [\delta C_a, \delta C_b] \longrightarrow F_k(\boldsymbol{C}_i),\ [\delta F_m, \delta F_n]$$

Component of X (input) Component of Y (output)

$$[\delta F_i, \delta F_j] =$$

$$= \sum_{k,l=1}^{T} \frac{1}{k!\,l!} \frac{\partial^k F_i}{\partial c_{\alpha_1} \ldots \partial c_{\alpha_k}} \{\delta c_{\alpha_1} \ldots \delta c_{\alpha_k}, \delta c_{\beta_1} \ldots \delta c_{\beta_l}\} \frac{\partial^l F_j}{\partial c_{\beta_1} \ldots \partial c_{\beta_l}}$$

Covariance matrix $[\delta F_m, \delta F_n]$ is non-degenerate and positive definite if dimensions $\dim(C_i)=m$, $\dim(F_k)=n$ and the order T of component of the Taylor polynomials approximated the measuring vector function F_k obey the inequality

$$n \le n_h = \frac{(m+T)!}{m!T!} - 1$$

Where:

m	1	1	1	2	2	2	3	3	3	4	4	4	5	5	5
T	1	2	3	1	2	3	1	2	3	1	2	3	1	2	3
n_{th}	1	2	3	2	5	6	3	9	19	4	14	4	5	20	55

Commentary on fundamental physical constancies

The adjustments of the fundamental physical constants (FPC) are regularly performed by the Fundamental Constants Data Center at NIST and recommended by CODATA as the unique source of the current FPC values. There are 325 adjusted quantities, from which 79 are called basic algebraically independent constants C_α^B. As an example there are listed in Table 4 last values of four FPC given by CODATA 2010 [9] in SI units and below – their correlation matrix.

Table 4

Elementary charge	e	C	$1.602\ 176\ 565(35) \cdot 10^{-19}$	e	h	me
Planck constant	h	J s	$6.626\ 069\ 57(29) \cdot 10^{-34}$	1.0000		
Electron mass	me	kg	$9.109\ 382\ 91(40) \cdot 10^{-31}$	0.9998	0.9999	
1/fine structure const	$1/\alpha(0)$		137.035 999 074(44)	− 0.0145	− 0.0072	0.0075

Eigenvalues of above correlation matrix are: [2.99942, 1.00006, 0.000719993, **- 0.000202165**]. The last eigenvalue is non-negligible negative.

Only two of constancies FPC can be used together in joint precision calculations. More about negative eigenvalues in CODATA publications is given in [3], [7], [8].

6. Application of e-publishing in multi-variable measurements

The traditional form of the scientific communication based on the paper oriented e-publications is now not the proper way to present and to exchange the multidimensional experimental data. After V. Ezhela in [3] the standardization of two-component forms of the scientific publication is unavoidable. First component will be the traditional descriptive scientific text already well formalized by publishers. The second part should be computer readable file with all numerical input data and results to allow "fast" numerical peer review of the publication reporting new results. It is discussed in detail in [3] together with given four dozen examples of "bad practice" of physical publications in journals of high "impact factor" and in other sources. Some particular problems connected with that proposal are in [8] and [10].

7. General conclusion

Some of the presented problems of the evaluation of results and uncertainty of the indirect multivariable measurements still need farer investigations to obtain clear enough backgrounds for the common international acceptance of the rounding and digital presentation methods of multivariate data results and of the calculation uncertainty of highly nonlinear related multivariate data.

These problems are not yet fully included in just finished the first version of Supplement 2 to GUM [1] about the extension of expressing uncertainty to any number of quantities [11]. Then its recommendations should be corrected and included in the next upgraded version of Supplement 2 and taken into account also in other post-GUM documents.

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