METHOD FOR CONTROL OF TECHNICAL OBJECTS OPERATION PROCESS WITH THE USE OF SEMI-MARKOV DECISION PROCESSES

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Abstract

In this article there is presented a description of a method for control of the operation process in a complex operation system of technical objects operation where semi-Markov decision processes are used for control. The study has been based on the operation process of a real bus municipal transportation system. For complex systems of technical objects operation, it is possible to achieve the set goal – appropriate accomplishment of the assigned task (tasks) with required efficiency – only when the control decisions, made by the system decision-makers, are rational. In systems where the complex operation process of technical objects is carried out, the choice of rational control decisions (from possible variants) is an issue of great importance and difficulty. In real complex operation systems of technical objects operation, the process of making control decisions should be carried out using mathematical procedures and methods, rather than according to the system decision makers' intuition or experience. Use of appropriate mathematical methods for control of the operation process facilitates the choice of rational control decisions providing correct and efficient accomplishment of the assigned tasks. The presented method involves elaborating a mathematical operation model (semi-Markov model) and choosing the best control strategy (from possible decision variants), according to the accepted criterion (criteria) of technical objects operation system assessment.

Keywords: *semi-Markov processes, control decisions, transport system*

1. Introduction

Obtaining high efficiency in operation of complex operation systems is possible when decisions made while controlling processes utilized in these systems are rational. In order to assure the appropriate running of decision-making process support tools are used, including all kinds of decision-making models, an important element of which is a mathematical model of technical object operation process (transport means). Due to the random nature of the factors influencing the running and efficiency of the transport means operation process introduced in a complex system, most often in the process mathematical modelling of the operation process, stochastic processes are used. From among the random processes, both Markov and semi-Markov processes [3, 5, 8, 11, 12] are widely used in the modelling of technical objects. Implementing semi-Markov processes makes it possible to create and analyze the mathematical model of the operation process in the case of the variables characteristic of the defined process states being distributions other than exponential. The creation of a mathematical model of technical object operation process makes it possible to perform an analysis of the process, which in turn constitutes the basis for evaluation and rational control of the system [9, 13, 15].

In this article there is presented a description of a method for control of the operation process in a complex operation system of technical objects operation where semi-Markov decision processes are used for control. The presented method involves elaborating a mathematical operation model (semi-Markov model) and choosing the best control strategy (from possible decision variants), according to the accepted criterion (criteria) of technical objects operation system assessment. The study has been based on the operation process of a real bus municipal transportation system.

2. Event-based model of transport means operation process

The model of operation process was created on the basis of the analysis of state space as well as operational events pertaining to technical objects (municipal buses) used in the analyzed authentic transport system. Due to the identification of the analyzed transport system and the multi-state process of technical object operation utilized in it, crucial operation states of the process as well as possible transfers between the defined states were designated. Based on this, a graph was created, depicting the changes of operation process states, shown in Fig. 1.

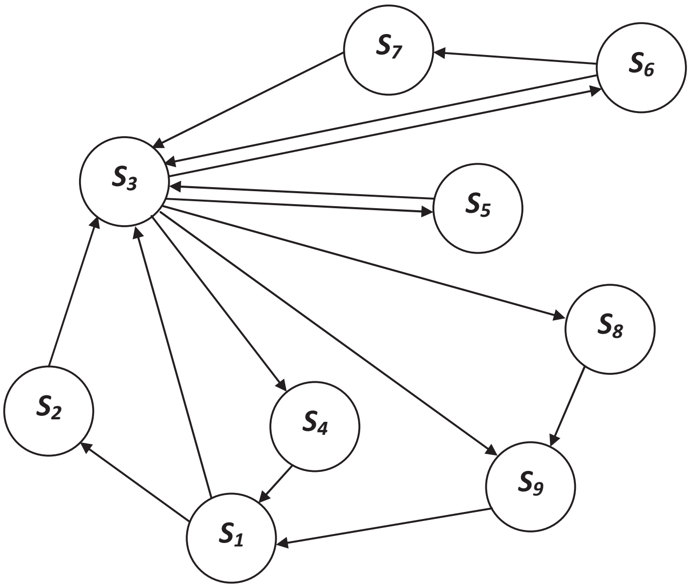

Fig. 1. Directed graph representing the transport means operation process; S_1 – refuelling, S_2 – awaiting the carrying out of the task at the bus depot parking space, S_3 – carrying out of the transport task, S_4 – awaiting the carrying out of the task between transport peak hours, S_5 – repair by technical support unit without losing a ride, S_6 – repair by the emergency service with losing a ride, S_7 – awaiting the start of task realization after technical support repair, S_8 – repair in the serviceability assurance subsystem, S_9 – maintenance check on the operation day

3. Mathematical model of transport means operation process

The mathematical model of transport means operation process implemented in the utilization subsystem of the analyzed transport system was built with the use of the semi-Markov processes theory. The semi-Markovian $X(t)$ process is one, where periods of time between the changes of consecutive process states have arbitrary probability distributions and a transfer to the consecutive state depends on the current process state. Using the semi-Markov processes in mathematical modelling of the operation process, the following assumptions were put forward:

- the modelled operation process has a finite number of states S_i, $i = 1,2,\ldots,9$,
- if technological object at moment t is in state S_i, then $X(t) = i$, where $i = 1,2,\ldots,9$,
- the random process $X(t)$ being the mathematical model of the operation process is a homogenous process,
- at moment $t = 0$, the process finds is in state S_3 (the initial state is state S_3), i.e. $P\{X(0) = 3\} = 1$.

The homogenous semi-Markovian process is unequivocally defined when initial distribution and its kernel are given [1, 5, 6]. Form our assumptions and based on the directed graph shown in Fig. 1, the initial distribution $p_i(0)$, $i = 1,2,\ldots,9$ takes up the following form:

$$p_i(0) = \begin{cases} 1 \quad when \quad i = 3, \\ \\ 0 \quad when \quad i \neq 3, \end{cases} \tag{1}$$

where:

$$p_i(0) = P\{X(0) = i\}, \quad i = 1,2,...,9, \tag{2}$$

whereas the kernel of process *Q(t)* takes up the form:

$$Q(t) = \begin{bmatrix} 0 & Q_{12}(t) & Q_{13}(t) & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & Q_{23}(t) & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & Q_{34}(t) & Q_{35}(t) & Q_{36}(t) & 0 & Q_{38}(t) & Q_{39}(t) \\ Q_{41}(t) & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & Q_{53}(t) & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & Q_{63}(t) & 0 & 0 & 0 & Q_{67}(t) & 0 & 0 \\ 0 & 0 & Q_{73}(t) & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & Q_{89}(t) \\ Q_{91}(t) & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}, \tag{3}$$

where:

$$Q_{ij}(t) = P\{X(\tau_{n+1}) = j, \tau_{n+1} - \tau_n \leq t | X(\tau_n) = i\}, \quad i, j = 1,2,...,9, \tag{4}$$

means that the state of semi-Markovian process and the period of its duration depends solely on the previous state, and does not depend on earlier states and periods of their duration, where τ_1, τ_2, ..., τ_n, ... are arbitrary moments in time, so that $\tau_1 < \tau_2 < ... < \tau_n < ...$, as well as:

$$Q_{ij}(t) = p_{ij} \cdot F_{ij}(t), \tag{5}$$

where:

$$p_{ij} = \lim_{t \to \infty} p_{ij}(t), \tag{6}$$

p_{ij} – means that the conditional probability of transfer from state S_i to state S_j,

$$p_{ij}(t) = P\{X(t) = j | X(0) = i\}, \tag{7}$$

as well as:

$$F_{ij}(t) = P\{\tau_{n+1} - \tau_n \leq t | X(\tau_n) = i, X(\tau_{n+1}) = j\}, \quad i, j = 1,2,...,9, \tag{8}$$

is a distribution function of random variable Θ_{ij} signifying period of duration of state S_i, under the condition that the next state will be state S_j.

Limit probability p_i^* of staying in states of semi-Markov process were assigned on the basis of limit theorem for semi-Markovian processes [5, 8]:

If hidden Markov chain in semi-Markovian process with finite state S set and continuous type kernel contains one class of positive returning states such that for each state $i \in S, f_{ij} = 1$ *and positive expected values* $E(\Theta_i), i \in S$ *are finite, limit probabilities:*

$$p_i^* = \lim_{t \to \infty} p_i(t) = \frac{\pi_i \cdot E(\Theta_i)}{\sum_{i \in S} \pi_i \cdot E(\Theta_i)}, \tag{9}$$

exist where probabilities $\pi_i, i \in S$ *constitute a stationary distribution of a hidden Markov chain, which fulfils the simultaneous linear equations:*

$$\sum_{i \in S} \pi_i \cdot p_{ij} = \pi_j, \quad j \in S, \quad \sum_{i \in S} \pi_i = 1. \tag{10}$$

In order to assign the values of limit probabilities p_i^* of staying in the states of semi-Markovian model of transport means operation, based on the directed graph shown in Fig. 1, the following were created: matrix P of the states change probabilities and matrix Θ of conditional periods of duration of the states in process $X(t)$:

$$P=\begin{bmatrix} 0 & p_{12} & p_{13} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & p_{34} & p_{35} & p_{36} & 0 & p_{38} & p_{39} \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & p_{63} & 0 & 0 & 0 & p_{67} & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}, \tag{11}$$

$$\Theta=\begin{bmatrix} 0 & \overline{\Theta}_{12} & \overline{\Theta}_{13} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \overline{\Theta}_{23} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \overline{\Theta}_{34} & \overline{\Theta}_{35} & \overline{\Theta}_{36} & 0 & \overline{\Theta}_{38} & \overline{\Theta}_{39} \\ \overline{\Theta}_{41} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \overline{\Theta}_{53} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \overline{\Theta}_{63} & 0 & 0 & 0 & \overline{\Theta}_{67} & 0 & 0 \\ 0 & 0 & \overline{\Theta}_{73} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \overline{\Theta}_{89} \\ \overline{\Theta}_{91} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}. \tag{12}$$

Based on the matrix of the states change probabilities $P = [p_{ij}]$ and on the matrix of average values $\Theta = \left[\overline{\Theta}_{ij}\right]$ of random variables Θ_{ij} (random variable Θ_{ij} means that the period of duration of technological object in i-th state, under the condition that the next state will be j-th state), average values $\overline{\Theta}_i$ of non-conditional duration periods of process states were defined, according to the dependence:

$$\overline{\Theta}_i = \sum_{i=1}^{9} p_{ij} \cdot \overline{\Theta}_{ij}, \quad i, j = 1,2,...,9 . \tag{13}$$

Non-conditional periods of duration of process states:

$$\overline{\Theta}_1 = p_{12} \cdot \overline{\Theta}_{12} + p_{13} \cdot \overline{\Theta}_{13}, \tag{14}$$

$$\overline{\Theta}_2 = p_{23} \cdot \overline{\Theta}_{23} = \overline{\Theta}_{23}, \tag{15}$$

$$\overline{\Theta}_3 = p_{34} \cdot \overline{\Theta}_{34} + p_{35} \cdot \overline{\Theta}_{35} + p_{36} \cdot \overline{\Theta}_{36} + p_{38} \cdot \overline{\Theta}_{38} + p_{39} \cdot \overline{\Theta}_{39}, \tag{16}$$

$$\overline{\Theta}_4 = p_{41} \cdot \overline{\Theta}_{41} = \overline{\Theta}_{41}, \tag{17}$$

$$\overline{\Theta}_5 = p_{53} \cdot \overline{\Theta}_{53} = \overline{\Theta}_{53}, \tag{18}$$

$$\overline{\Theta}_6 = p_{63} \cdot \overline{\Theta}_{63} + p_{67} \cdot \overline{\Theta}_{67}, \tag{19}$$

$$\overline{\Theta_7} = p_{73} \cdot \overline{\Theta_{73}} = \overline{\Theta_{73}}, \tag{20}$$

$$\overline{\Theta_8} = p_{89} \cdot \overline{\Theta_{89}} = \overline{\Theta_{89}}, \tag{21}$$

$$\overline{\Theta_9} = p_{91} \cdot \overline{\Theta_{91}} = \overline{\Theta_{91}}. \tag{22}$$

Then, with the use of the MATHEMATICA software, limit distribution of semi-Markov process were defined:

$$p_1^* = \frac{(p_{34}+p_{38}+p_{39})\cdot\overline{\Theta_1}}{\left[(p_{34}+p_{38}+p_{39})\cdot\left(\overline{\Theta_1}+p_{12}\cdot\overline{\Theta_2}\right)\right]+\overline{\Theta_3}+p_{34}\cdot\overline{\Theta_4}+p_{35}\cdot\overline{\Theta_5}+\left[p_{36}\cdot\left(\overline{\Theta_6}+p_{67}\cdot\overline{\Theta_7}\right)\right]+p_{38}\cdot\overline{\Theta_8}+(p_{38}+p_{39})\cdot\overline{\Theta_9}}, \tag{23}$$

$$p_2^* = \frac{p_{12}\cdot(p_{34}+p_{38}+p_{39})\cdot\overline{\Theta_2}}{\left[(p_{34}+p_{38}+p_{39})\cdot\left(\overline{\Theta_1}+p_{12}\cdot\overline{\Theta_2}\right)\right]+\overline{\Theta_3}+p_{34}\cdot\overline{\Theta_4}+p_{35}\cdot\overline{\Theta_5}+\left[p_{36}\cdot\left(\overline{\Theta_6}+p_{67}\cdot\overline{\Theta_7}\right)\right]+p_{38}\cdot\overline{\Theta_8}+(p_{38}+p_{39})\cdot\overline{\Theta_9}}, \tag{24}$$

$$p_3^* = \frac{\overline{\Theta_3}}{\left[(p_{34}+p_{38}+p_{39})\cdot\left(\overline{\Theta_1}+p_{12}\cdot\overline{\Theta_2}\right)\right]+\overline{\Theta_3}+p_{34}\cdot\overline{\Theta_4}+p_{35}\cdot\overline{\Theta_5}+\left[p_{36}\cdot\left(\overline{\Theta_6}+p_{67}\cdot\overline{\Theta_7}\right)\right]+p_{38}\cdot\overline{\Theta_8}+(p_{38}+p_{39})\cdot\overline{\Theta_9}}, \tag{25}$$

$$p_4^* = \frac{p_{34}\cdot\overline{\Theta_4}}{\left[(p_{34}+p_{38}+p_{39})\cdot\left(\overline{\Theta_1}+p_{12}\cdot\overline{\Theta_2}\right)\right]+\overline{\Theta_3}+p_{34}\cdot\overline{\Theta_4}+p_{35}\cdot\overline{\Theta_5}+\left[p_{36}\cdot\left(\overline{\Theta_6}+p_{67}\cdot\overline{\Theta_7}\right)\right]+p_{38}\cdot\overline{\Theta_8}+(p_{38}+p_{39})\cdot\overline{\Theta_9}}, \tag{26}$$

$$p_5^* = \frac{p_{35}\cdot\overline{\Theta_5}}{\left[(p_{34}+p_{38}+p_{39})\cdot\left(\overline{\Theta_1}+p_{12}\cdot\overline{\Theta_2}\right)\right]+\overline{\Theta_3}+p_{34}\cdot\overline{\Theta_4}+p_{35}\cdot\overline{\Theta_5}+\left[p_{36}\cdot\left(\overline{\Theta_6}+p_{67}\cdot\overline{\Theta_7}\right)\right]+p_{38}\cdot\overline{\Theta_8}+(p_{38}+p_{39})\cdot\overline{\Theta_9}}, \tag{27}$$

$$p_6^* = \frac{p_{36}\cdot\overline{\Theta_6}}{\left[(p_{34}+p_{38}+p_{39})\cdot\left(\overline{\Theta_1}+p_{12}\cdot\overline{\Theta_2}\right)\right]+\overline{\Theta_3}+p_{34}\cdot\overline{\Theta_4}+p_{35}\cdot\overline{\Theta_5}+\left[p_{36}\cdot\left(\overline{\Theta_6}+p_{67}\cdot\overline{\Theta_7}\right)\right]+p_{38}\cdot\overline{\Theta_8}+(p_{38}+p_{39})\cdot\overline{\Theta_9}}, \tag{28}$$

$$p_7^* = \frac{p_{36}\cdot p_{67}\cdot\overline{\Theta_7}}{\left[(p_{34}+p_{38}+p_{39})\cdot\left(\overline{\Theta_1}+p_{12}\cdot\overline{\Theta_2}\right)\right]+\overline{\Theta_3}+p_{34}\cdot\overline{\Theta_4}+p_{35}\cdot\overline{\Theta_5}+\left[p_{36}\cdot\left(\overline{\Theta_6}+p_{67}\cdot\overline{\Theta_7}\right)\right]+p_{38}\cdot\overline{\Theta_8}+(p_{38}+p_{39})\cdot\overline{\Theta_9}}, \tag{29}$$

$$p_8^* = \frac{p_{38}\cdot\overline{\Theta_8}}{\left[(p_{34}+p_{38}+p_{39})\cdot\left(\overline{\Theta_1}+p_{12}\cdot\overline{\Theta_2}\right)\right]+\overline{\Theta_3}+p_{34}\cdot\overline{\Theta_4}+p_{35}\cdot\overline{\Theta_5}+\left[p_{36}\cdot\left(\overline{\Theta_6}+p_{67}\cdot\overline{\Theta_7}\right)\right]+p_{38}\cdot\overline{\Theta_8}+(p_{38}+p_{39})\cdot\overline{\Theta_9}}, \tag{30}$$

$$p_9^* = \frac{(p_{38}+p_{39})\cdot\overline{\Theta_9}}{\left[(p_{34}+p_{38}+p_{39})\cdot\left(\overline{\Theta_1}+p_{12}\cdot\overline{\Theta_2}\right)\right]+\overline{\Theta_3}+p_{34}\cdot\overline{\Theta_4}+p_{35}\cdot\overline{\Theta_5}+\left[p_{36}\cdot\left(\overline{\Theta_6}+p_{67}\cdot\overline{\Theta_7}\right)\right]+p_{38}\cdot\overline{\Theta_8}+(p_{38}+p_{39})\cdot\overline{\Theta_9}}. \tag{31}$$

4. The choice of the optimal control strategy for the operation process of technical objects

Semi-Markov decision process is the stochastic process $\{X(t):\ t \geq 0\}$, that implementation depends on the decisions taken at the initial moment of the process t_0 and in the times of changes in process conditions $t_1, t_2, \ldots, t_n$. The analyzed semi-Markov process has a finite number of states $i = 1,2,\ldots,m$. Then:

$$D_i = \left\{d_i^{(1)}(t_n), d_i^{(2)}(t_n), \ldots, d_i^{(k)}(t_n)\right\}, \tag{32}$$

means set of all possible control decisions that can be applied in i-th state of the process, in the moment t_n, where $d_i^{(k)}(t_n)$ means k-th control decision undertaken in i-th state of the process, in the moment t_n.

In the case of use of semi-Markov decision processes taken at the time t_n, k-th control decision in i-th state of the process, means the choice of i-th line of the kernel of process, from the set:

$$\left\{Q_{ij}^{(k)}(t) : t \geq 0,\ \ d_i^{(k)}(t_n) \in D_i,\ \ i, j \in S\right\}, \tag{33}$$

where:

$$Q_{ij}^{(k)}(t) = p_{ij}^{(k)} \cdot F_{ij}^{(k)}(t). \tag{34}$$

The choice of i-th line of the kernel of process designates a probabilistic mechanism of evolution process in the time interval ‹t_n; t_{n+1}). It means that for semi-Markov process, in case of changing the state of the process from any on the i-th (entrance to i-th state process) in the moment t_n is made a decision $d_i^{(k)}(t_n) \in D_i$ and on schedule $\left(p_{ij}^{(k)} : j \in S\right)$ i-th state of the process is generated, to which the transition occurs at the moment t_{n+1}. At the same time according to a schedule determined by the cumulative distribution $F_{ij}^{(k)}(t)$ the length of time interval is generated ‹t_n; t_{n+1}) remain in the i-th state of the process, when the next state is the j-th state.

As the δ strategy is understood as a sequence, which terms are the vectors, made up of decisions $d_i^{(k)}(t_n)$ taken in consecutive moments t_n changes the states of the process $X(t)$:

$$\delta = \left\{ \left[d_1^{(k)}(t_n), d_2^{(k)}(t_n), \ldots, d_m^{(k)}(t_n) \right] : \quad n = 0,1,2,\ldots \right\}. \tag{35}$$

The strategy δ is called a stationary strategy, in case, when taking decisions on the next states of the process do not depend on the moment t_n, in which are taken, so $d_i^{(k)}(t_n) = d_i^{(k)}$. Then such defined the semi-Markov process is the homogeneous process, and formula (35) takes the form:

$$\delta = \left[d_1^{(k)}, d_2^{(k)}, \ldots, d_m^{(k)} \right]. \tag{36}$$

In the case of use of semi-Markov decision processes, the choice of appropriate control strategy δ called the optimal strategy δ, concerns the situation, when the function representing the selection criterion of the optimal strategy takes an extrene value (minumum or maximum).

$$\begin{gathered} f_C(\delta^*) = \min_{\delta}[f_C(\delta)] \\ or \\ f_C(\delta^*) = \max_{\delta}[f_C(\delta)]. \end{gathered} \tag{37}$$

Depending on needs, the semi-Markov decision processes can be used to mathematatically formulate and solve a wide range of optimization problems, concernig the control of the operation process of the technical objects, such as for example: control the availability and reliability, economic analysis, safety and risk management activities.

In the case, when it concerns solving problems related to the availability of the transport system to carry out the assigned to transport tasks, the criterion function may be a function describing the availability of the single technical object. Availability of a single technical object defined on the basis of the semi-Markovian model of operational process is determined as the sum of limit probabilities p_i^* of remaining at states belonging to the availability states set [4, 5, 17]:

$$G^{OT} = \sum_i p_i^*, \quad dla \quad S_i \in S_G. \tag{38}$$

In order to define availability of technical objects (means of transport) based on the semi-Markovian model of operational process, the operational states of the technical object should be divided into availability states S_G and non-availability states S_{NG} of the object for the carrying out of the assigned task. In the presented model, the following technical object availability states were enummerated:

– state S_2 – awaiting the carrying out of the task at the bus depot parking space,
– state S_3 – carrying out of the transport task,
– state S_4 – awaiting the carrying out of the task between transport peak hours,
– state S_5 – repair by technical support unit without losing a ride,
– state S_7 – awaiting the start of task realization after technical support rep air.

Then, with the use of the MATHEMATICA software, the limit probability p_i^* of staying in states of semi-Markov process and the availability of technical objects of the transport system were determined:

$$G^{OT} = \frac{p_{12} \cdot (p_{34} + p_{38} + p_{39}) \cdot \overline{\Theta_2} + \overline{\Theta_3} + p_{34} \cdot \overline{\Theta_4} + p_{35} \cdot \overline{\Theta_5} + p_{36} \cdot p_{67} \cdot \overline{\Theta_7}}{\left[(p_{34} + p_{38} + p_{39}) \cdot \left(\overline{\Theta_1} + p_{12} \cdot \overline{\Theta_2}\right)\right] + \overline{\Theta_3} + p_{34} \cdot \overline{\Theta_4} + p_{35} \cdot \overline{\Theta_5} + \left[p_{36} \cdot \left(\overline{\Theta_6} + p_{67} \cdot \overline{\Theta_7}\right)\right] + p_{38} \cdot \overline{\Theta_8} + (p_{38} + p_{39}) \cdot \overline{\Theta_9}} . \quad (39)$$

Then the optimal strategy δ^* process control operation is determined by the maximum value of the function describing the availability of technical objects (means of transport), presented in equation (39), so:

$$G^{OT}\left(\delta^*\right) = \max_{\delta}\left[G^{OT}(\delta)\right] . \quad (40)$$

The genetic algorithm [6, 14] constitutes the convenient tool for choosing the optimal strategy δ^* process control operation of technical objects on the base of developed semi-Markov model of the process. General scheme of choice of the optimal strategy using genetic algorithm is presented in Fig. 2.

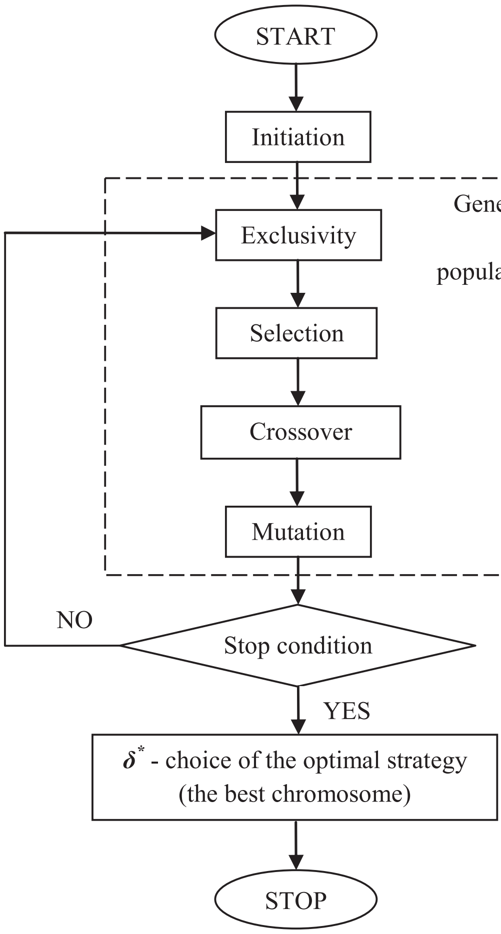

Fig. 2. General scheme of the genetic algorithm of choice of the optimal strategy δ^* [14]

In the case of using the genetic algorithm to determine the optimal strategy process control operation of technical objects, it should be adopted the following assumptions:
– the researched stochastic process is m-state semi-Markov decision process,
– strategies (stationary and deterministic) are functions of the process of transforming a set of states in a set of decisions likely to be used in each state,

- in each state can be used one of two ways of proceeding (called the decision),
- if decisions are labelled 0 and 1, the set of stationary and deterministic strategies will be the set of function:

$$\delta : S \to D,$$

where:
S – is the set of states of the process, $S = \{1, 2, \ldots, m\}$,
D – is the set of decisions making in the state of the process, $D = \{0,1\}$,
and to determine the possible control decisions in the state of the analyzed model of the process operation of technical objects, and to estimate the value of the items of the kernel process *Q(t)*, of the matrix *P* probabilities of transitions and the matrix Θ durations of states of the process. Tab. 1 shows the examples of control decisions in the states of the analyzed operation process of technical objects.

Tab. 1. The examples of control decisions in the states of the analyzed operation process

	Decision „0"	Decision „1"
S_3	The route marked code L („light" conditions of the delivery task)	The route marked code D („difficult" conditions of the delivery task)
S_5	Treatment by a PT type B (basic range)	Treatment by a PT type E (extended range)
S_6	Treatment by a PT type B (basic range)	Treatment by a PT type E (extended range)
S_8	Treatment in positions PZZ type N (normal)	Treatment in positions PZZ type I (intensive)
S_9	Operate in positions OC type N (normal)	Operate in positions OC type I (intensive)

Then each strategy can be represented as *n*-digit sequence consisting of 0 and 1. So it is a positional binary number (stored in the positional binary). For presented in the article the 9-state model of the operation process, an example of the strategy is determined as follows

$$\delta = [1,0,0,1,1,0,1,0,1].$$

5. Conclusions

The presented in the article method is the partial result of carrying out research, which purpose is to develop a comprehensive method of the control process for using the means of transport where semi-Markov decision processes are used for control. Semi-Markov decision processes are a convenient mathematical tool that application facilitates the complicated process of taking the rational control decisions in complex systems of technical object operation. In the literature, concerning the issue of control processes of technical objects operation, you can find many papers on both theoretical description and examples of practical applications of the semi-Markov decision processes [7, 16, 18]. Semi-Markov decision processes can also be used to mathematically formulate and solve a wide range of optimization problems concerning the operation of transport systems, such as for example: control the availability and reliability, analysis of the costs or profits and safe operation of such systems [1, 2, 4, 10].

The developed method for control of the operation process of technical objects is to determine an appropriate strategy (sequence of the decisions taken in individual states of the modelled process) for which the function representing measure reaches an extreme. For choosing the optimal strategy for controlling the operation, process of technical objects is proposed as a genetic algorithm. In the next stages of the conducted research, to determine the optimal strategy for process control operation, realized in the real system of operating the means of transport, it will be developed the input process model and computer program using genetic algorithm.

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