

## APPLICATION OF MULTIBODY SIMULATION FOR SEMITRAILER OPTIMIZATION

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### Abstract

*This paper presents an approach of optimization of a truck semitrailer suspension system, with utilization of multibody model; its purpose was to find the best values of operational parameters: stiffness and damping factors, in order to minimize the disadvantageous influence of force distribution in the high risk areas, where preceding strength analysis has pointed out dangerous load values. The model contains elements of two different types: flexible and rigid bodies, in purpose of increasing the accuracy level of conducted numerical calculations. A number of simulations with different parameters and under different load cases have been carried out, combined with a parametric and structural sensitivity analysis, what has enabled an estimation of individual factors influencing particular forces that have been the objectives of optimization procedure. The stiffness and damping coefficients of the construction suspension system have been adjusted by applying metamodeling techniques. Basing on the chosen design of experiment results, this procedure allows for an approximation of the behaviour of the analysed construction in the whole design space. In this process, two different approaches have been used: Kriging and polynomial regression, and both have been compared to the simulations results. Finally, using a desirability function, the most optimal solution has been found.*

**Keywords:** multibody dynamic system, optimization, response surface, design of experiment, sensitivity analysis

### 1. Introduction

The goal was to minimize forces at particular points of the construction, by means of using up-to-date techniques of multibody dynamic simulations, sensitivity analysis and optimization process based on surrogates and a desirability function.

A multibody analysis is a fast and reliable way of calculating kinematic and dynamic quantities in compound mechanisms. It helps to estimate all the forces acting on every part of the construction analyzed, which makes further FEM analysis more effective and reliable. Moreover, simulations of mechanisms containing flexible bodies, which deform under load and contain data from a modal analysis, makes all the achieved MD results more precise and accurate. Further information about this technique, readers can find in [18].

A sensitivity analysis is often used in designing process nowadays. Its purpose is to estimate the influence of a chosen changeable quantity on the mechanism. It also provides reliable information about the construction behaviour under particular conditions, which is an important element of the designing process. Further information can be found in [16, 17].

Metamodeling is an approach to approximate usually very complex and time-consuming nonlinear dynamic equations by replacing them with a simpler analytical model. For the past two decades, this technique has become a frequently chosen tool in designing and optimization processes due to its extreme processor capacity, hence time savings. More about it can be read in [3-5].

In the case of multi-parameter optimization, it is essential to choose a proper solution for the problem. One of the very popular approaches is to use a desirability function with weighted components. It guarantees reliable results and, what is principal, robustness. More information

about this method is provided in the paragraphs below. For more details and some variations of the chosen techniques, the reader can search in [6, 8, 9].

In the first part of the article, the problem considered and the conditions of the simulations being undertaken have been described in detail. Next, a brief theory of the optimization techniques that have been used, the comparison of the obtained metamodels and the differences between the experimental and theoretical data have been introduced to the reader. The last part is devoted to an overall solution and conclusions.

## **2. Problem Formulation**

The goal of the project described in this paper was to adjust the suspension parameters of the truck semitrailer, working under different load conditions and in diverse environments by means of minimizing forces at the crucial points of the construction. These points were specified as the kingpin (front bolt) connecting the truck and the articulated trailer; the bolt of the dump body at the rear end, and the mounting point of the lifting mechanism. They were defined as the most loaded and critical areas of the whole mechanism during the preliminary FEM analysis. Because a semitrailer can be used under different operational conditions, several types of simulations have been carried out. After gaining the data from a number of them, the parametric and structural sensitivity analysis has been conducted, in order to define the worst operating conditions. Based on the previously mentioned results the metamodels have been elaborated using Kriging and polynomial regression methods, with the purpose of creating a comparison of the surrogates built in both ways. Following the desirability function computations has led to achieving the most optimal stiffness  $k$  and damping ratio  $\alpha$  coefficients for the suspension system main elements: springs and shock absorbers. Those are responsible for maintaining optimal conditions of transferring excitations from wheel hubs to the body of the structure during operational process.

## **3. Model and simulation conditions**

A variance model, containing rigid and flexible bodies, has been created. In general, it consists of a tractor and semitrailer. The first element has been created with many simplifications, due to the fact that it is treated only as a part forcing the movement of the whole structure, with some specified parameters such as velocity, acceleration, jerk and an appropriate trajectory. The key part of the simulated model is the articulated trailer, which combines a suspension system and a flexible dump body, divided into box and frame. Those parts have been created using MSC Patran/Nastran software and prepared as files containing data about the geometry and modal analysis results, such as eigenvectors and natural frequencies. The principle of modal superposition has been used to combine the mode shapes at each time step to reproduce the total deformation of the flexible body. To achieve the most accurate results, modal analysis has been conducted for the parts under each kind of load case. During the multibody simulation, with the analysis conditions corresponding to the real ones, the flexible elements are excited, which results in the dynamic behaviour of the construction. This approach is more reliable and accurate thanks to taking into account the inevitable structure strains, which influence the values of the forces considered.

The maximum load that can be carried by the construction is that of 32 tonnes, and that was the mass taken under consideration. To fulfil the requirements, different load cases had to be specified, to make simulation as close to the real working conditions as possible. Therefore, except cargo uniformly distributed on the floor, the situation of concentrating it in one, smaller area has been examined. Hence, the assumption is that the bottom of the box would be divided into six smaller areas. To each of them the load of 25% of the maximum carriage capacity would be applied separately, when the remaining five areas would work under 4.708kN. Because of the construction symmetry, only three simulations of this case have been carried out, with the load concentrated on

one side only. The situation is illustrated in Fig. 1.



Fig. 1. Non-uniform cargo distribution. Simulations only for one-side changes, because of the construction symmetry

The simulations have been conducted under two different road conditions. The first was an uneven country road where the vehicle travelled with lower velocity, the second - a flat track with local asphalt pavement loss, nevertheless allowing the truck to travel faster. Only the second case has been used for further analysis, as it generates greater values of the forces being investigated, which is shown in Fig. 2.

#### 4. Results of the simulations

The main idea of the simulation was to investigate the influence of different load cases, velocities and road conditions on the specified forces. As mentioned above only one type of road was used to conduct the optimization process. The sensitivity analysis provided an answer to the question which parameter had the greatest effect on the semitrailer crucial points. The vehicle's speed, stiffness  $k$  and damping coefficient  $\alpha$  have been changed in a number of attempts, providing complete data for their influence estimation. An example of force-time characteristic for the tailgate bolt from one of the trials is shown in Fig. 3.

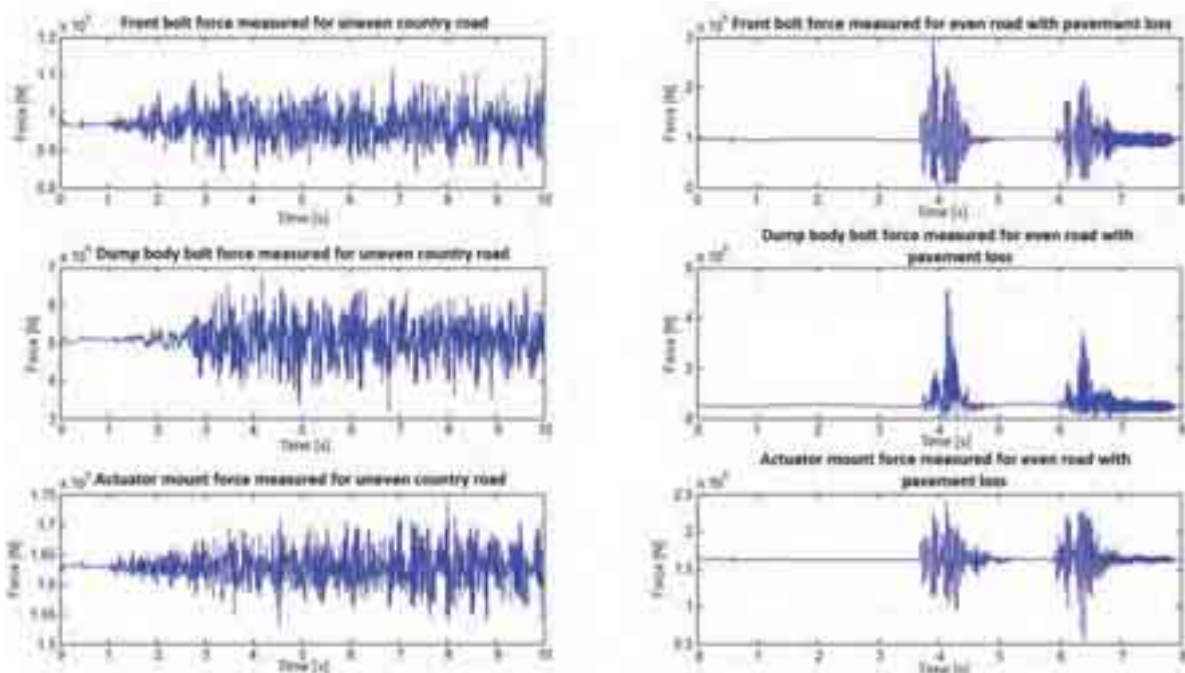


Fig. 2. The comparison of the acquired force values from the simulations on uneven country roads (left) and even roads with asphalt pavement loss (right)

The test was conducted on a flat road with double local pavement loss, with the truck velocity of 5.5 m/s, the uniform load case of 32 t and with suspension parameters corresponding to the original. Fig. 4 shows the same force measurements, but for the model with raised stiffness  $k$  coefficient.

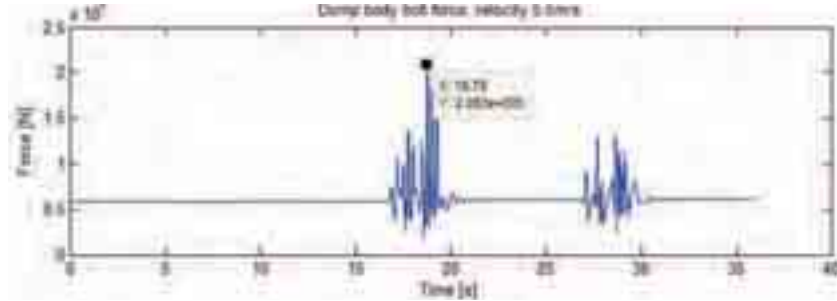


Fig. 3. The dump body bolt force value for the basic parameters

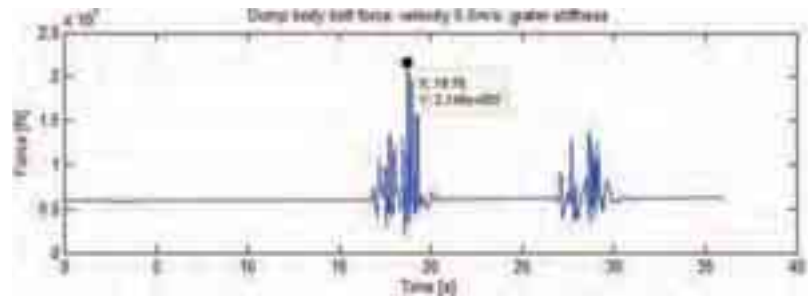


Fig. 4. Dump body bolt force value for greater stiffness coefficient

To perform sensitivity analysis [2] the finite difference principle was used, with a forward approximation approach. Equation (1) provides an absolute value, which cannot be compared with sensitivities for different types of parameters. If they are to be commensurable, equation (2) with normalization has to be used.

$$\frac{\Delta R_i}{\Delta P_j} = \frac{R_i(P_j + \Delta P_j) - R_i(P_j)}{\Delta P_j}, \quad (1)$$

where  $R_i$  is a response quantity, and  $P_j$  is the analyzed model property.

$$S_n = \frac{\Delta R_i}{\Delta P_j} \cdot \frac{P_j}{R_j}. \quad (2)$$

The results are presented on a Pareto graph. The examples of the specific parameters influence on the forces are shown in Fig. 5. They are related to the model with uniform distribution of total mass load of 32 tonnes.

Taking the above values into consideration, and assuming that greater velocity of the vehicle will result in higher force values, the decision was made to conduct further simulation on a flat road with the highest speed allowed for trucks: 90 km/h.

## 5. Metamodeling

To build the response surfaces (metamodels, also known as surrogates) 25 simulations have been conducted, for every model with different cargo distribution. In each case  $k$  and  $\alpha$  coefficients have been changed. The coefficients  $ik$  and  $i\alpha$  are used as follows: a new stiffness =  $ik \cdot$  basic stiffness, and new damping =  $i\alpha \cdot$  basic damping. The spectrum of factors has ranged from 0.5 to 1.5 with 0.25 step for each parameter, where the starting point was with both coefficients equal to 1.0 ( $ik = i\alpha = 1.0$ ). It

corresponds to the characteristics of springs and dampers typically used in this type of vehicles.

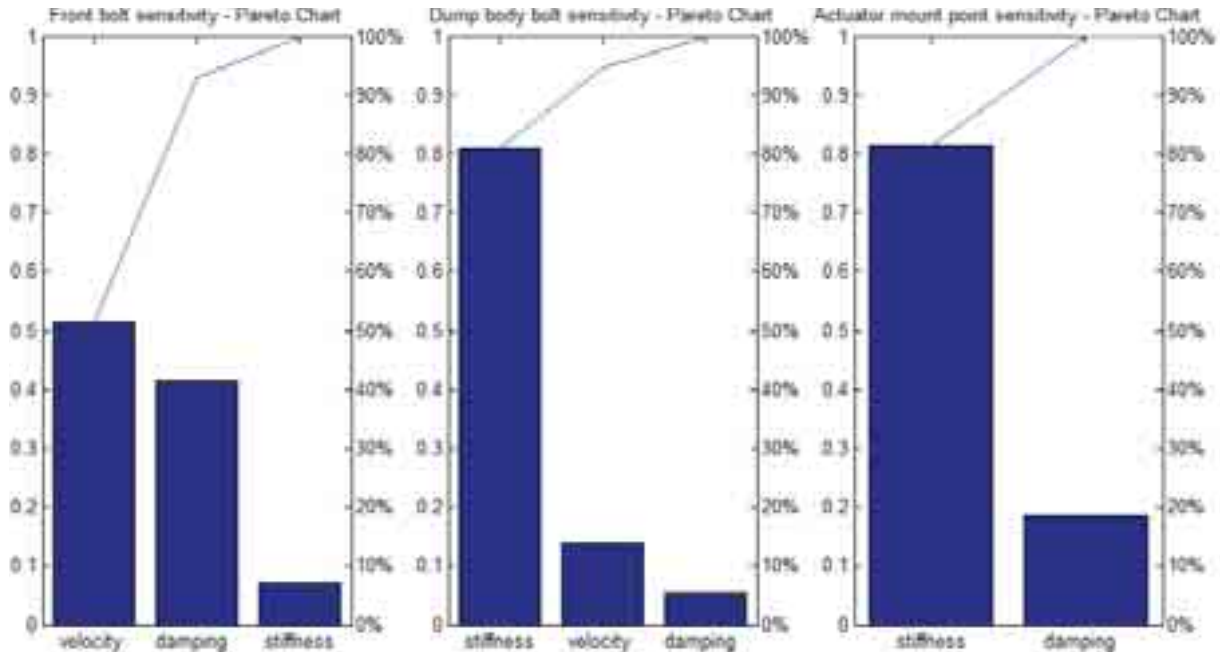


Fig. 5. Pareto chart showing the sensitivity of a loaded trailer. For actuator mount, the influence of velocity changes is negligibly small

The response surfaces have been created for every measured force and for every model separately, using two different methods: Kriging and polynomial regression. DACE Toolbox for Mathworks/Matlab has been used to create response surfaces applying the first method. A polynomial regression has been computed using the standard Matlab functions.

A Kriging model assumes that predicted values are a combination of a known function  $f_j(x)$  and departures from the form:

$$\hat{y} = \sum_{j=1}^k \beta_j f_j(x) + Z(x), \quad (3)$$

where  $Z(x)$  is a realization of a stochastic process with mean zero and a spatial correlation function given by:

$$\text{cov}[Z(x_i), Z(x_j)] = \sigma^2 R(x_i, x_j), \quad (4)$$

where  $\sigma^2$  is a process variance and  $R$  is a correlation. Many correlation functions can be chosen, however a Gaussian one is the most frequently adapted and has been used in this project.

The most common polynomial models of approximating a response function are first- and second-order. In general, they can be expressed as:

$$\hat{y} = \beta_0 + \sum_{i=1}^k \beta_i x_i \quad (5)$$

for the first order, and:

$$\hat{y} = \beta_0 + \sum_{i=1}^k \beta_i x_i + \sum_{i=1}^k \beta_{ii} x_i^2 + \sum_{i=1}^k \sum_{j=1, i < j}^k \beta_{ij} x_i x_j \quad (6)$$

for the second one.

According to Myers and Montgomery [4] in some situations, approximating polynomials of order greater than two are used. Because of its strong nonlinearity, a polynomial regression model of the fourth order has been adapted to fit the results obtained from the simulation.

The example results for the model with evenly distributed cargo, acquired from both methods are presented in Fig. 5 and 6. The black dots represent the values of the simulations of specified  $k$  and  $\alpha$  factor.

Each metamodel has been constructed on the basis of the maximum force value of a particular simulation.



## 6. Desirability function

Finding the optimal solution for all the measured forces is a typical multiresponse problem. In the example studied, there was a need of dealing with a double multiple response optimization, which was determined by three responses (forces) and four models taken into consideration simultaneously. Finding the most appropriate solution had to be preceded by reducing the problem to a simple function called desirability function. This method converts a multiresponse problem into a problem with a single aggregate measure.

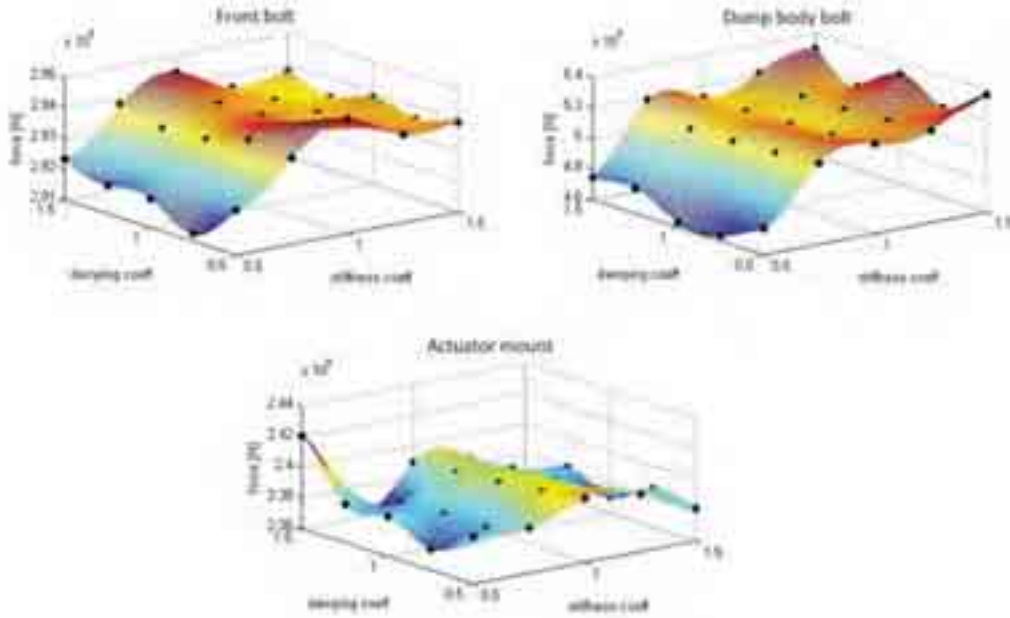


Fig. 5. Response surface acquired from Kriging method

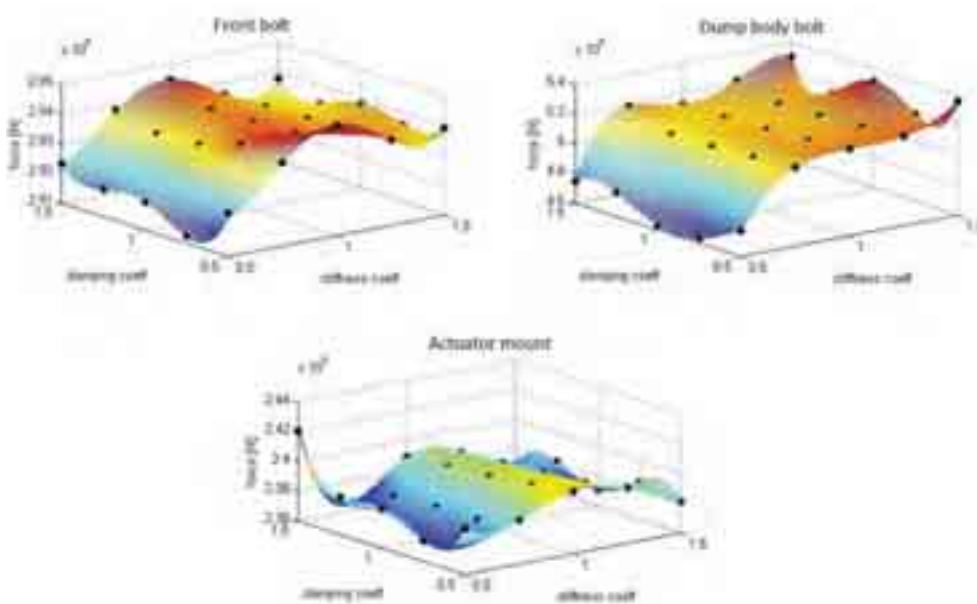


Fig. 6. The response surfaces acquired from the polynomial method

In similar cases, the most intuitive approach is to superimpose all the response plots and determine the optimal solution, by finding a global minimum (maximum). However, this method has very low robustness and can easily indicate wrong solution. The alternative approach, suggested in [15] and later modified in [7] assumes that a scale-free value  $d_j$  ( $0;1$ ) is assigned to a response  $j$ , and increases when the value of  $j$ -th response is getting more appropriate (desired). It

can be expressed as:

$$d_j(y_j(x)) = \begin{cases} 0 & \text{if } y_j(x) \leq y_j^{\min}, \\ \left( \frac{y_j - y_j^{\min}}{y_j^{\max} - y_j^{\min}} \right) & \text{if } y_j^{\min} \leq y_j(x) \leq y_j^{\max}, \\ 1 & \text{if } y_j(x) \geq y_j^{\max}, \end{cases} \quad (7)$$

where  $y_j^{\min}$  and  $y_j^{\max}$  are the lower and upper boundaries of the desired values of the response function  $y_j(x)$ . The overall desirability  $D$  is a weighted geometric mean, combining the  $d_j$  values:

$$D = (d_1^{w_1} d_2^{w_2} \dots d_n^{w_n})^{\frac{1}{\sum w_j}}, \quad l=1, 2, 3, 4, \quad (8)$$

where  $w_n$  are the responses weights.

In the case of desirability functions for the semitrailer, because of the need to minimize the forces at the crucial points of the construction, the target value was set to the global minimum  $y_j^{\min}$  of a particular response surface (forces values), therefore a global maximum, equal to the boundary value  $y_j^{\max}$  was the worst case, hence  $d_j(y_j^{\max}(x))=0$ . For each load case model the overall desirability function  $D_l$  have been expressed as:

$$D_l = (d_{fb} \cdot d_{dbb}^2 \cdot d_{amp})^{\frac{1}{4}}, \quad (9)$$

where  $l$  is a particular load case and  $d_{fb}, d_{dbb}, d_{amp}$  are the desirability values for the front bolt, dump body bolt and actuator mount point responses respectively. Because of the non-uniform cargo load case, the values of first and third forces decrease, when dump body bolt force increases, hence it is impossible to minimize all of them at the same time. Therefore, it has been assumed that to prevent an extreme build-up of the quantity mentioned,  $d_{dbb}$  weight is 2. The overall desirability for all the simulated models of the articulated trailer was expressed as:

$$D = (D_1^2 \cdot D_2 \cdot D_3 \cdot D_4)^{\frac{1}{5}}. \quad (10)$$

Because evenly distributed cargo is the most common load case in reality, it has weight of 2 in the optimization process. The same method has been used for Kriging and polynomial surrogates. An example of response surfaces for a non-uniformly loaded cargo and desirability surface is shown in the Fig. 8. The most optimal values of stiffness and damping coefficients have been computed for this particular case, therefore bigger points show forces corresponding to the chosen  $k$  and  $\alpha$ .

The common  $k$  and  $\alpha$  coefficient for the semitrailer have been based on the final desirability surface for all the models. The overall desirability function surfaces obtained from both Kriging and polynomials metamodels are shown in Fig. 9. The marked points are the global optimal solutions, indicating the coefficients compiled in the table in the section below.

## 7. Results check and metamodeling techniques comparison

For every model a local optimization has been carried out, which has pointed out the best parameters for the examined load scenario. In the Fig. 10 below, the surrogates for the non-uniform cargo load are shown, with the overall desirability surface modelled for this particular case and the bullet points indicating force values for the computed optimal coefficients. Finding the most optimal result has been achieved by searching for the global maximum of the created surface. The points of the analyzed desirability surface that equal zero represent the global maximums of the component forces, which, according to the previous assumptions, are the worst

possible evaluations.

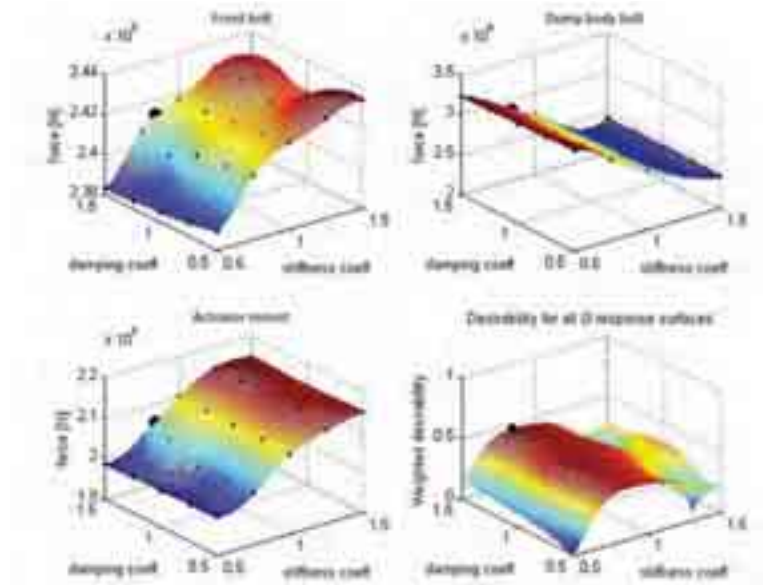


Fig. 8. The response surfaces for non-uniform cargo load with desirability surface for this particular case (Kriging metamodel)

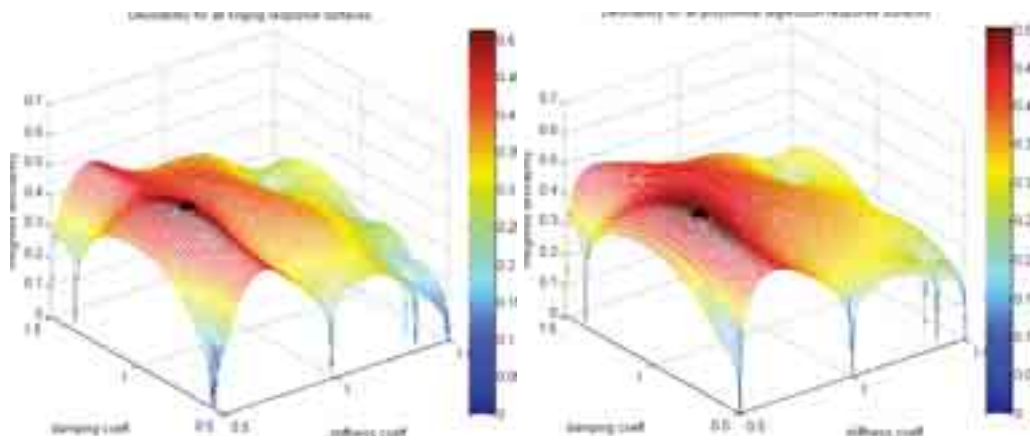


Fig. 9. The global desirability function obtained from kriging and polynomial surrogates. The marked points are the global optimums

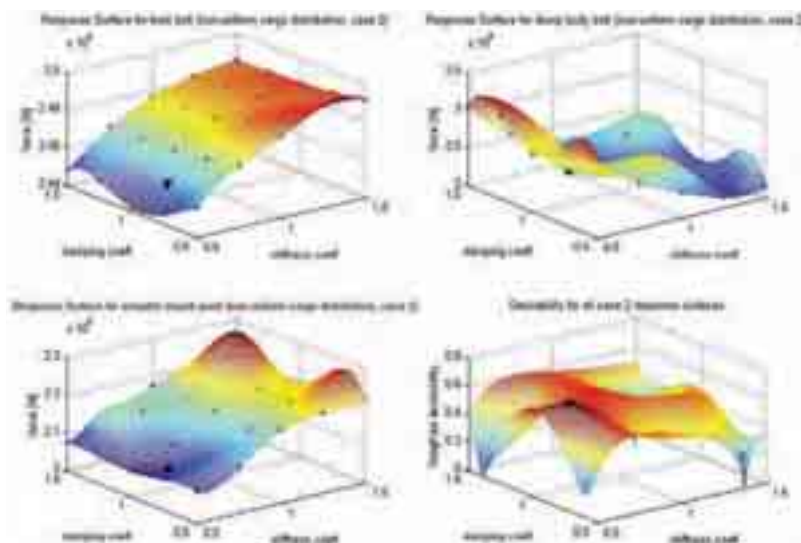


Fig. 10. Kriging surrogates and desirability surface for the non-uniform cargo load case, with marked force values



for the local optimal solution

To study the correctness and accuracy of the metamodells built, the force values indicated as local optimal solutions on both metamodells have been compared with the results obtained from the simulations. The comparative studies are shown in the table below. The differences expressed in percentage of the forces values obtained from the simulations are shown in Fig. 11.

Tab. 1. The comparison of the predicted and obtained from numerical experiments force values (for both metamodeling techniques), for the local optimal stiffness and damping solutions

		Local optimal $ik$ and $i\alpha$ values from kriging metamodel	Predicted force value from kriging metamodel	Simulation force value for $ik$ and $i\alpha$ from kriging metamodel	Local optimal $ik$ and $i\alpha$ values from polynomial metamodel	Predicted force value from polynomial metamodel	Simulation force value for $ik$ and $i\alpha$ from polynomial metamodel
Uniform load	Front bolt (1)	$ik = 0.5000$ $i\alpha = 0.7828$	292132.7	291251.5	$ik = 0.5000$ $i\alpha = 0.7323$	291258.5	291213.3
	Dump body bolt (2)		475427.8	462355.1		463782	464385.8
	Actuator mount (3)		238622.6	237451.9		238659	237322.5
Non-uniform load, case 1	Front bolt (4)	$ik = 0.8030$ $i\alpha = 1.5000$	262239.2	262385.3	$ik = 0.8030$ $i\alpha = 1.5000$	262239.2	262342.6
	Dump body bolt (5)		2605041	2608243.9		2605041	2614866.0
	Actuator mount (6)		228211.7	228020.3		228211.7	228042.4
Non-uniform load, case 2	Front bolt (7)	$ik = 0.6616$ $i\alpha = 0.9141$	246200.9	245682.3	$ik = 0.6111$ $i\alpha = 0.8636$	245784.7	245595.1
	Dump body bolt (8)		2795918	2630994.2		2893697	2589721.6
	Actuator mount (9)		212419.4	208284.2		210474.7	207688.9
Non-uniform load, case 3	Front bolt (10)	$ik = 0.8434$ $i\alpha = 1.5000$	241161.9	241272.3	$ik = 0.8939$ $i\alpha = 1.5000$	241393.2	241315.4
	Dump body bolt (11)		2879998	2881842.7		2825931	2853151.2
	Actuator mount (12)		206029	205472.5		207532.6	207005.8

A desirability function was used for defining the most optimal stiffness and damping coefficients of the articulated trailer suspension system. The requirements related to the importance level of measured forces have been formulated and taken into consideration. The global results obtained from Kriging and polynomial metamodells are shown below in Tab. 2:

Tab. 2. The optimal solutions obtained by using both metamodeling techniques

	Stiffness coefficient $ik$	Damping coefficient $i\alpha$
Kriging metamodel	0.6414	0.8939
Polynomial metamodel	0.6010	0.8636

For the results presented above a simulation for every load case has been conducted separately. The obtained force magnitudes have been compared to the values predicted by the surrogates of both types and to the forces resulting from the application of initial damping and stiffness ( $ik = 1$ ,

$\alpha = 1$ ). The comparison is shown in the table below and in Fig. 12.

Tab. 3. The comparison of predicted and obtained from numerical experiments force values (for both metamodeling techniques), for the global optimal stiffness and damping coefficients

		Simulation force value for optimal $\alpha$ and $\beta$ from kriging metamodel	Predicted force value from kriging metamodel for optimal $\alpha$ and $\beta$	Simulation force value for optimal $\alpha$ and $\beta$ from polynomial metamodel	Predicted force value from polynomial metamodel for optimal $\alpha$ and $\beta$	Simulation force for initial values of stiffness and damping
Uniform load	Front bolt (1)	292798.8	292714.7	292404.4	292822.0	293931.4
	Dump body bolt (2)	497655.4	486478.0	490953.7	490322.1	513380.0
	Actuator mount (3)	237054.2	237768.0	236832.7	237421.3	239400.0
Non-uniform load, case 1	Front bolt (4)	261446.7	261172.5	261089.3	260909.3	263608.5
	Dump body bolt (5)	2802732	2818412.4	2855024	2853162.8	2424200.2
	Actuator mount (6)	223453.1	223223.4	218765.6	222157.8	233909.5
Non-uniform load, case 2	Front bolt (7)	246041.6	245418.6	245683.3	245485.6	247841.9
	Dump body bolt (8)	2693055	2661307.2	2727617	2602322.0	2349900.2
	Actuator mount (9)	208122.4	207986.9	209591.8	207556.7	217715.4
Non-uniform load, case 3	Front bolt (10)	240114.7	239861.4	239176.5	239860.6	241910.8
	Dump body bolt (11)	3118629	3121793.7	3145833	3110604.7	2746970.7
	Actuator mount (12)	199574.7	199509.3	199464.1	198733.6	209959.4

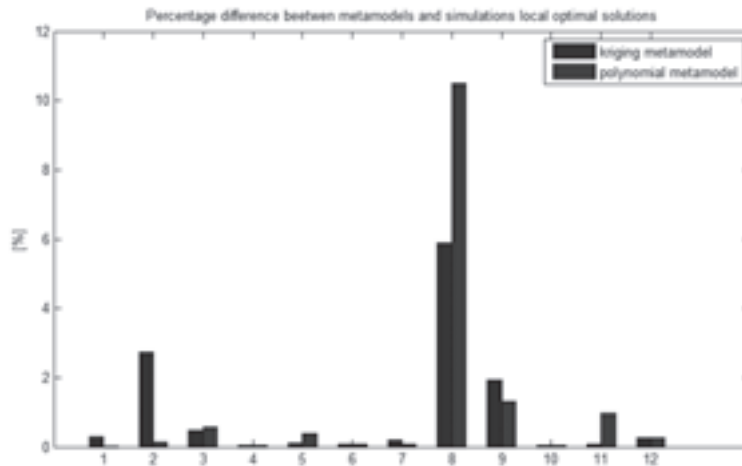


Fig. 11. The percentage differences between the local optimal values of the forces obtained from the surrogates and simulations. Quantities 1 – 12 (vertical axis) explained in Tab. 1

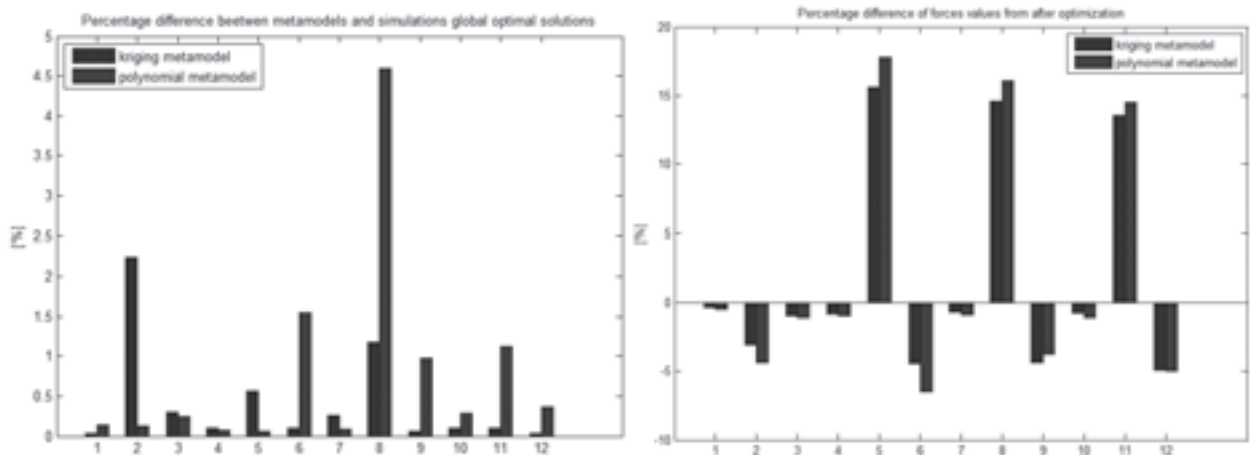


Fig. 12. The comparison of the results achieved using Kriging and polynomial metamodels. The tabular results show the predicted optimal values and the simulation values with optimal  $\alpha$  and  $\beta$ . The graph on the left - percentage differences between the results from the Tab. 3 (columns 1 - 4). The graph on the right – the comparison of the force values before and after the optimization process, for the optimal values from both metamodels (columns 2, 4, 5)

## 8. Conclusions

The target of the optimization has been partially achieved. Minimizing of the forces in the front bolt and mount point of the actuator has been achieved, however the forces in dump body bolt have been reduced only for the uniform load case. The increase of this force is significant and its correlation with the metamodeling technique being used is equal to 15 and 17%, 14 and 16% and 13 and 15% for the different non-uniform load cases. It is caused by the contrary slopes of response surfaces of the forces considered. When the forces at the front of the construction are approaching the global minimum, the force in the bolt at the back is growing. This leads to the conclusion that the construction is well designed and there is no need of changing the suspension parameters. Nevertheless, if there arises, a strong need to reduce forces at the front of the construction it can be done by applying the values achieved from the optimization procedure. In such a case, however, the strengthening of the dump body bolt construction should be recommended, because of the highly disadvantageous influence of the non-uniform cargo arrangement. Since this part of the construction is easily changeable, this operation is sure to succeed without exorbitant efforts. Changing weights in the desirability function may result in reducing forces in the area mentioned above, however it will increase loads in the other two crucial points, due to the trend of their responses.

Because of the high nonlinearities of the examined model, a polynomial of fourth order has been used. It has been shown that fitting and prediction of the interpolated values have been done with high accuracy, which has been confirmed by examination of the single points. The comparison of both metamodeling methods with the results obtained from the simulations has led to the estimation of divergence between them. Hence it has been proved that Kriging surrogates are more effective and precise, which is shown in Fig. 11 and 13.

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